CONTINUOUS MODULATION

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LECTURE OUTLINES

- 3.1 Basic Definitions
- 3.2 Phase Modulation
- 3.3 Frequency Modulation
- 3.4 Narrow Band Frequency Modulation
- 3.5 Wide Band Frequency Modulation

BASIC DEFINITION

- A general angle modulated signal is of the form:

$$s(t) = A_c \cos[\theta_i(t)]$$

The instantaneous phase $\theta_i(t) = 2\pi f_c t + \phi(t)$

Phase Deviation

Frequency Deviation

- The instantaneous frequency:

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} = \omega_c + \frac{d\phi(t)}{dt}$$

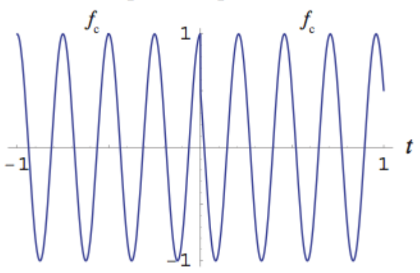
PHASE MODULATION (PM)

- Phase Modulation:

$$\phi(t) = k_p\, m(t)$$

Phase Deviation Constant [rad/volts]

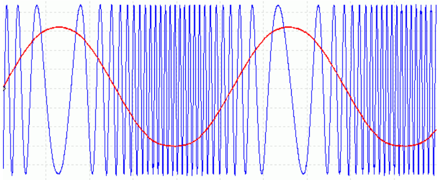
π/3 phase step at $t = 0$



- Which implies that the modulated signal is:

$$s(t) = A_c \cos[\theta_i(t)] = A_c \cos[2\pi f_c t + k_p m(t)]$$

FREQUENCY MODULATION (FM)



- Frequency Modulation

$$\frac{d\phi(t)}{dt} = k_f\, m(t)$$

Frequency Deviation Constant [Hz/volts]

$$\emptyset(t) = k_f \int_{t_0}^{t} m(t)dt$$

- Which implies that the modulated signal is:

$$s(t) = A_c \cos[\theta_i(t)] = A_c \cos\left[2\pi f_c t + k_f \int_{t_0}^{t} m(t)dt\right]$$

PROPERTIES OF FM AND PM SIGNALS

- Constancy of Transmitted Power: The average transmitted power of angle modulated wave is constant:

$$P_{av} = \frac{1}{2} A_c^2$$

- Nonlinearity of Modulation Process: The angle modulation process is nonlinear complicates the spectral analysis and noise analysis of PM and FM waves compared to AM. (offers superior noise performance).
- Irregularity of Zero-Crossing: A consequence of allowing the instantaneous angle $\theta_i(t)$ to become independent on the message signal, or even its integral, the zero-crossing of PM and FM waves is no longer exists. (The message is already resides in zero-crossing of the modulated waves).
- Difficulty in visualizing of message waveform.
- Higher bandwidth, improved noise performance.

NARROW BAND FREQUENCY MODULATION

- The FM signal is defined as:

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi f_d \int_{t_0}^{t} m(t)dt\right]$$

- Consider message signal of: $m(t) = A_m \cos(2\pi f_m t)$
- The instantaneous frequency

$$f_i(t) = \omega_c + \frac{d\phi(t)}{dt} = 2\pi f_c + \frac{d}{dt}\left[2\pi f_d \int_{t_0}^{t} m(t)\, dt\right]$$

$$\phi(t) = 2\pi f_d \int_{t_0}^{t} A_m \cos(2\pi f_m t)\, dt = \left(\frac{2\pi f_d}{2\pi f_m}\right) \sin[2\pi f_m t] = \frac{f_d}{f_m} \sin[2\pi f_m t]$$

NARROW BAND FM (NBFM)

$$\emptyset(t) = \frac{f_d}{f_m} \sin[2\pi f_m t]$$

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)] = A_c \cos[\,2\pi f_c t + \beta \sin(2\pi f_m t)]$$

$$s(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\, [\sin\beta \sin(2\pi f_m t)]$$

- For small value of $\beta \ll 1$, then $\sin\beta = \beta$:

$$s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

NARROW BAND FM (NBFM)

This formula is so similar to AM modulation signal given before.

