dc, High-, and Low-Frequency Effects on *L* and *C*

For dc circuits, the frequency is zero, and the reactance of a coil is

$$X_L = 2\pi fL = 2\pi(0)L = 0\ \Omega$$

The use of the short-circuit equivalence for the inductor in dc circuits (Chapter 12) is now validated. At very high frequencies, $X_L\uparrow = 2\pi f\uparrow L$ is very large, and for some practical applications the inductor can be replaced by an open circuit. In equation form,

$$\boxed{X_L = 0\ \Omega} \quad \text{dc, } f = 0\ \text{Hz} \tag{14.10}$$

and

$$\boxed{X_L \Rightarrow \infty\ \Omega \qquad \text{as } f \Rightarrow \infty\ \text{Hz}} \tag{14.11}$$

The capacitor can be replaced by an open-circuit equivalence in dc circuits since $f = 0$, and

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(0)C} \Rightarrow \infty\ \Omega$$

once again substantiating our previous action (Chapter 10). At very high frequencies, for finite capacitances,

$$X_C\downarrow = \frac{1}{2\pi f\uparrow C}$$

is very small, and for some practical applications the capacitor can be replaced by a short circuit. In equation form

$$\boxed{X_C \Rightarrow \infty\ \Omega \qquad \text{as } f \Rightarrow 0\ \text{Hz}} \tag{14.12}$$

and

$$\boxed{X_C = 0\ \Omega} \quad f = \text{very high frequencies} \tag{14.13}$$

14.5 AVERAGE POWER AND POWER FACTOR

For any load in a sinusoidal ac network, the voltage across the load and the current through the load will vary in a sinusoidal nature. The questions then arise, How does the power to the load determined by the product $v \cdot i$ vary, and what fixed value can be assigned to the power since it will vary with time?

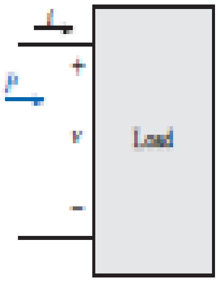


FIG. 14.28
Determining the power delivered in a sinusoidal ac network.

If we take the general case depicted in Fig. 14.28 and use the following for v and i:

$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$

then the power is defined by

$$p = vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i)$$
$$= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$

Using the trigonometric identity

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

the function $\sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$ becomes

$$\sin(\omega t + \theta_v)\sin(\omega t + \theta_i)$$
$$= \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2}$$
$$= \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2}$$

so that

$$p = \overbrace{\left[\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)\right]}^{\text{Time-varying (function of } t\text{)}}$$

A plot of v, i, and p on the same set of axes is shown in Fig. 14.29.

Note that the second factor in the preceding equation is a cosine wave with an amplitude of $V_m I_m/2$ and with a frequency twice that of

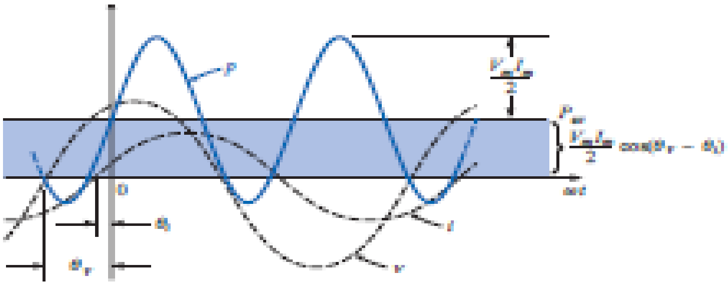


FIG. 14.29
Defining the average power for a sinusoidal ac network.

the voltage or current. The average value of this term is zero over one cycle, producing no net transfer of energy in any one direction.

The first term in the preceding equation, however, has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the average power, the reason for which is obvious from Fig. 14.29. The average power, or real power as it is sometimes called, is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for dc networks. The angle $(\theta_v - \theta_i)$ is the phase angle between v and i. Since $\cos(-\alpha) = \cos \alpha$,

the magnitude of average power delivered is independent of whether v leads i or i leads v.

Defining θ as equal to $|\theta_v - \theta_i|$, where $|\ \ |$ indicates that only the magnitude is important and the sign is immaterial, we have

$$P = \frac{V_m I_m}{2} \cos \theta \qquad \text{(watts, W)} \qquad (14.14)$$

where P is the average power in watts. This equation can also be written

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) \cos \theta$$

or, since $\qquad V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$

Equation (14.14) becomes

$$P = V_{\text{eff}} I_{\text{eff}} \cos \theta \qquad (14.15)$$

Let us now apply Eqs. (14.14) and (14.15) to the basic R, L, and C elements.

Resistor

In a purely resistive circuit, since v and i are in phase, $|\theta_v - \theta_i| = \theta = 0°$, and $\cos \theta = \cos 0° = 1$, so that

$$P = \frac{V_m I_m}{2} = V_{eff} I_{eff} \quad \text{(W)} \tag{14.16}$$

Or, since $I_{eff} = \frac{V_{eff}}{R}$

then

$$P = \frac{V_{eff}^2}{R} = I_{eff}^2 R \quad \text{(W)} \tag{14.17}$$

Inductor

In a purely inductive circuit, since v leads i by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2}(0) = 0 \text{ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Capacitor

In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos(90°) = \frac{V_m I_m}{2}(0) = 0 \text{ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

