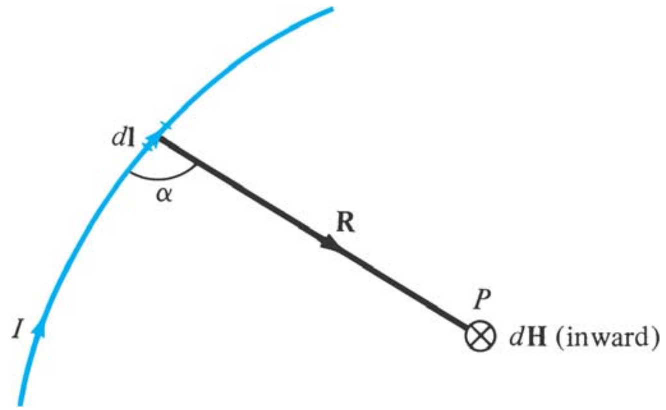
MAGNETICALLY COUPLED CIRCUITS

BIOT-SAVART'S LAW

The magnetic field intensity *dH* produced at a certain point P, by the differential current element *Idl* is proportional to the product *Idl* and the sine of the angle α between the element and the line joining P to the element and is inversely proportional to the square of the distance *R* between P and the element.

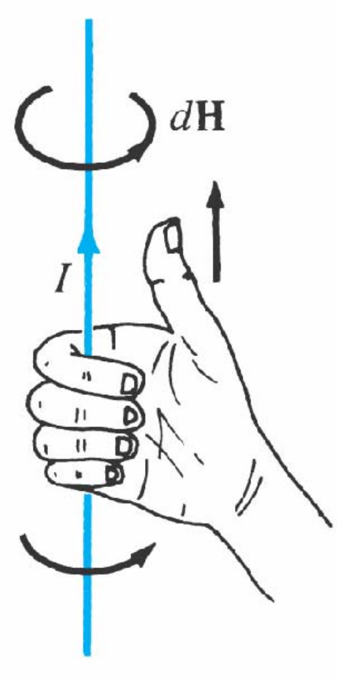
$$dH \propto \frac{I\, dl \sin\alpha}{R^2}$$

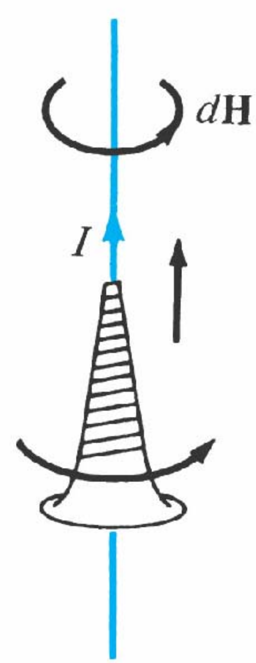
$$dH = \frac{I\ dl \sin\alpha}{4\pi R^2}$$

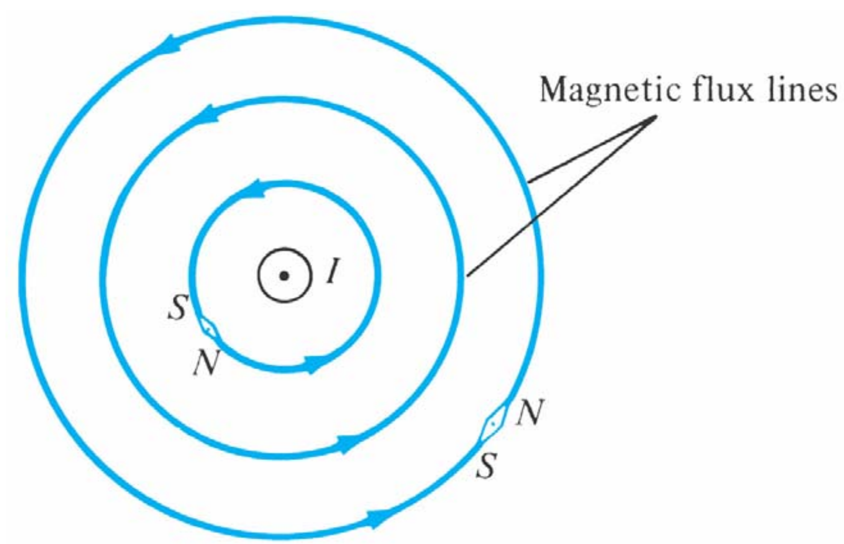


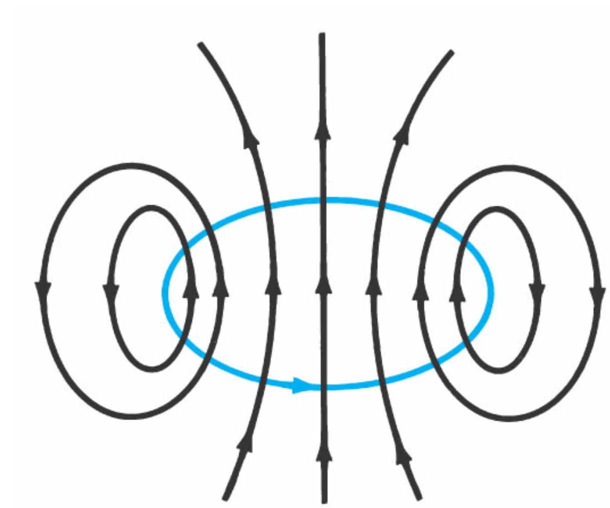
The direction of *dH* is determined by the right hand rule (RHR)

