Direct Current Circuits Analysis (DCCA)
LE104
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Doctor

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Chapter 5

SERIES-PARALLEL NETWORKS

SERIES-PARALLEL NETWORKS

A firm understanding of the basic principles associated with series and parallel circuits is a sufficient background to begin an investigation of **any single-source dc network** having a combination of series and parallel elements or branches. Multisource networks are considered in detail in Chapters 8 and 9. In general,

Series-Parallel networks are networks that contain both series and parallel circuit configurations.

One can become proficient in the analysis of series-parallel networks only through:

1) exposure, 2) practice, and 3) experience. In time the path to the desired unknown becomes more obvious as one recalls similar configurations and the frustration resulting from choosing the wrong approach. There are a few steps that can be helpful in getting started on the first few exercises, although the value of each will become apparent only with experience.

General Approach

1. **Take a moment to study the problem "in total" and make a brief mental sketch of the overall approach you plan to use**. The result may be time- and energy-saving shortcuts.

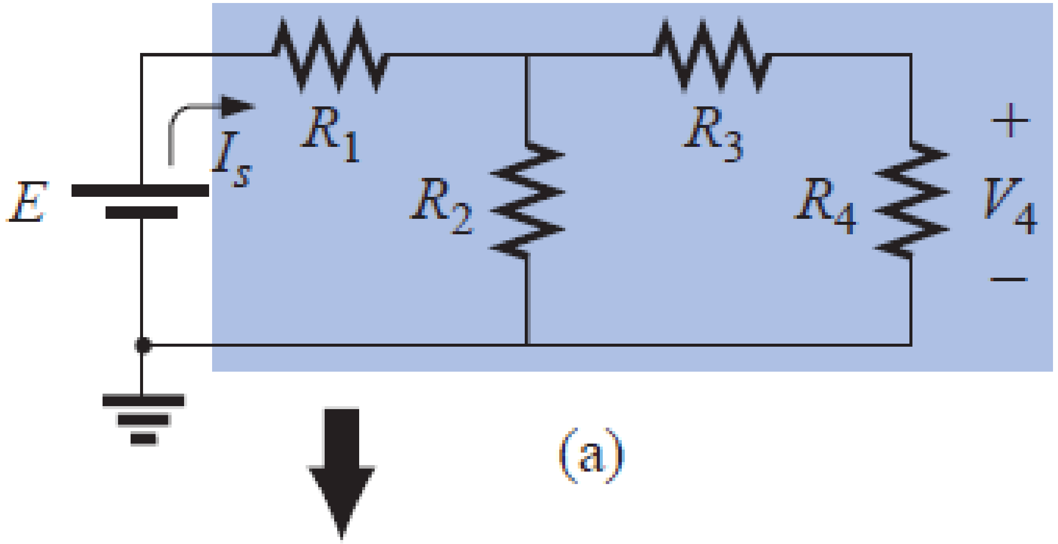
2. **Next examine each region of the network independently before tying them together in series-parallel combinations.** This will usually simplify the network and possibly reveal a direct approach toward obtaining one or more desired unknowns. It also eliminates many of the errors that might result due to the lack of a systematic approach.

3. **Redraw the network as often as possible** with the reduced branches and undisturbed unknown quantities to maintain clarity and provide the reduced networks for the trip back to unknown quantities from the source.

4. **When you have a solution, check that it is reasonable** by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve the circuit using another approach or check over your work very carefully.

Reduce and Return Approach

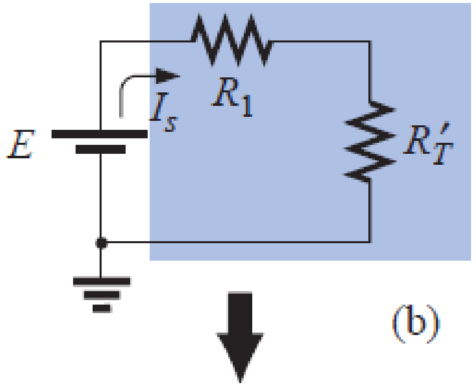
For many single-source, series-parallel networks, the analysis is one that works back to the source, **determines the source current**, and then finds its **way to the desired unknown.**



(a)

