

Exp No. (1) Malus' law

Figure 1: Experimental setup Malus` law

Related topics:

Electric theory of light, polarization, polarizer, analyzer, Brewster's law, Malus' law.

Principle:

Linear, polarized light passes through a polarization filter. Transmitted light intensity is determined as a function of the angular position of the polarization filter.

Equipment:

- Laser, He-Ne 1.0 mW, 220 V
- Optical profile bench, $l = 60$ cm
- Base for optical profile bench, adjustable
- Slide mount f. opt. pr.-bench, $h = 30$ mm
- Polarizing filter on stem
- Photocell, silicon on stem
- Digital multimeter

Object:

1. The plane of polarization of a linear, polarized laser beam is to be determined.

2. The intensity of the light transmitted by the polarization filter is to be determined as a function of the angular position of the filter.

3. Malus' law must be verified.

Theory and evaluation

Let AA' be the Polarization planes of the analyzer in Fig. 2. If linearly polarized light, the vibrating plane of which forms an angle φ with the polarization plane of the filter, impinges on the analyzer, only the part

$$
E_A = E_o \cdot \cos \varphi
$$

Will be transmitted. As the intensity I of the light wave is proportional to the square of electric field intensity vector E, the following relation (Malus' law) is obtained

$$
I_A = I_0 \cdot \cos^2 \varphi
$$

Fig. 2: Geometry for the determination of transmitted light intensity.

Fig.3 shows the photo cell current after background correction (this is a measure for the transmitted light intensity) as a function of the angular position of the polarization plane of the analyzer. The intensity peak for $\varphi = 50^{\circ}$ shows that the polarization plane of the emitted laser beam has already been rotated by this angle against the vertical.

Fig.3: Corrected photo cell current as a function of the angular position w of the polarization plane of the analyzer.

Fig.4 shows the normalized and corrected photo cell currents as a function of the angular position of the analyzer. Malus's law is verified by the initial line's 45° slope (Note: to determine Malus' line in Fig. 4, an angular setting of 50° of the analyzer must be considered for $\varphi = 0^{\circ}$).

Fig. 4: Normalized photo cell current as a function of $\cos^2\varphi$.

Set-up and procedure

The experiment is set up according to Figure 1. It must be made sure that the photocell is totally illuminated when the polarization filter is set up. If the experiment is carried out in a non darkened room, the disturbing background current I_0 must be determined with the laser switched off and this must be taken into account during evaluation. The laser should be allowed to warm up for about 30 minutes to prevent disturbing intensity fluctuations. The polarization filter is then rotated in steps of 5° between the filter positions $+/-90^\circ$ and the corresponding photo cell current (most sensitive direct current range of the digital multimeter) is determined.

Discussion:

1- Plot the experimental curve for the law $I_A = I_0 \cdot \cos^2 \varphi$ of Malus' on the same graph and calculate I as a percentage I_{Θ}

2- Describe in your own words what is mean by and draw a picture of

A- Unpolarized light.

B- Linearly polarized light.

3- Describe in words and figures what a transverse wave is and what a longitudinal wave is. Is laser transverse wave or longitudinal wave?

4- Discusses how intensity is become when the linearly polarized and un polarized light are passes through the polarizer filter.

5- Define polarization of light?

6- Define Brewster's law with equation?

7-State types of polarization?

Exp No. (2) Polarization by Reflection

Figure 1: Experimental arrangement

Object:

Study reflection of polarized light from a glass plate.

Equipment:

Sodium lamp, collimating lens, Mirror at 56.3 normal, Polaroid polarize Study.

Theory:

Probably the most familiar application of polarizing material is glare reducing sun glasses. These glasses work because the light reflected from non-conducting surfaces such as water or snow is at least partially polarized. By orienting the lens transmission axes in the correct direction, the polarized glare can be blocked. In order to distinguish between the polarization components of reflected light, the descriptions parallel (\parallel) and perpendicular (\perp) are often used (rather than x and y). These terms refer to the orientation of the electric field components with respect to the plane of incidence, the plane containing the incident and reflected rays and the normal to the surface. The plane of incidence contains the incident, reflected, and refracted rays and the normal to the surface $P \parallel$ (also called P polarization).

The Fresnel equations solve for the quantity called reflectivity, which is calculated by dividing the reflected power by the incident power. That is, reflectivity is the fraction of the incident light reflected by the surface. Since there are two reflected components, perpendicular and parallel to the plane of incidence, there are two Fresnel reflection equations:

$$
R_{\parallel} = \left(\frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_t + n_t \cos \theta_i}\right)^2
$$

$$
R_{\perp} = \left(\frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}\right)^2
$$

Figure 2: Polarization components for polarization by reflection

In this equation, n_i is the index of refraction of the incident material and n_t is the index of refraction of the transmitting material. The angle $\theta_{\rm i}$ is the incident angle at the material surface, and θ_t is the refracted angle in the material, which can be found by Snell's law. It is difficult to gain an intuitive picture of the situation by looking at these equations. The graphs of Fresnel equations, Plotted in (fig 3) for the case of light incident in air $(n_i=1)$ and reflecting from glass $(n_t = 1.5)$, better explain the situation. The graphs percentage for each polarization component reflected from the surface as a function of incident angle.

Figure 3: Fresnel reflection equation for reflection by glass in air

The graphs of reflectivity shown in fig 3 explain some everyday observations. First, note that the perpendicular component is almost always reflected more strongly than the parallel component, except the case of normal incidence where they are reflected equally. Therefore, light reflected from most surfaces tends to be at least partially polarized in the direction perpendicular to the plane of incidence.

Also note that at very large angles of incidence, light of both polarizations is reflected very strongly. You may have seen this effect looking down a long tiled corridor. The Fresnel equations also predict the existence of back reflection, a source of loss in optical systems. For normal incidence ($\theta_i=0$), cos $\theta_i=cos\theta_t=1$ and both of the Fresnel equations reduce to

$$
R = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2
$$

For an air- glass interface with n_t = 1.5, the reflectivity at normal incidence is approximately 4% as shown in fig 3. Although 4% optical system with many air glass interfaces, the total loss can be an important factor. Components may be

treated with anti-reflection losses. In an optical fiber system, the reflection loss in connectors can be minimized by the use of a refractive index matching gel. Making the values of n_i and n_t nearly equal has the effect of reducing the numerator in equation and thus reducing the amount of back reflected light. The 4% reflection from an air-glass interface also explain the behavior of a window that reveals the outside word during the day time and reflect the inside world at night. Light entering from outside during daylight hours is sufficiently bright that the Fresnel reflection cannot be seen. At night, with no external light, the Fresnel reflection gives the window the appearance of a mirror.

Procedure:

1- Change the setup to resemble figure 1.

2- Move the second optical bench so that light reflecting of the glass plate at

56.3 will go through the Polaroid and hit the screen.

3- By rotating the Polaroid, verify that the reflected light is polarized and determine the plane of polarization.

Discussion:

1- How would you classify sunlight?

2- After passing through polarizer, would you expect light to be more intense than before it passed through?

3- Is it possible that light, after passing through a polarizer, could remain at the same intensity? Explain.

4- What are the components of Fresnel reflection equation?

5- What's object of this experiment?

6- Light after passing through polarizer reduce, explain why?

7- Is the polarization can by occur in transverse wave or longitudinal wave?

Exp No. **(3)**

Optical Activity

Figure 1: Experimental arrangement

Object:

To observe the effect of optically active molecule on the plane polarized light.

Equipment:

(He-Ne laser, Polarizer, Sugar solution, Photometer)

Theory

there is one physical property in which enantiomers differ: their ability to rotate plane-polarized light.

Recall that [light consists of a series of vibrating waves.](http://www.sparknotes.com/physics/optics/intro/) The light that we typically see is unpolarized; that is, it consists of waves that are oriented in every possible direction in an even distribution. We can pass unpolarized light through a polarizing filter to obtain plane-polarized light, which consists of light waves oriented in only a single direction.

to filter

Figure 2: Plane-polarized light

different directions

Solutions of chiral compounds have the property of rotating plane-polarized light passed through them. That is, the angle of the light plane is tilted to the right or to the left after emerging from the sample. Achiral compounds do not have this property. The ability of a solution to rotate plane-polarized light in this fashion is called optical activity, and solutions which have this ability are said to be optically active.

Figure 3: Rotation of plane-polarized light by optically active compounds

Using a technique called polarimetry, optical activity is measured by a device called a polarimeter. Monochromatic light (light containing a single color) is filtered through a polarizer to produce plane-polarized light, and it is passed through the sample. A second filter is placed with its slits parallel to those of the first filter, and then the sample is rotated until light is transmitted through the second filter. The number of degrees the sample is rotated is called the optical rotation of the sample. If rotation occurs to the right (clockwise), the optical rotation is given $a + sign$ and the sample is considered dextrorotary. If rotation occurs to the left (counter-clockwise), the optical rotation is assigned a--sign and the sample is levorotary.

The optical rotation of a given sample varies with its concentration and the light's path length:

Figure 4: Dependency of the optical rotation on concentration and path length

The carbon atom is the fundamental building block of bimolecules. Typically carbon can have up to four different atoms (or functional groups) bonded to it. With four different atoms bonded, these molecules are called "optical active" or chiral since the atoms can be arranged to produce two structures which are chemically the same, but are mirror image of each other as shown in the figure below. One structural orientation of the molecule is called "right handed" since it rotate linearly polarized light in a right –handed corkscrew direction. The other orientation called left handed. In biological systems, typical only one configuration is produced. Therefore, if one measure the degree to which the polarization of linearly polarized light is rotations, one can infer the concentration of the "optically active" molecule solution.