$$d\vec{H} = \frac{I\, d\vec{l} \times \hat{R}}{4\pi R^2} = \frac{I d\vec{l} \times \vec{R}}{4\pi R^3} \rightarrow \vec{H} = \int_L \frac{I\, d\vec{l} \times \hat{R}}{4\pi R^2}$$ (line current)


dH
I

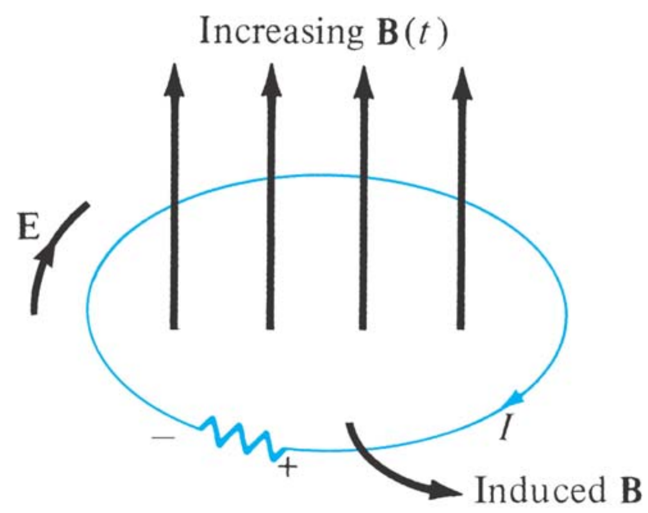

dH
I


Magnetic flux lines
I
S
N
N
S



FARADAY'S LAW

- The physical or experimental law governing the principle of magnetic induction .
- "The electromotive force (EMF) induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit."
- The EMF can either be produced by changing B (induced EMF) or by changing the area, e.g., by moving the wire (motional EMF).

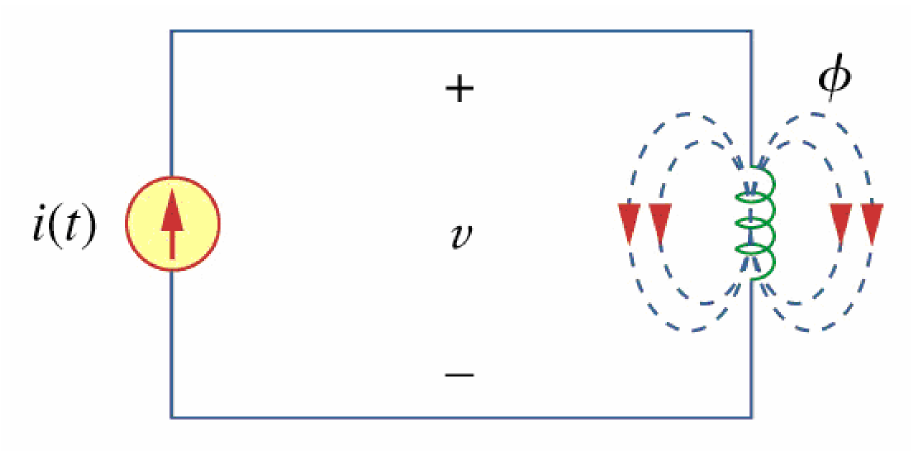


SELF INDUCTANCE

- According to Faraday's law, the voltage induced in a coil is proportional to the number of turns N and the time rate of change of the magnetic flux ϕ.

$$v = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

- L is the inductance of the inductor commonly called self-inductance (relating the induced voltage in a coil by a time-varying current in the same coil)