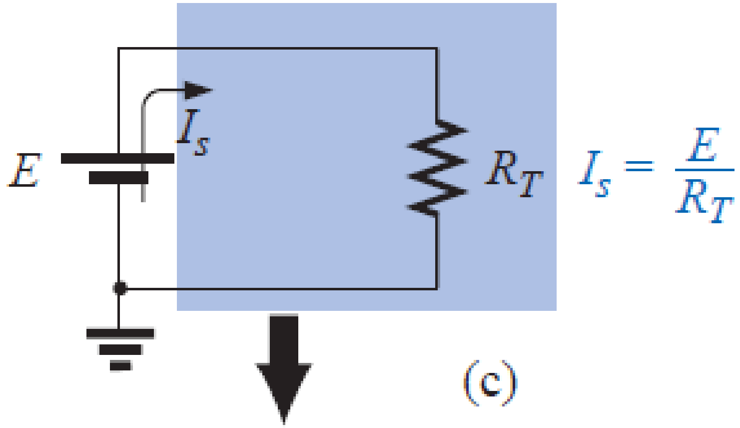
In Fig. 1(a), for instance, the voltage V_4 is desired. The absence of a single series or parallel path to V_4 from the source immediately:

First, series and parallel elements must be combined to establish the reduced circuit of Fig. 1(b).



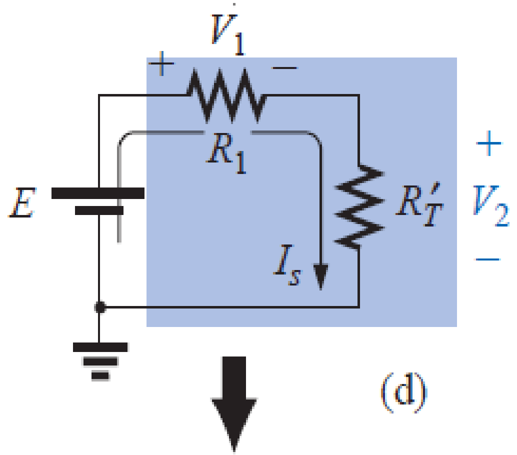
(b)

Second, series elements are combined to form the simplest of configurations in Fig. 1(c). The source current can now be determined using **Ohm's law**,



(c)

Third, and we can proceed back through the network as shown in Fig. 1(d). The voltage V_2 can be determined



(d)

Finally, and then the original network can be redrawn, as shown in Fig. 1(e). Since V_2 is now known, the **voltage divider rule (VDR)** can be used to find the desired voltage V_4.

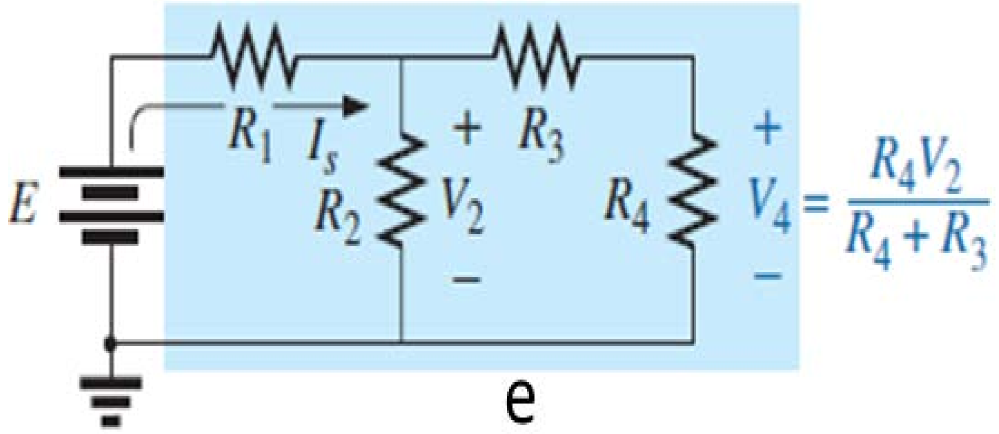


Fig. 1

Introducing the reduce and return approach.

Block Diagram Approach

The block diagram approach will be employed throughout to emphasize the fact that combinations of elements, not simply single resistive elements, can be in series or parallel.

