

2- Actual sourcesVoltage and Current sources

For the voltage source, if $R_s = 0 \Omega$ or is so small compared to any series resistor that it can be ignored, then we have an “**ideal**” voltage source. For the current source, if $R_s = \infty$ or is large enough compared to other parallel elements that it can be ignored, and then we have an “**ideal**” current source. See fig (2-4).

If the internal resistance is included with either source, then we have an “**actual**” voltage source or “**actual**” current source fig (2-5); then that source can be converted to the other type. Fig (2-6).

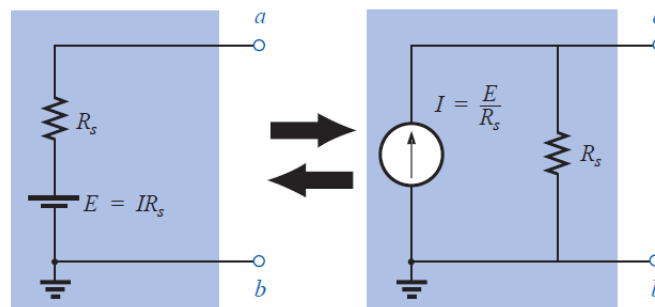


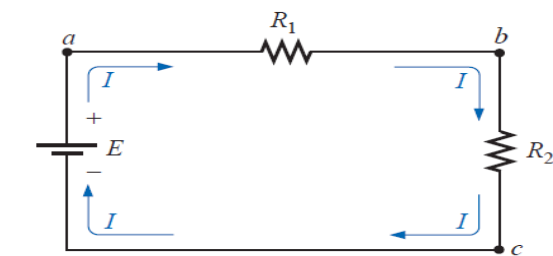
Fig (2-6)

Source conversion.

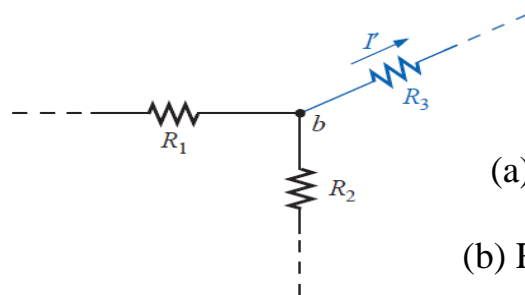
Voltage source to current source and vice versa

3- Network simplification3-1 SERIES CIRCUITS

A **circuit** consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 3-1 has three elements joined at three terminal points (a , b , and c) to provide a closed path for the current I .



(a) Series circuit



(b) R_1 and R_2 are not in series

Fig(3-1)

(a) Series circuit R_1 and R_2 and E

(b) R_1 and R_2 and R_3 are **not** in series.

In Fig.(3-1) the resistors R_1 and R_2 are in series , the battery E and resistor R_1 are in series, and the resistor R_2 and the battery E are in series .Since all the elements are in series, the network is called a **(series circuit.)**

😊 Note

1- The total resistance of a series circuit is the sum of the resistance levels.

$$R_T = R_1 + R_2 \quad (\text{ohm } \Omega)$$

2- The current is the same through each element and the current drawn from the source

(Total current I_T) of Fig. (3-1a) equal:

$$I = I_{R1} = I_{R2} = I_T \quad (\text{Amp})$$

I_T can be determined using Ohm's law.

$$I = I_T = \frac{E}{R_T} \quad (\text{Amp})$$

3- $V_1 = IR_1$ $V_2 = IR_2$ $V_T = V_1 + V_2$ (Volt)

4- The power delivered to each resistor can then be determined using any one of three equations:

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}$$

(watts, W)

The power delivered by the source is

$$P_{\text{del}} = EI \quad (\text{watts, W})$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

$$P_{\text{del}} = P_1 + P_2 + P_3 + \cdots + P_N$$

EXAMPLE 1

- Find the total resistance for the series circuit of Fig. 3-2
- Calculate the source current I_s .
- Determine the voltages V_1 , V_2 , and V_3 .
- Calculate the power dissipated by R_1 , R_2 , and R_3 .
- Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