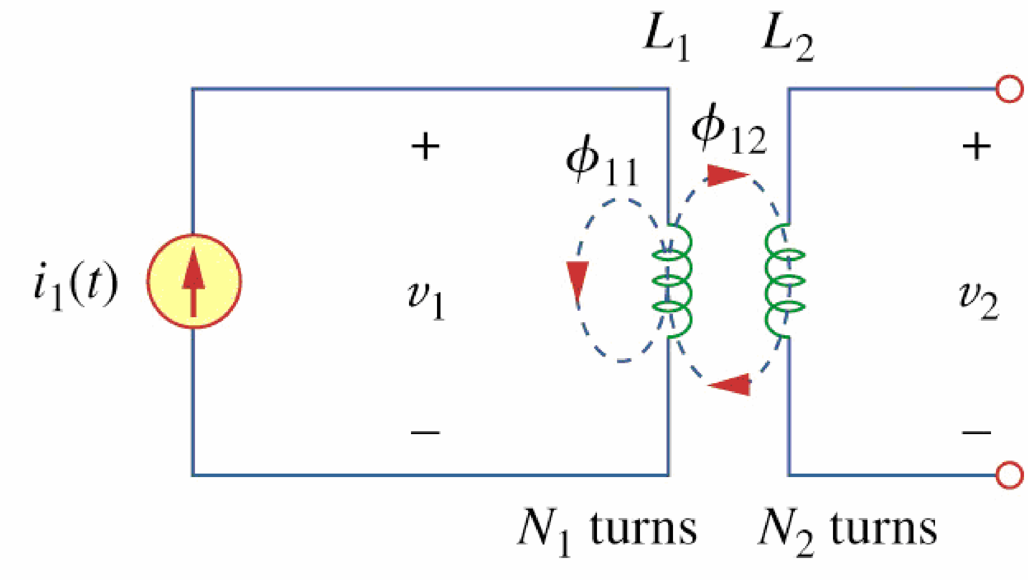
MUTUAL INDUCTANCE

- Two coils in a close proximity are linked together by the magnetic flux produced by current in one coil, therby inducing voltage in the other.

  → the two coils are said to be magnetically coupled although they are physically apart.

- MUTUAL INDUCTANCE is the ability of one inductor to induce a voltage across a neighbouring inductor, measured in henrys (H).

- Mutual coupling only exists when the coils are in close proximity, and the circuits are driven by time-varying sources.



$$v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L\frac{di_1}{dt}$$

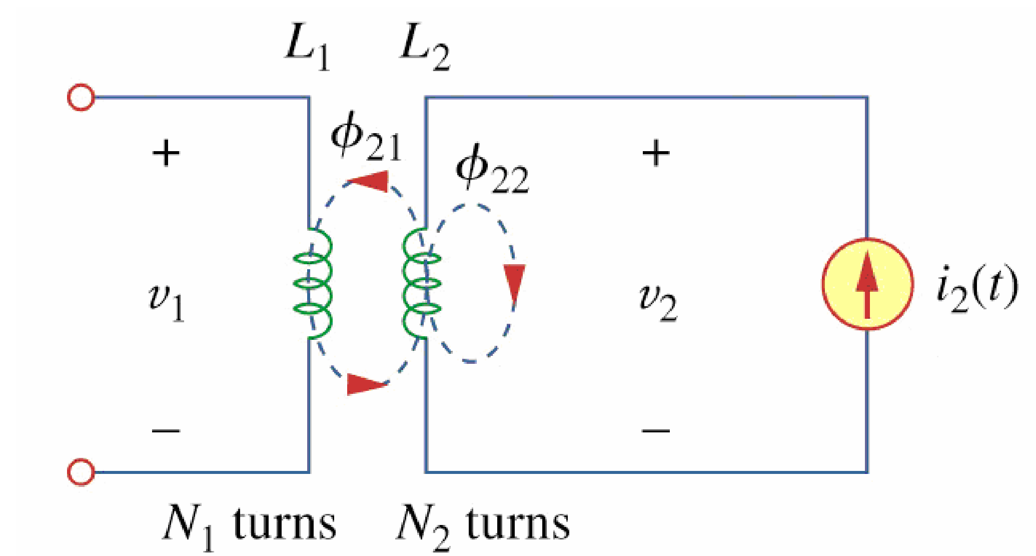
$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