Figure 5: atoms bonded

Procedure:

1- Place an empty box in between the polarizers. Rotate the second polarizer (p_2) until no light is transmitted. Record this position of the second polarizer. At this point you have "nulled out" any birefringence of the container. Fill the solution of glucose from your preparation in the box.

Rotate the second polarizer until you see a minimum in transmission. The amount you rotated the second polarizer is the angular rotation of the polarized light due to the glucose solution. Also note the direction of rotation.

2- Repeat this measurement 3 times to make sure that you are getting consistent angular rotations.

In order to find the relationship between the concentrations of glucose with the angular rotation, you can either dilute or concentrate the solution. The choice is yours. Does the optical rotation change with your adjustment?

3- Measure the path length through the solution (e.g. the inner length of the box).

4- Draw a graph showing the relationship between the concentration of the glucose and the angular orientation of the polarized light and red off the slope α = α L where α = specific rotation of glucose and L= path length through the solution.

5- From the angular rotation measurements, using the specific rotation of the glucose (α =45.62 degree/cm/(g/ml) for a wavelength of 633 nm) calculate the concentration of glucose in solution :

 Θ = α .L.C

Where Θ is the average measured angle of rotation, L is the path length through the solution, and c is the concentration (gm/ml) of the glucose solution. Does the calculation agree with the experimental results?

Discussion:

- 1- Discuss your results?
- 2- What is the optical activity?
- **3-** How can we increase the optical activity?

Exp No. **(4)**

Diffraction technique for measuring index of refraction

Figure 1: Experimental arrangement

Equipment:

- Diffraction grating
- Laser beam
- Tank of water

Theory:

When entering a new material refracts light, there is also a change in wavelength.

The frequency of light is fixed at the source, but as the speed of light changes in different materials the wavelength also changes. Determining this wavelength change with a diffraction grating provides a novel technique for measuring the index of refraction.

Procedure:

1- Aim a laser beam into an empty tank so the beam is normal (perpendicular) to the surface of glass. *Longer tanks* will give better resolution for this measurement.

2- Measure the angle that the beam makes between the central maximum and the first order maximum to the left or right of the central point.

Calculate the relationship:

$$
\lambda_a = d \, \sin \varphi_a
$$

Where:

 λ _a: is the wavelength of the laser beam in the tank.

d : is the distance between slits in diffraction grating.

 \varnothing _a : is the angle between the central maximum and the first order maximum with air in the tank.

3- Fill the tank with water and notice how the angle Ø decreases. The decrease is due to the refraction of the water.

4- Calculate λ the wavelength of the laser beam with water in the tank.

5- Find the index of refraction of the water by dividing the wavelength found in step B by the wavelength found in step D

$$
\lambda = d \, \sin \varphi
$$

Discussion:

1- Discuss the systematic and statistical errors that affect your determination of the index of refraction.

2- Compare between the wavelength in empty tank λ_a and the wavelength in the fill tank λ .

3-How many kind of diffraction grating? State them?

Exp No. (5) Measurement of laser wavelength by using Michelson interferometer

Fig (1) Setup of experiment

Principle:

An interferometer is a device that can be used to measure lengths or changes in length with great accuracy by means of interference fringes. In this experiment, it will be used to measure the wavelength of He-Ne laser.

Theory:

Figure 2 shows the basic design of the interferometer. The beam splitter is a glass plate that is half silvered so that light from the source splits at the first surface. Half of the incoming beam is transmitted to the mirror $M₁$ (passing through the glass compensator plate on the way) and the other half is reflected toward the mirror M_2 .

Mirrors M_1 and M_2 reflect the light back to the beam splitter. And half of each reaches viewing screen. The reminder being directed back to the source and lost.

Mirror M_2 , mounted on a carriage which slides on a track, can be translated toward or away from the observer by means of a precision micrometer. So the displacement of mirror can be measured accurately by reading the micrometer.

Fig (1) Michelson interferometer

The original beam of light has now been split and portions of the resulting beams brought back together. Since the beams are from the same source, they are highly correlated. When a lens is placed between the laser source and the beam splitter, the light ray spreads out, and an interference pattern of dark and bright rings or fringes is seen on the viewing screen.

The interference pattern occurs because of the phase difference between the two beams when they reach the viewing screen. When they were initially split, they were in phase. The phase difference arises from the fact that the beams traveled different optical paths before reaching the screen. Moving M_2 varies the path length of one of the beams. Since the beam traverses the path between M_2 and beam splitter twice, moving $M_2 \frac{\lambda}{4}$ $\frac{\pi}{4}$ nearer the beam splitter reduces the optical path of that beam by $\frac{\lambda}{4}$. This will change the interference pattern: the radii of the maxima will be reduced so that they now occupy the position of the former minima. If M_2 is moved an additional distance of $\frac{\pi}{4}$ closer to the beam splitter. The maxima and minima will again trade positions with the new arrangement indistinguishable from the original pattern. By slowly moving the mirror a measured distance a measured distance $d_{\rm m}$ and counting **m**, the number of times the fringe pattern is restored to its original state, the wavelength of light can be calculated as:

$$
\lambda = \frac{2d_m}{m} \qquad \qquad \text{---} \
$$

The compensator plate is needed if white light fringes are to be obtained. Note that the light rays going to mirror M_2 traverse the beam splitter three times before reaching the observer, where as the rays

going to mirror M_1 traverses it only once. In order to achieve exact quality of path through glass, therefore, the compensator plate of exactly the same thickness as the beam splitter is added. Generally, the compensator plate is not needed when the source is a laser.

Procedure:

Adjust the laser beam so that its approximately parallel with the top of the base. The beam should strike the centre of M_2 and be reflected back into the laser aperture. Attach the viewing screen to its magnetic backing. Position the beam splitter at a 45 degree angle to the laser beam. Within the crop mark, so that the beam is reflected to the adjustable mirror M_1 . Adjust the angle of the beam splitter so that the reflected beam hits M_1 near its centre.

There should now be two sets of bright dots on the viewing screen; one set comes from M_1 and the other from M_2 . Each set of dots should include a bright dot with two or more dots of laser brightness due to multiple reflections. Adjust the angle of the beam splitter again until the two sets of dots are as close together as possible, and then tighten the thumbscrew to secure the beam splitter. Using the thumbscrews on the back of M_2 , adjust the mirror's tilt until the two sets of dots on the viewing screen coincide.

Attach the 18 mm FL lens to the magnetic backing of the component holder in front of the laser and adjust its position until the diverging beam is centered on the beam splitter. You should now see circular fringes on the viewing screen. If not, carefully adjust the tilt of M_2 until the fringes appear. (One can also remove the viewing screen and

project the fringe pattern onto a wall if the set up is in a location at which this would be convenient). When the interference pattern is clearly visible on your viewing screen, or wall, adjust the micrometer knob to a medium reading (approximately 50 μm). Turn the micrometer knob one full turn counterclockwise.

Continue turning counterclockwise until the zero on the knob is aligned with the index mark. Record the micrometer reading. Adjust the position of the viewing screen so that one of the marks on the millimeter scale is aligned with one of the fringes in the interference pattern. It will be easier to count the fringes if the reference mark is one or two fringes from the center of the pattern.

Rotate the micrometer knob slowly counter-clockwise, counting the fringes as they pass the reference mark. Record the micrometer reading for every tenth fringe up to the 190th. Tabulate your result as indicated in table (1) using equation (1) to calculate the wavelength of laser light λ . Notice that table 1 is set up so that for each row m=100. Remember that each small division on the micrometer knob responds to 1 μ m (10⁻⁶ m) of mirror movement. Determine your percentage error relative to the accepted value λ = 632.8 \pm 0.1 nm.

reading		Fringe no. Interferometer Fringe No. Interferometer Difference reading	Deviation
	100		

Table (1): Data and results

Discussion:

1- What is the purpose of the compensator plate G_2 ?

2- If mirror M_1 is moved a distance d, what will be the change in the total path length for light travelling in arm 1 of the interferometer?

3- Define coherence length and coherence time. What is the relationship between these quantities?

4- Discuss the systematic and statistical errors that affect your determination of the wavelength of He-Ne laser?

5- What's the purpose of the compensator plate G2?

6- How the interference pattern occurs?

Exp No. **(6)**

Measure of refractive index of glass

Aim of experiment:-

Find the refractive index of glass.

Apparatuses:-

- 1- Bread board
- 2- Diode laser with power supply
- 3- Laser mount
- 4- Rotation stage with class.
- 5- Screen
- 6- Mirrors with mount (2 no.)
- 7- Beam splitter with mount

Fig (1) Setup of experiment

Theory:-

The light passes through a greater length of glass as the plate is rotated. The change in the path length of the light beam as the glass plate is rotated and relates the change in path length with the laser beam through air.

The reflective index of glass side, $n = \frac{(2t - N\lambda)(1 - cos\theta)}{2t(1 - cos\theta)(N\lambda)}$ $2t(1-\cos\theta)-$

Where, t: the thickness of the glass slide N: number of fringes counted Ɵ: angle of rotation λ: wavelength of laser beam.

