1.1 PARALLEL RESISTANCE

Two elements, branches, or networks are in parallel if they have two points in common.

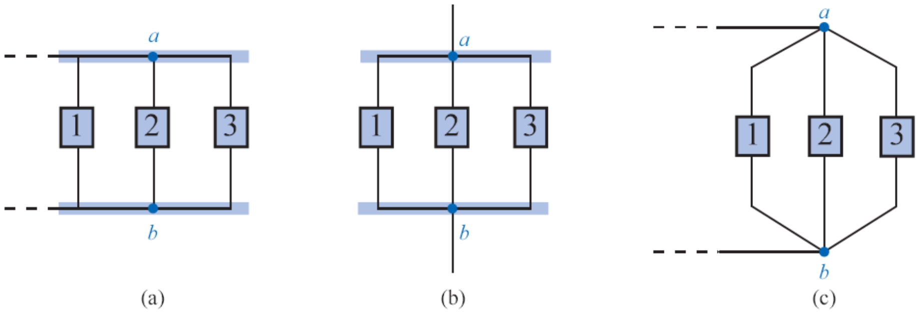

Fig. 1

Fig. 1 Different ways of three parallel elements.

For resistors in parallel as shown in Fig. 2, the total resistance is determined from the following equation:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

Since G = 1/R, the equation can also be written in terms of conductance levels as follows:

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N \qquad \text{(siemens, S)}$$

In general, however, when the total resistance is desired, the following format is applied:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

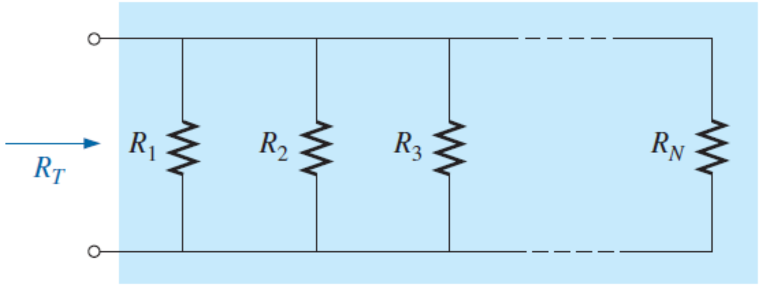

Fig (2) Parallel combination of resistors.

EXAMPLE 1 Find the total resistance of the configuration in Fig. 3.

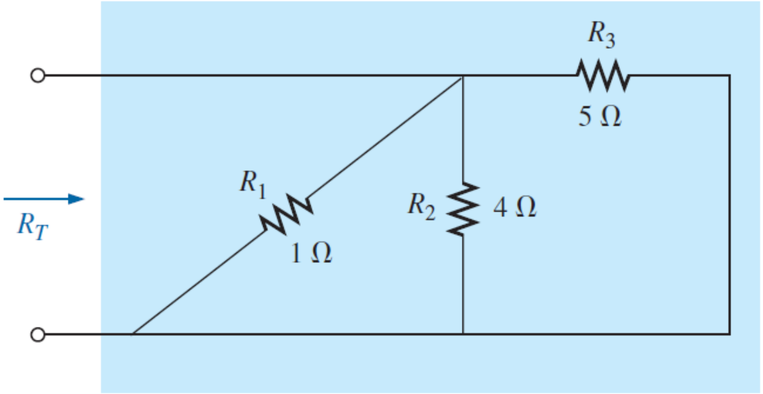

Fig. 3

Solution: First, the network is redrawn as shown in Fig. 4 to clearly demonstrate that all the resistors are in parallel.

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{1\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{5\,\Omega}}$$

$$= \frac{1}{1\text{ S} + 0.25\text{ S} + 0.2\text{ S}} = \frac{1}{1.45\text{ S}} \cong \mathbf{0.69\,\Omega}$$

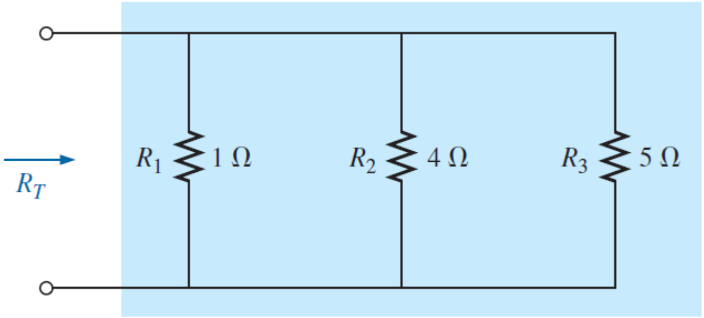

Fig .4 Redrawing the network

Note: **the total resistance of parallel resistors is always less than the value of the smallest resistor.**

For equal resistors in parallel, the equation for the total resistance becomes significantly easier to apply. For *N* equal resistors in parallel, the total resistance becomes:

$$R_T = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R_N}}$$

$$= \frac{1}{N\left(\frac{1}{R}\right)} = \frac{1}{\frac{N}{R}}$$

$$\boxed{R_T = \frac{R}{N}}$$

In other words,
the total resistance of N parallel resistors of equal value is the resistance of one resistor divided by the number (N) of parallel resistors.

EXAMPLE 2 Find the total resistance of the parallel resistors in Fig. 5

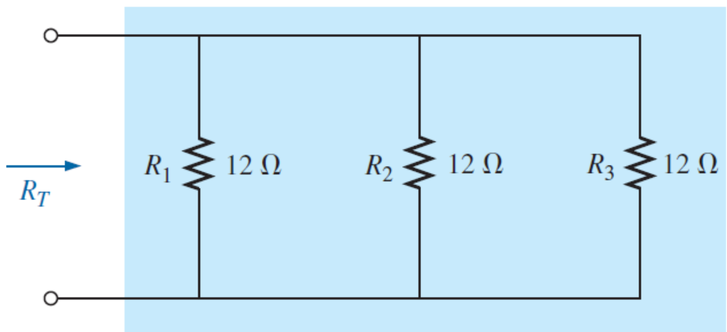

Fig. 5 Three equal parallel resistors to be investigated.

Solution:

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

EXAMPLE 3 Find the total resistance for the configuration in Fig. 6.

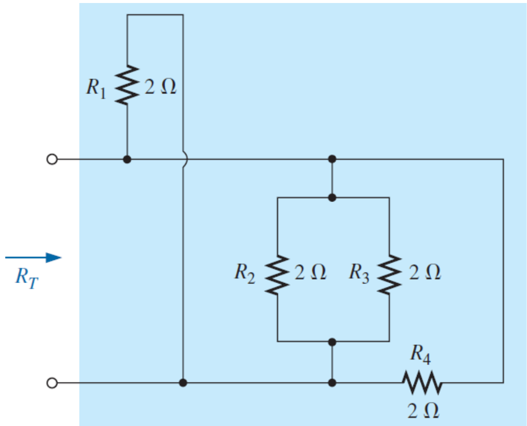

Fig. 6

Solution: Redrawing the network results in the parallel network in Fig. 7.

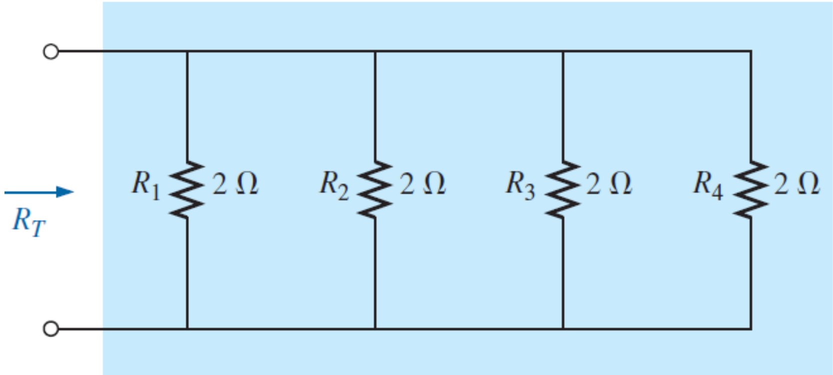

Fig. 7 Network in Fig. 6 redrawn.

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Special Case: Two Parallel Resistors

In the vast majority of cases, only two or three parallel resistors will have to be combined. With this in mind, an equation has been derived for two parallel resistors that is easy to apply and removes the need to continually worry about dividing into 1 and possibly misplacing a decimal point. For three parallel resistors, the equation to be derived here can be applied twice, or Equation of Total Resistance can be used.

For two parallel resistors, the total resistance is determined:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying the top and bottom of each term of the right side of the equation by the other resistor results in

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

$$R_T = \frac{R_1R_2}{R_1 + R_2}$$

In words, the equation states that
the total resistance of two parallel resistors is simply the product of their values divided by their sum.

Note:
Recall that series elements can be interchanged without affecting the magnitude of the total resistance. In parallel networks,

parallel resistors can be interchanged without affecting the total resistance.

EXAMPLE 4 Determine the total resistance of the parallel elements in Fig. 8.