M_{21} is mutual inductance of coil 2 with respect to coil 1

$$v_2 = N_2 \frac{d\phi_2}{di_2}\frac{di_2}{dt} = L\frac{di_2}{dt}$$

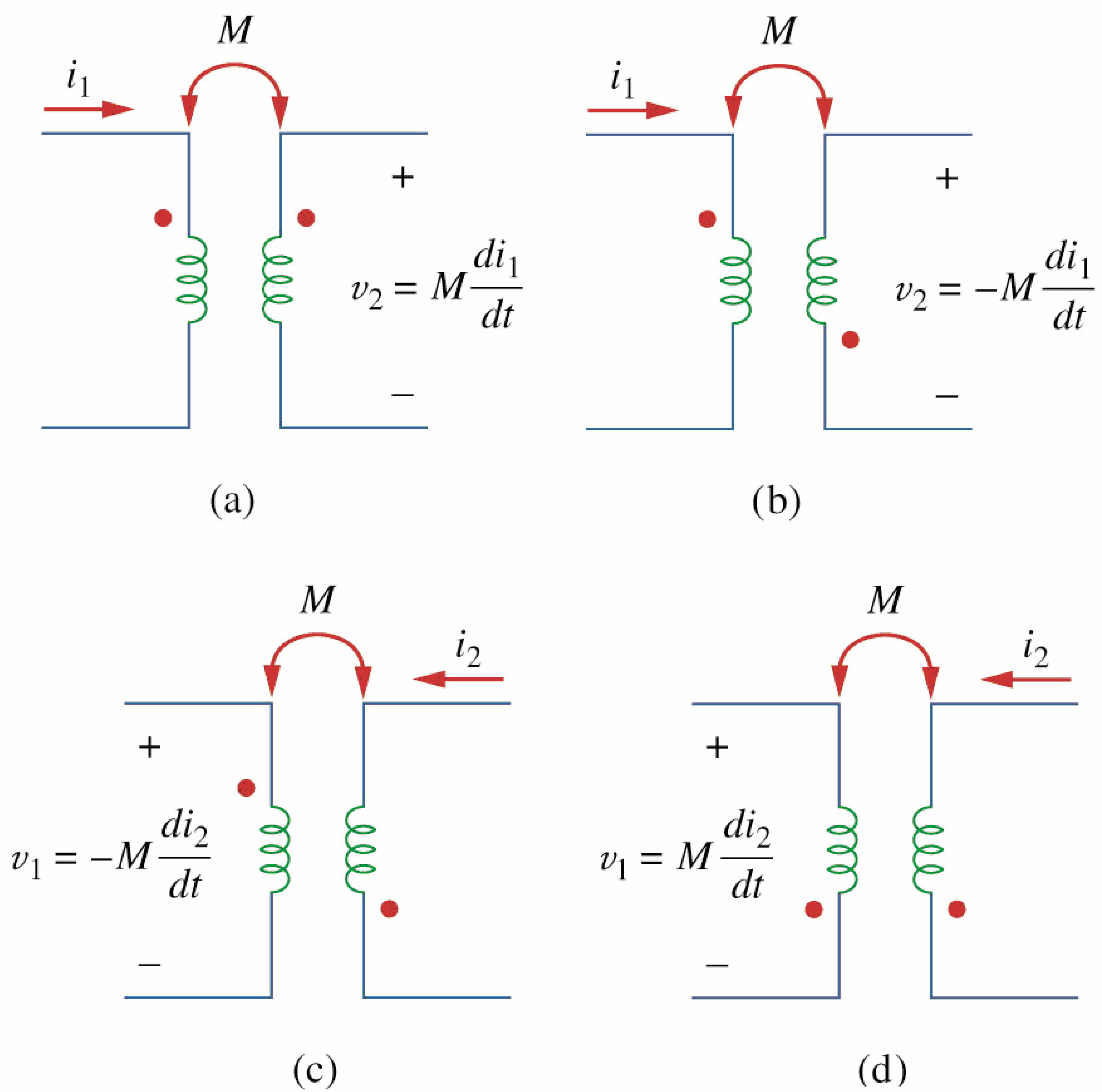
$$v_1 = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_1}{dt}$$

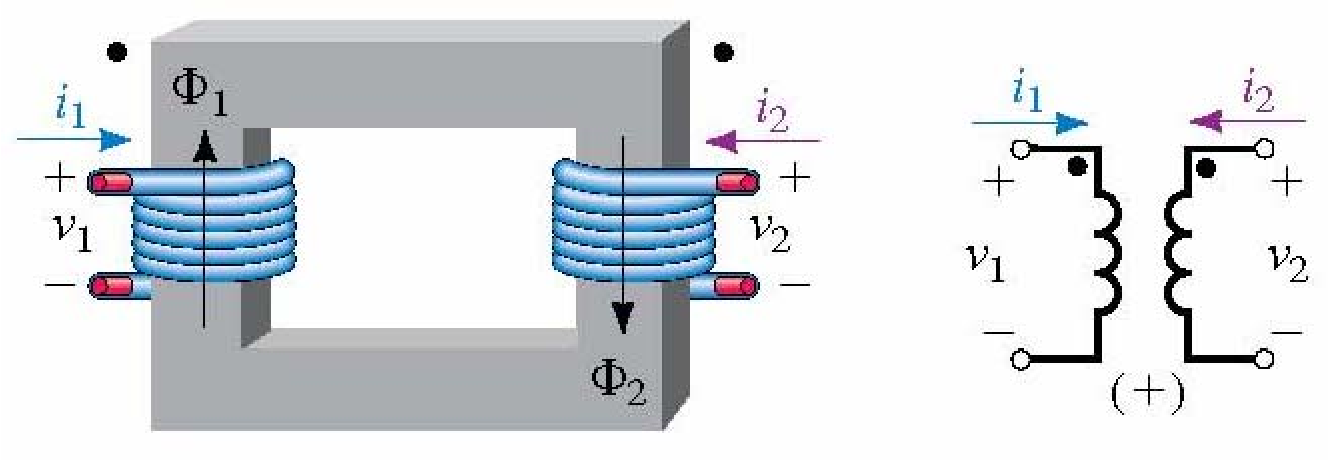
M_{12} is mutual inductance of coil 1 with respect to coil 2