Procedure:-

- 1. Align the laser and interferometer in the Michelson mode.
- 2. Place the rotation stage between the beam splitter and movable mirror, perpendicular to the optical path.
- 3. Mount the glass plate on the rotation stage.
- 4. Position the stage & glass such that glass slide is perpendicular to the optical path.
- 5. When glass plate is introduced in the optical path of Michelson interferometer, the fringe will be shifted & will become blur. To make the fringes sharpen again, move mirror mount to & fro till the clear set of fringes is achieved on the viewing screen.
- 6. Slowly rotate rotation stage. Count the number of fringes translations that occur as you rotate that table to an angle 0 (at least 10 degrees). As shown in fig (2).

Discussion:-

1- Discuss your results?

2- Derive Equation

$$
n = \frac{(2t - N\lambda)(1 - \cos\theta)}{2t(1 - \cos\theta) - N\lambda}
$$

3- Plot N versus θ as elicit by Equation above?

4- Is it possible in this experiment measuring the refractive index for any types of plastic? If yes, how?

Exp No. **(7)**

Refractive index of air by using Michelson

Aim of experiment:-

Find the refractive index of air.

Apparatuses:-

- 1- Bread board
- 2- Diode laser with power supply
- 3- Laser mount
- 4- Beam splitter with mount
- 5- Screen
- 6- Pressure cell
- 7- Mirrors with mount (2 no.)

Fig (1) setup of experiment

Theory:-

When a piece of material of thickness d is placed in one arm of the Michelson interferometer, the change in optical path length is given by 2d Δn where Δn is the difference in refractive index between the sample and the material it replaced (usuall y air). In other words 2d $(n_m-n_{air})/\lambda$ extra wavelengths are introduced if air is replaced by a sample of refractive index n_m .

Let λ be the wavelength of light, n the refractive index of air at atmospheric pressure, d the length of the air cell, P_{atm} the current atmospheric pressure and ΔP the pressure change.

The relationship between the pressure change ΔP and the number of fringes shift $m_{\Delta P}$ is given by:

$$
m_{\Delta P} = \left(\frac{2d (n-1)}{\lambda}\right) \left(\frac{\Delta P}{P_{atm}}\right)
$$

Procedure:-

Arrange the Michelson interferometer experimental set up. Introduce the pressure cell in any one of arms of the interferometer. Tune the micrometer in order to get the interference pattern in a good manner.

Now pressurize the cell up to 300 mmHg. Slowly release the air and count the number of fringes. Reading of pressure gauge may be tabulated up to the total release of air from the cell.

Plot a graph between number of fringes and corresponding pressure. The slope of the graph will give the value $m_{\Delta P}$ / ΔP . Put this value in the equation given below:

$$
m_{\Delta P} / \Delta P = \left(\frac{2d (n-1)}{\lambda}\right) \left(\frac{1}{P_{atm}}\right)
$$

From the above equation we can calculate the value of refractive index of air n

We have :

 $(m_{\Delta P} / \Delta P) = (2d(n-1)/\lambda)(1/P_{atm})$ Length of pressure cell $= d$ Wavelength of light = λ $(m\Delta P / \Delta P)$ =from graph $(1/P_{atm})$ = from graph

Substituting the dates in the above equation, we can find the refractive index of air n as shown in figure (2)

Fig (2) Measure of refractive index of air

Observation:

Discussion:-

- 1- Discuss your results?
- 2- The cell length was measured between the inside surfaces of the glass ends. What is it?
- 3- Is it possible in this experiment measuring the refractive index of gases? If yes, how?
- 4- Calculate the accepted value for dry air is at 15 C and 760 Torr and compare with your result?

Exp No. (8) Diffraction of light at a slit and an edge

Figure 1: Experimental setup for the diffraction of light at a slit and an edge

Related topics:

Intensity, Fresnel integrals, Fraunhofer diffraction.

Principle:

Monochromatic light is incident on a slit or an edge. The intensity distribution of the diffraction pattern is determined.

Equipment:

- Laser, He-Ne 1.0 mw, 220 V
- Photocell, selenium, on stem
- Lens holder
- Lens, mounted, f -50 mm
- Slit, adjustable
- -Screen, metal, 300-300 mm
- $-$ Meter scale, demo. $l = 1000$ mm
- Measuring tape, $l = 2$ m

Object:

1- Measurement of the width of a given slit.

2- Measurement of the intensity distribution of the diffraction pattern of the slit and of the edge.

Theory and evaluation:

If light of wavelength λ falls onto a slit of width b, each point of the slit acts as the starting point of a new spherical wave. The diffraction pattern is formed on a screen behind the slit as a result of the interference of these new waves. If this diffraction is treated according to the Fraunhofer approximation, the intensity at point P on a screen parallel to the slit, using the symbols of Figure 2, is:

$$
I = c \cdot \left(\frac{\sin\frac{b\pi}{\lambda}}{\frac{b\pi}{\lambda}\sin\theta}\sin\theta\right)^2
$$

Where: C is a constant which depends on the wavelength and the geometry. Intensity maxima occur for:

$$
\tan\frac{b\pi}{\lambda}\sin\theta = \frac{b\pi}{\lambda}\sin\theta
$$

The first maximum is thus obtained for $\theta = 0$. The following maxima occur if the argument of the tangent assumes the values:

1.43 π, 2.459 π, 3.47π, 4.479 π, ...

Intensity minima occur when:

 $\frac{b\pi}{\lambda}$ sin $\theta = n\pi$; $n = 1, 2, ...$

Figure 3: Intensity distribution on diffraction at the slit, as a function of the position along a straight line parallel to the plane of the slit, standardised on the intensity without the slit.

Where $a \gg x$, the minima are approximately equidistand, and

$$
x = n \cdot \frac{a\lambda}{b}
$$

If light falls on to a slit formed by a straight edge (parallel to the y axis), it is diffracted. If the origin of coordinates is placed at the intersection of the connecting line PQ between the light source and the point of incidence with the plane of the diffraction screen, the intensity distribution of the diffraction pattern behind the diffracting edge is:

$$
I=\frac{I_o}{2}\left\{\left(U(\omega)+\frac{1}{2}\right)^2+\left(V(\omega)+\frac{1}{2}\right)^2\right\}
$$

Figure 4: Diffraction at the edge.

Using the symbols of Fig. 4 we have:
$$
I_o = \frac{1}{(R_0 + R)^2}
$$

$$
\omega = x \cdot \cos \delta \sqrt{\frac{2}{\lambda} \left(\frac{1}{R_o} + \frac{1}{R}\right)}
$$

U and V are the Fresnel integrals, defined as follows:

$$
U(\omega) = \int_{0}^{w} \cos\left(\frac{\pi}{2}n^2\right) dn
$$

$$
V(\omega) = \int_{0}^{w} \sin\left(\frac{\pi}{2}n^2\right) dn
$$

The intensity on the shadow side decreases regularly. On the light side the intensity exhibits maxima and minima, while the total intensity according to (3) decreases quadratically with the distance between the light source and the point of incidence.

Fig. 5: Intensity distribution on diffraction at the edge, as a function of the position on a straight line at right angles to the line connecting the light source and the edge, standardised on the intensity without the edge.

Set-up and procedure:

The experimental set up is as shown in Fig. 1. The divergent lens of focal length - 50 mm is placed in front of the laser to expand the beam. An inner edge of the slit which is fully open serves as the edge. The distance between lens and slit is 75 mm. The laser power is adjusted to

1 mW. For diffraction at the slit the laser beam is directed symmetrically onto the vertical closed slit edges. The metal screen with the tape scale stuck to the middle, is set up at a certain distance (e. g. 3m). The slit is opened and the slit width is calculated from

$$
b = \frac{2m+1}{2 \sin \alpha_m} \cdot \lambda
$$

$$
\sin \alpha_m = \frac{x_m}{\sqrt[2]{x_m^2 + r^2}}
$$

Where:

 $b =$ slit width

 $m =$ serial order of the maximum from the centre outwards

 α_m = distance of the m_{th} maximum

 $r =$ distance between the slit and the screen

 λ = wavelength of the laser light

To ensure glare-free reading at the screen it is necessary to cover up the intensely bright centre of the pattern (e. g. with a pencil in a barrel base). For diffraction at the edge, the screen with a single slit (vertical) is stuck to the photocell with adhesive tape. The meter scale, on which a barrel base with the photocell can be moved at right angles to the laser beam, is secured at a certain distance (e. g. 3 m).

The photocell is connected to the multirangemeter with amplifier (mV $=$ nA). First of all, the intensity I_0 is measured without the edge – initially without the laser (dark value) and then with it (light value). These values must be taken into account in the evaluation. The edge (an edge of the slit) is moved into the laser beam so that half of it is

masked. This requires some care. In some circumstances, an intensity measurement can be carried out more rapidly with the slit screen lying horizontally. In this case the edge is moved into the beam until only half the voltage is recorded.

Discussion:

1- Explain the difference between Fresnel and Franhofer diffraction.

2- Explain the difference between diffraction pattern of a single slit and edge.

3- Plot the graph relation on the paper and discuss the experimental mistake.

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Exp No. (9) Intensity of diffractions due to pinhole diaphragms and circular obstacles

Figure 1: Experimental set-up: Intensity of diffractions due to pin hole diaphragms and circular obstacles.

Related topics:

Huygens principle, interference, Fraunhofer and Fresnel diffraction, Fresnel's zone construction, coherence, laser, Airy disk, Airy ring, Poisson's spot, Babinet's theorem, Bessel functions, resolution of optical instruments.