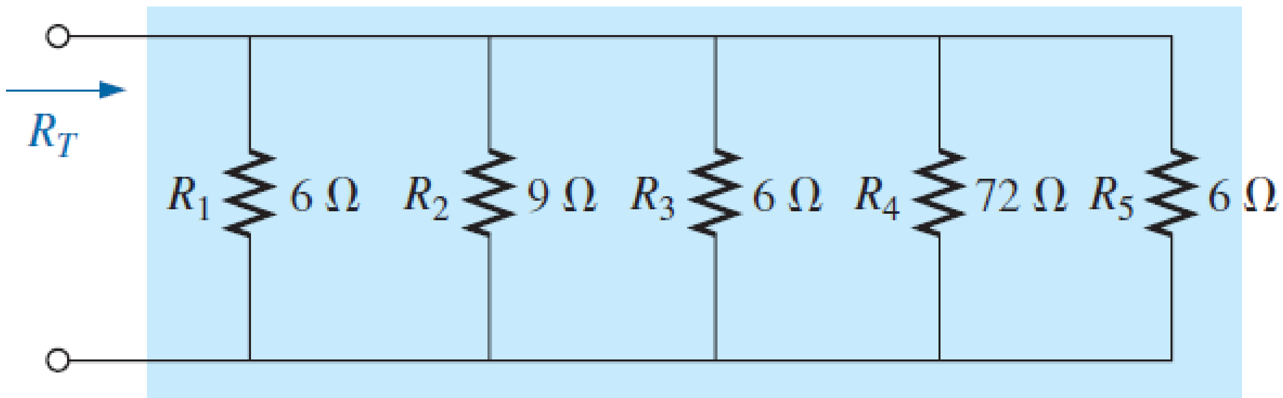

Fig. 8.

Solution: The network is redrawn in Fig. 9.

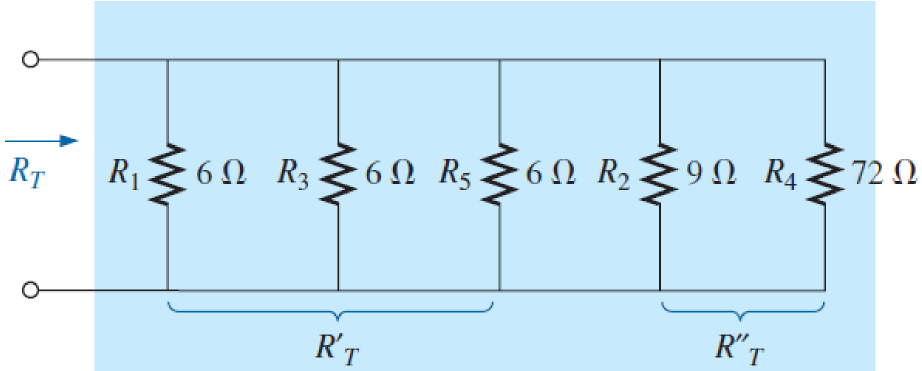

Fig. 9 Redrawing the network

$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648}{81}\ \Omega = 8\ \Omega$$

$$R_T = \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16}{10}\ \Omega = \mathbf{1.6\ \Omega}$$

EXAMPLE 5 Determine the value of R_2 in Fig. 10 to establish a total resistance of 9 kilo ohms.

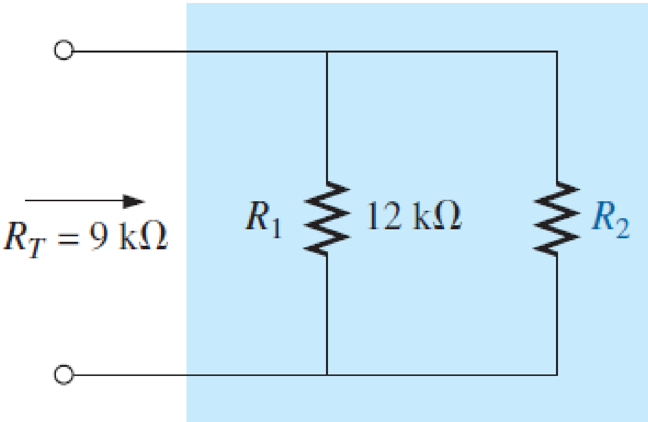

Fig. 10

Solution:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_T(R_1 + R_2) = R_1 R_2$$

$$R_T R_1 + R_T R_2 = R_1 R_2$$

$$R_T R_1 = R_1 R_2 - R_T R_2$$

$$R_T R_1 = (R_1 - R_T) R_2$$

and

$$R_2 = \frac{R_T R_1}{R_1 - R_T}$$

Substituting values:

$$R_2 = \frac{(9\ \text{k}\Omega)(12\ \text{k}\Omega)}{12\ \text{k}\Omega - 9\ \text{k}\Omega} = \frac{108}{3}\ \text{k}\Omega = \mathbf{36\ k\Omega}$$

EXAMPLE 6 Determine the values of R_1, R_2 and R_3 in Fig. 11 if $R_2 = 2R_1$, $R_3 = 2R_2$, and the total resistance is 16 kilo ohms.

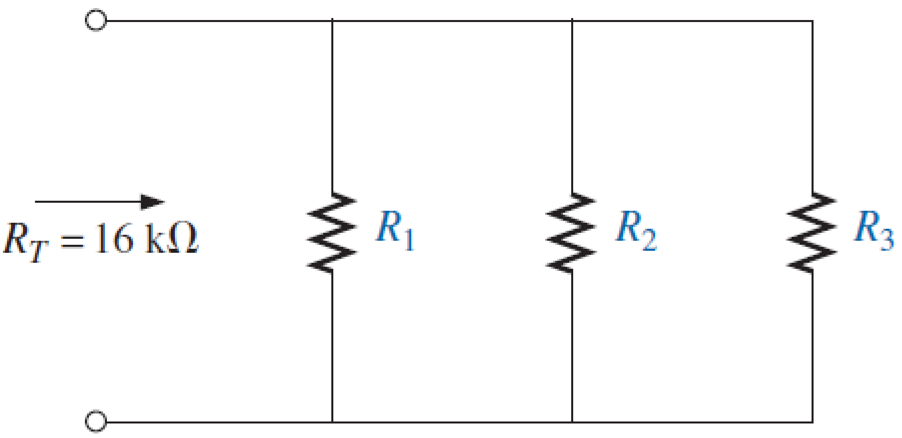

Fig. 11

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

However, $R_2 = 2R_1$ and $R_3 = 2R_2 = 2(2R_1) = 4R_1$

so that
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

and
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$$

or
$$\frac{1}{16\ \text{k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

resulting in $R_1 = 1.75(16\ \text{k}\Omega) = \mathbf{28\ k\Omega}$

so that $R_2 = 2R_1 = 2(28\ \text{k}\Omega) = \mathbf{56\ k\Omega}$

and $R_3 = 2R_2 = 2(56\ \text{k}\Omega) = \mathbf{112\ k\Omega}$

1.2 PARALLEL CIRCUITS

A **parallel circuit** can now be established by connecting a supply across a set of parallel resistors as shown in Fig.12. The positive terminal of the supply is directly connected to the top of each resistor, while the negative terminal is connected to the bottom of each resistor. Therefore, it should be quite clear that the applied voltage is the same across each resistor.

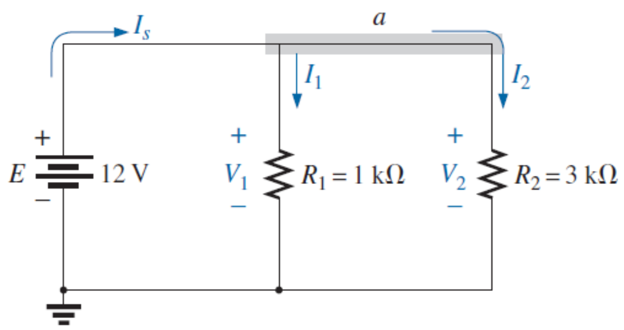

Fig. 12

In general,

the voltage is always the same across parallel elements.

For the voltages of the circuit in Fig. 12, the result is that

$$V_1 = V_2 = E$$

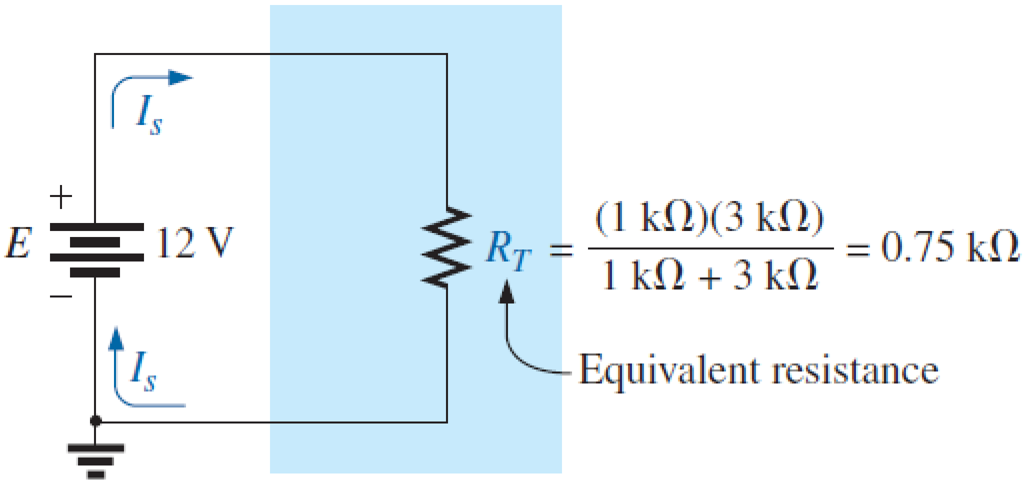

Fig. 13

Replacing the parallel resistors in Fig. 13 with the equivalent total resistance.