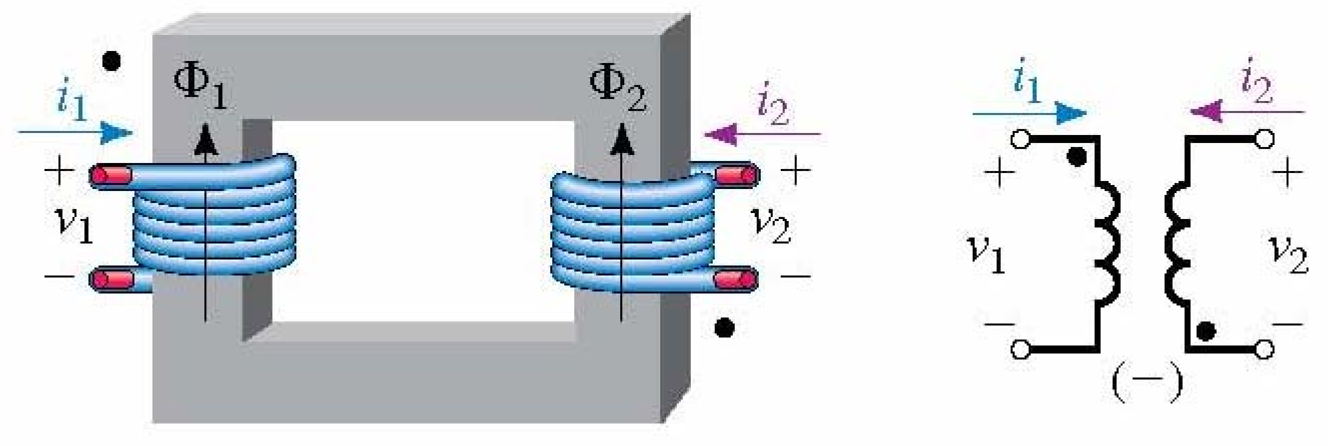
DOT CONVENTION

- $M_{12} = M_{21} = M$ and is always a positive quantity.
- The induced voltage $M\frac{di}{dt}$ may be positive or negative.
- The choice of polarity is made by examining the way in which both coils are physically wound and applying Lenz's law in conjunction with the right-hand-rule.
- The procedure is inconvenient in circuit analysis since it is difficult to show the construction details of the coil in circuit schematics. $\rightarrow$ use the dot convention (often predetermined)
- If a current enters (leaves) the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive (negative) at the dotted terminal of the second coil.