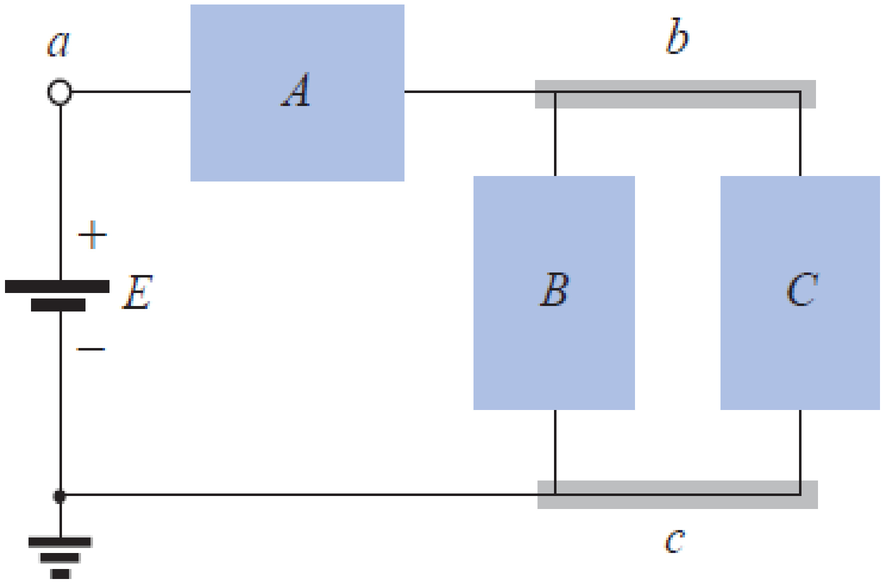


Fig. 2

Introducing the block diagram approach.

1) In Fig. 2, blocks *B* and *C* are in parallel (points *b* and *c* in common),
2) and the voltage source *E* is in series with block *A* (point *a* in common).

3) The parallel combination of *B* and *C* is also in series with *A* and the voltage source *E* due to the common points *b* and *c*, respectively.

To ensure that the analysis to follow is as clear and uncluttered as possible,

For series resistors R_1 and R_2, a **comma** will be inserted between their **subscript notations**, as shown here:

$$\boldsymbol{R_{1,2} = R_1 + R_2}$$

For parallel resistors R_1 and R_2, **the parallel symbol** will be inserted between their **subscript notations**, as follows:

$$R_{1\|2} = R_1 \parallel R_2 = \frac{R_1R_2}{R_1 + R_2}$$

Example 1: Calculate the source current (I_S); (I_B) ;(I_C) for the cct. of fig (3):

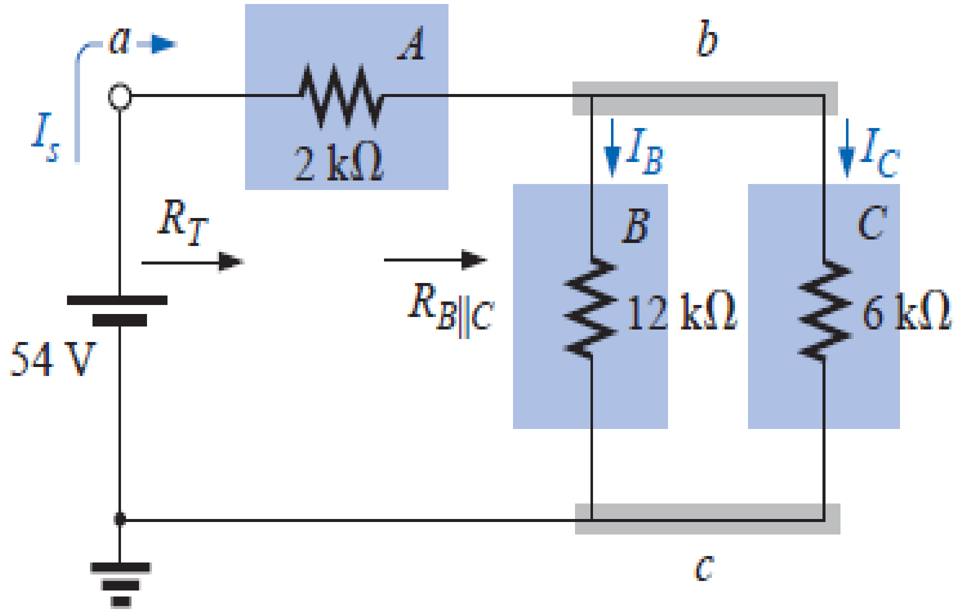


Fig. (3)

Sol: The parallel combination of R_B and R_C results in

$$R_{B\|C} = R_B \,||\, R_C = \frac{(12\text{ k}\Omega)(6\text{ k}\Omega)}{12\text{ k}\Omega + 6\text{ k}\Omega} = 4\text{ k}\Omega$$

The equivalent resistance $R_{B\|C}$ is then in series with R_A, and the total resistance "seen" by the source is

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 2\text{ k}\Omega + 4\text{ k}\Omega = 6\text{ k}\Omega \end{aligned}$$

The result is an equivalent network, as shown in Fig. 4, permitting the determination of the source current I_s.