The source current can then be determined using Ohm's law:

$$I_s = \frac{E}{R_T}$$

Since the voltage is the same across parallel elements, the current through each resistor can also be determined using Ohm's law. That is,

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

The direction for the currents is dictated by the polarity of the voltage across the resistors. Recall that for a resistor, current enters the positive side of a potential drop and leaves the negative. The result, as shown in Fig. 12, is that the source current enters point *a*, and currents I_1 and I_2 leave the same point.

An excellent analogy for describing the flow of charge through the network of Fig. 12 is the flow of water through the parallel pipes of Fig. 14.

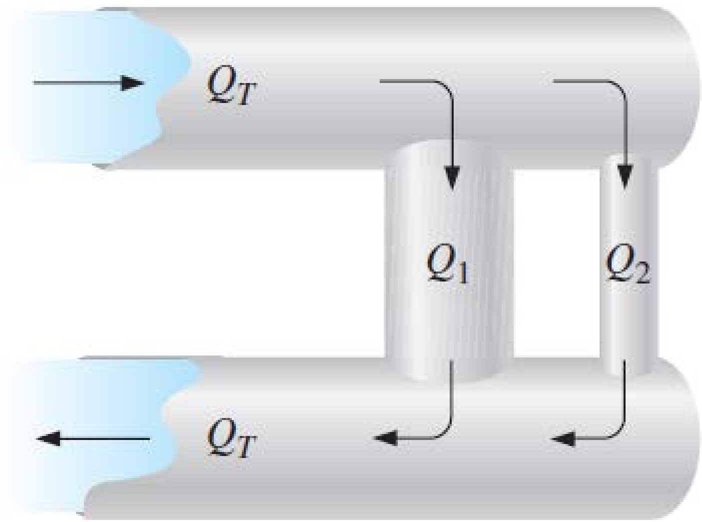

Fig. 14

The larger pipe with less "resistance" to the flow of water will have a larger flow of water through it. The thinner pipe with its increased "resistance" level will have less water through it. In any case, the total water entering the pipes at the top Q_T must equal that leaving at the bottom, with $Q_T = Q_1 + Q_2$.

The relationship between the source current and the parallel resistor currents can be derived by simply taking the equation for the total resistance in equation of total resistance.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying both sides by the applied voltage:

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

resulting in

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

Then note that $E/R_1 = I_1$ and $E/R_2 = I_2$ to obtain

$$\boxed{I_s = I_1 + I_2}$$

The result reveals a very important property of parallel circuits:

For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.

The duality that exists between series and parallel circuits continues to surface as we proceed through the basic equations for electric circuits.

for a parallel circuit, the source current equals the sum of the branch currents, while for a series circuit, the applied voltage equals the sum of the voltage drops.

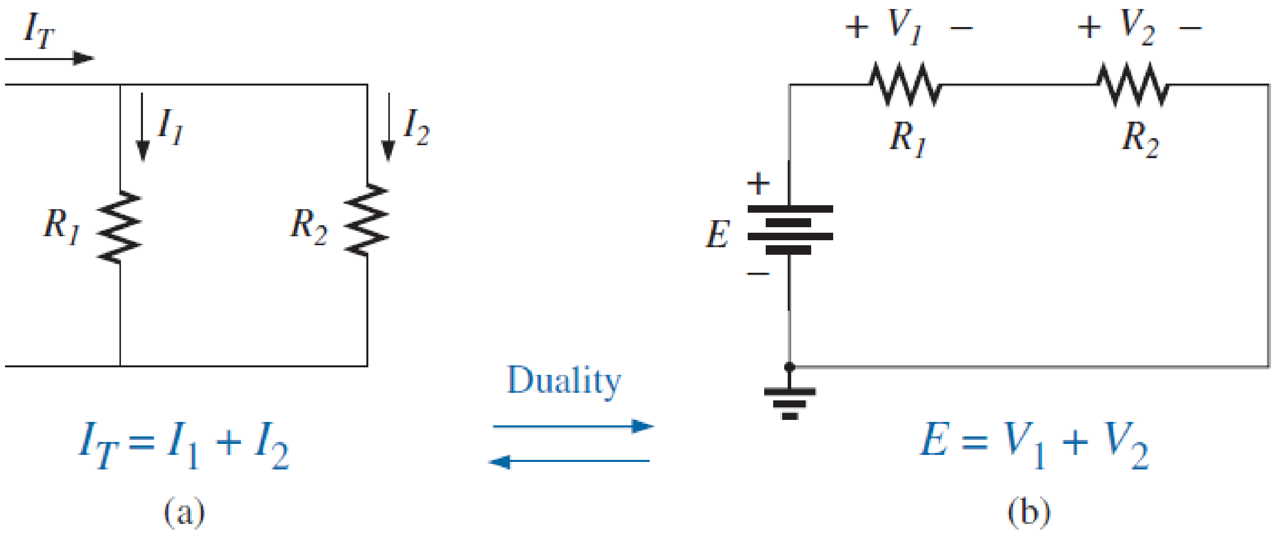

Fig. 15 Demonstrating the duality that exists between series and parallel circuits.

EXAMPLE 7 For the parallel network in Fig. 16.

a. Find the total resistance.
b. Calculate the source current.
c. Determine the current through each branch.

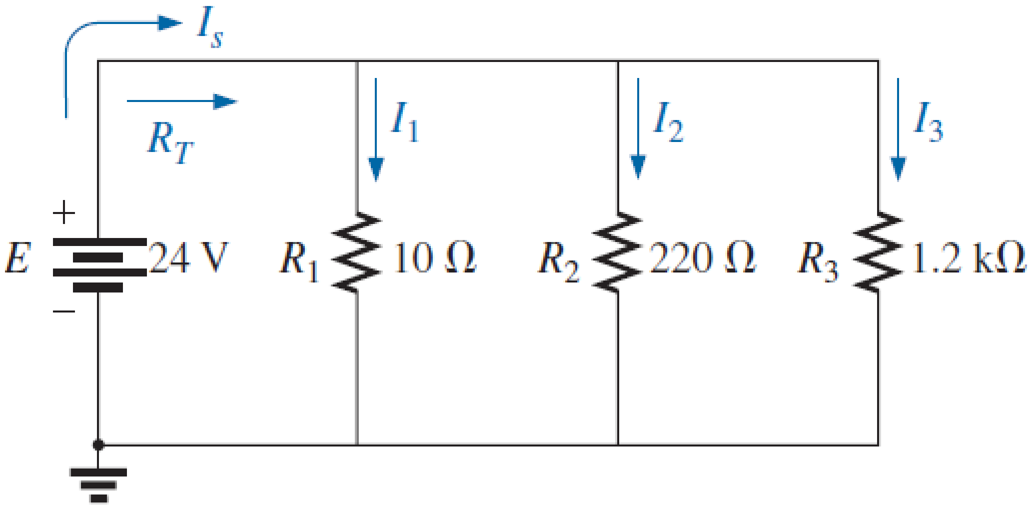

Fig. 16

Solutions:

a.

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{10\ \Omega}+\frac{1}{220\ \Omega}+\frac{1}{1.2\ \text{k}\Omega}}$$

$$= \frac{1}{100\times10^{-3}+4.545\times10^{-3}+0.833\times10^{-3}} = \frac{1}{105.38\times10^{-3}}$$

$$R_T = \mathbf{9.49\ \Omega}$$

Note that the total resistance is less than the smallest parallel resistor.

b. Using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{9.49\ \Omega} = \mathbf{2.53\ A}$$

c. Applying Ohm's law:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{24\ \text{V}}{10\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{24\ \text{V}}{220\ \Omega} = \mathbf{0.11\ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{24\ \text{V}}{1.2\ \text{k}\Omega} = \mathbf{0.02\ A}$$

for parallel resistors, the greatest current will exist in the branch with the least resistance.

A more powerful statement is that
current always seeks the path of least resistance.

1.3 POWER DISTRIBUTION IN A PARALLEL CIRCUIT

for any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements.

For the parallel circuit in Fig. 17:

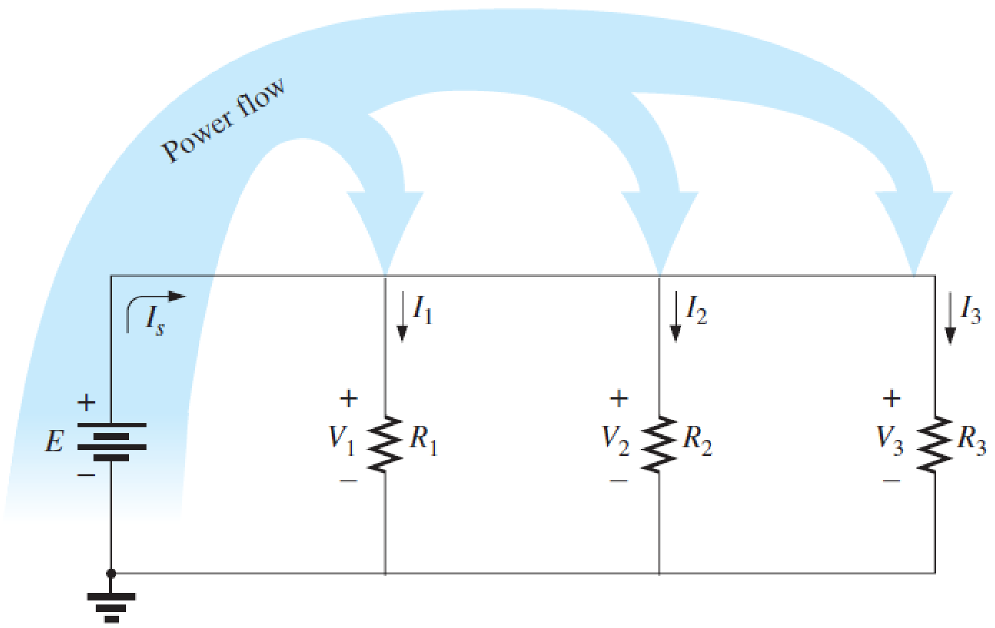

Fig. 17
Power flow in a dc parallel network.