EXAMPLE 14.10 Find the average power dissipated in a network whose input current and voltage are the following:

$$i = 5 \sin(\omega t + 40°)$$
$$v = 10 \sin(\omega t + 40°)$$

Solution: Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10 \text{ V})(5 \text{ A})}{2} = 25 \text{ W}$$

or

$$R = \frac{V_m}{I_m} = \frac{10 \text{ V}}{5 \text{ A}} = 2 \ \Omega$$

and

$$P = \frac{V_{eff}^2}{R} = \frac{[(0.707)(10 \text{ V})]^2}{2} = 25 \text{ W}$$

or

$$P = I_{eff}^2 R = [(0.707)(5 \text{ A})]^2(2) = 25 \text{ W}$$

EXAMPLE 14.11 Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 70°)$
b. $v = 150 \sin(\omega t - 70°)$
   $i = 3 \sin(\omega t - 50°)$

Solutions:

a. $V_m = 100, \quad \theta_v = 40°$
   $I_m = 20, \quad \theta_i = 70°$
   $\theta = |\theta_v - \theta_i| = |40° - 70°| = |-30°| = 30°$

and

$$P = \frac{V_m I_m}{2} \cos\theta = \frac{(100\text{ V})(20\text{ A})}{2} \cos(30°) = (1000\text{ W})(0.866)$$
$$= \mathbf{866\ W}$$

b. $V_m = 150\text{ V}, \quad \theta_v = -70°$
   $I_m = 3\text{ A}, \quad \theta_i = -50°$
   $\theta = |\theta_v - \theta_i| = |-70° - (-50°)|$
   $= |-70° + 50°| = |-20°| = 20°$

and

$$P = \frac{V_m I_m}{2} \cos\theta = \frac{(150\text{ V})(3\text{ A})}{2} \cos(20°) = (225\text{ W})(0.9397)$$
$$= \mathbf{211.43\ W}$$

Power Factor

In the equation $P = (V_m I_m/2)\cos\theta$, the factor that has significant control over the delivered power level is the $\cos\theta$. No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero; if $\cos\theta = 1$, the power delivered is a maximum. Since it has such control, the expression was given the name power factor and is defined by

$$\text{Power factor} = F_p = \cos\theta \qquad (14.18)$$

For a purely resistive load such as the one shown in Fig. 14.30, the phase angle between v and i is 0° and $F_p = \cos\theta = \cos 0° = 1$. The power delivered is a maximum of $(V_m I_m/2)\cos\theta = ((100\text{ V})(5\text{ A})/2)\cdot(1) = 250\text{ W}$.

For a purely reactive load (inductive or capacitive) such as the one shown in Fig. 14.31, the phase angle between v and i is 90° and $F_p = \cos\theta = \cos 90° = 0$. The power delivered is then the minimum value of zero watts, **even though the current has the same peak value as that encountered in Fig. 14.30.**

For situations where the load is a combination of resistive and reactive elements, the power factor will vary between 0 and 1. The more resistive the total impedance, the closer the power factor is to 1; the more reactive the total impedance, the closer the power factor is to 0.

In terms of the average power and the terminal voltage and current,

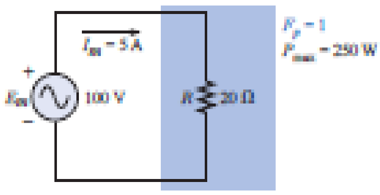


FIG. 14.30
Purely resistive load with $F_p = 1$.

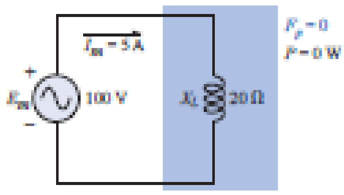


FIG. 14.31
Purely inductive load with $F_p = 0$.

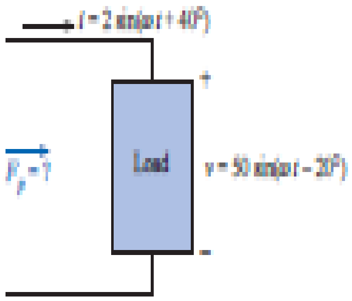


FIG. 14.32

Example 14.12(a).

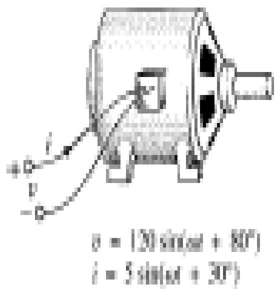
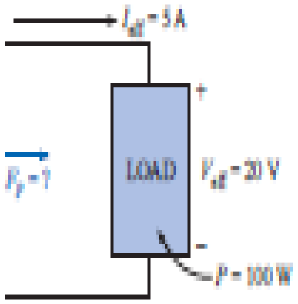


FIG. 14.33

Example 14.12(b).



$$F_p = \cos\theta = \frac{P}{V_{eff}I_{eff}} \tag{14.19}$$

The terms *leading* and *lagging* are often written in conjunction with the power factor. *They are defined by the current through the load.* If the current leads the voltage across a load, the load has a leading power factor. If the current lags the voltage across the load, the load has a lagging power factor. In other words,

capacitive networks have leading power factors, and inductive networks have lagging power factors.

The importance of the power factor to power distribution systems is examined in Chapter 19. In fact, one section is devoted to power-factor correction.

EXAMPLE 14.12 Determine the power factors of the following loads, and indicate whether they are leading or lagging:

a. Fig. 14.32
b. Fig. 14.33
c. Fig. 14.34

Solutions:

a. $F_p = \cos\theta = \cos|40° - (-20°)| = \cos 60° = 0.5$ **leading**
b. $F_p = \cos\theta\,|80° - 30°| = \cos 50° = 0.6428$ **lagging**
c. $F_p = \cos\theta = \dfrac{P}{V_{eff}I_{eff}} = \dfrac{100\text{ W}}{(20\text{ V})(5\text{ A})} = \dfrac{100\text{ W}}{100\text{ W}} = 1$

The load is resistive, and F_p is neither leading nor lagging.