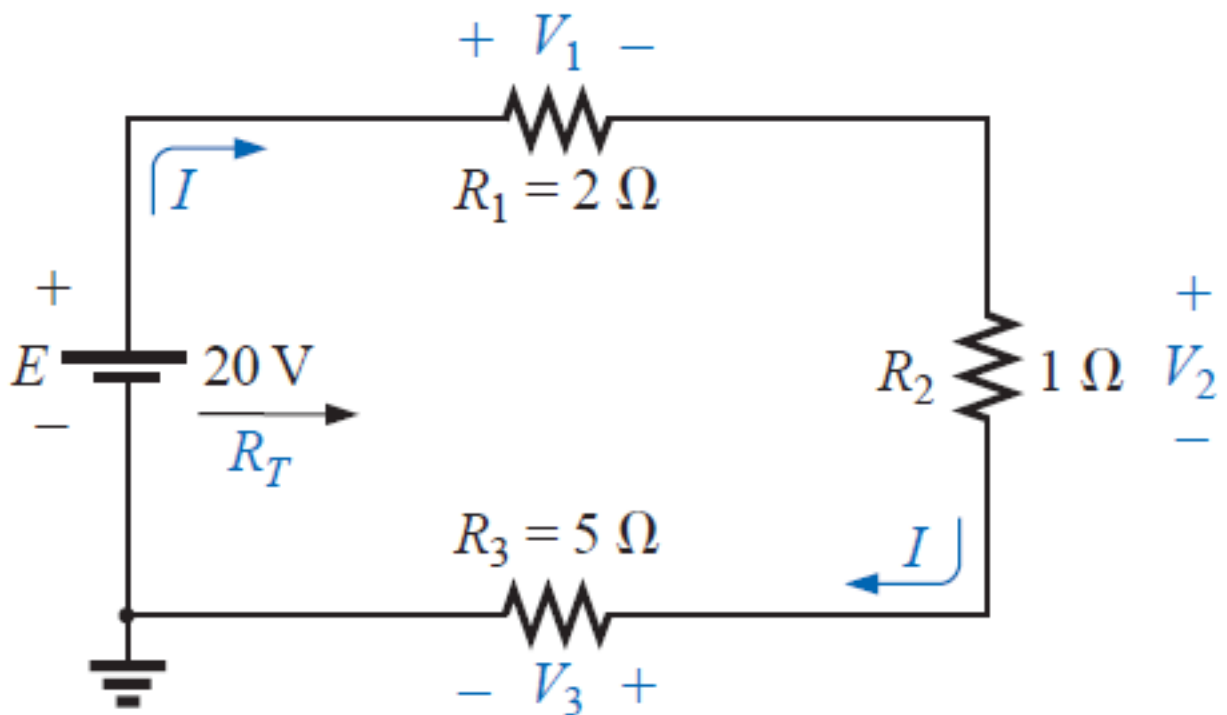


Fig (3-2)

Solutions:

$$a. R_T = R_1 + R_2 + R_3 = 2 \Omega + 1 \Omega + 5 \Omega = 8 \Omega$$

$$b. I_s = \frac{E}{R_T} = \frac{20 \text{ V}}{8 \Omega} = 2.5 \text{ A}$$

$$c. V_1 = IR_1 = (2.5 \text{ A})(2 \Omega) = 5 \text{ V}$$

$$V_2 = IR_2 = (2.5 \text{ A})(1 \Omega) = 2.5 \text{ V}$$

$$V_3 = IR_3 = (2.5 \text{ A})(5 \Omega) = 12.5 \text{ V}$$

$$d. P_1 = V_1 I_1 = (5 \text{ V})(2.5 \text{ A}) = 12.5 \text{ W}$$

$$P_2 = I_2^2 R_2 = (2.5 \text{ A})^2 (1 \Omega) = 6.25 \text{ W}$$

$$P_3 = V_3^2 / R_3 = (12.5 \text{ V})^2 / 5 \Omega = 31.25 \text{ W}$$

$$e. P_{\text{del}} = EI = (20 \text{ V})(2.5 \text{ A}) = 50 \text{ W}$$

$$P_{\text{del}} = P_1 + P_2 + P_3$$

$$50 \text{ W} = 12.5 \text{ W} + 6.25 \text{ W} + 31.25 \text{ W}$$

$$50 \text{ W} = 50 \text{ W} \quad (\text{checks})$$

EXAMPLE 2 Determine R_T , I , and V_2 for the circuit of Fig. 3-3

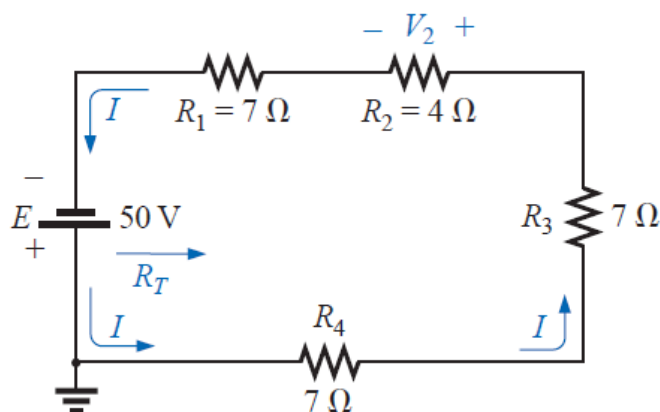


Fig 3-3

Solution. Note the current direction as established by the battery and the polarity of the voltage drops across R_2 as determined by the current direction

$$R_T = R_1 + R_2 + R_3 + R_4$$

$$R_T = 7 + 4 + 7 + 7 = 25 \Omega$$

$$I = \frac{E}{R_T} = \frac{50 \text{ V}}{25 \Omega} = 2 \text{ A}$$

$$V_2 = IR_2 = (2 \text{ A})(4 \Omega) = 8 \text{ V}$$

EXAMPLE 3 Given R_T and I , calculate R_1 and E for the circuit of Fig.3-4 .