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

which is exactly the same as obtained for the series combination.

The power delivered by the source in the same:

$$P_E = EI_s \qquad \text{(watts, W)}$$

as is the equation for the power to each resistor.

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)}$$

in a parallel resistive network, the larger the resistor, the less the power absorbed.

EXAMPLE 8 For the parallel network in Fig. 18

a. Determine the total resistance R_T.

b. Find the source current and the current through each resistor.

c. Calculate the power delivered by the source.

d. Determine the power absorbed by each parallel resistor.

e. Verify the power applied by the battery will equal that dissipated by the resistive elements.

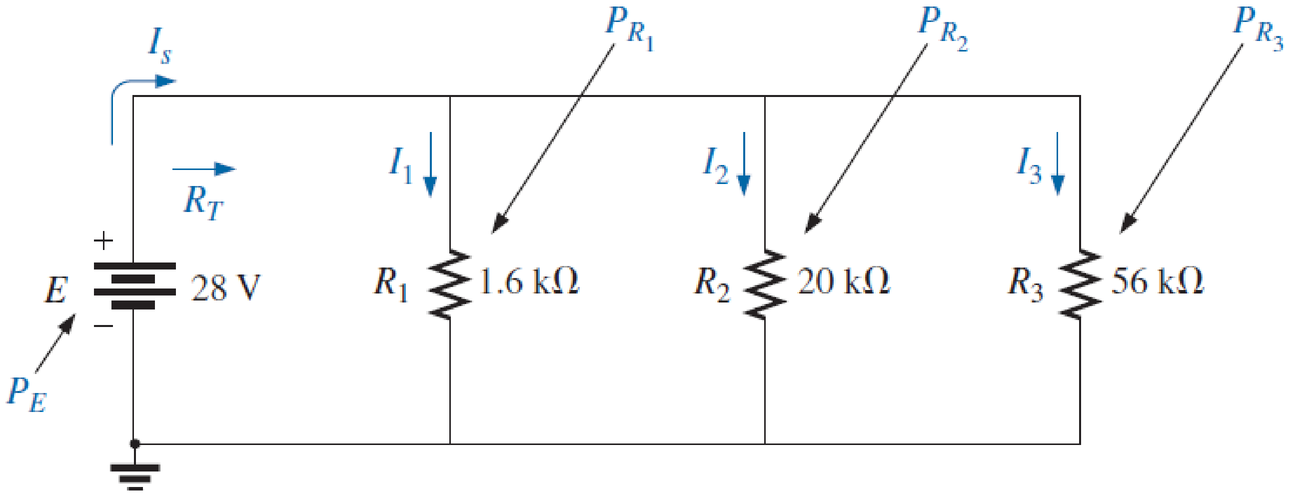

Fig. 18

Solutions:

a.

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1.6\text{ k}\Omega} + \frac{1}{20\text{ k}\Omega} + \frac{1}{56\text{ k}\Omega}}$$

$$= \frac{1}{625 \times 10^{-6} + 50 \times 10^{-6} + 17.867 \times 10^{-6}} = \frac{1}{692.867 \times 10^{-6}}$$

and $R_T = \mathbf{1.44\ k\Omega}$

b. Applying Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{28\text{ V}}{1.44\text{ k}\Omega} = \mathbf{19.44\ mA}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\text{ V}}{1.6\text{ k}\Omega} = \mathbf{17.5\ mA}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\text{ V}}{20\text{ k}\Omega} = \mathbf{1.4\ mA}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\text{ V}}{56\text{ k}\Omega} = \mathbf{0.5\ mA}$$

c.

$$P_E = EI_s = (28\text{ V})(19.4\text{ mA}) = \mathbf{543.2\text{ mW}}$$

d. Applying each form of the power equation:

$$P_1 = V_1 I_1 = EI_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\text{ mW}}$$

$$P_2 = I_2^2 R_2 = (1.4\text{ mA})^2 (20\text{ k}\Omega) = \mathbf{39.2\text{ mW}}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{E^2}{R_3} = \frac{(28\text{ V})^2}{56\text{ k}\Omega} = \mathbf{14\text{ mW}}$$

A review of the results clearly substantiates the fact that the larger the resistor, the less the power absorbed.

e.

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

$$\mathbf{543.2\text{ mW}} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\text{ mW}} \quad \text{(checks)}$$

1.4 KIRCHHOFF'S CURRENT LAW (KCL)

The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

In equation form, the above statement can be written as follows:

$$\Sigma I_i = \Sigma I_o$$

with I_i representing the current entering, or "in," and I_o representing the current leaving, or "out."

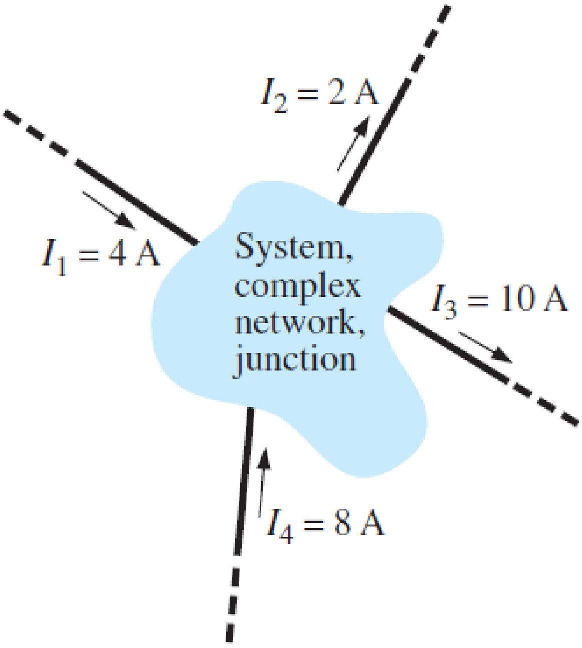

Fig. 19

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A} = 12\text{ A}} \quad \text{(checks)}$$

In technology, the term **node** is commonly used to refer to a junction of two or more branches. Therefore, this term is used frequently in the analyses to follow.

EXAMPLE 9 Determine currents I_3 and I_4 in Fig. 20 using Kirchhoff's current law.

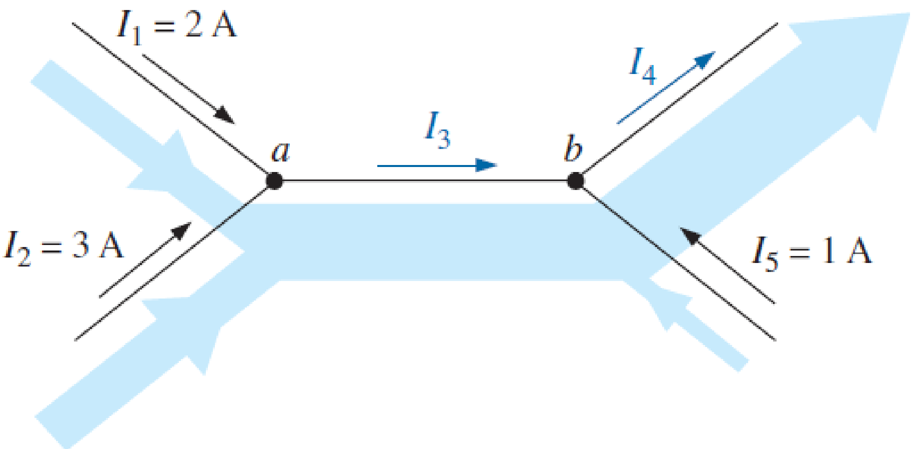

Fig. 20

Solution: There are two junctions or nodes in Fig. 20. Node *a* has only one unknown, while node *b* has two unknowns. Since a single equation can be used to solve for only one unknown, we must apply Kirchhoff's current law to node *a* first.

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\text{ A}}$$

At node *b*, using the result just obtained:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\text{ A}}$$

Note that in Fig. 20, the width of the blue shaded regions matches the magnitude of the current in that region.

EXAMPLE 10 Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 21.

Solution: In this configuration, four nodes are defined. Nodes *a* and *c* have only one unknown current at the junction, so Kirchhoff's current law can be applied at either junction.

