

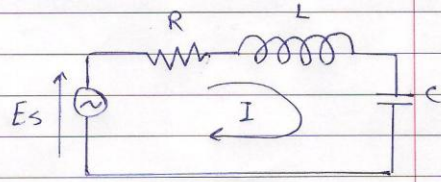
## Series Resonance

$$Z_T = R + j(X_L - X_C)$$

at resonance

$$X_L = X_C \dots (1)$$

$$\therefore Z_T = R \dots (2)$$



$$\omega L = \frac{1}{\omega C}$$

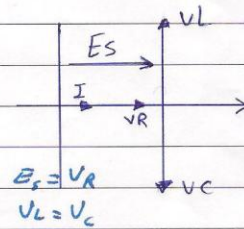
$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \dots (3)$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \dots (4)$$

$$E_s = V_R$$

$$V_L = V_C$$



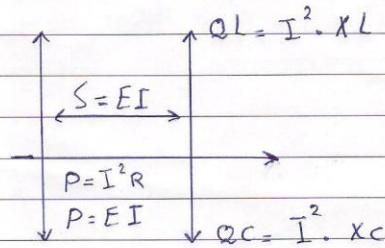
\* Average power of resonance  $\Rightarrow$

$$P = I^2 \cdot R \dots (5)$$

\* Reactive power at resonance  $\Rightarrow$

$$\begin{aligned} Q_L &= I^2 \cdot X_L \\ &= Q_C = I^2 \cdot X_C \dots (6) \end{aligned}$$

\* power triangle at resonance :=



\* power factor :=

$$F_p = \cos \theta = \frac{P}{S}$$

$$\theta = 0$$

$$\therefore F_p = 1$$

\* The quality factor Q :=

$$Q_s = \frac{\text{reactive power}}{\text{active power}} \dots \textcircled{7}$$

at resonance  $X_L = X_C$

$$Q_s = \frac{I^2 \cdot X_L}{I^2 \cdot R} = \frac{\omega L}{R} \dots \textcircled{8}$$

IF (R) is resistance of coil only  $\rightarrow R = R_L$

$$Q_{\text{coil}} = Q_s = \frac{X_L}{R} \dots \textcircled{9}$$

$R = R_L$

$$\omega = 2\pi f$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}, \quad Q_s = \frac{\omega_s \cdot L}{R}$$

$$\therefore Q_s = \frac{1}{R} \sqrt{\frac{L}{C}} \dots (10)$$

$Q_s$  must be  $> 1$   
at resonance

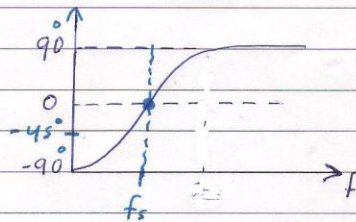
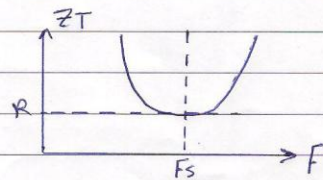
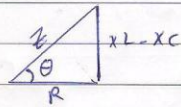
$$V_L = Q_s \cdot E \dots (11)$$

$$V_C = Q_s \cdot E \dots (12)$$

### 20.4 ZT vs Frequency $\Rightarrow$

$$Z_T(f) = \sqrt{(R)^2 + [(X_L(f)) - X_C(f)]^2} = \sqrt{R^2 + (X(f))^2}$$

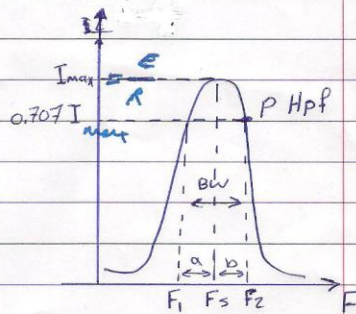
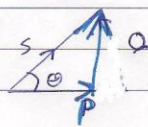
$$\theta = \tan^{-1} \frac{X_L - X_C}{R} \dots (13)$$



### 20.5 Selectivity $\Rightarrow$

HPF = Half power Frequency

$$P_{HPF} = \frac{1}{2} P_{max} \dots (14)$$



$$P_{\max} = I^2 \cdot R$$

$$\begin{aligned} \text{IF } (R) \downarrow & \therefore Q_S \uparrow \\ (R) \uparrow & \therefore Q_S \downarrow \end{aligned}$$

$$\text{When } Q_S \geq 10 \quad \therefore a = b$$

and small (BW) with high selectivity.

