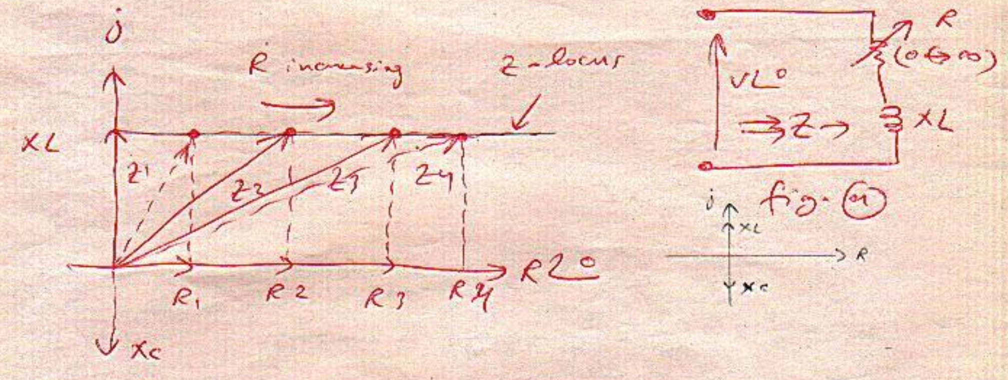
المحل الهندسي

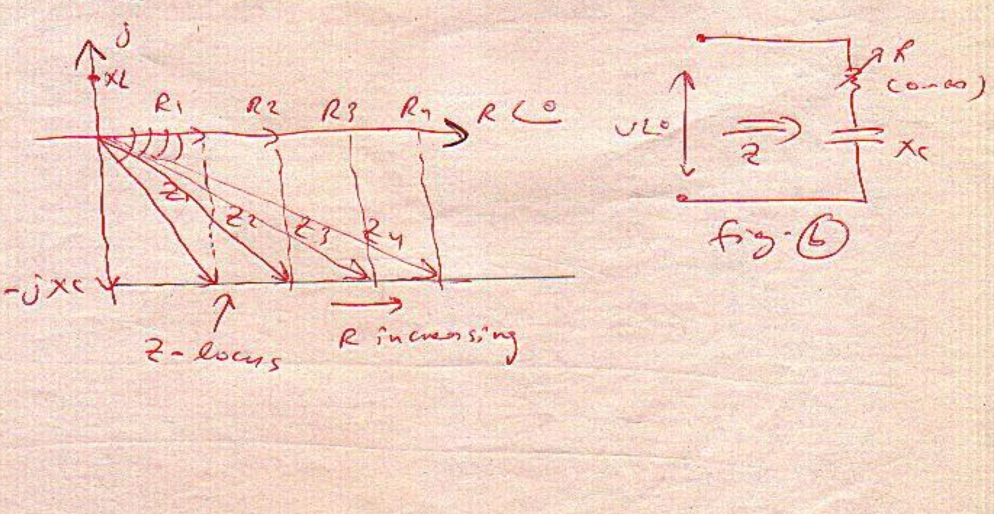
# Locus Diagram ①

find the Locus diagram of the total current, or, Z, Y when single elements of cct. is changed.

① (R-L) series cct. with variable (R)

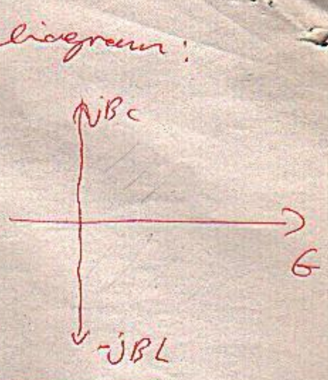
ⓐ impedance Z-locus diagram:



(b) admittance Y-locus diagram:

for fig. (a):




$$Z = R + jX_L = \frac{1}{Y} = \frac{1}{G+jB} \times \frac{G-jB}{G-jB}$$

$$= \frac{G}{G^2+B^2} - j\frac{B}{G^2+B^2}$$

$$\therefore \boxed{R = \frac{G}{G^2+B^2}} \text{ — (1)}, \quad \boxed{X_L = \frac{-B}{G^2+B^2}} \text{ — (2)}$$

$$X_L = \frac{-B}{G^2+B^2} \Rightarrow \left(G^2X_L + B^2X_L = -B\right) / X_L$$

$$\boxed{G^2 + B^2 + \frac{B}{X_L} = 0} \text{ — (3)}$$

for fig. (b) $\quad Z = R - jX_C = \frac{1}{Y} = \frac{1}{G+jB} \times \frac{G-jB}{G-jB}$

$$Z = R - jX_C = \frac{1}{Y} = \frac{G-jB}{G^2+B^2} = \frac{G}{G^2+B^2} - j\frac{B}{G^2+B^2} \quad \therefore X_C = \frac{B}{G^2+B^2}$$

$$X_C = \frac{B}{G^2+B^2} \Rightarrow \boxed{G^2 + B^2 - \frac{B}{X_C} = 0} \text{ — (4)}$$

from eq. (3)

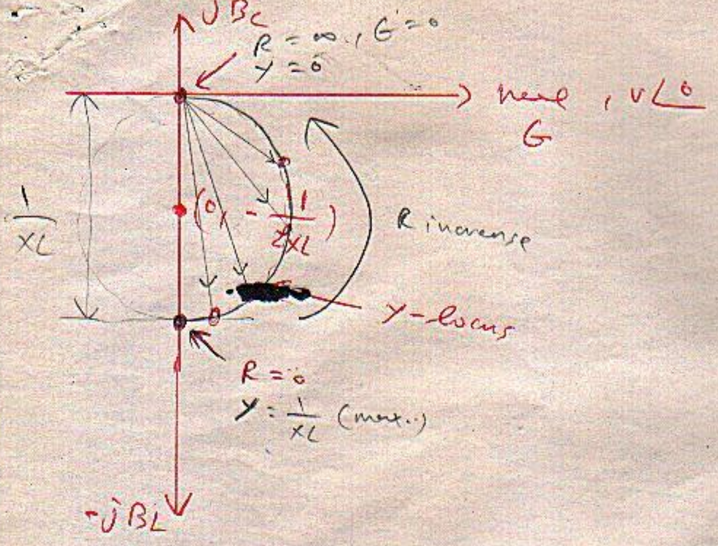
$$\left(G^2 + B^2 + \frac{B}{X_L} = 0\right)$$ ~~and~~ $\left(\frac{1}{4X_L^2}\right)$ بإضافة للطرفين

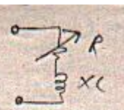
$$(G-0)^2 + \left(B^2 + \frac{B}{X_L}\right) + \left(\frac{1}{2X_L}\right)^2 = \left(\frac{1}{2X_L}\right)^2$$

$$\boxed{(G-0)^2 + \left(B + \frac{1}{2X_L}\right)^2 = \left(\frac{1}{2X_L}\right)^2 \quad \text{معادلة دائرة} \quad (5)}$$

* locus of (Y) is circle of center $\left(0, -\frac{1}{2X_L}\right)$, radius is $\left(\frac{1}{2X_L}\right)$

$$(x-k)^2 + (y-m)^2 = r^2$$




(2)

* for fig (6)

from eq. (4)

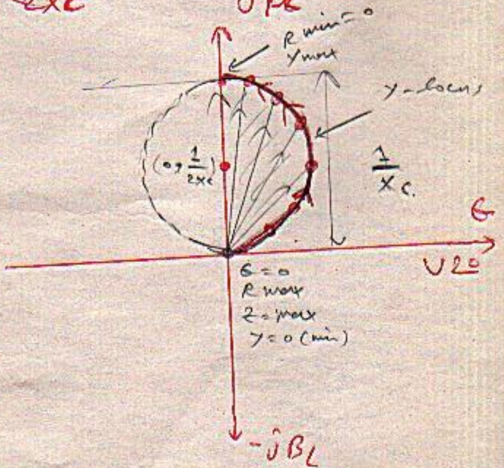

$$\boxed{G^2 + B^2 - \frac{B}{X_C} = 0} \quad \text{--- (4)}$$

باضافة $\left(\frac{1}{4X_C^2}\right)$ للطرفين

$$\boxed{(G-0)^2 + \left(B - \frac{1}{2X_C}\right)^2 = \left(\frac{1}{2X_C}\right)^2 \quad \text{--- (6)}}$$

معادلة الدائرة

* locus of (Y) is a circle of center $(0, \frac{1}{2X_C})$, radius is $\left(\frac{1}{2X_C}\right)$.