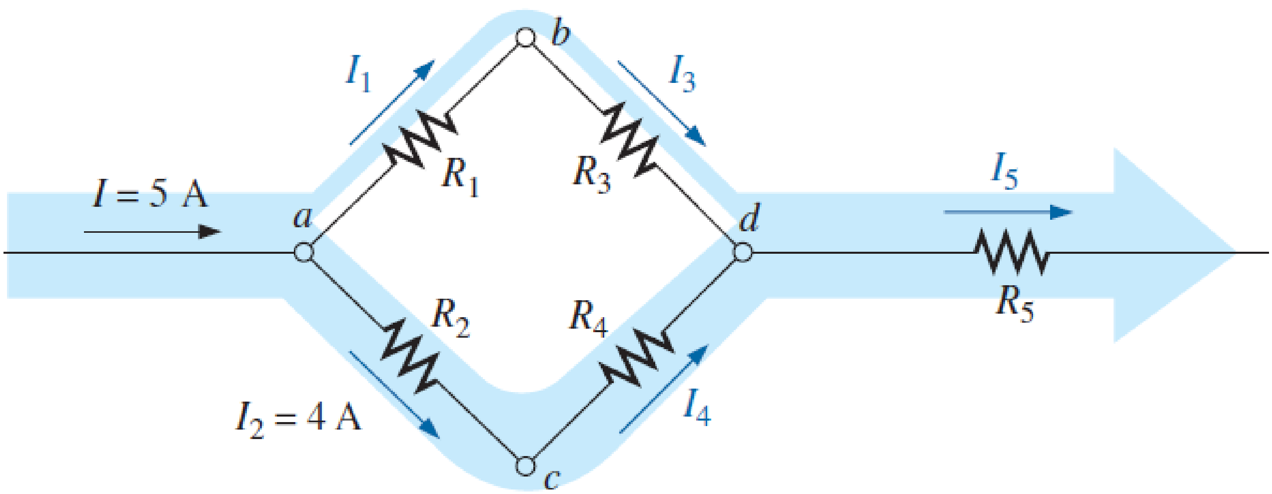

Fig. 21 Four-node configuration for the Example.

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$
$$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\ A}$$

At node *c:*

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$
$$I_4 = I_2 = \mathbf{4\ A}$$

Using the above results at the other junctions results in the following.

At node *b:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$
$$I_3 = I_1 = \mathbf{1\ A}$$

At node *d:*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\ A}$$

1.5 CURRENT DIVIDER RULE (CDR)

For series circuits we have the powerful voltage divider rule for finding the voltage across a resistor in a series circuit. We now introduce the equally powerful **current divider rule (CDR)** for finding the current through a resistor in a parallel circuit.

For two parallel elements of equal value, the current will divide equally. For parallel elements with different values, the smaller the resistance, the greater the share of input current. For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

EXAMPLE 11

a) Determine currents I_1 and I_3 for the network in Fig. 22.
b) Find the source current I_s.

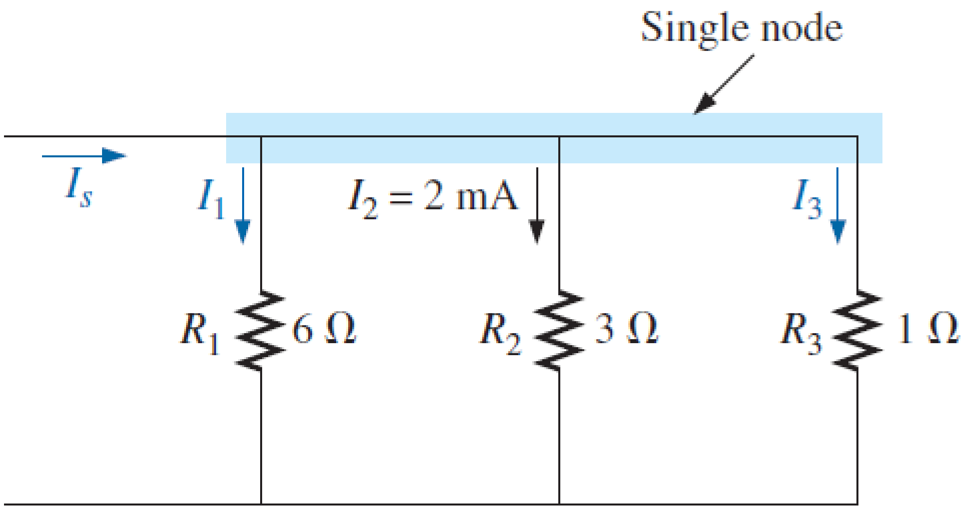

Fig. 22

Solutions:

a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

$$I_1 = \frac{I_2}{2} = \frac{2\text{ mA}}{2} = \mathbf{1\text{ mA}}$$

Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2\text{ mA}) = \mathbf{6\text{ mA}}$$

b. Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1\text{ mA} + 2\text{ mA} + 6\text{ mA} = \mathbf{9\text{ mA}}$$

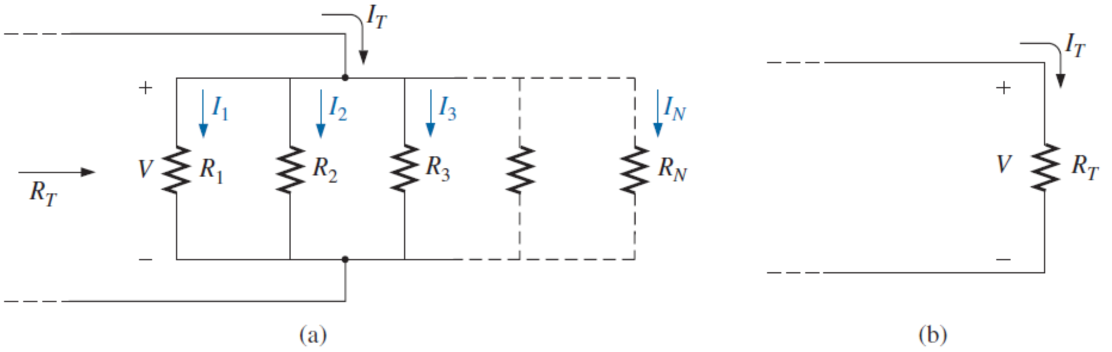

Fig. 23 Deriving the current divider rule: (a) parallel network of N parallel resistors; (b) reduced equivalent of part (a).

Although the above discussions and examples allowed us to determine the relative magnitude of a current based on a known level, they do not provide the magnitude of a current through a branch of a parallel network if only the total entering current is known. The result is a need for the current divider rule which will be derived using the parallel configuration in Fig. 23(a). The current I_T (using the subscript T to indicate the total entering current) splits between the N parallel resistors and then gathers itself together again at the bottom of the configuration.

In Fig. 23(b), the parallel combination of resistors has been replaced by a single resistor equal to the total resistance of the parallel combination as determined in the previous sections.
The current I_T can then be determined using Ohm's law:

$$I_T = \frac{V}{R_T}$$

Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

where the product I_xR_x refers to any combination in the series. Substituting for V in the above equation for I_T, we have

$$I_T = \frac{I_xR_x}{R_T}$$

Solving for I_x , the final result is the **current divider rule:**

$$I_x = \frac{R_T}{R_x}I_T$$

which states that
the current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.
Since R_T and I_T are constants, for a particular configuration the larger the value of R_x (in the denominator), the smaller the value of I_x for that branch, confirming the fact that current always seeks the path of least resistance.

EXAMPLE 12 For the parallel network in Fig. 24, determine current I_1 using Eq. CDR.

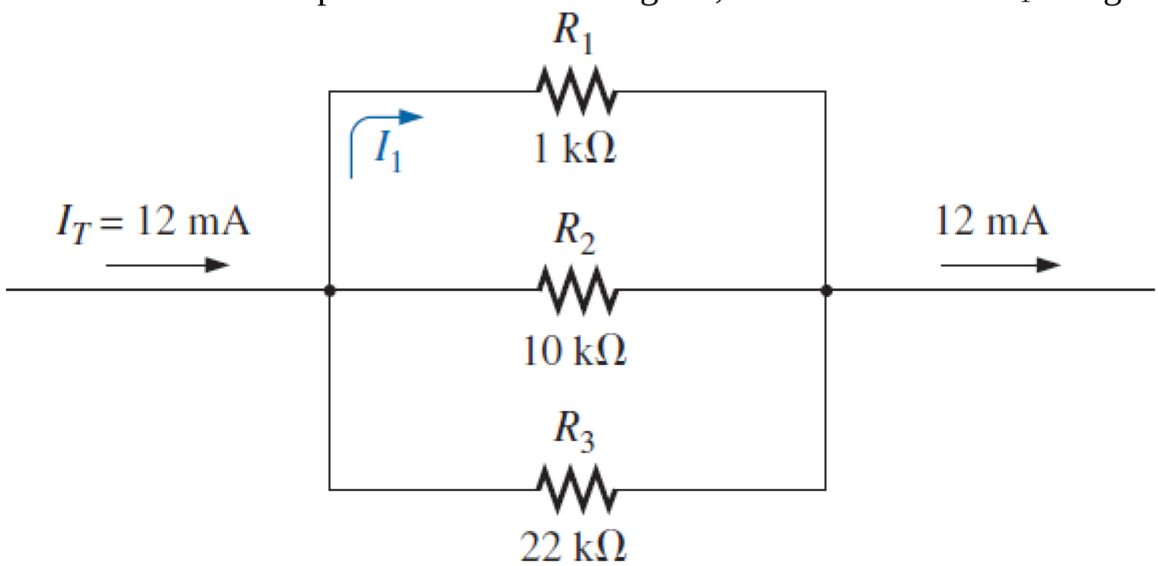

Fig. 24 Using the current divider rule to calculate current I_1

Solution:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{1\text{ k}\Omega} + \frac{1}{10\text{ k}\Omega} + \frac{1}{22\text{ k}\Omega}}$$

$$= \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}}$$

$$= \frac{1}{1.145 \times 10^{-3}} = \mathbf{873.01\ \Omega}$$

$$I_1 = \frac{R_T}{R_1} I_T$$

$$= \frac{(873.01\ \Omega)}{1\text{ k}\Omega}(12\text{ mA}) = (0.873)(12\text{ mA}) = \mathbf{10.48\ mA}$$

and the smallest parallel resistor receives the majority of the current.