$$I_s = \frac{E}{R_T} = \frac{54\ \text{V}}{6\ \text{k}\Omega} = \mathbf{9\ mA}$$

and, since the source and R_A are in series,

$$I_A = I_s = 9\ \text{mA}$$

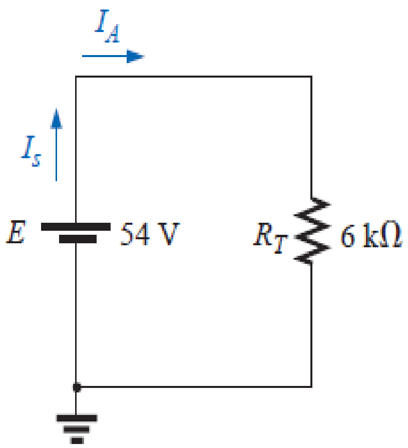


Fig. (4)

We can then use the equivalent network of Fig. (5) to determine I_B and I_C using the **current divider rule (CDR):**

$$I_B = \frac{6\ \text{k}\Omega(I_s)}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = \frac{6}{18}I_s = \frac{1}{3}(9\ \text{mA}) = \mathbf{3\ mA}$$

$$I_C = \frac{12\ \text{k}\Omega(I_s)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = \frac{12}{18}I_s = \frac{2}{3}(9\ \text{mA}) = \mathbf{6\ mA}$$

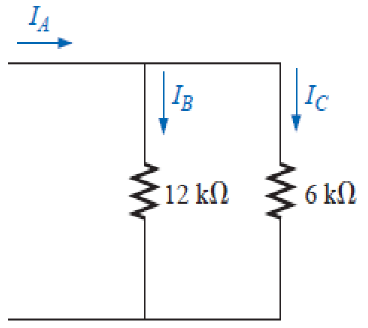


Fig. (5)

or, applying Kirchhoff's current law,

$$I_C = I_s - I_B = 9\ \text{mA} - 3\ \text{mA} = \mathbf{6\ mA}$$

Note that in this solution, we worked back to the source to obtain the source current or total current supplied by the source.

Example 2: for the cct. of fig. (6) calculate: R_A , R_B , R_C , R_T , I_T , I_A , I_B , I_C , I_{R1} , I_{R2} , I_{R3} , I_{R4} , I_{R5}

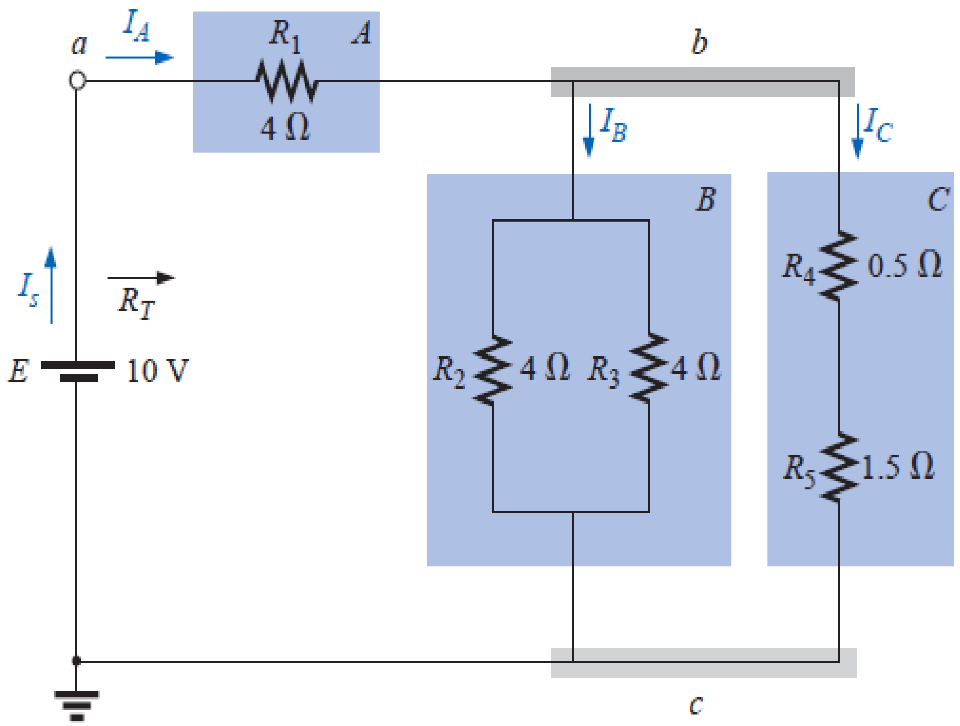


Fig.(6)