Principle:

Pin hole diaphragms and circular obstacles are illuminated with laser light. The resulting intensity distributions due to diffraction are measured by means of a photo diode.

Equipment:

- Laser, He-Ne 1.0 mw, 220V
- Optical profile bench $l = 60$ cm
- Base f.opt.profile-bench, adjust.
- Slide mounts f. opt. pr.-bench, h 80 mm
- Slide mount, lateral. Adjust. Cal.
- Object holder, 535 cm
- Screen, metal, 300-300 mm
- Screen, with diffracting elements
- Photo element f. opt. base plt.
- Multi-range meter

Object:

1. The complete intensity distribution of the diffraction pattern of a pin hole diaphragm ($D1 = 0.25$ mm) is determined by means of a sliding photo diode. The diffraction peak intensities are compared with the theoretical values. The diameter of the pin hole diaphragm is determined from the diffraction angles of peaks and minima.

2. The positions and intensities of minima and peaks of a second pin hole diaphragm ($D2 = 0.5$ mm) are determined. The diffraction peak intensities are compared with the theoretical values. The diameter of the pin hole diaphragm is determined.

3. The positions of minima and peaks of the diffraction patterns of two complementary circular obstacles $(D^*1= 0.25$ mm and $D^*2= 0.5$ mm) are determined. Results are discussed in terms of Babinet's Theorem.

Theory and evaluation:

If a plane wave impinges on a diaphragm of diameter D, due to reasons of symmetry, the corresponding diffraction pattern consists of a central bright circle

(Airy disk) surrounded by alternating concentric bright and dark rings (Airy rings). The intensity distribution is given by:

$$
I_{\varphi} = I_o \left(\frac{2J_1(\frac{\pi D}{\lambda} \sin \varphi)}{\frac{\pi D}{\lambda} \sin \varphi} \right)^2 \quad (J_1 = \text{Bessel function first order})
$$

The expected theoretical values for the diffraction angle w of the bright and dark rings (peaks and minima) as well as for the normalized intensities of the peaks derived from (1) are given in Table 1.

n	Minima	Peaks	I_n/I_o
	$\sin \theta_1 = 1.220 \frac{\lambda}{D}$	$\sin \theta'_1 = 1.638 \frac{\lambda}{D}$	0.0175
	$\sin \theta_2 = 2.232 \frac{\lambda}{D}$	$\sin \theta_2 = 2.679 \frac{\lambda}{D}$	0.00416
3	$\sin \theta_3 = 3.238 \frac{\lambda}{D}$	$\sin \theta_3^{\prime} = 3.699 \frac{\lambda}{D}$	0.00160
	$\sin \theta_4 = 4.241 \frac{\lambda}{D}$	$\sin \theta_4 = 4.710 \frac{\lambda}{D}$	0.00078

Table 1:

Figure 2 shows the intensity distribution of the diffraction pattern of a diaphragm with diameter $D1 = 0.25$ mm obtained empirically. The distance of the extremes of the nth order situated left and right of the central maximum is represented through D_{x_n} . Their arithmetic averages are taken for evaluation as well as the averages of the corresponding intensity peaks. The values indicated in table 2 for the pin diaphragm with diameter D1 as well as for intensity ratios I_n/I_0 are obtained from sin φ – tan $\varphi = \frac{\Delta}{\Delta}$ $\frac{\Delta x}{L}$ (L = 120.5 cm = distance between the diffracting object and the slit diaphragm) and λ = 632.8 nm.

Table 2:

Minima		Peaks				
					I_n/I_0	I_n/I_0
n	$\Delta x/mm$		$D_1/mm \Delta x/mm D_1/mm$		(exp.)	(theor.)
$\mathbf{0}$						
	3.78	0.246	4.93	0.254	2.6 Skt/160 Skt = 0.016	0.01750
$\overline{2}$	6.75	0.252	7.95	0.257	0.64 Skt/160 Skt = 0.004	0.00416
3	9.75	0.253	10.95	0.257	0.22 Skt/160 Skt = 0.0014	0.00160
	12.60	0.257				

 $D1 = (0.254 \pm 0.004)$ mm; $\Delta D1 / D1 = \pm 1.5\%$

Figure 2: Diffracted intensity I vs position x of the photodiode, using a diaphragm with $D1 = 0.25$ mm.

Table 3 shows the corresponding results of diffraction at the diaphragm with $D2 = 0.5$ mm (without figure).

Peaks Minima I_n/I_0 $\frac{1}{2}$ $\Delta x/mm$ $D_2/mm \Delta x/mm$ D_2/mm $(exp.)$ (theor.) n $\overline{0}$ $\mathbf{1}$ \blacksquare 2.18 2.7 0.01750 $\overline{\mathbf{1}}$ 0.427 0.463 17 Skt/840 Skt = 0.020 3.59 3.3 Skt/840 Skt = 0.0039 0.00416 $\overline{2}$ 0.474 4.17 0.491 3 5.00 1.4 Skt/840 Skt = 0.0017 0.00160 0.493 5.65 0.499 0.507 6.38

If the pin hole diaphragms are substituted by complementary circular obstacles, (e. g. $D^*1 = 0.25$ mm and $D^*2 = 0.5$ mm), the observed diffraction patterns are very similar (Babinet's theorem). The positions of the minima and peaks coincide with those of the corresponding complimentary pin hole diaphragms. Only the central peak is symmetrically crossed by two further minima. If Fraunhofer observation is used, the brightness peak always lies in the geometrical shadow of the circular obstacle.

Set-up and procedure:

The complete measurement set-up is shown in Figure 1. The slide mount for the laser and the slide mount for measurement are situated at the extremities of the optical bench, the diffracting object is situated at a distance of 19 cm. Before starting measurements, it must be made sure that the laser beam impinges at the centre of the diffracting objects. This is the case if the diffraction rings on the metal screen set immediately before the slit diaphragm for this purpose are in the centre of the screen and have a homogeneous distribution.

Table 3

This is obtained by carefully adjusting the diaphragm with the diffraction objects laterally and in height. Furthermore, it must be assured that the displacement trajectory of the photo cell diaphragm coincides with the meridian of the diffraction pattern.

In order to assure satisfactory local resolution during the determination of the diffracted intensity, the length of the diaphragm slit must be reduced symmetrically in such a way that the height of the slit is equal to its width. This can be done for example using two strips of black cardboard fixed with adhesive tape to the diaphragm.

Before carrying out measurements, the laser and the measuring amplifier should be allowed to warm up for about 15 minutes. Photovoltaic current is directly proportional to the intensity of impinging light. The current, converted to voltage, is measured by means of the measurement amplifier. The typical adjustment parameters for the measuring amplifier are: "Low-Drift Mode", amplifying factor 104–105; time constant 0.1 s. In order to determine the intensity of the diffraction pattern, the photo diode is shifted in steps of 0.3 mm, using the measurement shifting slide mount. During this procedure, the positions of the extremes must be determined with particular care.

Discussion:

1- Plot the experimental curve on the paper and compare with the theoretical curve.

2- Explain the difference between square and circular aperture to the pin hole.

3- What is the Babinet`s theorem? Explain and discuss the experimental mistake.

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University of Technology Laser & Optoelectronics Department Optoelectronic engineering branch

Exp No. (10) Fourier optics – 2f Arrangement

Figure 1: Experimental setup for fundamental principle of Fourier optics

Related topics:

Fourier transforms lenses, Fraunhofer diffraction, index of refraction, Huygens ' Principle.

Principle:

The electric field distribution of light in a specific plane (object plane) is Fourier transformed into the 2 f configuration.

Equipment

- Optical base plate w. rubber
- Laser, He-Ne 0.2/1.0 mW, 220 VAC*
- Adjusting support 35-35 mm
- Surface mirror 30-30 mm
- Magnetic foot f. opt. base plt.
- Holder f. diaphr./beam splitter
- Lens, mounted, f +150 mm
- Lens, mounted, f +100 mm
- Lens holder f. optical base plate
- Screen, white, 150-150 mm
- Diffraction grating, 50 lines/mm
- Screen, with diffracting elements
- Qbjective 20 N.A. 0.45
- Pin hole 30 um
- Rule, plastic, 200 mm

Alternative:

He/Ne Laser, 5 mW with holder Power supply f. laser head 5 mW

Objective:

Investigation of the Fourier transform by a convex lens for different diffraction objects in a 2 f set-up.

Set-up and procedure:

- In the following, the pairs of numbers in brackets refer to the coordinates on the optical base plate in accordance with Fig. 1b. These coordinates are intended to help with coarse adjustment.

- Perform the experimental set-up according to fig. 1a or 1b. The recommended set-up height (beam path height) is 130 mm.

- The E25x beam expansion system and the lens L_0 are not to be used for the first beam adjustment.

- When adjusting the beam path with the adjustable mirrors M_1 and M_2 , the beam is set along the 1. x and 1. y coordinates of the base plate.

- Now place the E25x beam expansion system without its objective and pinhole, but equipped instead only with the adjustment diaphragm, in the beam path. Orient it such that the beam passes through the circular stops without obstruction. Now replace these diaphragms with the objective and the pinhole diaphragm. Move the pinhole diaphragm toward the focus of the objective. In the process, first ensure that a maximum of diffuse light strikes the pinhole diaphragm and later the expanded beam. Successively adjust the lateral positions of the objective and the pinhole diaphragm while approaching the focus in order to ultimately provide an expanded beam without diffraction phenomena. The L_0 (f = +100 mm) is now positioned at a distance exactly equal to the focal length behind the pinhole diaphragm

- Such that parallel light now emerges from the lens. No divergence of the light spot should occur with increasing separation. (Testing for parallelism via the light spot's diameter with a ruler at various distances behind the lens L_0 in a range of approximately 1 m).