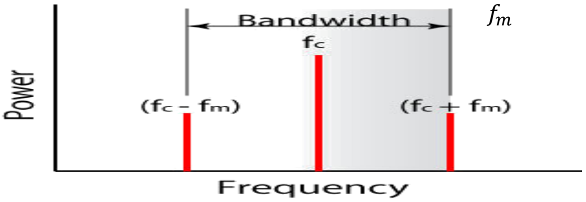
$$s(t) = \boxed{A_c \cos(2\pi f_c t)} - \frac{1}{2}\boxed{\beta} A_c \{\cos[2\pi \boxed{(f_c + f_m)} t] - \cos[2\pi \boxed{(f_c - f_m)} t]$$

Carrier Signal

Modulation index $\frac{f_d}{f_m}$

Upper Sideband

Reversed Lower Sideband



$$BW = 2f_m$$

WIDE-BAND FM (WBFM)

- Assume that f_c is large enough compared to the FM bandwidth, hence, we can rewrite the NBFM signal again in exponential form as:

$$s(t) = \mathcal{R}e[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))]$$

$$s(t) = \mathcal{R}e[s'(t) \exp(2\pi f_c t)]$$

- We assumed that:

$$s'(t) = A_c \exp[j\beta \sin(2\pi f_c t)]$$

- Unlike the original FM signal, the $s'(t)$ represents the complex envelope of the FM signal, and it is a periodic function of time with fundamental frequency of f_m.

WBFM AND SIGNAL SPECTRA

$$c_n = \frac{A_c}{2\pi}\int_{-\pi}^{+\pi} \exp[j\beta\sin(2\pi f_m t) - j2\pi n f_m t]\, dt$$

- Put: $x = 2\pi f_m t$

$$c_n = \frac{A_c}{2\pi}\int_{-\pi}^{+\pi} \exp[j(\beta\sin x - nx)]\, dx$$

- The above formula is recognized as nth order Bessel function, except the scaling factor, which is given by:

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \exp[j(\beta\sin x - nx)]\, dx$$

- Hence, $c_n = A_c J_n(\beta)$

WBFM AND SIGNAL SPECTRA

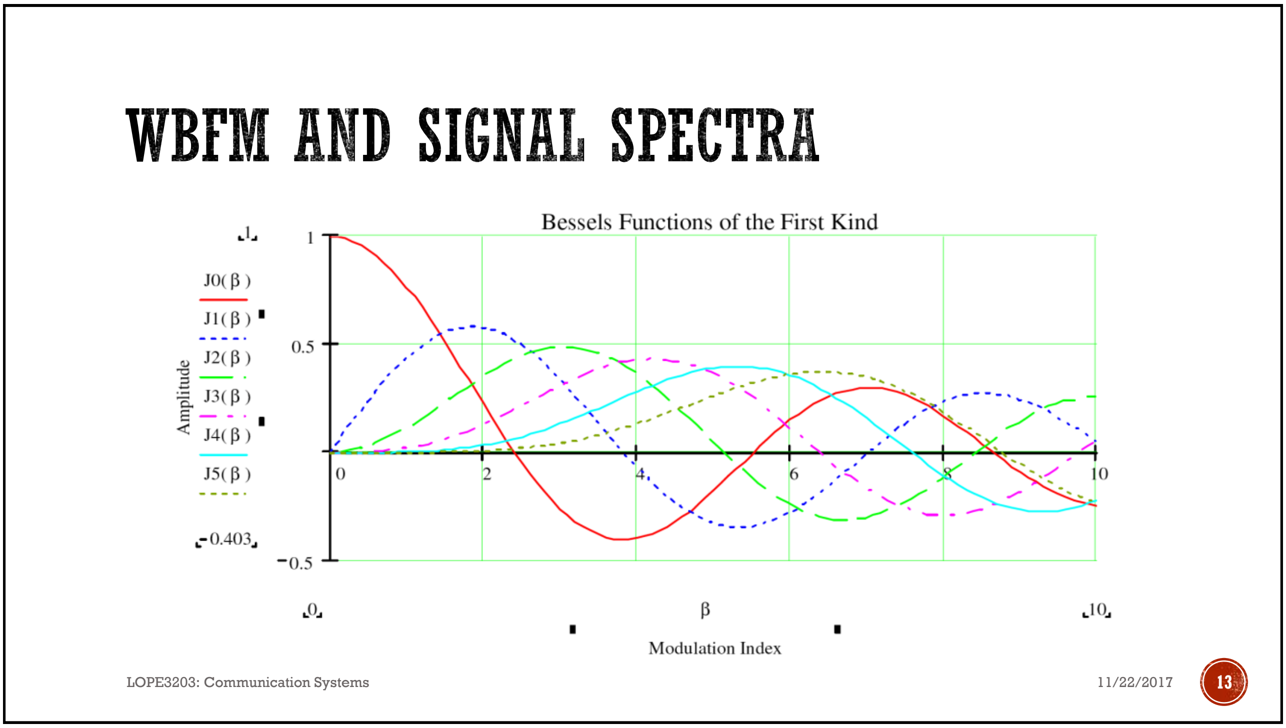
- We can now rewrite the WBFM signal as:

$$s(t) = \boxed{A_c}\, \mathcal{R}e\{\sum_{n=-\infty}^{+\infty} \boxed{J_n(\beta)} \exp[j2\pi \boxed{(f_c + nf_m)} t]\}$$

Amplitude of WBFM signal

nth order Bessel function

Bandwidth of WBFM



WBFM AND SIGNAL SPECTRA

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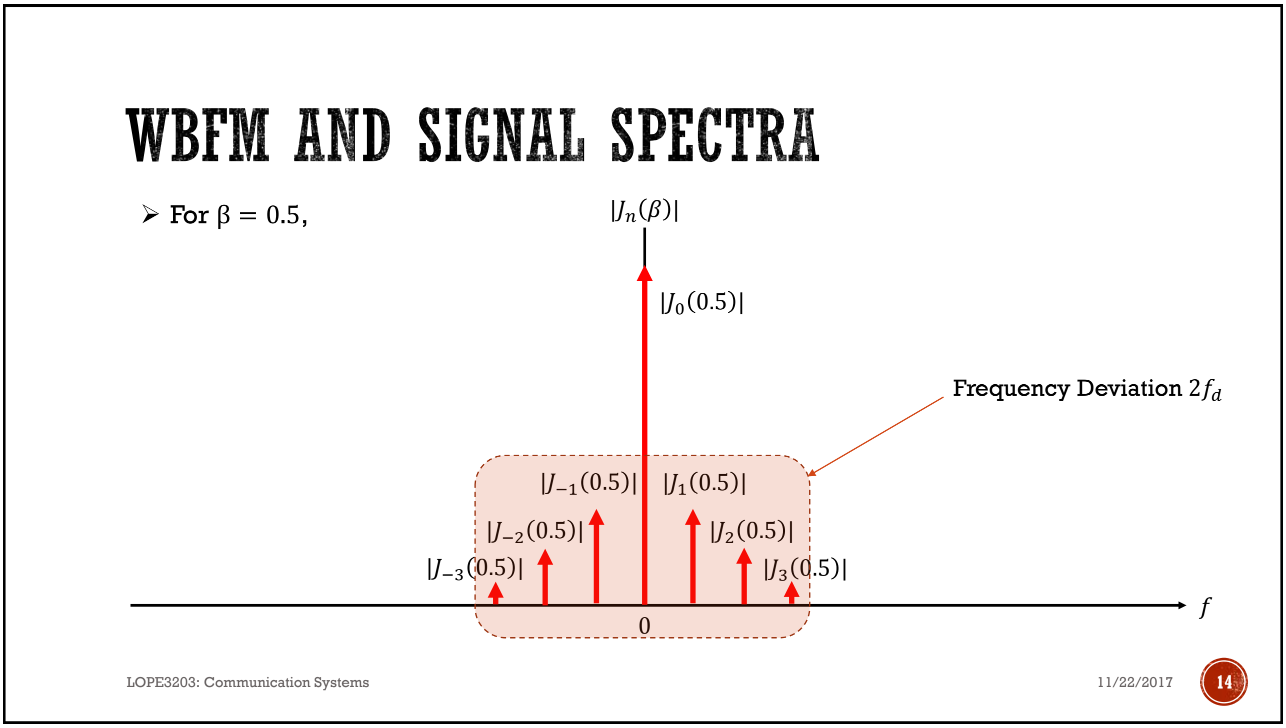