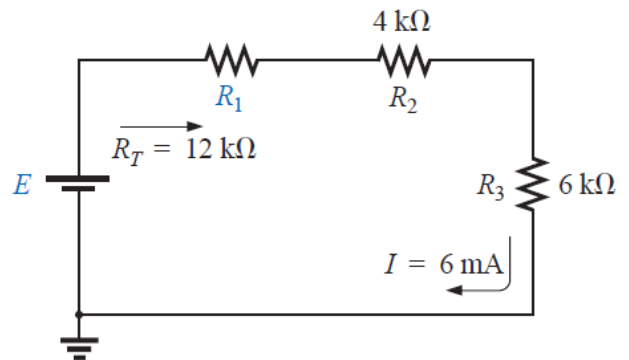
Solution:

$$R_T = R_1 + R_2 + R_3$$

$$12 \text{ k}\Omega = R_1 + 4 \text{ k}\Omega + 6 \text{ k}\Omega$$

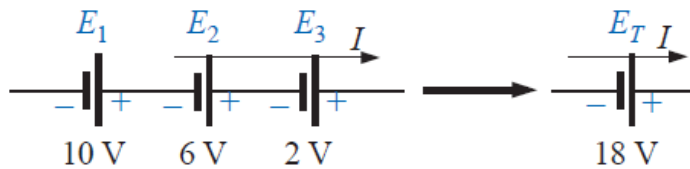
$$R_1 = 12 \text{ k}\Omega - 10 \text{ k}\Omega = 2 \text{ k}\Omega$$

$$E = IR_T = (6 \times 10^{-3} \text{ A})(12 \times 10^3 \Omega) = 72 \text{ V}$$



Fig(3-4)

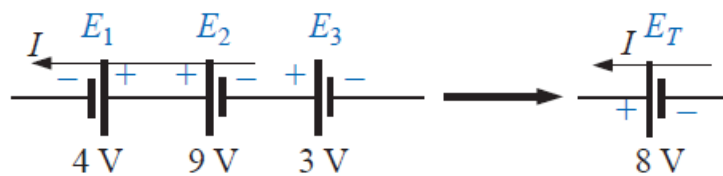
3-2 VOLTAGE SOURCES IN SERIES



(a)

$$E_T = E_1 + E_2 + E_3 = 10 \text{ V} + 6 \text{ V} + 2 \text{ V} = 18 \text{ V}$$

and the polarity shown in the figure



(b)

$$E_T = E_2 + E_3 - E_1 = 9 \text{ V} + 3 \text{ V} - 4 \text{ V} = 8 \text{ V}$$

and the polarity shown in the figure

Fig3.5

(a ,b) Reducing series dc voltage sources to a single source.

3-3 KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

A closed loop is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

The clockwise (CW) direction will be used throughout the text for all applications of Kirchhoff's voltage law. Be aware, however, that the same result will be obtained if the counterclockwise (CCW) direction is chosen and the law applied correctly. A plus sign is assigned to a potential rise (- to +), and a minus sign to a potential drop (+ to -). If we follow the current in Fig. (3-6) from point a, we first encounter a potential drop V_1 (+ to -) across R_1 and then another potential drop V_2 across R_2 .

Continuing through the voltage source, we have a potential rise E (- to +) before returning to point a. In symbolic form, where Σ represents (summation), the closed loop, and V the potential **drops** and **rises**, we have :

$$\sum_{\text{C}} V = 0$$

(Kirchhoff's voltage law
in symbolic form)

Which for the circuit of Fig. (3-6) yields (clockwise direction, following the current I and starting at point d):