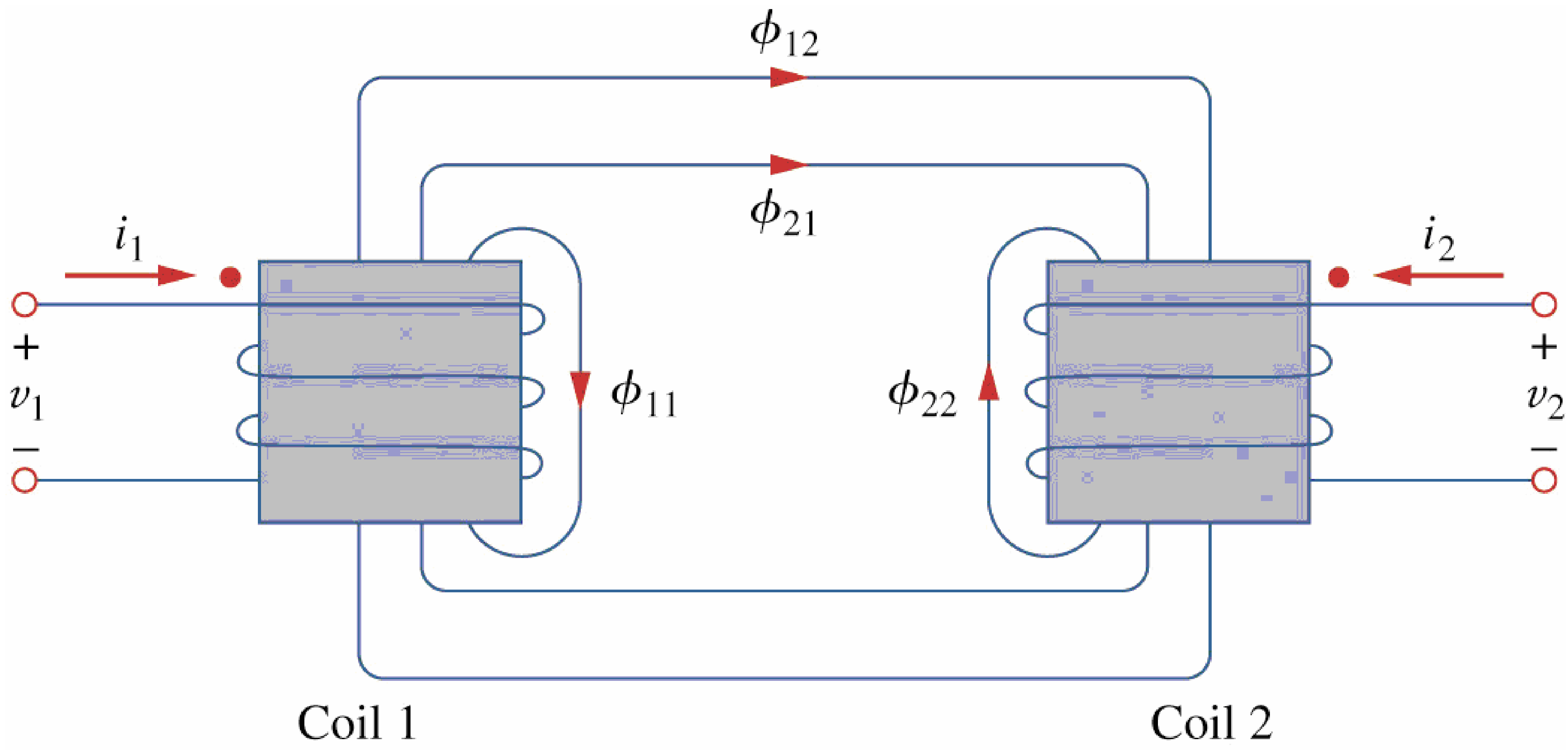

M
i_1
+
$v_2 = M \frac{di_1}{dt}$
−
(a)
M
i_1
+
$v_2 = -M \frac{di_1}{dt}$
−
(b)
M
i_2
+
$v_1 = -M \frac{di_2}{dt}$
−
(c)
M
i_2
+
$v_1 = M \frac{di_2}{dt}$
−
(d)



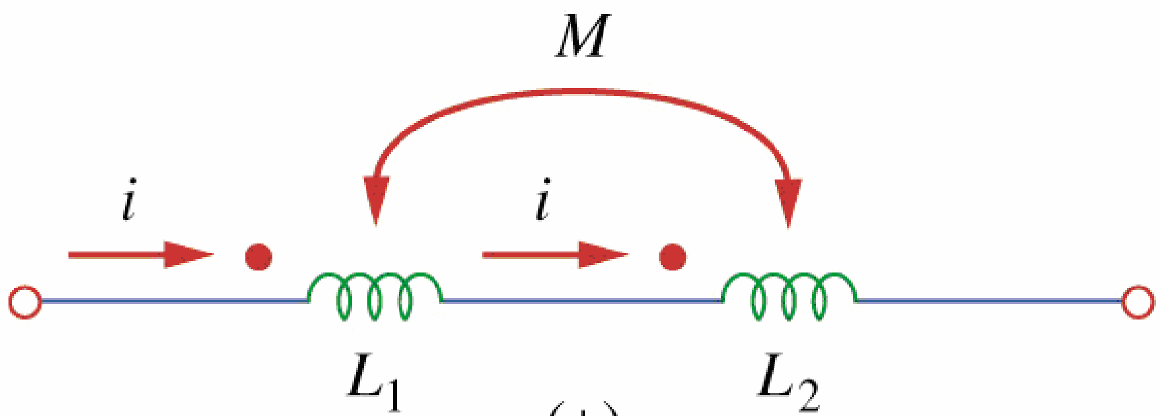
Φ1
Φ2
i1
i2
+
−
v1
v2
(+)



Φ1
Φ2
i1
i2
+
−
v1
v2
(−)

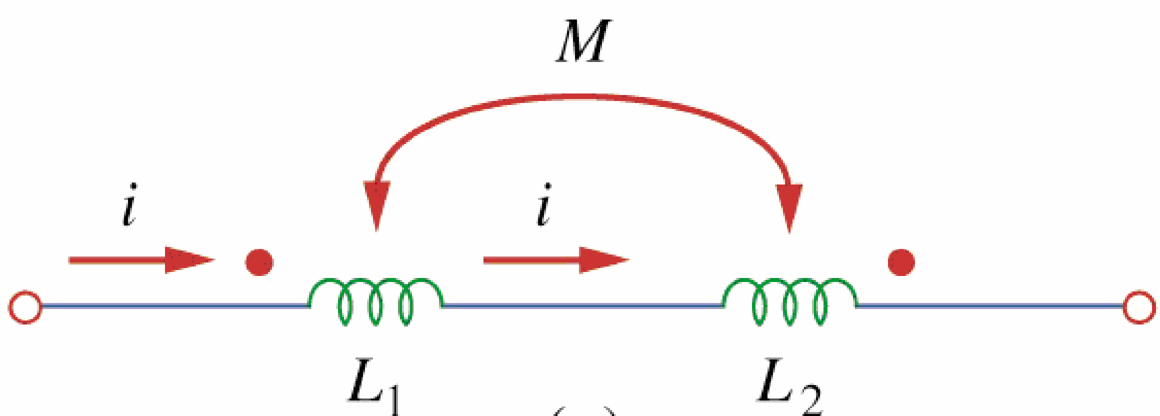


ϕ_{12}
ϕ_{21}
i_1
i_2
+
v_1
−
+
v_2
−
ϕ_{11}
ϕ_{22}
Coil 1
Coil 2



(+)