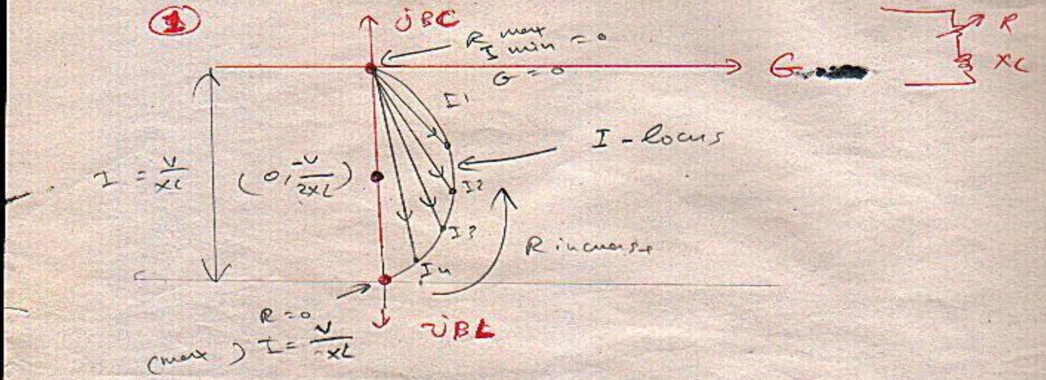


١) صفا ثؤ، + بحريا =

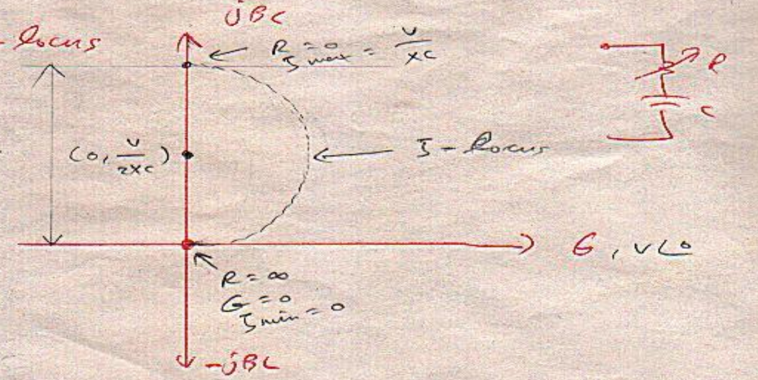
## (c) Current (I- locus) diagram.

$$I = \frac{V}{Z} = Y \cdot V$$

(1)




(2) I-locus

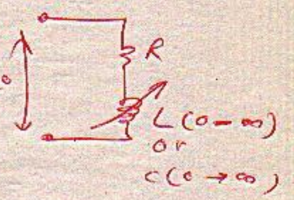


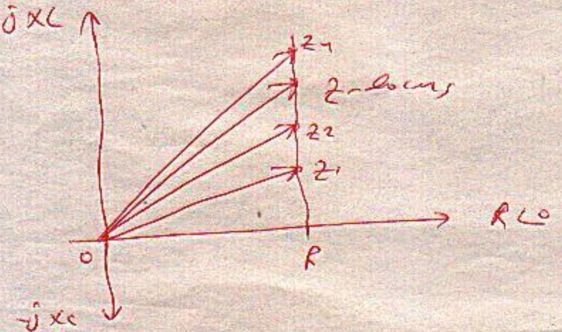

(Hidp)

## (2) (R-L) series cct with variable (L) :-

(a) Z - locus ($R\cancel{\angle}$)