$$+E - V_1 - V_2 = 0$$

$$E = V_1 + V_2$$

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

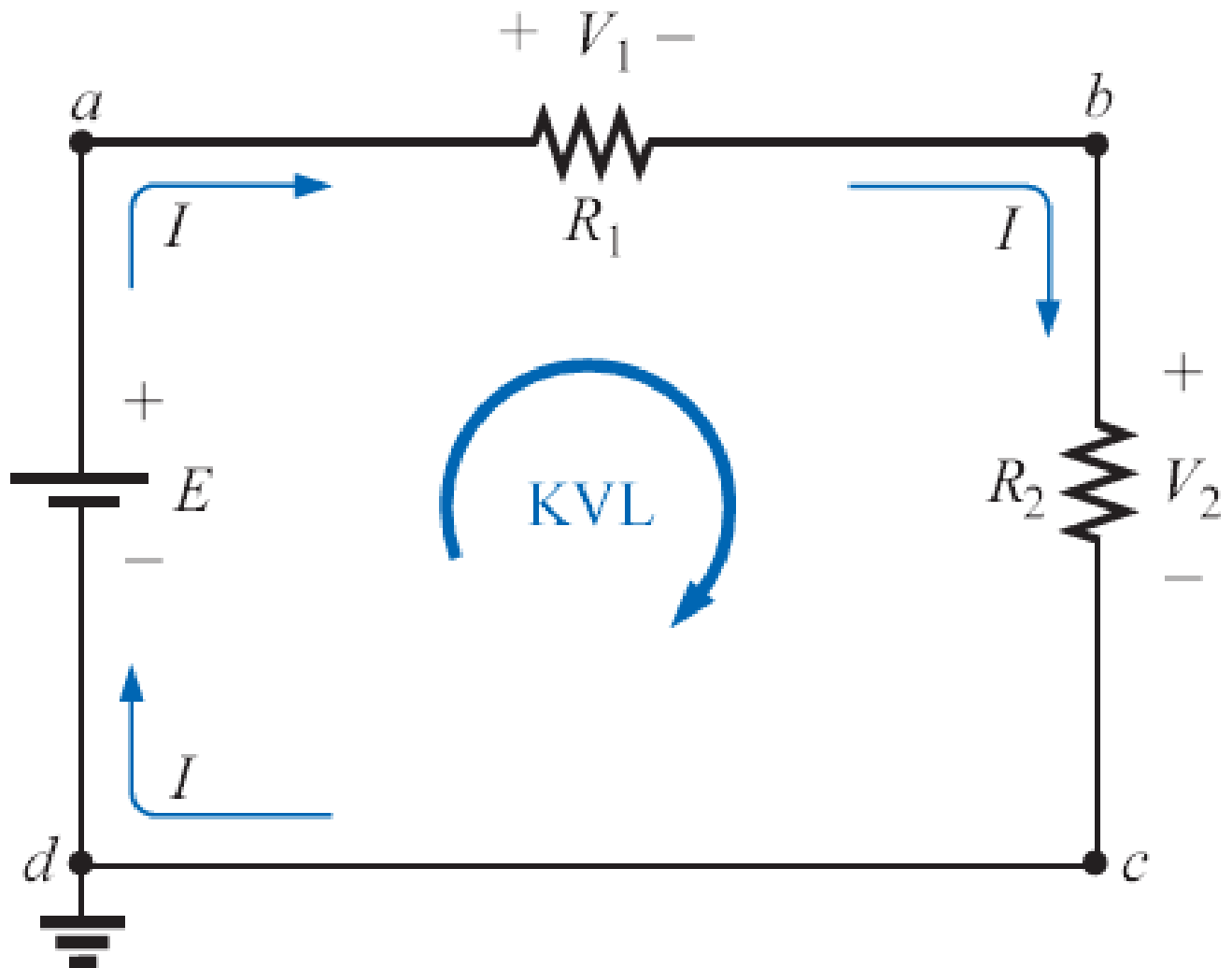


Fig (3-6) Applying Kirchhoff's voltage law to a series dc circuit.

Kirchhoff's voltage law can also be stated in the following form:

$$\sum_{\text{clockwise}} V_{\text{rises}} = \sum_{\text{clockwise}} V_{\text{drops}}$$

EXAMPLE 4 Determine the unknown voltages for the networks of Fig. (3-7)

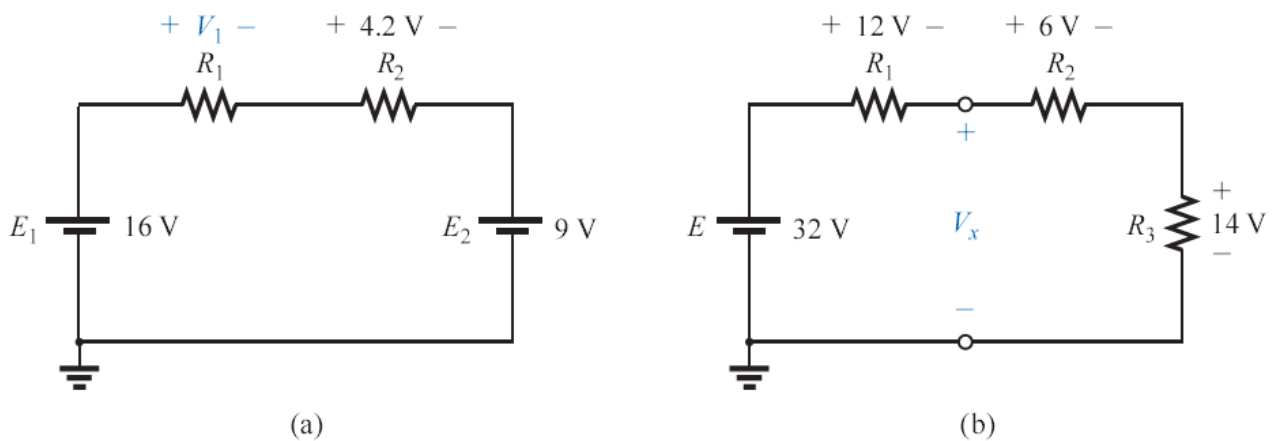


Fig3-7

Sol:

a-

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and
$$V_1 = E_1 - V_2 - E_2 = 16 \text{ V} - 4.2 \text{ V} - 9 \text{ V} = 2.8 \text{ V}$$

b-

$$+E - V_1 - V_x = 0$$

and

$$\begin{aligned} V_x &= E - V_1 = 32 \text{ V} - 12 \text{ V} \\ &= \mathbf{20 \text{ V}} \end{aligned}$$

Using the clockwise direction for the other loop involving R_2 and R_3 will result in

$$+V_x - V_2 - V_3 = 0$$

and

$$\begin{aligned} V_x &= V_2 + V_3 = 6 \text{ V} + 14 \text{ V} \\ &= \mathbf{20 \text{ V}} \end{aligned}$$

EXAMPLE 5 For the circuit of Fig. 3-8

- Find R_T .
- Find I .
- Find V_1 and V_2 .
- Find the power to the 4- Ω and 6- Ω resistors.
- Find the power delivered by the battery, and compare it to that dissipated by the 4- Ω and 6- Ω resistors combined.
- Verify Kirchhoff's voltage law (clockwise direction).