$$\text{at } Q_S < 10 \quad \therefore a \neq b$$

large (BW) and small selectivity.

$$F_2 = \frac{1}{2\pi} \left[ \frac{R}{2L} + \frac{1}{2} \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right] \dots (15)$$

$$F_1 = \frac{1}{2\pi} \left[ \frac{-R}{2L} + \frac{1}{2} \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right] \dots (16)$$

$$BW = F_2 - F_1 = \frac{R}{2\pi L} \dots (17)$$

$$\text{or } BW = \frac{F_S}{Q_S} \dots (18)$$

$$\text{or } \frac{F_2 - F_1}{F_S} = \frac{1}{Q_S} \dots (19)$$

*fractional Bandwidth*

$$F_S = \sqrt{F_2 \cdot F_1} \dots (20)$$

\* solve the examples at text Book.  
20.7 example series.

- parallel Resonance -

$$\textcircled{1} R_p = \frac{R_i^2 + X_L^2}{R_i} \dots \textcircled{1}$$

$$X_{LP} = \frac{R_i^2 + X_L^2}{X_L} \dots \textcircled{2}$$

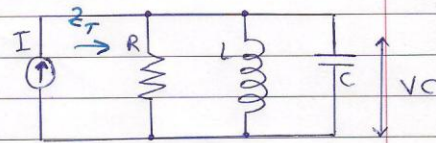


Fig (1)

$$R = R_s // R_p \dots \textcircled{3}$$

Fig (2)

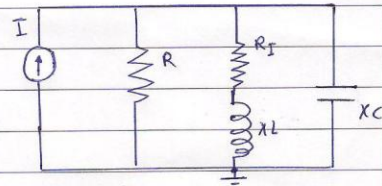


Fig (2)

② unite factor power = 0

$$F_p = 1 \quad \text{Fig (4)}$$

$\cos \phi = 1$  or reactive component = 0  
 $\phi = 0$

$$Y_T = \frac{1}{R} + j \left( \frac{1}{X_C} - \frac{1}{X_{LP}} \right)$$

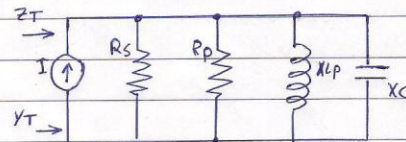


Fig (3)

at resonance imaginary part = zero

$$\therefore X_C = X_{LP} \dots \textcircled{4}$$

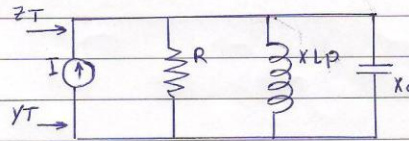


Fig (4)  $R = R_s // R_p$

$$X_C = \frac{R_i^2 + X_L^2}{X_L} \dots \textcircled{5}$$

From ⑤

$$F_p = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R_i^2 C}{L}}$$

$$F_p = f_s \sqrt{1 - \frac{R_i^2 C}{L}} \dots \textcircled{6}$$

resonance Frequency at  $F_p = 1$

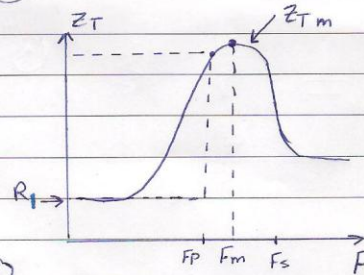
or  
 $X_{LP} = X_C$ ,  $R_1 \approx 0$ ,  $F_p = F_s$ , but not at  $Z_{Tm}$

\* Max. Impedance,  $F_m$  :=

$$F_m = F_s \sqrt{1 - \frac{1}{4} \left( \frac{R_1^2 C}{L} \right)} \dots (7)$$

$F_m$  = resonance frequency.

$$F_s > F_m > F_p$$



$$Z_{Tm} = R // X_{LP} // X_C \dots (8)$$

$$R = R_s // R_p, f = F_m \text{ Fig (5)}$$

\* Selectivity Curve :=

from Fig (4)

$$V_c = V_p = I \cdot Z_T$$

$$Q_p = \frac{\text{reactive power}}{\text{real power}} = \frac{V_p^2 / X_{LP}}{V_p^2 / R}$$

