DIGITAL BASEBAND MODULATION

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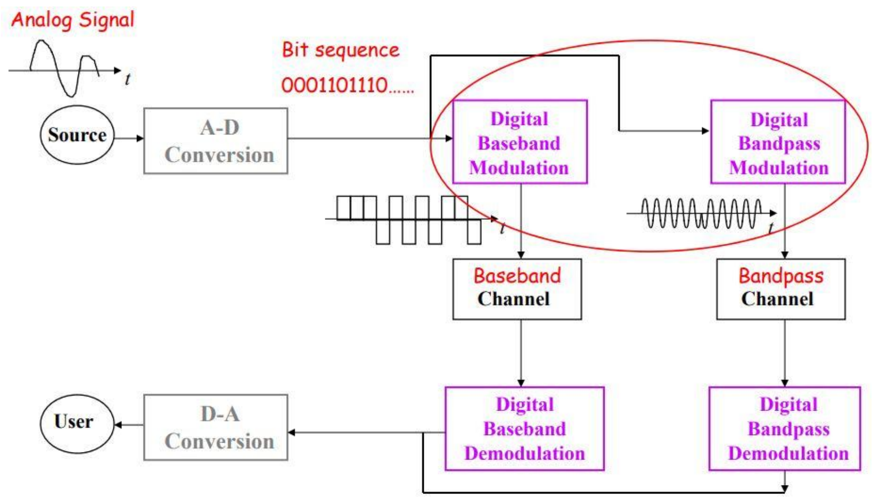
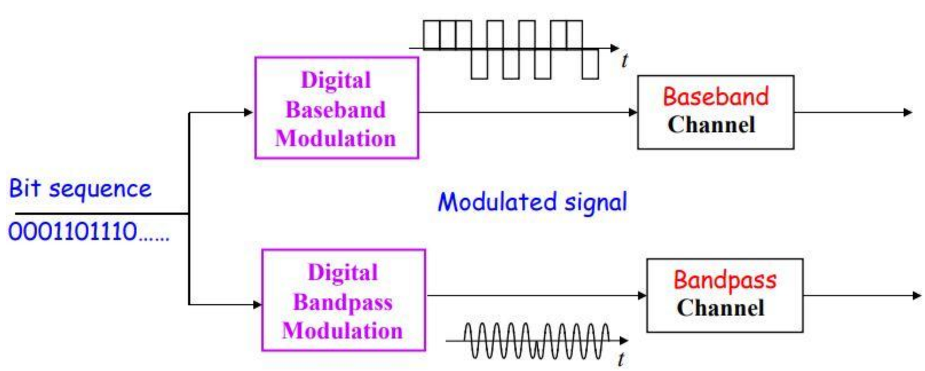
LECTURE LAYOUT

- Digital Modulation Block Diagram
- Digital Baseband Modulation
- Binary Pulse Amplitude Modulation
- 4 – Ary Amplitude Modulation
- Bandwidth Efficiency of M-Ary

INTRODUCTION

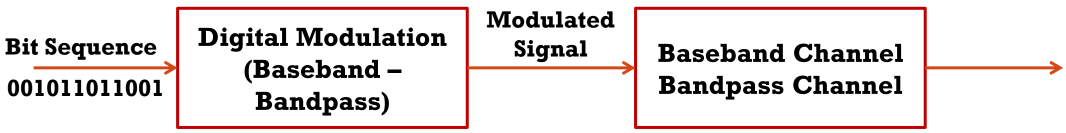


DIGITAL MODULATION



- How to choose proper digital waveform to carry the digits?

DIGITAL MODULATION



- **Bite Rate (R_b):** Number of bits transmitted per unit of time (e.g. 10kbps : Ten Thousands bits per one second transmitted over the channel).
- **Channel Bandwidth (B):** based on the modulated signal.
- **Bandwidth Efficiency:**

$$\epsilon = \frac{R_b}{B}$$

DIGITAL BASEBAND MODULATION

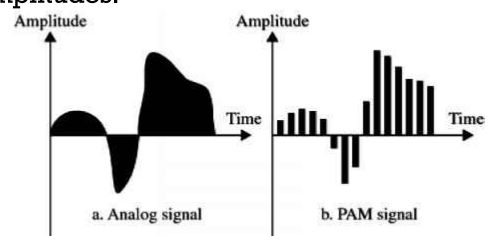
- Choose ***baseband*** signals to carry the digits.
- Each baseband signal can carry multiple bits.
- Base on the number of bits carried by the baseband signal, the modulation can be:

  - ***Binary:***
    - Carrying single bit only per baseband signal.
    - The bit Rate $= 1/\tau$.
    - Total number of baseband signals required for transmission is TWO.
  - ***M – ary:***
    - Carrying $M - symbols$ per single baseband signal.
    - Number of bits per symbol is $\log_2 M$ bps (Bits per Symbol).
    - Symbol rate $= 1/\tau$ Bit rate $= \log_2 M/\tau$
    - Total numbers of baseband signals required for transmission is M.