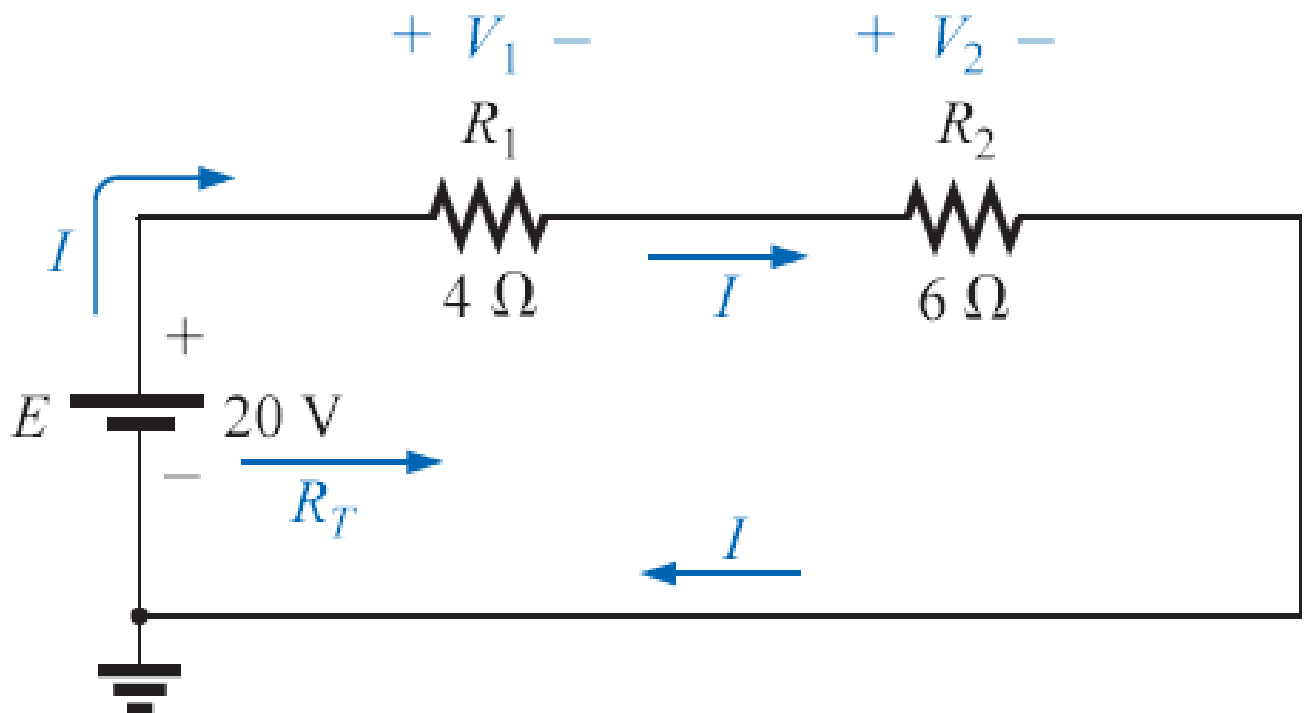


Fig 3-8

Solutions:

a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = 10\ \Omega$

b. $I = \frac{E}{R_T} = \frac{20\ \text{V}}{10\ \Omega} = 2\ \text{A}$

- c. $V_1 = IR_1 = (2 \text{ A})(4 \ \Omega) = \mathbf{8 \text{ V}}$
 $V_2 = IR_2 = (2 \text{ A})(6 \ \Omega) = \mathbf{12 \text{ V}}$
- d. $P_{4\Omega} = \frac{V_1^2}{R_1} = \frac{(8 \text{ V})^2}{4} = \frac{64}{4} = \mathbf{16 \text{ W}}$
 $P_{6\Omega} = I^2 R_2 = (2 \text{ A})^2(6 \ \Omega) = (4)(6) = \mathbf{24 \text{ W}}$
- e. $P_E = EI = (20 \text{ V})(2 \text{ A}) = \mathbf{40 \text{ W}}$
 $P_E = P_{4\Omega} + P_{6\Omega}$
 $40 \text{ W} = 16 \text{ W} + 24 \text{ W}$
 $40 \text{ W} = 40 \text{ W} \quad (\text{checks})$
- f. $\sum_{\text{C}} V = +E - V_1 - V_2 = 0$
 $E = V_1 + V_2$
 $20 \text{ V} = 8 \text{ V} + 12 \text{ V}$
 $20 \text{ V} = 20 \text{ V} \quad (\text{checks})$

EXAMPLE 6 For the circuit of Fig. 3-9

- Determine V_2 using Kirchhoff's voltage law.
- Determine I .
- Find R_1 and R_3 .