- Now set-up the additional optical components.

Figure.1b: Experimental set-up for the fundamental principles of Fourier optic (2 f set-up).

Set-up and procedure: (in accordance with Fig.1a and 1b)

Place a plate holder P1 [2,1] in the object plane. Position the lens L1 at the focus $(f = 150 \text{ mm})$ and the screen at the same distance behind the lens.

- (a) As a first partial experiment observe the plane wave itself (the light spot), i. e. no diffracting structures are placed in the object plane. According to the theory, a point should appear in the Fourier plane Sc behind the lens.

This is also the focus; this fact can be checked by changing the screen's distance from the lens.)

- (b) Now clamp the diaphragm with diffraction objects into the plate holder P1 in the object plane. While doing so, adjust its height and lateral position in such a manner that the light spot strikes the slit which has a slit width of 0.2 mm. The Fourier transform of the slit can be seen on the screen as the typical diffraction pattern of a slit (compare with the theory).

- (c) The diffraction grating (50 lines/mm) now serves as a diffracting structure; clamp it in the plate holder P1.Conclusions about the slit separation can be made from the separation of the diffraction maxima in the Fourier plane Sc behind the lens L1 (see theory).

Fig.2: A plane wave E_e (x,y) is diffracted in the plane with t (x,y) for $z=0$

Theory and evaluation:

The Fourier transform plays a major role in the natural sciences. In the majority of cases, one deal with Fourier transforms in a time range, which supplies us with the spectral composition of a time signal. This concept can be extended in two aspects:

1- In our case a spatial signal and not a temporal signal is transformed.

2. A two-dimensional transform is performed. From this, the following is obtained:

 $\begin{array}{l} \widetilde{E}(\nu_{\mathsf{x}},\,\nu_{\mathsf{y}})\cdot \widetilde{F}\,\left[E(x,y)\right]\,(\nu_{\mathsf{x}},\,\nu_{\mathsf{y}})=\\ \overset{\leftharpoonup}{\left|\vphantom{\int\limits_{\int}}\right|^{+\infty}} E(x,y)\mathrm{e}^{-\,2\pi i\,(\nu_{\mathsf{x}}\,\cdot\,x\,+\,\nu_{\mathsf{y}}\,\cdot\,y)}d_{\mathsf{x}}d_{\mathsf{y}} \end{array}$ -------------------------------1

Where $\mathsf{U} \mathsf{x}$ and v_y are spatial frequencies.

Scalar diffraction theory

In Fig. 2 we observe a plane wave which is diffracted in one plane. For this wave in the xy plane directly behind the plane $z = 0$ with the following transmission distribution t (x,y): $E(x,y) = t(x,y) E_e(x,y)$ where $E_e(x,y)$: electric field distribution of the incident wave. The further expansion can be described by the assumption that a spherical wave emanates from each point $(x,y,0)$ behind the diffracting structure (Huygens' principle). This leads to Kirchhoff's diffraction integral:

$$
E(x', y', z) = \frac{1}{i\lambda} \iint_{-\infty}^{+\infty} E(x, y) \cos(n, r) dx dy
$$

- With λ = spherical wavelength
- $n =$ normal vector of the (x, y) plane

$$
k
$$
 = wave number = $\frac{2\pi}{\lambda}$

Equation (2) corresponds to a accumulation of spherical waves, where the factor $\frac{1}{i\lambda}$ is a phase and amplitude factor and cos(n, r) a directional factor which results from the Maxwell field equations. The Fresnel approximation (observations in a remote radiation field) considers only rays which occupy a small angle to the optical axis (z axis), i. e. $|x|, |y| \ll z$ and $|x'|, |y'| \ll z$. In this case, the directional factor can be neglected and the $1/r$ dependence becomes: $1/r = 1/z$. In the exponential function, this cannot be performed as easily since even small changes in r result in large phase changes. To achieve this, the roots in

$$
r = \sqrt{(x - x)^2 + (y - y)^2 + z^2} = z \sqrt{1 + \frac{(x - x)^2}{z^2} + \frac{(y - y)^2}{z^2}}
$$

are expanded into a series and one obtains:

Figure 3: Relationships between spatial frequencies and the diffraction angle.

This results in the Fresnel approximation of the diffraction integral

$$
E(x', y', z) = \frac{e^{ikz}}{i\lambda}
$$

$$
\iint_{-\infty}^{+\infty} E(x, y) \cdot e^{i\frac{k}{2z}((x - x)^2 + (y - y)^2)} dxdy
$$
 (3)

For long distances from the diffracting plane with concurrent finite expansion of the diffracting structure, one obtains the Fraunhofer approximation:

$$
E(x', y', z) = C(x', y', z)
$$

\n
$$
\iint_{-\infty}^{+\infty} E(x, y) \cdot e^{-2\pi i \left(\frac{x}{\lambda z}x + \frac{y}{\lambda z}y\right)} dx dy
$$

\nwith $c(x', y', z) = \frac{e^{ikz}}{i\lambda z} \cdot e^{i\frac{\pi}{\lambda z}(x'^2 + y'^2)}$

With the spatial frequencies as new coordinates:

$$
v_x = \frac{x}{\lambda z} \; ; \; v_y = \frac{y}{\lambda z} \tag{5}
$$

Consequently, the field distribution in the plane of observation (x', y', z) is shown by the following:

$$
E\left(x,y,z\right) = C\left(\lambda z v_x, \lambda z v_y, z\right) \tilde{F}\left[E(x,y)\right]\left(v_x, v_y\right)
$$

$$
= \tilde{E}\left(v_x, v_y\right) \tag{6}
$$

The electric field distribution in the plane (x' , y') for $z =$ const is thus established by a Fourier transform of the field strength distribution in the diffracting plane after multiplication with a quadratic phase factor $\exp((i_p/1z)(x^2+y^2))$. The spatial frequencies are proportional to the corresponding diffraction angles (see Fig. 3), where:

Fig. 4: Experimental set-up with supplement for direct measurement of the initial velocity of the ball

Through the making of a photographic recording or through observation of the diffraction image with one eye, the intensity formation disappears due to the phase information of the light in the plane (x', y', z) . As a consequence, only the intensity distribution (this corresponds to the power spectrum) can be observed. As a result the phase factor C (Equation 6) drops out of the operation. Therefore, the following results:

$$
I(v_x, v_y) = \frac{1}{\lambda^2 z^2} |\tilde{F}[E(x, y)](v_x, v_y)|^2
$$
\n(7)

Fourier transform by a lens

A biconvex lens exactly performs a two-dimensional Fourier transform from the front to the rear focal plane if the diffracting structure (entry field strength distribution) lies in the front focal plane (see Fig. 4). In this process, the coordinates v and u correspond to the angles β and α with the following correlations:

$$
v_x = \frac{\dot{x}}{\lambda z} = \frac{\alpha}{\lambda} = \frac{u}{\lambda f_B}
$$

$$
v_y = \frac{y}{\lambda z} = \frac{\beta}{\lambda} = \frac{v}{\lambda f_B}
$$
 (8)

This means that the lens projects the image of the remote radiation field in the rear focal plane:

$$
\tilde{E}(u, v) = A(u, v, f_B)
$$
\n
$$
\iint_{-\infty}^{+\infty} E(x, y) \cdot e^{-2\pi i \left(\frac{u}{\lambda f_B} x + \frac{v}{\lambda f_B} y\right)} d_x d_y
$$
\n(9)

The phase factor A becomes independent of u and v, if the entry field distribution is positioned exactly in the front focal plane. Thus, the complex amplitude spectrum results:

$$
\tilde{E}(x,y) \sim \tilde{f}[E(x,y)](u,v)
$$

Again the power spectrum is recorded or observed:

$$
I(u, v) = |\tilde{E}(u, v)|^{2} \sim |\tilde{F}[E(x, y)]|^{2}
$$
\n(10)

It, too, is independent of the phase factor A and thus becomes independent of the position of the diffraction structure in the front focal plane. Additionally, equation 8 shows that the larger the focal length of the lens is, the more extensive the diffraction image in the (u ,v) plane is.

Figure 5: Spectra of a plane wave. (a) for the direction of light propagation parallel to the optical axis. (b) for slanted incidence of the plane wave with reference to the optical axis.

Examples of Fourier spectra

(a) Plane wave:

A plane wave which propagates itself in the direction of the optical axis (z axis) (Figure 5) is distinguished in the object plane – (x,y) plane – by a constant amplitude. Thus, the following results for the Fourier transform:

$$
E(x, y) = E_o \tag{11}
$$

and

$$
\tilde{F}[E(x, y)] = \iint_{-\infty}^{+\infty} E_0 e^{-2\pi i (v_x x + v_y y)} d_x d_y
$$

$$
= E_0 \cdot \delta(v_x) \delta(v_y)
$$

This is a point on the focal plane at $(nx,ny) = (0,0)$, which shifts at slanted incidence by an angle a to the optical axis on the rear focal plane (see Fig. 5) with $nx = \sin \frac{\pi}{2}$.