Sol. : By re-drawing the cct. of fig.(6) we get an equivalent cct. of fig(7):

$$A: \quad R_A = 4\ \Omega$$

$$B: \quad R_B = R_2 \parallel R_3 = R_{2\parallel 3} = \frac{R}{N} = \frac{4\ \Omega}{2} = 2\ \Omega$$

$$C: \quad R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$$

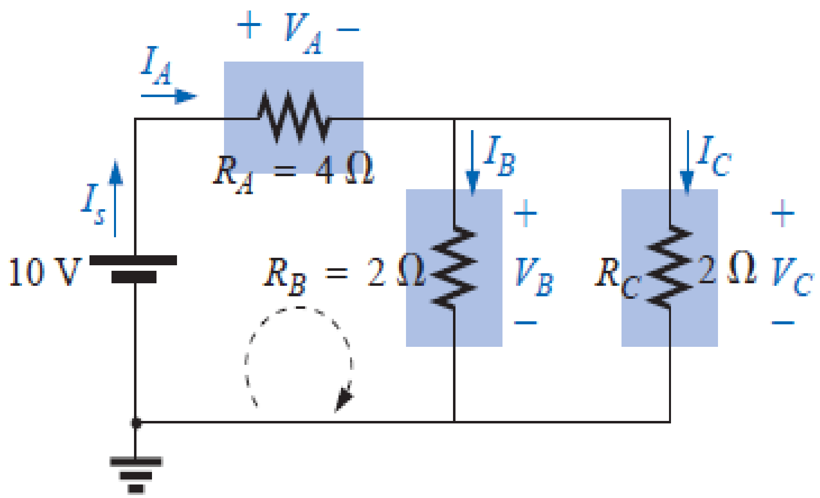


Fig. (7)

Blocks B and C are still in parallel, and

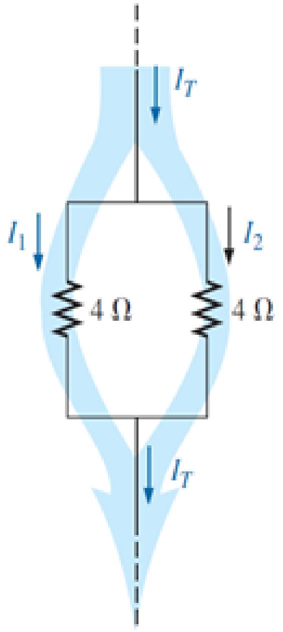
$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega} \end{aligned}$$

$$I_s = \frac{E}{R_T} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

$$I_A = I_s = \mathbf{2\ A}$$

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$



$I_1 = I_2 = \dfrac{I_T}{2}$

Returning to the network of Fig. 6, we have:

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$$
$$V_B = I_B R_B = (1\text{ A})(2\ \Omega) = \mathbf{2\ V}$$
$$V_C = V_B = \mathbf{2\ V}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. (7), we obtain:

$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$
$$E = V_A + V_B = 8\text{ V} + 2\text{ V}$$
$$\underline{10\text{ V} = 10\text{ V} \quad \text{(checks)}}$$

Example 3 : for the circuit in the fig(8) find: R_T , R_A , R_B , R_C , I_S , I_A , I_B , I_C , I_1 , I_2