$Z = R + j\cancel{wL}$



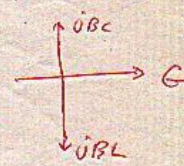




$R\cancel{\angle}$ (b) Y - locus :

$$Z = R + jXL = \frac{1}{Y} = \frac{1}{G + jB}$$

$$\therefore \frac{1}{Y} = \frac{G}{G^2+B^2} - j\frac{B}{G^2+B^2}$$

$$\therefore R = \frac{G}{G^2+B^2} \quad , \quad XL = \frac{-B}{G^2+B^2}$$




$\uparrow R =$ (ثابت ، نأخذ)

$$\therefore RG^2 + RB^2 = G \Rightarrow G^2 + B^2 = \frac{G}{R}$$

$$\therefore \left(G^2 + B^2 - \frac{G}{R} = 0\right)\left(\frac{1}{4R_1^2}\right)$$ بإضافة إلى الطرفين

$$\boxed{\left(G - \frac{1}{2R}\right)^2 + B^2 = \left(\frac{1}{2R_1}\right)^2} \quad (2)$$

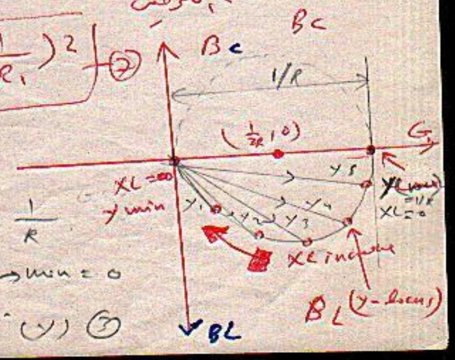
* (Y) على شكل

$$\boxed{Z = R + jXL = \frac{1}{Y}}$$

1) $XL = 0 \quad \therefore Y = \frac{1}{R_1 + 0} \quad \therefore Y \to max = \frac{1}{R}$

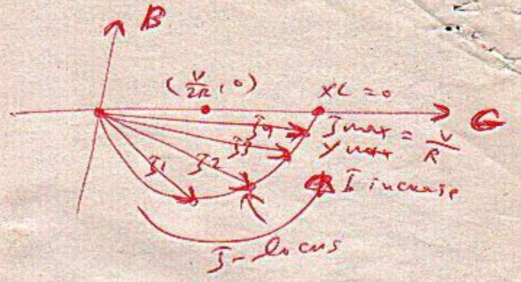
2) $XL = \infty \quad Y = \frac{1}{R + \infty} = 0 \quad \therefore Y \to min = 0$

(3) (Y) تزداد حسب السهم لأن الرسم



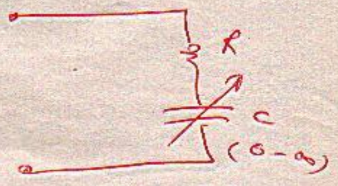
I - locus

$$I = \frac{V}{Z} = V \cdot y$$

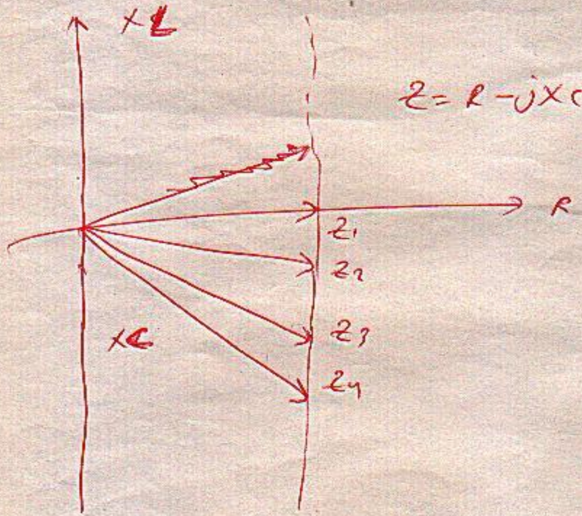



(b) R - C

b-1) Z - locus




$Z = R - jXc$




b-2) y - locus

$$Z = R - jXc = \frac{1}{y} = \frac{1}{G+jB} \times \frac{G-jB}{G-jB}$$

$$= \frac{G-jB}{G^2+B^2}$$

$$\therefore Z = R - jXc = \frac{1}{y} = \frac{G}{G^2+B^2} - j\frac{B}{G^2+B^2}$$

$$\therefore R = \frac{G}{G^2+B^2}, \quad \cancel{-j}Xc = \cancel{j}-\frac{B}{G^2+B^2}$$

$$\therefore Xc = \frac{B}{G^2+B^2}$$

* take the constant (R) (4)