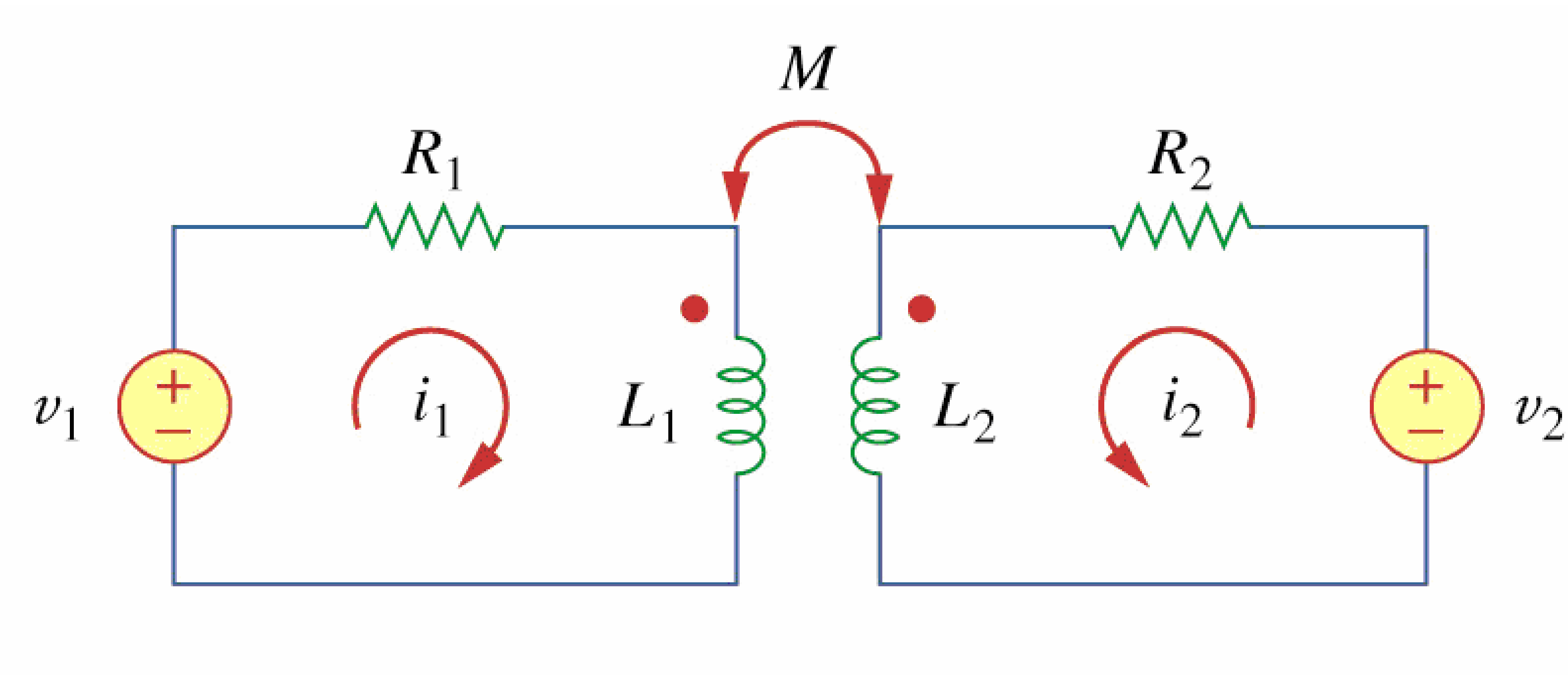
Series-aiding connection : $L = L_1 + L_2 + 2M$



(−)