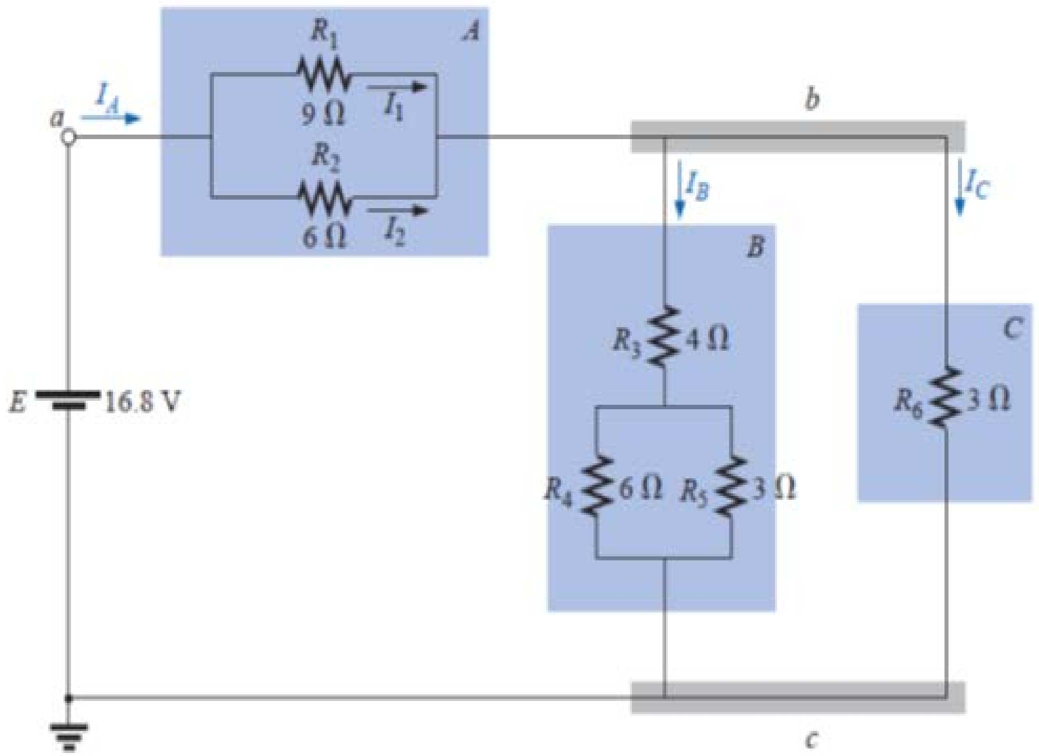


Fig. (8)

SOL.

$$R_A = R_{1\|2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$

$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

The network of Fig. (8) can then be redrawn in reduced form, as shown in Fig. (9).

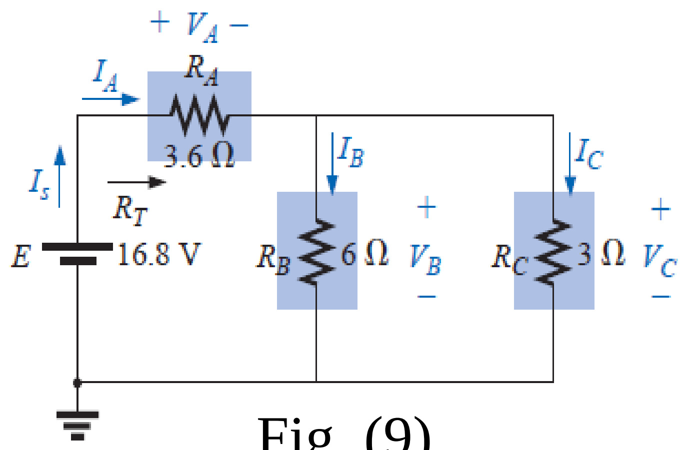


Fig. (9)

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \mathbf{5.6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = \mathbf{1\ A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\ \text{A})(3.6\ \Omega) = \mathbf{10.8\ V}$$

$$V_B = I_B R_B = V_C = I_C R_C = (2\ \text{A})(3\ \Omega) = \mathbf{6\ V}$$

Returning to the original network (Fig. 8) and applying the current divider rule:

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ \text{A})}{6\ \Omega + 9\ \Omega} = \frac{18\ \text{A}}{15} = \mathbf{1.2\ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\ \text{A} - 1.2\ \text{A} = \mathbf{1.8\ A}$$

EXAMPLE 4: Find the indicated currents and voltages for the network of Fig.(8)