DIGITAL BASEBAND MODULATION

- We focus on "Amplitude Modulation".
- The signal of the same shape having different amplitudes.
- The *time-domain* representation of PAM:

$$s(t) = \sum_{n=-\infty}^{\infty} Z_n . v(t - nT)$$



- Z_n: discrete random variable with $\Pr\{Z_n = a_i\} = 1/M$ $\quad i = 1, 2, \dots, M$
- $v(t)$: is a unit baseband signal

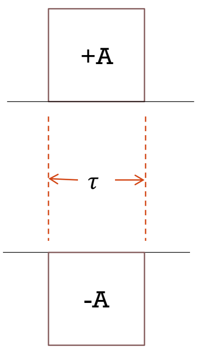
DIGITAL BASEBAND MODULATION

- The power spectrum of the modulated signal of the PAM is:

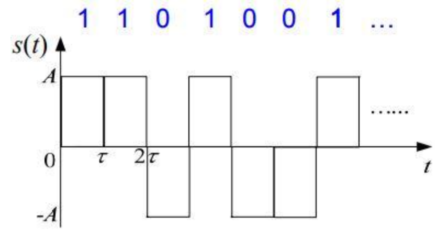
$$G_s(f) = \frac{1}{\tau} |V(f)|^2 \left(\sigma^2 + \frac{\mu^2}{\tau} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{\tau}\right) \right)$$

- σ: is the variance.
- μ: is the mean value.

PULSE AMPLITUDE MODULATION (PAM)



- Bit "1" to represents the +ve rectangular pulse of amplitude +A and width τ
- Bit "0" to represents the -ve rectangular pulse of amplitude -A and width τ



- $s(t) = \sum_{n=-\infty}^{\infty} Z_n . v(t - n\tau)$
- $\Pr\{Z_n = \mp 1\} = 0.5$
- $v(t) = \begin{cases} A & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$

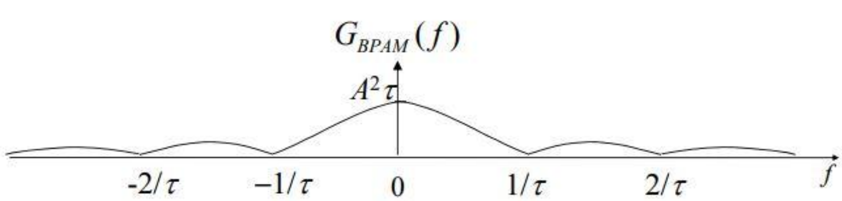
POWER SPECTRAL DENSITY

$$G_s(f) = \frac{1}{\tau}|V(f)|^2 \cdot \left(\sigma_Z^2 + \frac{\mu_Z^2}{\tau}\sum_{m=-\infty}^{\infty}\delta\left(f - \frac{m}{\tau}\right)\right)$$

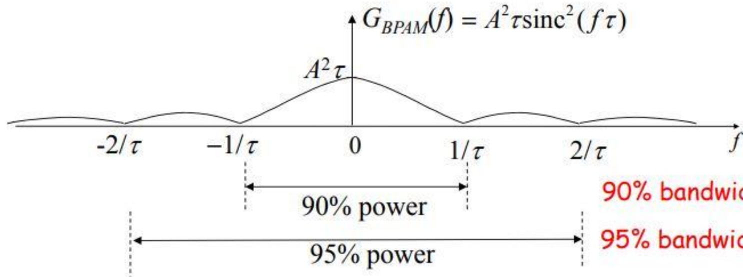
With Binary PAM: $V(f) = A\tau \mathrm{sinc}(f\tau)$

$\mu_Z = 0, \ \sigma_Z^2 = 1$

$$G_{BPAM}(f) = A^2\tau \mathrm{sinc}^2(f\tau)$$



EFFECTIVE BANDWIDTH OF BINARY PAM



90% bandwidth: $1/\tau$

95% bandwidth: $2/\tau$

- Suppose 90% of signal power must pass through the channel (90% in-band power):

  Required Channel Bandwidth: $B_{h\_90\%} = 1/\tau$

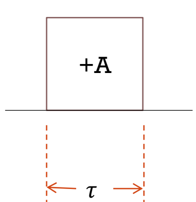
  Bit rate: $R_b = 1/\tau$

  $\Rightarrow B_{h\_90\%} = R_b$

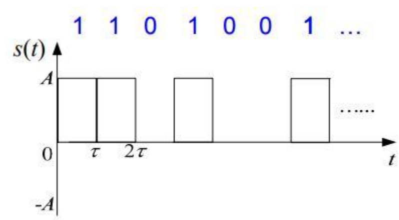
- Suppose 95% of signal power must pass through the channel (95% in-band power):

  Required Channel Bandwidth: $B_{h\_95\%} = 2/\tau = 2R_b$

BINARY ON – OFF KEYING



- Bit "1" to represents the +ve rectangular pulse of amplitude +A and width τ



- $s(t) = \sum_{n=-\infty}^{\infty} Z_n . v(t - n\tau)$

- Bit "0" to represents the ZERO rectangular pulse of amplitude ZERO and width τ

- $\Pr\{Z_n = 1\} = \Pr\{Z_n = 0\} = 0.5$
- $v(t) = \begin{cases} A & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$