Series-opposing connection : $L = L_1 + L_2 - 2M$

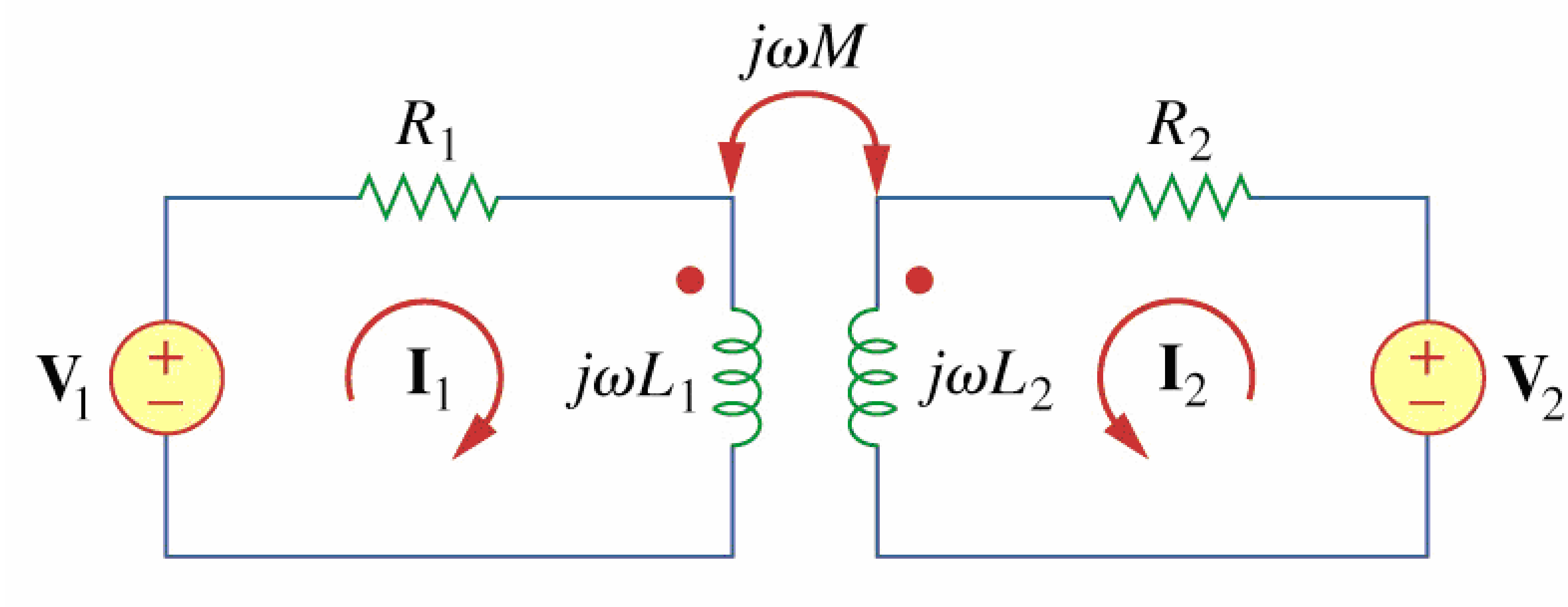
TIME-DOMAIN ANALYSIS



Applying KVL to coils 1,
$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Applying KVL to coils 2,
$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

FREQUENCY-DOMAIN ANALYSIS

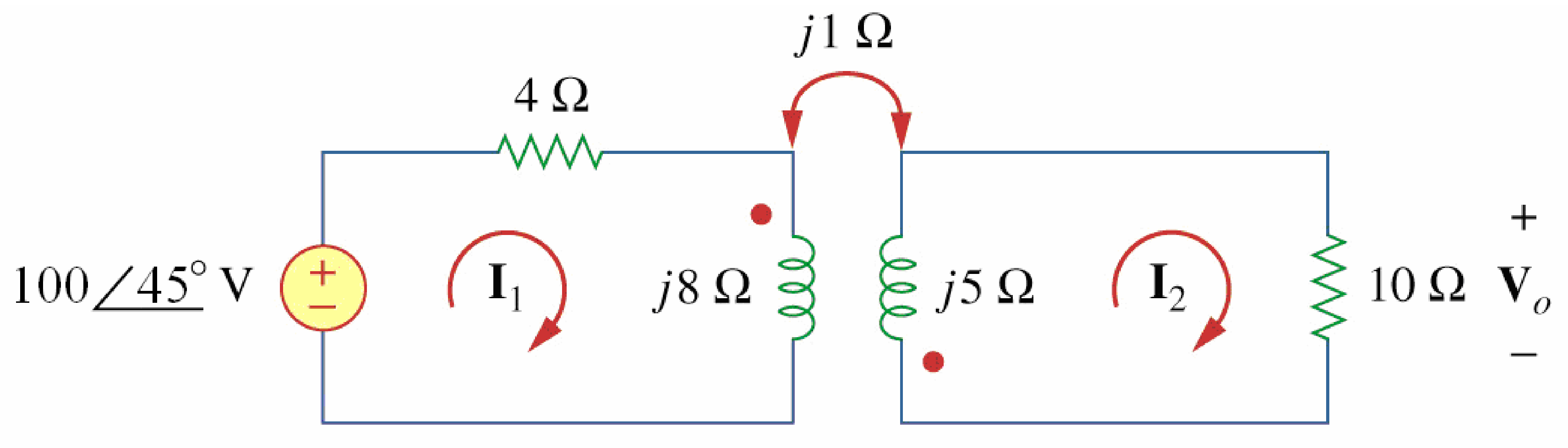


$$\underline{V}_1 = (R_1 + j\omega L_1)\underline{I}_1 + j\omega M\,\underline{I}_2$$

$$\underline{V}_2 = j\omega M\, I_1 + (R_2 + j\omega L_2)\underline{I}_1$$

Ex. Practice Problem 13.1

Determine the voltage $\underline{V}_o$ in the circuit shown.



Ex. Practice Problem 13.2

Determine the phasor currents $\underline{I}_1$ and $\underline{I}_2$ in the circuit shown.