$R = \frac{G}{G^2+B^2}$ (the same as $R_L$)

$$\boxed{\left(G-\frac{1}{2R}\right)^2 + B^2 = \left(\frac{1}{2R_1}\right)^2} \quad \text{— (8)}$$

ونرسم محل (Y) الكامل

1) $Z = R - jX_C = \frac{1}{Y}$

at $X_C = 0 = B_C$

$Y = +\frac{1}{R}$

2) $X_C = \infty = B_C$

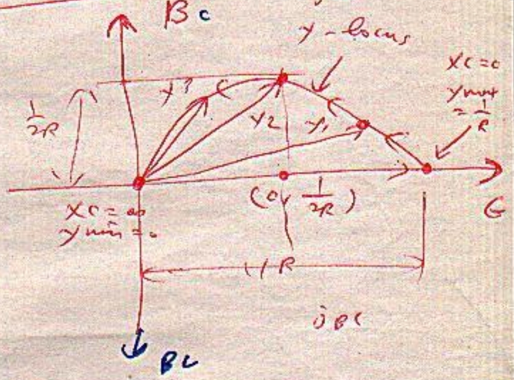
$Y = 0$

الرسم الكامل

3) at $G = \frac{1}{2R}$ from (8)

$\left(\frac{1}{2R} - \frac{1}{2R}\right)^2 + B^2 = \left(\frac{1}{2R_1}\right)^2$

$\therefore B = \frac{1}{2R}$



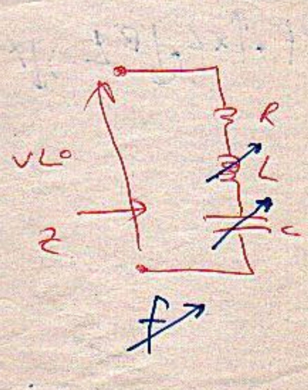

* I - locus : ???

(3) (R - L - C) ckt :

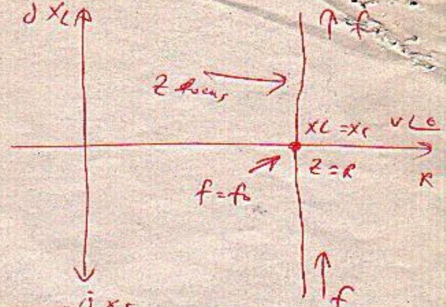
$Z = R + j\underbrace{(X_L - X_C)}_{X}$

(a) Z - locus

$f = 0$ | $X_L$ 0 | $X_C$ $\infty$ | $X$ $-\infty$




$f = f_0$ (resonance) $Z = R$




(b) Y-locus : f

$$\boxed{Z = R + j(X_L - X_C) = \frac{1}{Y}}$$

$$= \frac{G}{G^2+B^2} - j\frac{B}{G^2+B^2}$$


$$\therefore\ R = \frac{G}{G^2+B^2}$$

center $\left(\frac{1}{2R}, 0\right)$
Radius $= \frac{1}{2R}$ } eq. (8)

1) $f = 0 \Rightarrow X_L = 0, X_C = \infty, X = \infty, Y = 0 \rightarrow$ min (capacitive)
2) $f = f_0 \Rightarrow X_L = X_C \Rightarrow X = 0, Y = \frac{1}{R} = Y_{max}$
3) $f = \infty \Rightarrow X_L = \infty, X_C = 0, X = \infty, \therefore Y = 0$ (min) (inductive)

$\uparrow f, \uparrow X_L, \downarrow B_L, \downarrow X_C, \uparrow B_C$