(b) Infinitely long slit with finite width

If the diffracting structure is an infinite slit which is transilluminated by a plane wave, this slit is mathematically described by a rectangular function react perpendicular to the slit direction and having the same width a:

$$
E(x, y) = \text{rect}\left(\frac{x}{a}\right) = E_o \begin{cases} 1 \text{ for } |x| < \frac{a}{2} \\ 0 \text{ else} \end{cases}
$$

In the rear focal plane the following spectrum then results:

$$
\tilde{F}[E(x, y)] = E_o \iint_{-\infty}^{+\infty} e^{-2\pi i (v_x x + v_y y)} d_x d_y
$$
\n
$$
= E_o \cdot \delta(v_x) \frac{\sin(\pi \cdot v_x a)}{\pi v_x} (v_y)
$$
\n
$$
= E_o \cdot a \delta(v_y) \operatorname{sinc}(av_x)
$$
\n(12)

With the definition of the slit function sinc:

$$
\operatorname{sinc}(x) = \frac{\sin(\pi \cdot x)}{\pi \cdot x}
$$

For infinitely long extension of the slit, one obtains on extension in the slit direction in the spectrum. This changes for a finite length of the slit.

The zero points of the Sinc function are located at $\dots - 2/a$, $-1/a$, $1/a$, $2/a$, \dots (see Figure 6).

(c) Grid:

A grid is a composite diffracting structure. It consists of a periodic sequence (to be represented by a so-called comb function comb) of individual identical slit functions sinc.

Fig. 6: Infinitely long slit with the width a and its Fourier spectrum.

The grid consists of M slits having a width a and a slit separation d ($>a$) in the x direction. As a result, the field strength distribution can be in the front focal plane can be represented as follows:

$$
E(x, y) = E_o \sum_{m=1}^{M} \text{rect}\left(\frac{x}{a} - \frac{m \cdot d}{a}\right) = E_o \left[\sum_{m=1}^{M} \delta(x - m \cdot d)\right] * \text{rect}\left(\frac{x}{a}\right)
$$

Where the Fourier transform of a convolution product (E1*E2) is given by:

$$
\tilde{F}[(E_1 * E_2)(x, y)](v_x, v_y) = \tilde{F}[E_1(x, y)](v_x, v_y) \cdot \tilde{F}[E_2(x, y)](v_x, v_y)
$$

Using the calculation rules for Fourier transforms, the following spectrum results in the rear focal plane of the lens:

$$
\tilde{F}(E) = E_o \cdot \delta(v_x) \cdot \frac{\sin(\pi a v_x)}{\pi v_x} \sum_{m=1}^{\infty} e^{-2\pi i m \cdot d v_x}
$$
\n(13)

$$
= E_o \, \delta(v_y) \, a \, .\text{sinc}(av_x) \, . \, e^{-\pi i dv_x(M+1)} \frac{\sin(\pi M dv_x)}{\sin(\pi . dv_x)}
$$

Due to the intensity formation the phase factor is cancelled:

$$
I(v_x, v_y) = |E_o|^2 \delta(v_y) a^2 \operatorname{sinc}^2(a v_x) \frac{\sin^2(\pi . M . dv_x)}{\sin^2(\pi . dv_x)}
$$
(14)

In Figure 7, a grid with its corresponding spectrum (and the corresponding intensity distributions) is presented. One sees on the spectrum that the envelope curve is formed by the spectrum of the individual slit which has a width a. The finer structure is produced by the periodicity, which is determined by the grid constant Md.

Figure 7: Grating consisting of M slits and its Fourier spectrum.

Discussion:

1- Is it possible that beam laser after passing through a lens could become collimated without using beam expanding? Explain?

- 2- What is the aperture function in the Fourier transform?
- 3- Why output beam from the lens is collimated and not normal beam?

University of Technology Laser & Optoelectronics Department Optoelectronic engineering branch

Exp No. (11) Fourier optics – 4f arrangement

Figure 1: Experimental setup for Fourier optics (4f set-up).

Related topics:

Fourier transforms, lenses, Fraunhofer diffraction, index of refraction, Huygens' Principle, fog technique.

Principle and task:

The electric field distribution of light in a specific plane (object plane) is Fourier transformed into the 4f configuration by 2 lenses and optically filtered by appropriate diaphragms.

Equipment:

- Optical base plate w. rubber ft.
- Laser, He-Ne 0.2/1.0 mW, 220 V
- Adjusting support 35-35 mm
- Surface mirror 30-30 mm
- Magnetic foot f. opt. base plt.
- Holder f. diaphr./beam splitter
- Lens, mounted, $f = +100$ mm
- Lens holder f. optical base plate
- Screen, white, 150-15 mm
- Slide -Emperor Maximilian-
- Screen, with arrow slit
- Diffraction grating, 4 lines/mm
- Diffraction grating, 50 lines/mm
- Diaphragms, d 1, 2, 3, 5 mm
- Screen, with diffracting elements
- Objective 25- N.A. 0.45
- Sliding device, horizontal
- Pin hole 30 micron

Object:

- 1. Optical Filtration of diffraction objects in 4f set-up.
- 2. Reconstruction of a filtered image.

Fig. 1b: Experimental set-up for Fourier optics, 4f set-up, filtering and reconstruction.

Set-up and procedure

- In the following, the pairs of numbers in brackets refer to the coordinates on the optical base plate in accordance with Fig. 1b. These coordinates are only intended to help with coarse adjustment.

- Perform the experimental set-up according to Fig. 1a and 1b. The recommended set-up height (beam path height) is 130 mm.

- The E25x beam expansion system and the lens L_0 are not to be used for the first beam adjustment.

- When adjusting the beam path with the adjustable mirrors M1 and M2, the beam is set along the $1st$ x and $1st$ y coordinates of the base plate.

Now, place the E25x beam expansion system without its objective and pinhole, but equipped instead only with the adjustment diaphragms, in the beam path. Orient it such that the beam passes through the circular stops without obstruction. Now, replace these diaphragms with the objective and the pinhole diaphragm. Move the pinhole diaphragm toward the focus of the objective. In the process, first ensure that a maximum of diffuse light strikes the pinhole diaphragm and later the expanded beam. Successively adjust the lateral positions of the objective and the pinhole diaphragm while approaching the focus in order to ultimately provide an expanded beam without diffraction phenomena. The L_0 (f = +100mm) is now positioned at a distance exactly equal to the focal length behind the

pinhole diaphragm such that parallel light now emerges from the lens. No divergence of the light spot should occur with increasing separation. (Test for parallelism via the light spot's diameter with a ruler at various distances behind the lens L_0 in a range of approximately 1 m).

- Now set-up the additional optical components.

4-f-set-up:

- Place a plate holder P1 in the object plane. Position the lens L1 at the focus $(f = 100 \text{ mm})$ and the second plate holder P2 at the same distance behind the lens. Additionally, place another lens L2 at a distance equal to the focus $f = 100$ mm and at the same distance of 10 cm set up the screen. The parallel light beam that strikes the lens L1 must appear on the screen Sc at the same height and with the same extension (Check with the ruler!). To begin with, observations without optical filtration

a- Clamp the diaphragm with the arrow (arrow pointing upwards) as the first diffracting structure in plate holder P1 in the object plane and shifted laterally in such a manner that the light from the mirror M2 strikes the arrow head. An arrow also appears on the observation screen (compare with theory!). The arrow is now turned 90° so that it points in a horizontal direction (shift the diaphragm laterally until the arrow head is will illuminate). In this case also, compare the image on the screen with the theory.

b- The photographic slide of Emperor Maximilian serves as the next diffracting structure. It is placed in plate holder P1 and laterally shifted until the light beam illuminates significant contours of the face (e.g. the nose) (in a recognizable form!). Observe the image in the observation plane (what is different than the original?).

c- Place the grid (4 lines/mm) in the object plane P1 as a further diffracting structure. In the process, observe the Fourier-transformed image in the Fourier plane P2 with the screen and subsequently examine it in the observation plane.

One ascertains that the lines of the grid cannot be resolved in the image (but they cannot be resolved in the real grid either!).

By turning the screen around its vertical axis, the image can be expanded in a distorted manner such that the grid lines can be discerned.

Observations with optical filtration:

a- Clamp the grid (50 lines/mm) in the object plane P1 in a plate holder. Nowperform a low-pass filtration in the Fourier plane P2, by positioning a pinhole diaphragm (diameter:1–2 mm) in such a manner that only a single, arbitrary diffraction maximum passes through Observe the image on the screen.

b- Clamp the slide (Emperor Maximilian) and the grid (4 lines/mm in vertical direction) together in the optical plane in the plate holder P1. To begin with, observe the Fourier spectrum in P2 on the screen and subsequently the image on the screen at position. In the Fourier plane, one sees sharply focused diffraction structures. This can be checked by moving a black diaphragm (blackened part of a pinhole diaphragm) in the direction of light propagation. The filter plane P2 (intended position of the plate holder) should correspond with the thusdetermined Fourier plane (most sharply focused diffraction structures): If the Fourier plane (or the focal length of the lens) has shifted, remove the grid and the slide in order to readjust the second lens L2, since this lens also should have the same focal length fB as the first lens.

- Move the lens L2 in the direction of the optical axis along the 1st y coordinate of the base plate such that behind this lens (according to Fig. 1b to the right of lens L2) parallel light (constant diameter of the light spot) is present (check with a ruler).

- The grid with slide can then again be placed in the object plane and the screen should be set up at the same distance from the 2nd lens L2 as from the 2nd lens to the Fourier plane P2.