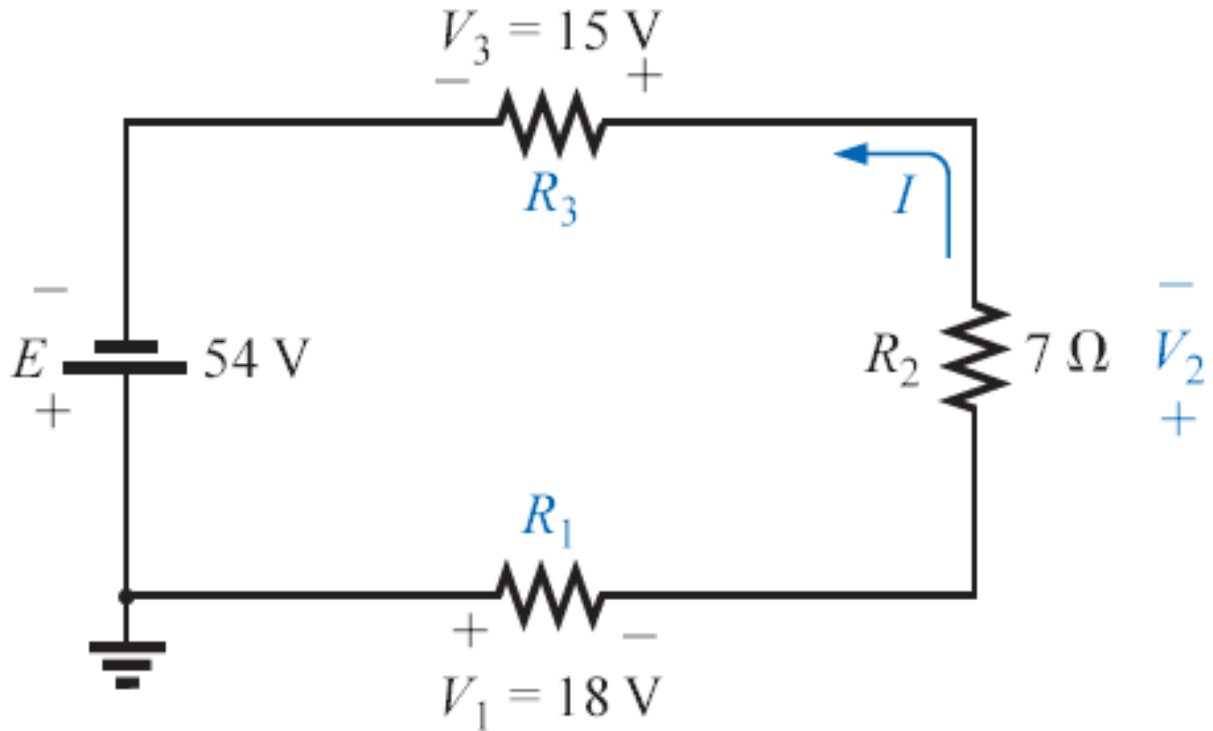


Fig 3-9

Solutions:

a. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or
$$E = V_1 + V_2 + V_3$$

and
$$V_2 = E - V_1 - V_3 = 54 \text{ V} - 18 \text{ V} - 15 \text{ V} = 21 \text{ V}$$

b.
$$I = \frac{V_2}{R_2} = \frac{21 \text{ V}}{7 \Omega} = 3 \text{ A}$$

c.
$$R_1 = \frac{V_1}{I} = \frac{18 \text{ V}}{3 \text{ A}} = 6 \Omega$$

$$R_3 = \frac{V_3}{I} = \frac{15 \text{ V}}{3 \text{ A}} = 5 \Omega$$

3-4 INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element. For instance, the network of Fig. (3-10) can be redrawn as shown in Fig.(3-11)

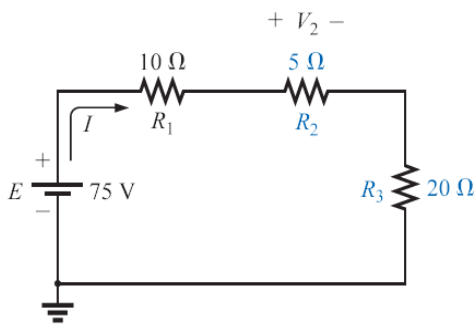


Fig. (3-10)

Series dc circuit with elements to be interchanged

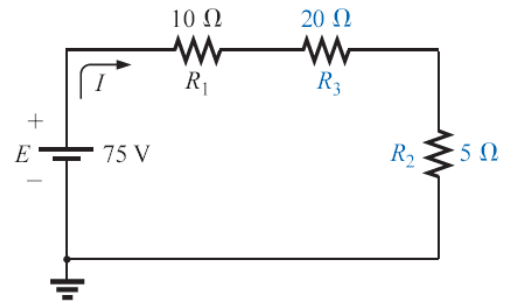
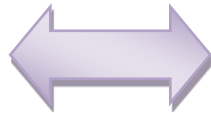


Fig. (3-11)

Circuit of with R2 and R3 interchanged.