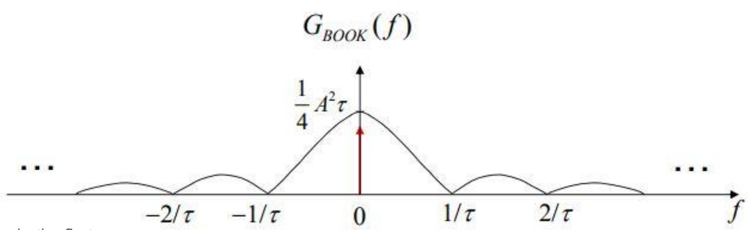
POWER SPECTRAL DENSITY

$$G_s(f) = \frac{1}{\tau}|V(f)|^2 \cdot \left(\sigma_Z^2 + \frac{\mu_Z^2}{\tau}\sum_{m=-\infty}^{\infty}\delta\left(f-\frac{m}{\tau}\right)\right)$$

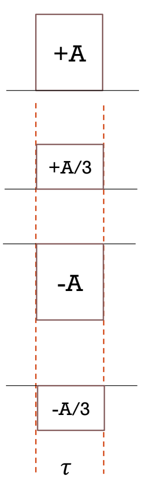
With Binary OOK: $V(f) = A\tau\text{sinc}(f\tau)$

$\mu_Z = 1/2, \ \sigma_Z^2 = 1/4$

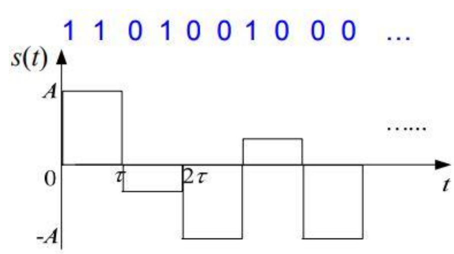
$$G_{BOOK}(f) = \frac{1}{\tau}(A\tau\text{sinc}(f\tau))^2 \cdot \left(\frac{1}{4} + \frac{1}{4\tau}\sum_{m=-\infty}^{\infty}\delta\left(f-\frac{m}{\tau}\right)\right)$$



4 – ARY PULSE AMPLITUDE MODULATION



- Bit "11" to represents the +ve rectangular pulse of amplitude +A and width τ
- Bit "10" to represents the ZERO rectangular pulse of amplitude +A/3 and width τ
- Bit "00" to represents the ZERO rectangular pulse of amplitude –A and width τ
- Bit "0q" to represents the ZERO rectangular pulse of amplitude -A/3 and width τ



- $s(t) = \sum_{n=-\infty}^{\infty} Z_n . v(t - n\tau)$
- $\Pr\{Z_n = 1\} = \Pr\{Z_n = 1/3\} = \Pr\{Z_n = -1\} = \Pr\{Z_n = -1/3\} = 0.25$
- $v(t) = \begin{cases} A & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$

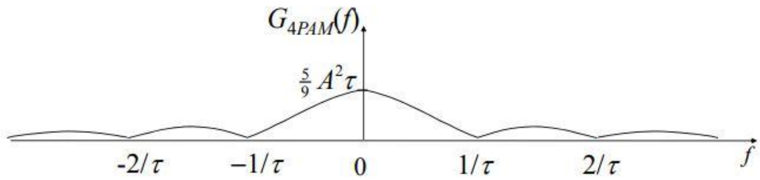
POWER SPECTRUM OF 4-ARY PAM

$$G_s(f) = \frac{1}{\tau}|V(f)|^2 \cdot \left(\sigma_Z^2 + \frac{\mu_Z^2}{\tau}\sum_{m=-\infty}^{\infty}\delta\left(f - \frac{m}{\tau}\right)\right)$$

With 4-ary PAM: $V(f) = A\tau\mathrm{sinc}(f\tau)$

$\mu_Z = 0,\ \sigma_Z^2 = 5/9$

$$G_{4PAM}(f) = \tfrac{5}{9}A^2\tau\mathrm{sinc}^2(f\tau)$$



- Required channel bandwidth with 90% in-band power: $B_{h\_90\%} = 1/\tau$
- Required channel bandwidth with 95% in-band power: $B_{h\_95\%} = 2/\tau$

BANDWIDTH EFFICIENCY OF M-ARY PAM

- Suppose there are totally M distinct amplitude (power) levels.
- How many bits are carried by each symbol?

$$M = 2^k \Rightarrow k = \log_2 M$$

- What is the relationship between symbol rate R_S and bit rate R_b?

$$R_S = R_b / k \quad \text{or} \quad R_b = kR_S$$

- What is the required channel bandwidth with 90% in-band power?

$$B_{h\_90\%} = R_S = R_b / k$$

- Bandwidth Efficiency of M-ary PAM

Tradeoff between bandwidth efficiency and fidelity performance

$$\gamma_{MPAM} = k = \log_2 M \quad \text{with 90\% in-band power}$$

- A larger M also leads to a smaller minimal amplitude difference – higher error probability (to be discussed).