Special Case: Two Parallel Resistors

For the case of two parallel resistors as shown in Fig 25, the total resistance is determined by

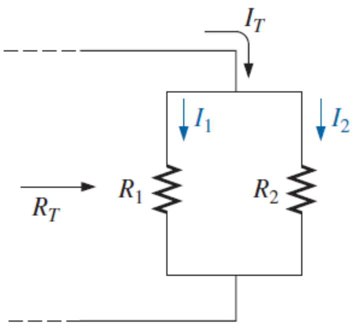

Fig. 25 Deriving the current divider rule for the special case of only two parallel resistors.

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Substituting R_T into Eq. CDR for current I_1 results in

$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

And

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T$$

Similarly, for I_2,

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

Equation I_1, I_2 states that

for two parallel resistors, the current through one is equal to the other resistor times the total entering current divided by the sum of the two resistors.

Since the combination of two parallel resistors is probably the most common parallel configuration, the simplicity of the format for Equation (I_1, I_2) suggests that it is worth memorizing. Take particular note, however, that the denominator of the equation is simply the *sum*, not the total resistance, of the combination.

EXAMPLE 13 Determine current I_2 for the network in Fig. 26 using the current divider rule.

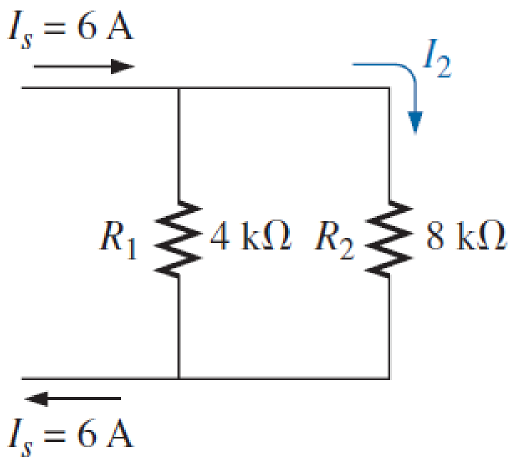

Fig. 26

Solution:

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

$$= \left(\frac{4\ \text{k}\Omega}{4\ \text{k}\Omega + 8\ \text{k}\Omega}\right) 6\ \text{A} = (0.333)(6\ \text{A}) = \mathbf{2\ A}$$

$$I_2 = \frac{R_T}{R_2} I_T$$

with $$R_T = 4\ \text{k}\Omega \parallel 8\ \text{k}\Omega = \frac{(4\ \text{k}\Omega)(8\ \text{k}\Omega)}{4\ \text{k}\Omega + 8\ \text{k}\Omega} = 2.667\ \text{k}\Omega$$

and $$I_2 = \left(\frac{2.667\ \text{k}\Omega}{8\ \text{k}\Omega}\right) 6\ \text{A} = (0.333)(6\ \text{A}) = \mathbf{2\ A}$$

matching the above solution.

EXAMPLE 14 Determine resistor R_1 in Fig. 27 to implement the division of current shown.

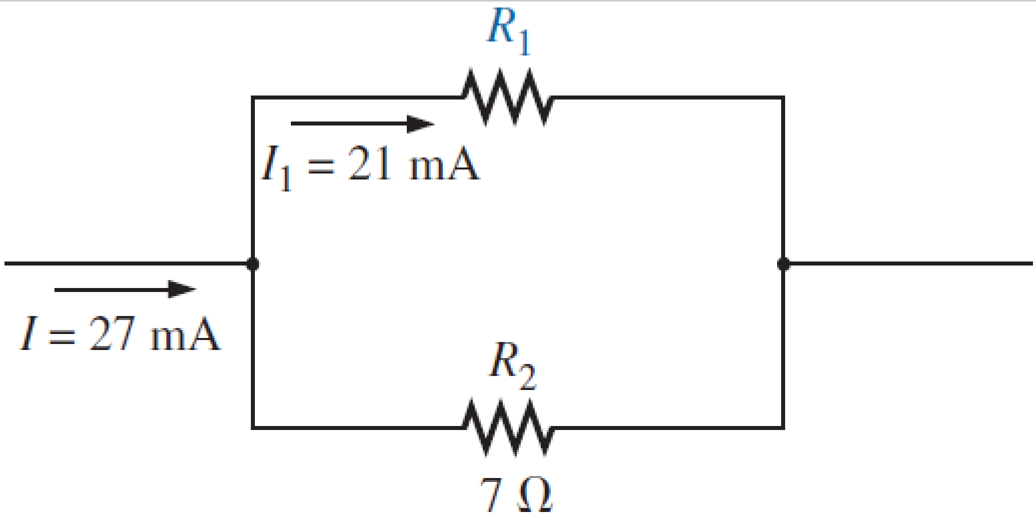

Fig. 27 A design-type problem for two parallel resistors.

Solution: There are essentially two approaches to this type of problem. One involves the direct substitution of known values into the current divider rule equation followed by a mathematical analysis. The other is the sequential application of the basic laws of electric circuits.

First we will use the latter approach.

Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$

$$I = I_1 + I_2$$

$$27\text{ mA} = 21\text{ mA} + I_2$$

and $$I_2 = 27\text{ mA} - 21\text{ mA} = 6\text{ mA}$$

The voltage V_2: $$V_2 = I_2R_2 = (6\text{ mA})(7\ \Omega) = 42\text{ mV}$$

so that $$V_1 = V_2 = 42\text{ mV}$$

Finally, $$R_1 = \frac{V_1}{I_1} = \frac{42\text{ mV}}{21\text{ mA}} = \mathbf{2\ \Omega}$$

Now for the other approach using the current divider rule:

$$I_1 = \frac{R_2}{R_1 + R_2} I_T$$

$$21\text{ mA} = \left(\frac{7\ \Omega}{R_1 + 7\ \Omega}\right) 27\text{ mA}$$

$$(R_1 + 7\ \Omega)(21\text{ mA}) = (7\ \Omega)(27\text{ mA})$$

$$(21\text{ mA})R_1 + 147\text{ mV} = 189\text{ mV}$$

$$(21\text{ mA})R_1 = 189\text{ mV} - 147\text{ mV} = 42\text{ mV}$$

and

$$R_1 = \frac{42\text{ mV}}{21\text{ mA}} = \mathbf{2\ \Omega}$$

In summary, therefore, remember that current always seeks the path of least resistance, and the ratio of the resistor values is the inverse of the resulting current levels, as shown in Fig 28. The thickness of the blue bands in Fig. 28 reflects the relative magnitude of the current in each branch.

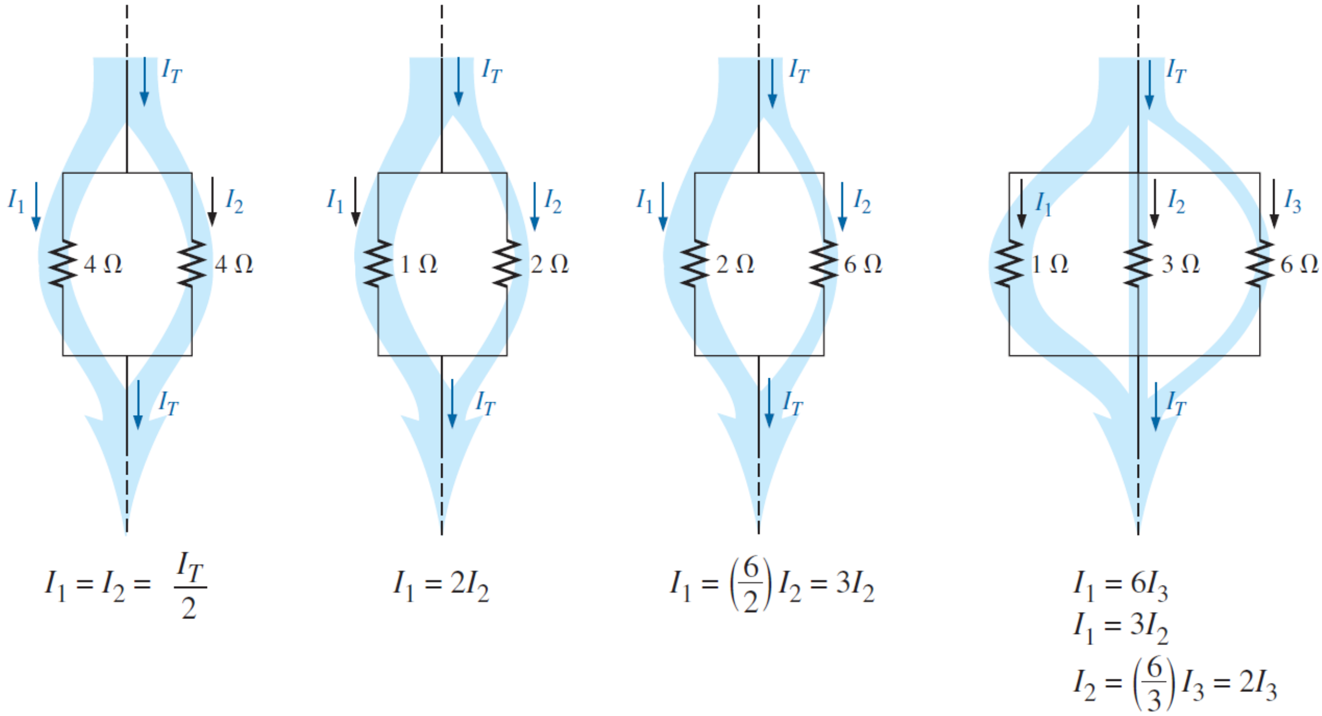

Fig. 28 Demonstrating how current divides through equal and unequal parallel resistors.