EXAMPLE 7 Determine I and the voltage across the 7Ω resistor for the network of Fig. 3-12

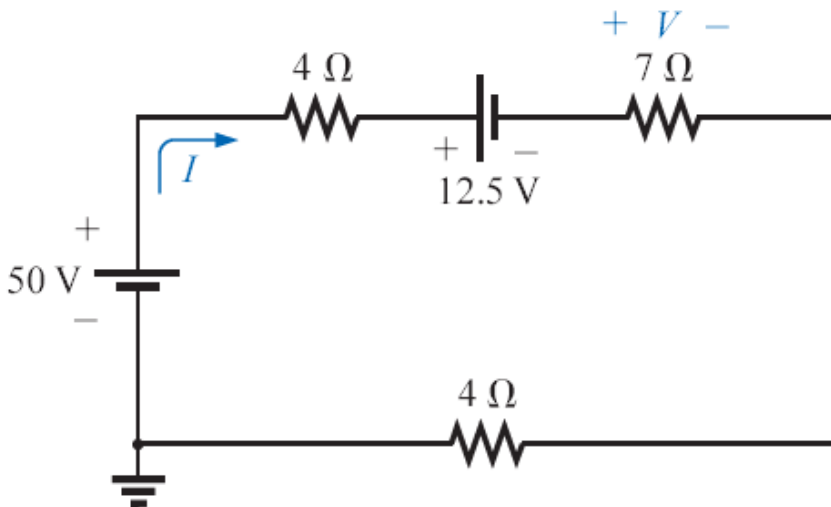


Fig 3-12

Solution:

The network is redrawn in Fig. 3-13

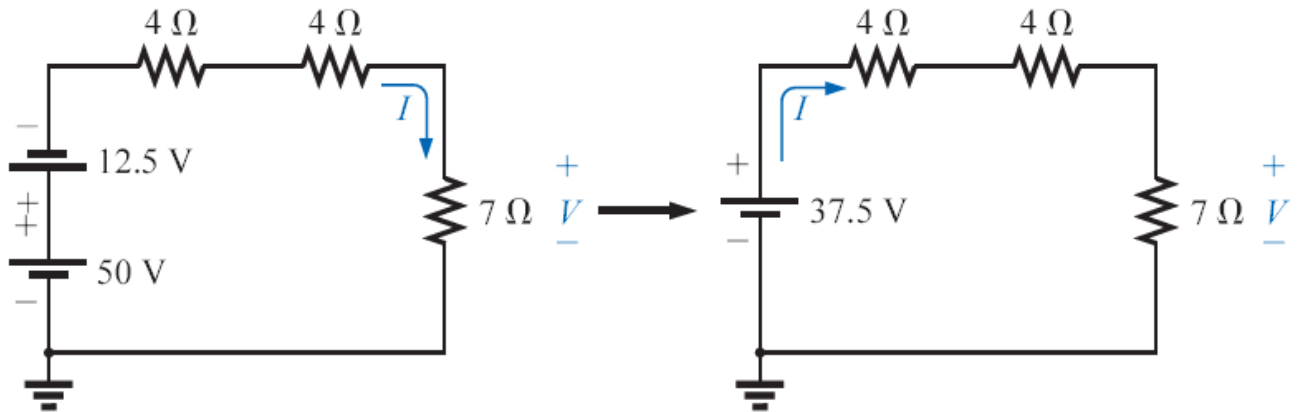


Fig 3-13

$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\ \text{V}}{15\ \Omega} = 2.5\ \text{A}$$

$$V_{7\Omega} = IR = (2.5\ \text{A})(7\ \Omega) = 17.5\ \text{V}$$

3-5 VOLTAGE DIVIDER RULE (V.D.R.)

The voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

$$V_x = \frac{R_x E}{R_T}$$

(voltage divider rule)

EXAMPLE 8 Determine the voltage V_1 for the network of Fig. 3-14

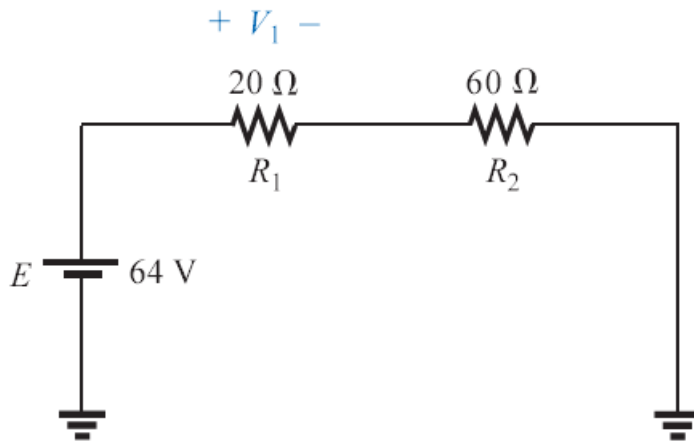


Fig (3-14)