WBFM AND SIGNAL SPECTRA

- For β = 0.5,

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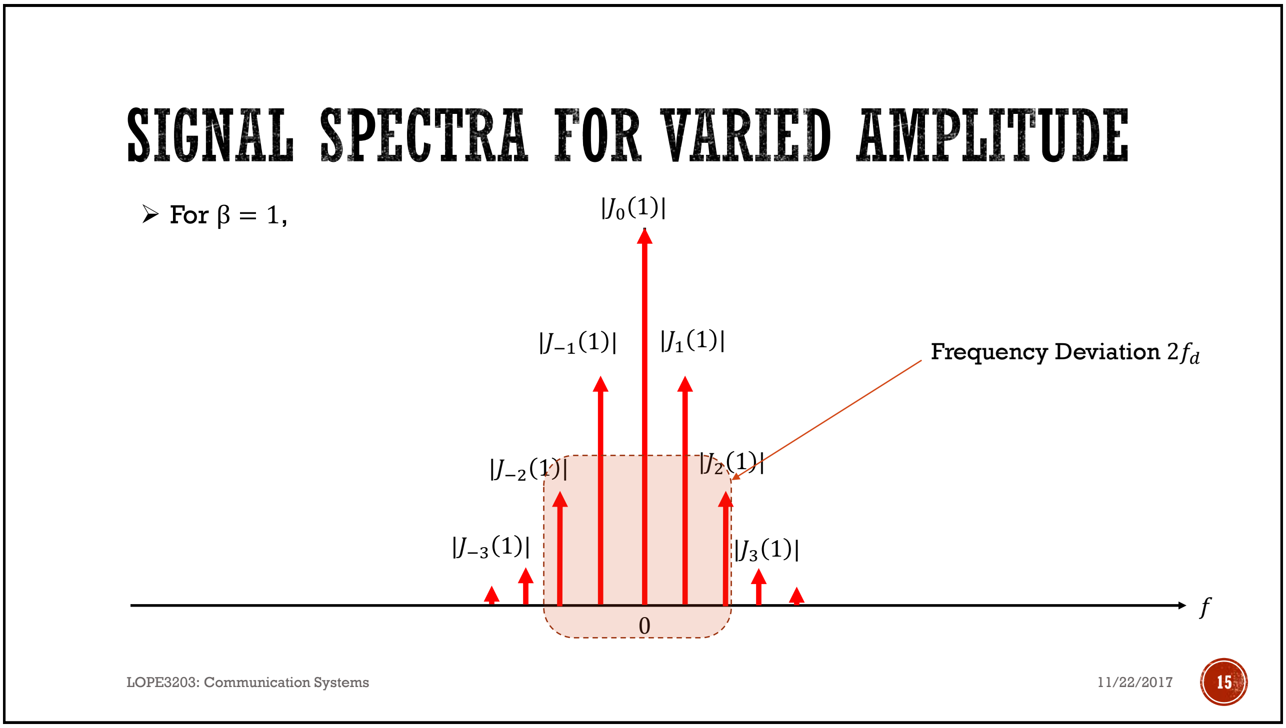
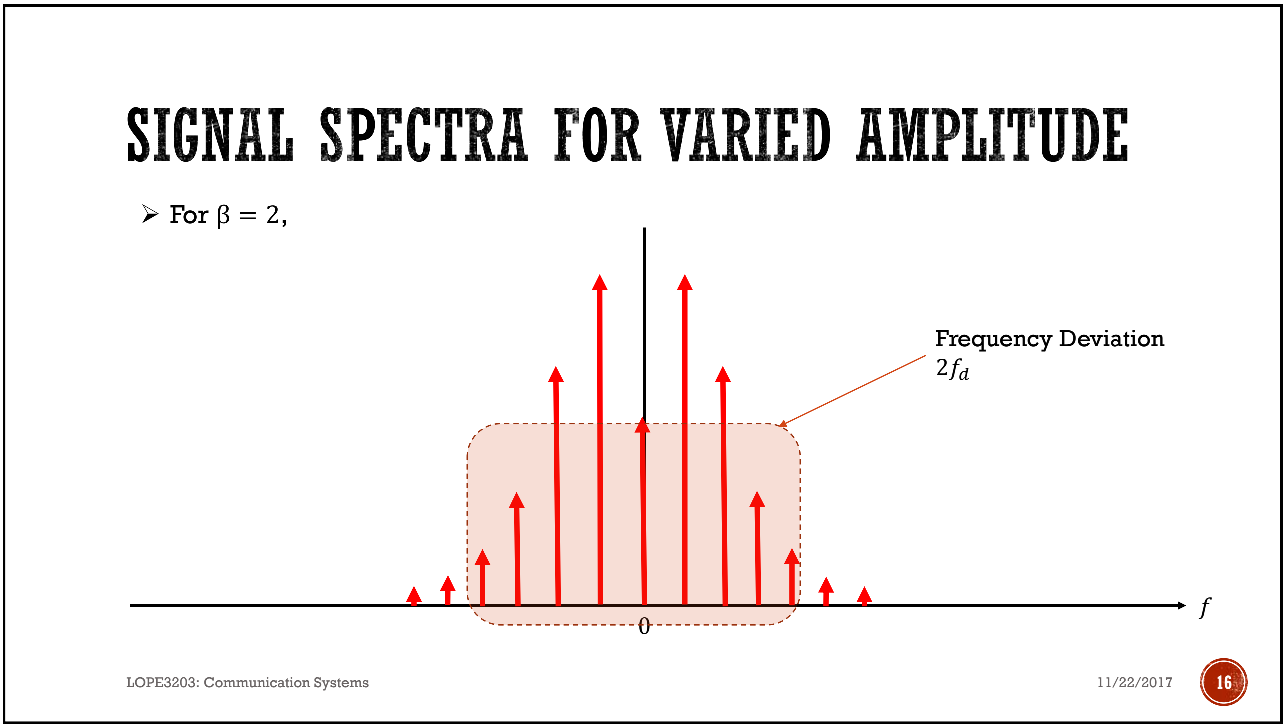