1.6 OPEN AND SHORT CIRCUITS

An **open circuit** is two isolated terminals not connected by an element of any kind, as shown in Fig. 29(a).

Since a path for conduction does not exist, the current associated with an open circuit must always be **zero**. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to. In summary, therefore,

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

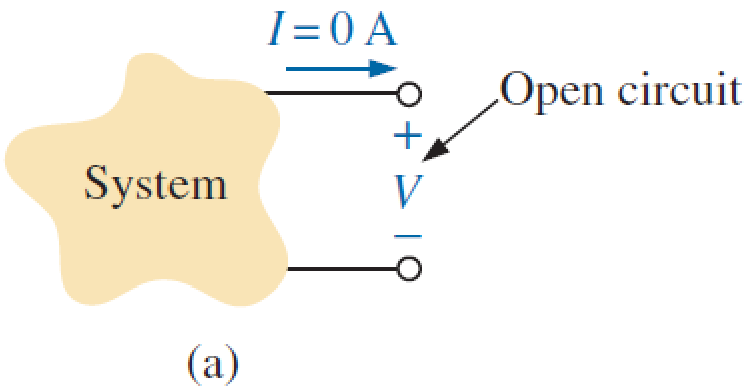

Fig. 29-a

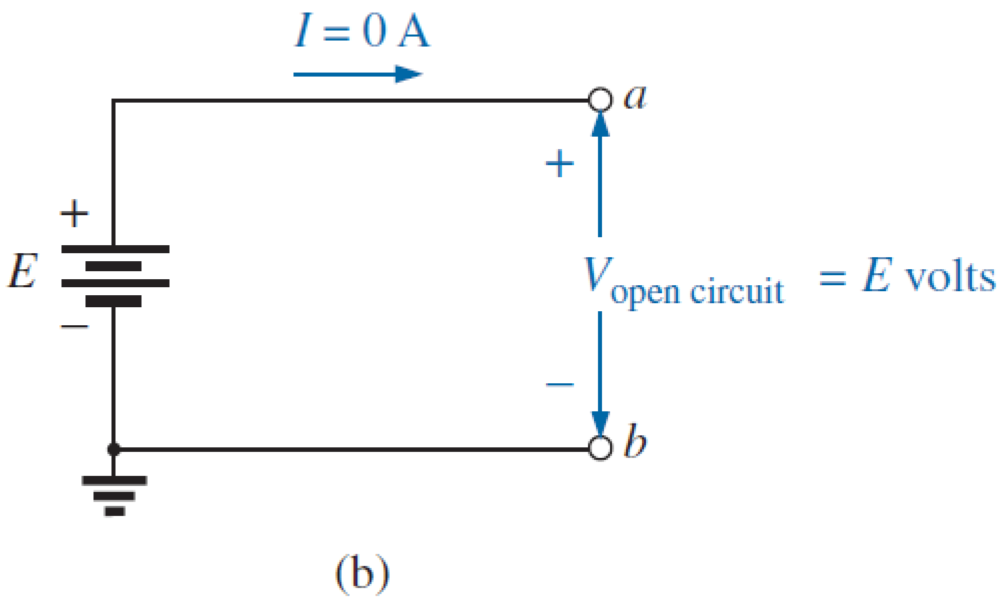

Fig. 29-b

In Fig. 29(b), an open circuit exists between terminals *a* and *b*. The voltage across the open-circuit terminals is the **supply voltage**, but the current is **zero** due to the absence of a complete circuit.

A **short circuit** is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 30. The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit is always zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and

$$V = IR = I(0\ \Omega) = 0\ \text{V}.$$

In summary, therefore,

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

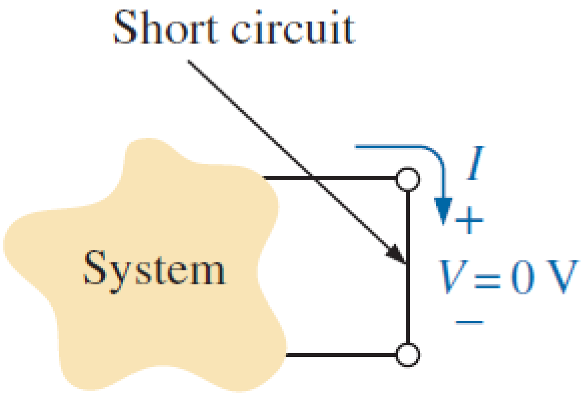

Fig. 30

EXAMPLE 15 Determine voltage V_{ab} for the network in Fig. 31.

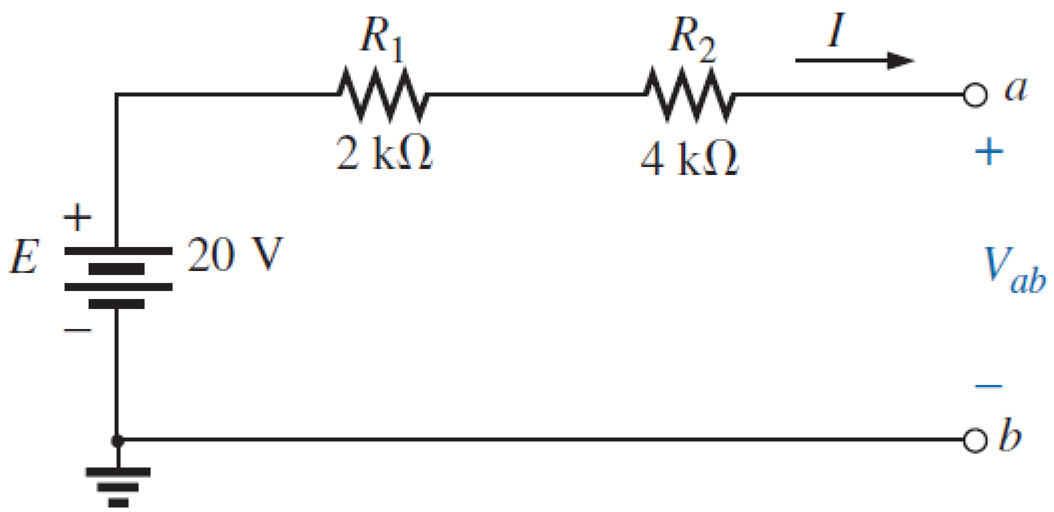

Fig. 31

Solution: The open circuit requires that I be zero amperes.
The voltage drop across both resistors is therefore zero volts since

$$V = IR = (0)R = 0\ \text{V}.$$

Applying Kirchhoff's voltage law around the closed loop,

$$V_{ab} = E = \mathbf{20\ V}$$

EXAMPLE 16 Determine voltages V_{ab} and V_{cd} for the network in Fig. 32.