Observation:

In the Fourier plane, one sees primarily the grid spectrum with discrete diffraction maxima. On the observation screen, the slide (Emperor Maximilian) can be seen. It is possible that the fine grid cannot be seen due to the marked speckle formation.

To still be able to resolve this grid, the screen can be turned around its vertical axis to expand the image. It is necessary to turn the screen to such an extent that it is nearly parallel to the direction of light propagation. In this manner, the fine structures will be enlarged and readily visible. Now perform a low-pass filtration with a pinhole diaphragm (diaphragm with diffraction objects) in the Fourier plane P2 by selectively filtering out all but a single arbitrary diffraction maximum. It is

advisable to use the central zero diffraction maximum, as the greatest light intensity is located there. And accordingly, the image on the screen is the brightest.

On using the pinhole diaphragm with a diameter of 0.25 mm, the grid structure disappears. The image of the slide (Emperor Maximilian) is not affected when the screen is in a perpendicular position (no distortion). If the screen is turned nearly parallel to the direction of light propagation, the grid structure cannot be seen.

However, as soon as the pinhole diaphragm has a diameter equal to 0.5 mm, it is impossible to filter out the grid structure. When the screen is turned to the horizontal position, the image of the slide always has superimposed grid lines.

c- Fog technique: This procedure makes phase gradients visible, such as those, for example, which occur in the flow of gases having different densities (air flow of a candle). Marked phase changes also occur at the edge of a pane of glass.

- To observe this, place the glass pane (as phase object), diaphragm with phase objects, in the object plane P1 in such a manner that the left or right edge (without blackening!) is reproduced on the screen with only slight light reflections. In the Fourier plane a half-plane is now covered with a black diaphragm (e.g. arrow diaphragm). To achieve this, the diaphragm is fixed in a plate holder and then the

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total unit with magnetic base is shifted in such a manner that one diaphragm edge, which is parallel to the glass plate's edge, is cautiously moved into the region of the main diffraction maximum (brightest point in the Fourier spectrum). Half of the diffraction maximum must be covered so that the elevated intensity at the margin of the pane of glass can be seen in the image on the screen (The edge appears as a black stripe!).

- Density oscillations in water as a phase object: The experimental set-up remains the same as in (c) (see Fig. 2). The only difference is that in the object plane the plate holder with the diaphragm is removed. Instead of it, place two magnetic bases in the positions which are each provided with a small table with rod. Place the cell which has been filled with water on this small table such that the light spot remains visible on the screen without additional reflections. Clamp the handle of the sonic transducer in a universal clamp and fix it onto the optical plate using a right-angle clamp to connect it to a support rod clamped into a magnetic base. Now position the sonic transducer with its surface plane parallel to the water's surface such that it is immersed in the liquid by about 5 mm without bubble formation. (Caution: Do not immerse the transducer too deeply!) Connect the sonic transducer with the ultrasonic generator. In its switched-off state, one does not observe any changes on the screen.

- The transducer is driven by a sinusoidal excitation, i.e. depress the right pushbutton on the ultrasound device. When the device is switched on and the excitation level is appropriately elevated (Do not select an excessively strong excitation, as otherwise there is danger of bubble formation and of evaporation of the liquid!) – Intermediate level adjustment – a standing sound field is generated in the glass cell. It serves as a phase grid for the light wave which is incident perpendicular to it (see theory). In this state also, one notices no change of any kind on the observation screen Sc. Now, cover a half-plane of the Fourier spectrum in the Fourier plane with the black diaphragm (arrow diaphragm). To do this, fix the diaphragm in a plate holder and then shift the entire unit with the plate holder in a magnetic base vertically in such a manner that one edge of the

diaphragm, which is parallel to the direction of sound propagation, is cautiously moved into the region of the main diffraction maximum (brightest point in the Fourier spectrum). Half of the diffraction maximum must be covered, so that intensity elevation in the region of the ultrasonic wave can be seen in the image on the screen Sc (parallel, horizontal stripes). If the sonic excitation is reduced, these stripes disappear.

Figure 2: Experimental set-up for the fog method. Making the phase grid visible with an ultrasonic wave

Theory and Evaluation

For information on the fundamentals of Fourier optics and the Fourier transformation by a lens, see the "Fourier optics – 2f Arrangement" experiment.

Coherent optical filtration

By intervening in the Fourier spectrum, optical filtration can be performed which can result in image improvement, etc. The appropriate operation for making the original image visible again, ins the inverse Fourier transform, which however cannot be used due to diffraction. The Fourier transform is again used; this leads to the 4f set-up (see Fig.3). Using the 1st lens (L1), the spectrum with the appropriate spatial frequencies is generated in the Fourier plane from the original diffracting structure t (x, y) (see the "Fourier optics – 2f Arrangement" experiment). In this plane, the spectrum can be altered by fading out specific spatial frequency fractions. A modified spectrum is created, which is again Fourier transformed by the 2nd lens (L2). If the spectrum is not altered, one obtains the original image in the inverse direction in the image plane (right focal plane of the 2nd lens) (partial experiment (a) with the arrow diaphragm). This follows from the calculation of the twofold Fourier transformation:

$$
\tilde{F}\left[\tilde{F}[f(x,y)]\right] = f(-x, -y) \tag{1}
$$

The simplest applications for optical filtration are the high- and low-pass filtration

Figure 3: Principle of the set-up for coherent optical filtration

Low-pass raster elimination

In the experiment, the photographic slide was provided with a raster by superimposing grid lines on it in one direction. The scanning theory states that a non-raster image (in this case: Emperor Maximilian) can be exactly reconstructed if the image is band-limited in its spectrum, i. e. if it only contains spatial frequencies in the Fourier plane up to an upper limiting frequencies. The raster image can be described mathematically as follows:

$$
b(x, y) = \text{comb}_{b}^{\mathcal{Y}} \cdot g(x, y) \tag{2}
$$

With the comb function

$$
comb\frac{y}{b} = \sum_{-\infty}^{+\infty} \delta(y - nb) \tag{3}
$$

This describes the grid lines (the grid) and $g(x, y)$ the non-raster image (in this case: the slide). The slit separation of the grid is b. The Fourier spectrum *B* (v_x , v_y) of the entire image becomes the following with the convolution law:

$$
\tilde{F}[b(x, y)] = B(v_x, v_y) = b \text{ .comb}(b \cdot v_y) \cdot G(v_x, v_y)
$$
\n(4)

Where G (v_x, v_y) is the Fourier transformation of the non-raster image. In addition, the fact that the Fourier transformations of a comb function is also a comb function. This means that the Fourier spectrum once again a grid which is formed by the reiteration of the spectrum of the non-raster image (see Fig. 4). Each grid point with its immediate surroundings contains the total information of the non-raster image $g(x, y)$. It is important that the grid points in the Fourier plane are sufficiently far apart that the spectra of the unlined image do not overlap. Only in this case is it possible to filter out a single image point with a pinhole diaphragm. This spatial frequency filtration can be considered as multiplication of the spectrum by an aperture function A (v_x, v_y) (pinhole diaphragm in the Fourier spectrum). In this case, an appropriate measuring dimension would be a diameter of $\approx 1/b$. Therefore:

$$
B_{filtered}(v_x, v_y) = B(v_x, v_y) . A(v_x, v_y)
$$

= $b(-x, -y) * \tilde{F}[A(v_x, v_y)]$ (5)

Figure 4: Composite spectrum, e.g. raster image.

Fog procedure

The fog procedure makes it possible to see phase objects (e.g. phase grids through a standing ultrasonic wave). In this case, the Fourier spectrum is filtered with a half-plane filter. This amplitude filter should fade out exactly one half-plane of the spatial frequency spectrum including half of the zero order! The intensity of the image in the observation plane (in this case at screen) is then proportional to the gradient of the phase, where the direction of the observed gradient is a function of the position of the half-plane:

$$
I(-x, -y) \sim \left| \frac{\partial \varphi(x, y)}{\partial x} \right| \tag{6}
$$

With φ: phase of light.

The principle can be described as follows: The total image information is contained in a half-plane of the spatial frequency spectrum. In order to obtain the original intensity distribution of the diffracting structure by repeated Fourier transform, the superimposition of the two spatial frequency half-planes is necessary. If this

superimposition is prevented, the phase information of the diffracting structure is made visible.

Information on the ultrasonic wave:

The ultrasonic wave forms a standing sound wave between the ultrasonic transducer and the cell's bottom. This is a periodically oscillating pressure (and density) variation of the water with spatially fixed pressure nodes. Since the optical refraction index is proportional to the density of the medium, the propagation velocities of the light which is perpendicularly incident to the direction of sonic propagation are different in the various pressure regions. This results in a phase modulation of the light behind the ultrasonic wave (see Figure 5) Since the ultrasonic wave is a standing wave, the location of the phase gradients does not change (it disappears only in the pressure maxima and minima of the sonic field), but the intensity changes periodically. Due to the sluggishness of perception, a temporal average is taken here as the sound frequency is approximate 800 kHz

Fig 5: Schematic diagram of a standing ultrasonic wave based

Application areas of optical filtration:

Low-pass filtration can be used as a spatial frequency filter to eliminate the disturbances of the wave front, which result from soiling of the lenses, in a beam expansion by a microscope's objective (as E25x in this experiment). Another

direct application of low-pass filtration is the elimination of raster lines in composite images. For example, in astronomy, composite satellite images are freed from their raster. In Fourier optics, numerous procedures exist for making phase objects or modulations visible:

1- The phase-contrast procedure effects the transformation of a phase modulation on a diffraction object into an amplitude modulation by inserting a l/4 wafer into the zero diffraction order. This modulation can then be observed in the observation plane. The phase-contrast microscope is a widespread application of this procedure.