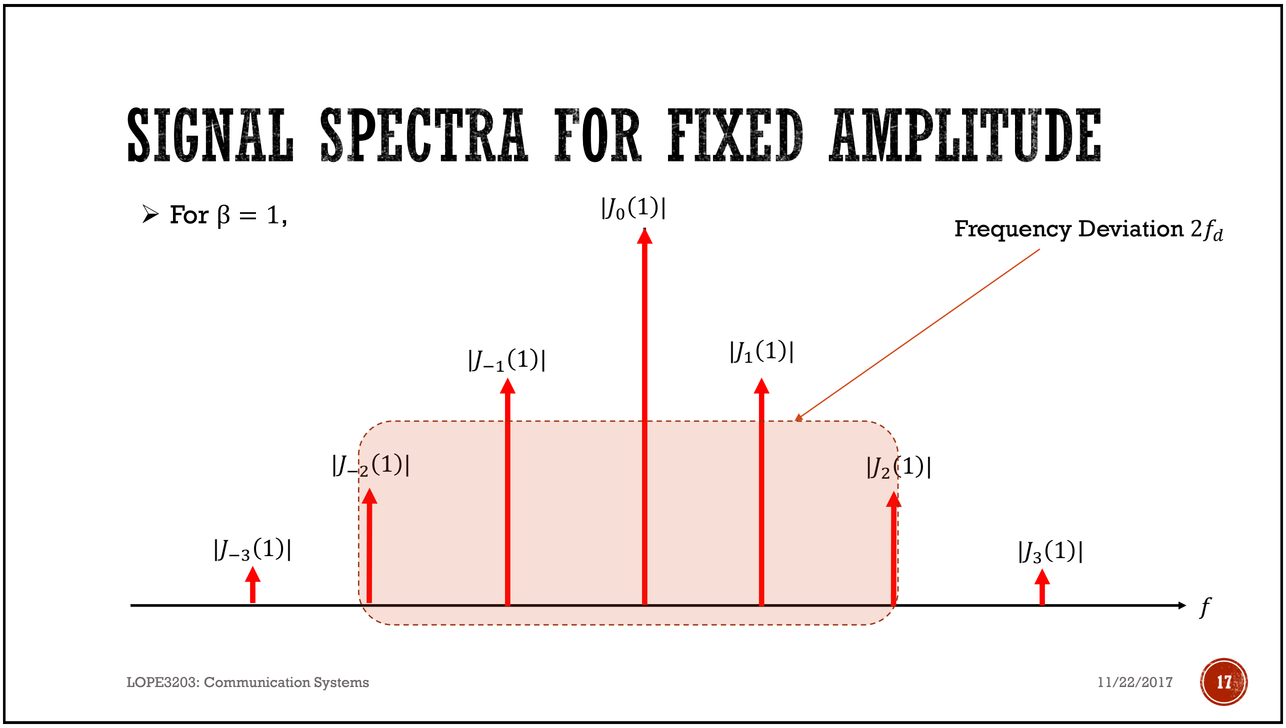
SIGNAL SPECTRA FOR VARIED AMPLITUDE

- For β = 1,



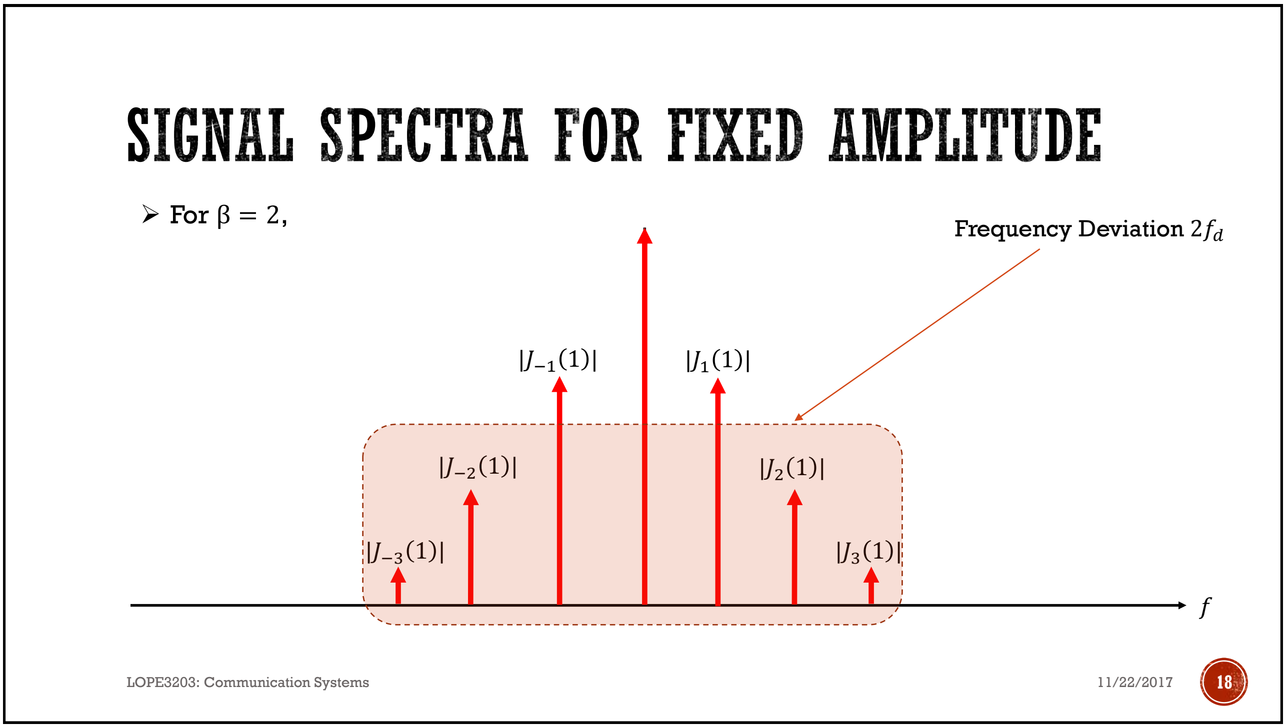
SIGNAL SPECTRA FOR VARIED AMPLITUDE

- For β = 2,



SIGNAL SPECTRA FOR FIXED AMPLITUDE

- For β = 1,



SIGNAL SPECTRA FOR FIXED AMPLITUDE

- For β = 2,

BANDWIDTH OF WBFM

- For very large values of β, the frequency deviation of the FM modulated signals is limited. Therefore, the bandwidth is:

$$B = 2f_d$$

- For very small values of β, the bandwidth is defined:

$$B = 2f_m$$

- For values between $\beta = 1$ and $\beta = 20$, the bandwidth is defined as:

$$B = 2f_d\left(1 + \frac{1}{\beta}\right)$$