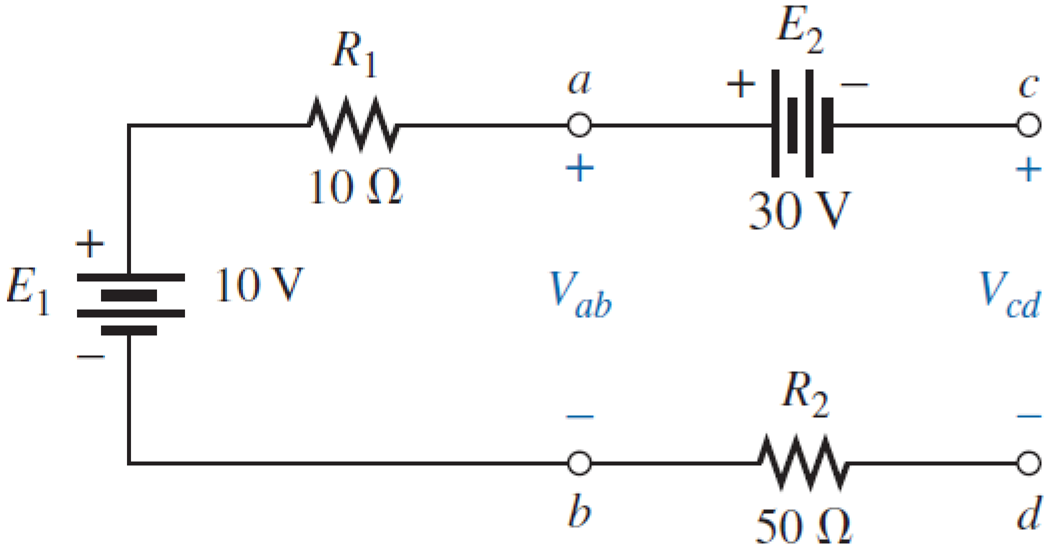

Fig. 32

Solution: The current through the system is zero amperes due to the open circuit, resulting in a 0 V drop across each resistor.
Both resistors can therefore be replaced by short circuits, as shown in Fig. 33. Voltage V_{ab} is then directly across the 10 V battery, and

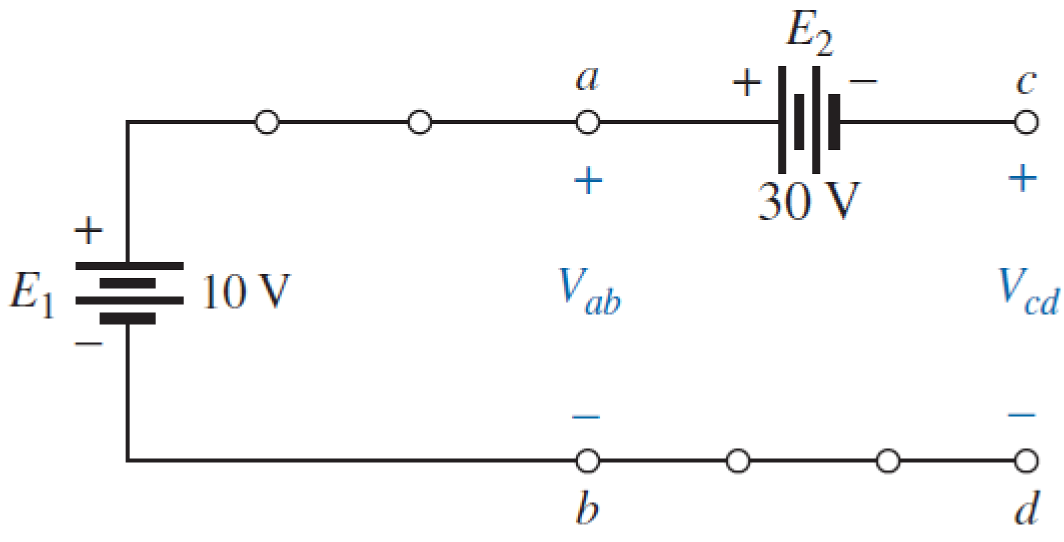

Fig. 33 Circuit in Fig. 32 redrawn.

$$V_{ab} = E_1 = \mathbf{10\ V}$$

Voltage V_{cd} requires an application of Kirchhoff's voltage law:

$$+E_1 - E_2 - V_{cd} = 0$$

$$V_{cd} = E_1 - E_2 = 10\text{ V} - 30\text{ V} = \mathbf{-20\text{ V}}$$

The negative sign in the solution indicates that the actual voltage V_{cd} has the opposite polarity of that appearing in Fig. 32.

EXAMPLE 17 Determine V and I for the network in Fig. 34 if resistor R_2 is shorted out.

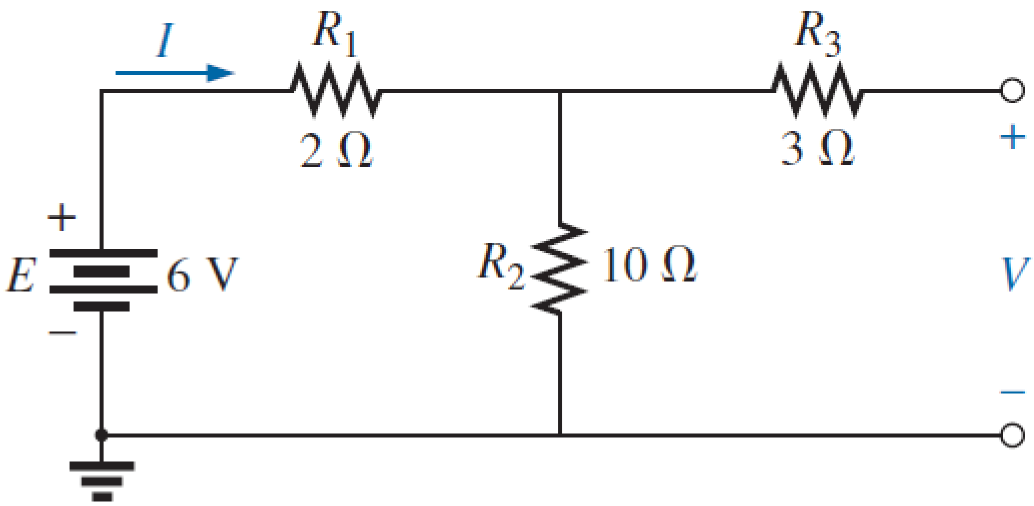

Fig. 34

Solution: The redrawn network appears in Fig. 35.

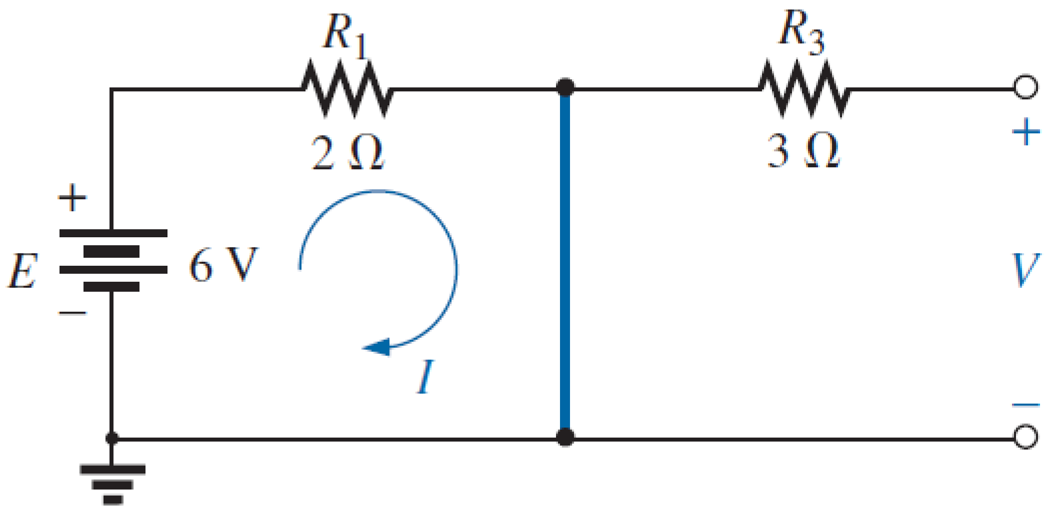

Fig. 35 Network in Fig. 34 with R_2 replaced by a jumper.

The current through the 3 ohm resistor is zero due to the open circuit, causing all the current *I* to pass through the jumper. Since

$$V_{3\Omega} = IR = (0)R = 0\ \text{V}$$

, the voltage *V* is directly across the short, and

$$V = \mathbf{0\ V}$$

$$I = \frac{E}{R_1} = \frac{6\ \text{V}}{2\ \Omega} = \mathbf{3\ A}$$

1.7 SUMMARY TABLE

Now that the series and parallel configurations have been covered in detail, we will review the salient equations and characteristics of each. The equations for the two configurations have a number of similarities. In fact, the equations for one can often be obtained directly from the other by simply applying the **duality** principle.

TABLE 1

Summary table.

Series and Parallel Circuits		
Series	**Duality**	**Parallel**
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \rightleftarrows G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \rightleftarrows G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements	$R \rightleftarrows G$	$G_T = G_1 + G_2$
$R_T = R_1 + R_2$		and $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{R_1R_2}{R_1 + R_2}$
I the same through series elements	$I \rightleftarrows V$	*V* the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \rightleftarrows I$	$I_T = I_1 + I_2 + I_3$
Largest *V* across largest *R*	$V \rightleftarrows I$ and $R \rightleftarrows G$	Greatest *I* through largest *G* (smallest *R*)
$V_x = \dfrac{R_x E}{R_T}$	$E, V \rightleftarrows I$ and $R \rightleftarrows G$	$I_x = \dfrac{G_x I_T}{G_T} = \dfrac{R_T I_T}{R_x}$
		with $I_1 = \dfrac{R_2 I_T}{R_1 + R_2}$ and $I_2 = \dfrac{R_1 I_T}{R_1 + R_2}$
$P = EI_T$	$E \rightleftarrows I$ and $I \rightleftarrows E$	$P = I_T E$
$P = I^2R$	$I \rightleftarrows V$ and $R \rightleftarrows G$	$P = V^2G = V^2/R$
$P = V^2/R$	$V \rightleftarrows I$ and $R \rightleftarrows G$	$P = I^2/G = I^2R$

Please read and try understand in the first reference Chapter 6.

References:

1- Introductory Circuits Analysis, By Boylested, Tenth (10th) Edition.

2- Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.

3- Any reference that has Direct Current Circuits Analysis (DCCA).