2- The fog technique in which a half-plane filter is used in the Fourier plane makes the phase gradients visible (as described in this experiment).

3- The dark-field technique is a high-pass filtration in the Fourier plane, i.e. the zero diffraction order is filtered out by a small disk of the appropriate size.

Using this filtration method, thin sections, organic preparation, currents of air and pressure waves in fluid dynamics research are made visible.

Additional fields of application result from the use of holographic filters.

Images of Fourier spectra of certain diffracting structures on holographic photo material can be used as filters in the Fourier plane of the 4f set-up.

These then are used for pattern recognition, i. e. for the repeated recognition of fundamental diffracting structure. on a pressure change (or density change).

Discussion:

1- Explain the advantages of the low pass filtration.

2- How can we remove the dust on image?

3- Discuss the systematic and statistical errors that affect your determination of the optical filtration

Faraday Effect

Aim of Experiment:

To determine the Verdet's constant of the material for a given wavelength of light.

Equipment:

Diode laser with power supply , Detector out put measurement unit , Electromagnet with glass rod , Polarizer and analyzer.

Principle:

The Verdet constant (**V)** is the proportionality constant between the angle of rotation **θ** of plain polarized light and the product of the path length **l** through the sample and the applied magnetic filed **B**, ie $\theta = V \cup B$ or $V = \theta / I \cup B$

Fig (1) Influence the magnetic filed

Theory :

The Faraday Effect is a magnetoptical effect in which a plane of polarized light is rotated as it passes through a medium that is in a magnetic filed. the amount of rotation is dependent on the amount of sample that the light passes through the strength of the magnetic filed and a proportionality constant called the Verdet Constant .

 θ = **V l B** , Strength of magnetic field **B** = π NI , Where N present number of turn per unite length of the coil , I present the current through the coil

Fig (2) components setup

Procedure :

1- Arrange the components as shown in figure (2)

2- Align the laser beam in order to pass through the central axis of the coil containing the glass rod and the center of the pin hol detector .

3- Rotate the polarizer up to the position of minimum intensity .

4- Switch on the power supply of the electro magnet .

5- increase the current through the electro magnet to an intervals of 0.4 A .

6- Now rotate the lead screw of the analyzer in order to get a minimum intensity position and note the corresponding reading from the dial of the analyzer , this is the angle of rotation **θ**

note: Number of turns in the coil = 2508, Length of the coil = 15 cm, Length of the glass rod $(l=10cm)$.

Discussion

- 1- Determine the magnetic B flux-density between the pole pieces for different coil current and draw the figure (discuses the result) .
- 2- Determine the minimum position angle **θ** for different current and draw the figure (discuses the result) .
- 3- Determine the Verdet Constant (**V**)

Kerr Effect

Aim of Experiment:

Verifying the Kerr effect and calculating Kerr's constant. *Equipment:*

1 Prism table, 1 Small optical bench, 1 Pair of polarizing filters, 1 Halogen lamp with holder, 1 Picture slider, 2 Lens in frame (100mm), Bench clamp riders, 1Kerr cell, 1 High-voltage power supply unit, Connecting leads, 1 Translucent screen, nitrobenzene.

THEORY:

When a substance (especially a liquid or a gas) is placed in an electric field its moleculesmay become partly oriented making the substance anisotropic and birefringent ; that means it can refract light differently in two directions. This effect is called electro-optical Kerr effect or simply Kerr effect and was discovered in 1875.

This birefringence increases quadratically with the electric field strength. For reasons of symmetry, the optical axis of birefringence lies in the direction of the electric field. The normal refractive index of the substance is changed to *n*e for the direction of oscillation parallel to the applied field, and to *n*o for the direction of oscillation perpendicular to it. The change in index is given by:

 $ne - no = K \lambda E^2$

$$
\begin{array}{cc} (1) \end{array}
$$

K is the Kerr constant for the medium λ Is the wavelength of the light used

E is the electric field strength

This difference in index of refraction causes the material to act like a waveplate when light is incident on it in a direction perpendicular to the electric field. If the material is placed between two "crossed" (perpendicular) linear polarizers, no light will be transmitted when the electric field is turned off, while nearly all of the light will be transmitted for some optimum value of the electric field. Higher values of the Kerr constant allow complete transmission to be achieved with a smaller applied electric field. Some polar liquids, such as nitrotoluene (C7H7NO2) and nitrobenzene (C6H5NO2) exhibit very large Kerr constants. A glass cell filled with one of these liquids is called a *Kerr cell*. These are frequently used to modulate light, since the Kerr effect responds very quickly to changes in electric field. Light can be modulated with these devices at frequencies as high as 10 GHz. Because the Kerr effect is relatively weak, a typical Kerr cell may require voltages as high as 30 kV to achieve complete transparency. This is in contrast to Pockels cells, which can operate at much lower voltages. The Kerr effect is usually demonstrated by placing a Kerr cell containing nitrobenzene between two flat parallel plates spaced several millimeters and applying a high voltage on the plates.

If the field is such that the cell retards the extraordinary ray by half wavelength,

the polarization rotation will be 90о. If a pair of polarizers is put around the cell, oriented at 45о, the assembly acts as a shutter . The voltage needed to do this is called the halfwavelength voltage. At the halfwavelength voltage, the following is true:

$$
(\text{ne} - \text{no})L = \lambda/2 \tag{2}
$$

Where , L is the distance in the Kerr cell covered by the light λ is the wavelength of the light.

In general the optical difference in paths for 2 waves is $(ne - no)$ L and this corresponds to a phase displacement of:

$$
\Delta = (ne - no) L/\lambda \tag{3}
$$

It can also be shown that the phase displacement is proportional to length L and the square of polarization P. If we anticipate that the polarization is a linear function of the electric field strength E and the proportionality factor is designated by K, then the following relation is obtained:

$$
\Delta = KE^2 L \tag{4}
$$

Thus if the phase difference between the ordinary ray and the extraordinary ray equals to λ /2 then we have $\Delta = (\lambda/2) / \lambda = 1/2$

In addition if $E=V/d$, where V=applied Voltage and d the inter-electrode distance we can say that:

 Δ = K V² L / d² and V² = d² Δ / K L

Figure 1 (Kerr effect experiment setup)

PROCEDURE:

IMPORTANT SAFETY NOTES:

Nitrobenzene is an extremely strong liver poison. It can enter the body if the vapor is breathed in the event of skin contact of by swallowing. Spark discharge may cause the nitrobenzene to explode. It is therefore important that the following safety notes are taken:

- Carry out the experiment in a well-ventilated room.
- Avoid skin contact with nitrobenzene.
- Do not breather in the vapor.
- Fill and empty the Kerr cell under the extraction hood.
- Fill the Kerr cell that the liquid level is several millimeters above the condenser gap in order to avoid spark discharge.
- Activate the cell as follows before starting measurements:
- Fill the cell with nitrobenzene so that it is 2/3 full and connect to the high voltage power supply unit. Use the 5 kV, 2 mA output. Set the voltage selector switch to the scale center (2.5 kV).
- Allow the cell to stand for some time, occasionally looking at the voltage display on the high-voltage power supply unit. The cell is ready for measurement when the voltage has risen to approximately 2 kV.
- Identify the polarity of the cell. Always use the cell with the same polarity because there is otherwise a danger of spark discharge. Setting up the optical apparatus:
- 1. Set up the apparatus on the optical bench as shown in Figure 1 and make the electrical connections. Equip the halogen lamp and the picture slider with heat insulation filter (to prevent the nitrobenzene from being heated up) and switch on.
- 2. Set both polarizing filters to 0°.
- 3. Set the high-voltage power supply unit to zero. Mount the Kerr cell on the prism table and place the capacitor gap on the optical axis. Form an image of the light coil in the opening of the 100 mm lens. Move the lens so that a sharp image of the capacitor gap is formed on the screen.

Carrying out the experiment:

1. Set the polarizer to 45° and the analyzer to -45° so that the field of view is darkened.

Note: The field of view is lit up as soon as a high voltage is applied to the capacitor gap.

2. Starting from 0 kV, increase the voltage gradually until you reach 5 kV noting how the intensity changes. Record (in Table 1) the voltage values at each maximum and minima where the maximum intensities correspond to relative path difference $\Delta = 1/2$, $\frac{3}{2}$, $\frac{5}{2}$... and the minimum intensities correspond to relative path difference $\Delta=0$, 1, 2, 3... ($\Delta=0$ at 0 kV).

Figure 2: Optical setup (1) Light source (2) Ploarizer (3) Kerr cell (4) Lens (5)

Analyzer (6) Screen.

3. Plot Δ vs. V^2

4. Using the slope, calculate Kerr constant. (Note: $d = 1$ mm and $l = 2$ cm)

Discussion

1. The phase-shift between the normal and the extra-ordinary light beam is to be recorded for different voltages applied to the nitrobenzene-element respectively for different electric field

strengs. Determine the half-wave voltage $V(\lambda/2)$?

- 2. Plot the square of the applied voltage versus the phase shift between normal and extraordinary beam? Discuss your results?
- 3. Calculate the Kerr constant?

4. Demonstrate that
$$
V^2 = \frac{d^2}{\pi K l} arc \sin \sqrt{\frac{l}{10}}
$$
 ?