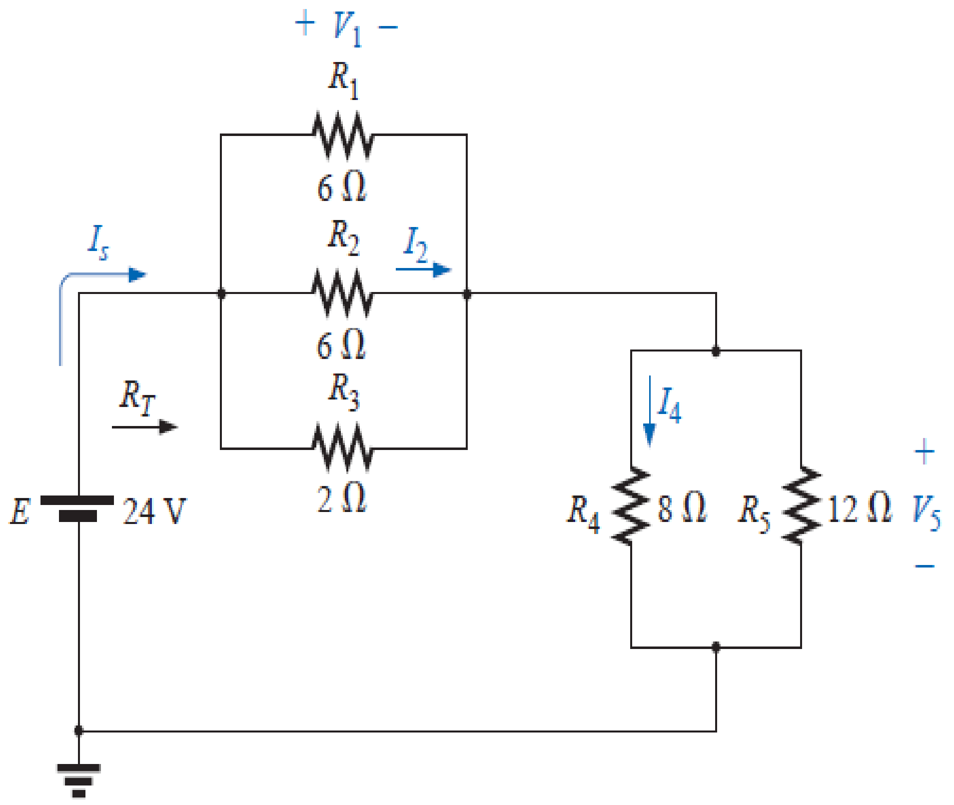
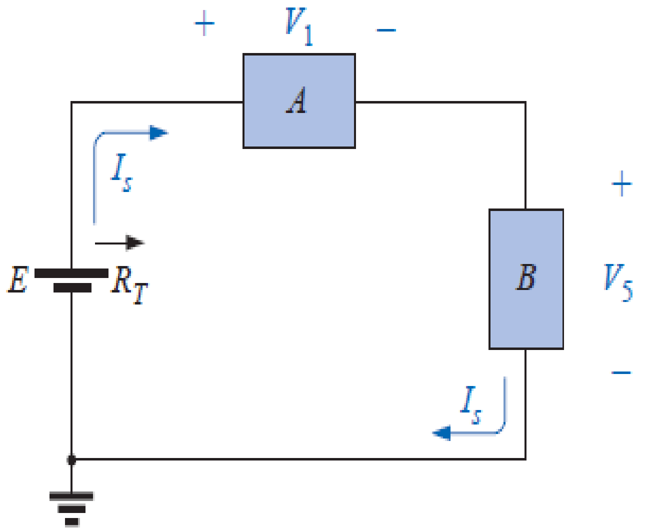


Fig.(8)



Block diagram for Figure (9)

$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

The reduced form of Fig. (8)will then appear as shown in Fig.(10) :

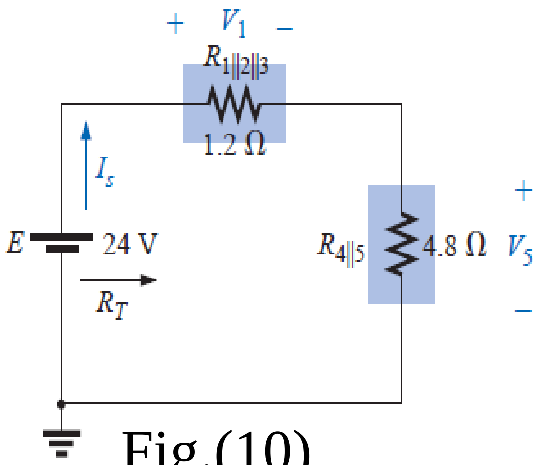


Fig.(10)

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \Omega} = \mathbf{4\ A}$$

with
$$V_1 = I_s R_{1\|2\|3} = (4\ \text{A})(1.2\ \Omega) = \mathbf{4.8\ V}$$
$$V_5 = I_s R_{4\|5} = (4\ \text{A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ \text{V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$

Example 5 :

a. Find the voltages V_1, V_3, and V_{ab} for the network of Fig. (11).

b. Calculate the source current I_S.