\* for coil  $Q_1 = \frac{X_L}{R_1}$

$$Q_p = \frac{R}{X_{LP}} = \frac{R_s // R_p}{X_{LP}} = \frac{R_s // R_p}{X_C}$$

\* When  $R_s = \infty$

$$\text{or } R_s \gg R_p \Rightarrow R = R_s // R_p \approx R_p$$

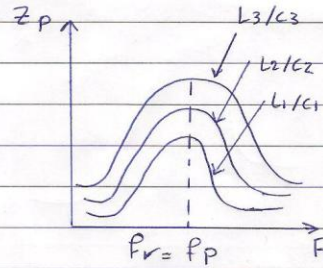
$$Q_P = \frac{R_P}{X_{LP}} = \frac{X_L}{R_I} = Q_I \quad R_S \gg R_P$$

quality factor of coil

$$BW = F_2 - F_1 = \frac{F_P = F_r}{Q_P} \quad \text{from Fig (11)}$$

$$F_1 = \frac{1}{4\pi C} \left[ \frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

$$F_2 = \frac{1}{4\pi C} \left[ \frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}} \right]$$

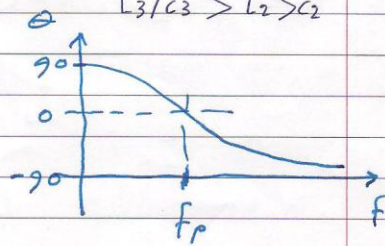


$$L_3/C_3 > L_2/C_2$$

\* Effect of  $Q_I \geq 10$  :-

$$X_{LP} \approx X_L$$

$$X_L \approx X_C$$



$$F_P \approx F_S = \frac{1}{2\pi\sqrt{LC}} \approx F_m$$

So :-

$$F_P \approx F_S \approx F_m$$

$$R_P \approx Q_I^2 \cdot R_I \quad Q_I \geq 10$$

$$R_P \approx \frac{L}{R_I C} \quad Q_I \geq 10$$

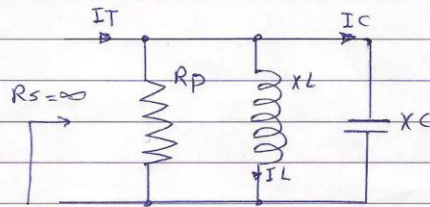
At  $R_s = \infty \Omega$

$$BW = P_2 - P_1 \approx \frac{RI}{2XL} \quad \begin{array}{l} R_s = \infty \\ Q_i \geq 10 \end{array}$$

←  $V_C$  and  $V_L$  :

$$V_C = V_L = V_R = I_T \cdot Z_{TP} = I_T \cdot Q_i^2 R_i \quad \begin{array}{l} Q_i \geq 10 \\ R_s \geq R_p \end{array}$$

$$I_C \approx Q_i \cdot I_T \quad Q_i \geq 10$$
$$I_L \approx Q_i \cdot I_T \quad Q_i \geq 10$$



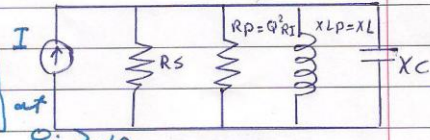
$$Z_{TP} = R_p \approx Q_i^2 \cdot R_i$$



\* Input impedance at resonance ( $Z_{TP}$ ) =

\*  $Z_{TP} \approx R_S // R_P$

$Z_{TP} = Z_{TM} = R_S // Q_I^2 \cdot R_I$



\* For ideal current source =  
( $R_S = \infty \Omega$ ) or  $R_S \gg R_P$

$Q_I \geq 10$

$Z_{TP} \approx Q_I^2 \cdot R_I$      $Q_I \geq 10$   
 $R_S \gg R_P$

\*  $Q_P \approx \frac{R}{X_{LP}} \approx \frac{R_S // Q_I^2 R_I}{X_L}$      $Q_I \geq 10$

When  $R_S \gg R_P$

$Q_P \approx Q_I$      $R_S \gg R_P$   
 $\therefore R_S \approx \infty$

\* BW =

$BW = f_2 - f_1 = \frac{f_p}{Q_P}$      $Q \geq 10$

$f_p = f_s = f_m$

or =

$BW \approx \frac{1}{2\pi} \left[ \frac{R_I}{L} + \frac{1}{R_S C} \right]$      $Q_I \geq 10$