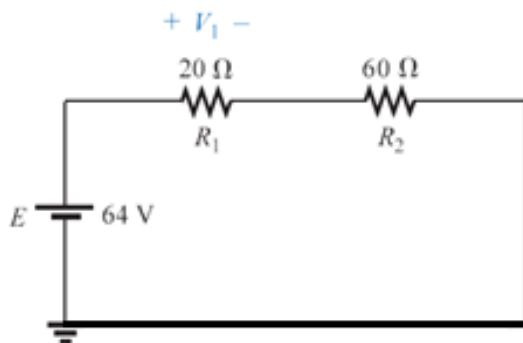


Fig (3-15)

Sol.

The circuit is simplified to fig (3-15)

$$V_1 = \frac{R_1 E}{R_T} = \frac{R_1 E}{R_1 + R_2} = \frac{(20\ \Omega)(64\text{ V})}{20\ \Omega + 60\ \Omega} = \frac{1280\text{ V}}{80} = \mathbf{16\text{ V}}$$

EXAMPLE 9 Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Fig. 3-16

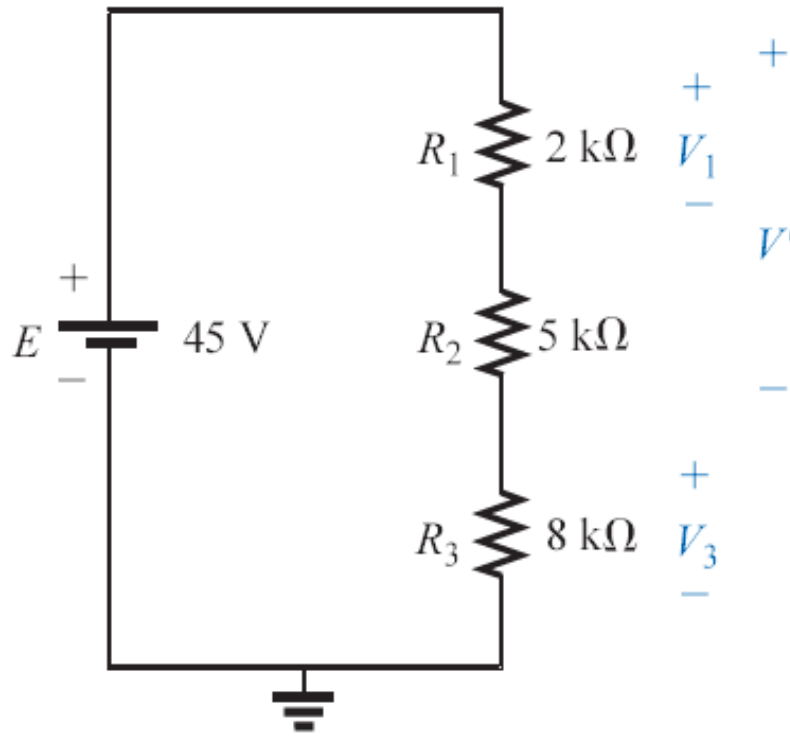


Fig 3-16

Solution:

$$V_1 = \frac{R_1 E}{R_T} = \frac{(2 \text{ k}\Omega)(45 \text{ V})}{2 \text{ k}\Omega + 5 \text{ k}\Omega + 8 \text{ k}\Omega} = \frac{(2 \text{ k}\Omega)(45 \text{ V})}{15 \text{ k}\Omega}$$

$$= \frac{(2 \times 10^3 \Omega)(45 \text{ V})}{15 \times 10^3 \Omega} = \frac{90 \text{ V}}{15} = \mathbf{6 \text{ V}}$$

$$V_3 = \frac{R_3 E}{R_T} = \frac{(8 \text{ k}\Omega)(45 \text{ V})}{15 \text{ k}\Omega} = \frac{(8 \times 10^3 \Omega)(45 \text{ V})}{15 \times 10^3 \Omega}$$

$$= \frac{360 \text{ V}}{15} = \mathbf{24 \text{ V}}$$

Note The rule can be extended to the voltage across two or more series elements.

$$V' = \frac{R' E}{R_T} \quad (\text{volts})$$

EXAMPLE 10 Determine the voltage V' in Fig.(3-16) across resistors R_1 and R_2 .

Solution:

$$V' = \frac{R' E}{R_T} = \frac{(2 \text{ k}\Omega + 5 \text{ k}\Omega)(45 \text{ V})}{15 \text{ k}\Omega} = \frac{(7 \text{ k}\Omega)(45 \text{ V})}{15 \text{ k}\Omega} = 21 \text{ V}$$

Please read and try understand in the first reference Chapter 5.

References:

- 1- **Introductory Circuits Analysis, By Boylested, Tenth (10th) Edition.**
- 2- **Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.**
- 3- **Any reference that has Direct Current Circuits Analysis (DCCA).**