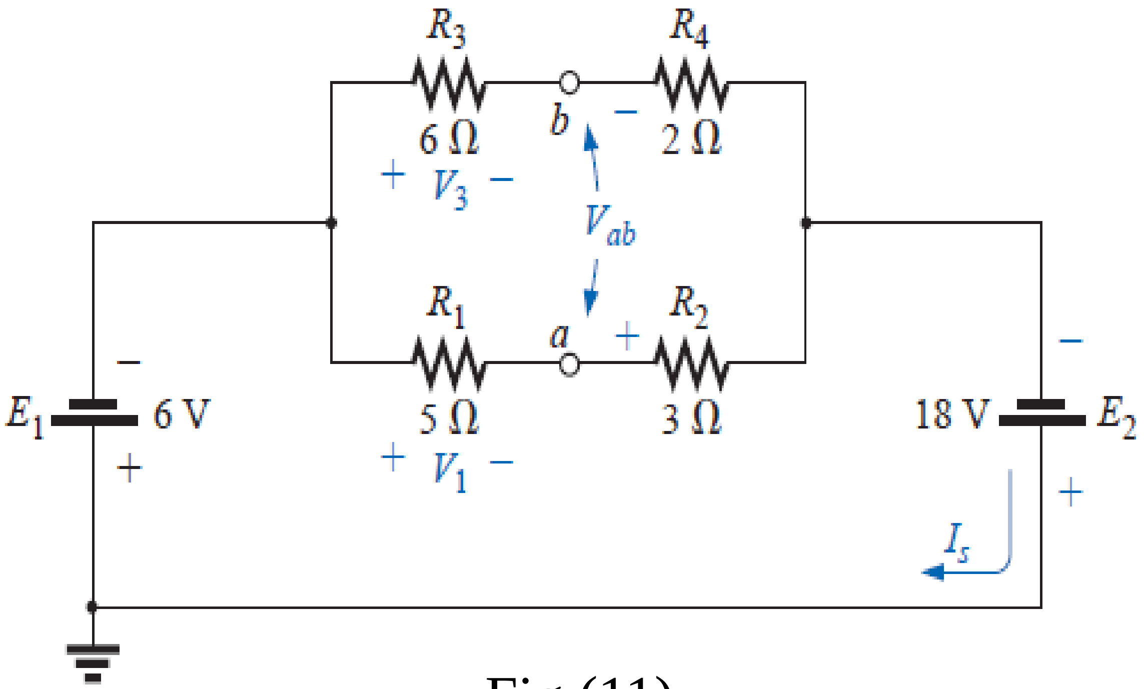


Fig.(11)

Sol: Since combining both sources will not affect the unknowns

The cct. is re-drawn as in fig(12):

The net source voltage is the difference between the two with the polarity of the larger.

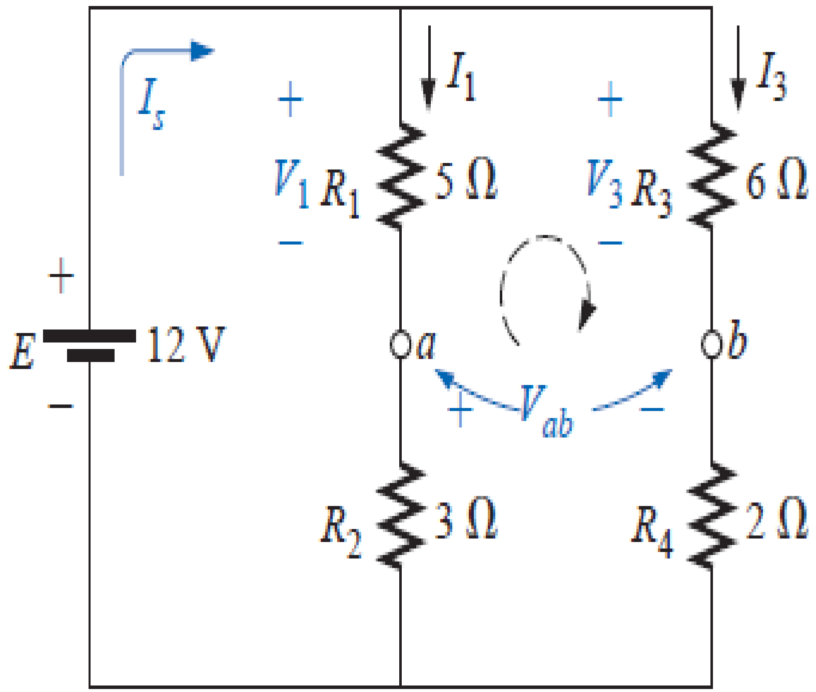


Fig.(12)

a.

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop of Fig.(12) in the clockwise direction starting at terminal *a*.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\text{ V} - 7.5\text{ V} = \mathbf{1.5\ V}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\text{ V}}{5\ \Omega} = 1.5\text{ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\text{ V}}{6\ \Omega} = 1.5\text{ A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\text{ A} + 1.5\text{ A} = \mathbf{3\ A}$$

Please read and try understand in the first reference Chapter 7.

References:

1- Introductory Circuits Analysis, By Boylested, Tenth (10th) Edition.

2- Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.

3- Any reference that has a Direct Current Circuits Analysis (DCCA).

Thank you for listening