الجامعة التكنولوجية

قسم هندسة الليزر والالكترونيات البصرية

Laser & Optoelectronics Eng. Department

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Direct Current Circuits Analysis (DCCA)

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Chapter 6
Methods of Analysis of the dc circuit

Methods of Analysis of the dc circuit

The methods to be discussed in detail in this chapter include:

1. **branch-current analysis,**
2. **mesh analysis,** and
3. **nodal analysis.**

Each can be applied to the same network. The “best” method cannot be defined by a set of rules but can be determined only by acquiring كسب a firm understanding فهم راسخ of the relative advantages of each.

1-BRANCH-CURRENT ANALYSIS:

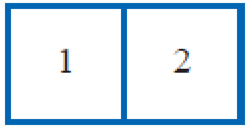
There are four steps, as indicated below. Before continuing, understand that this method will produce the current through each branch of the network, the branch current.

1. Assign a distinct current of arbitrary direction to each branch of the network.

2. Indicate the polarities for each resistor as determined by the assumed current direction.

3. Apply Kirchhoff's voltage law around each closed, independent loop of the network.

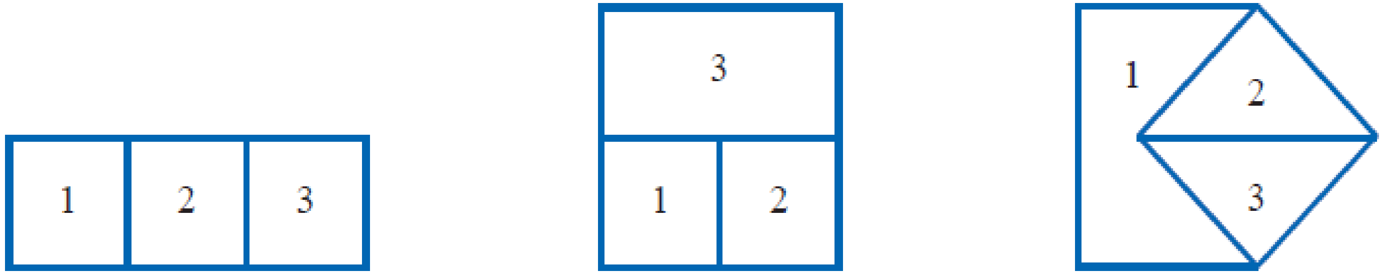
The best way to determine how many times Kirchhoff's voltage law will have to be applied is to **determine the number of "windows"** in the network. The network of Example has a definite similarity to the two-window configuration of Fig. 1(a). The result is a need to apply Kirchhoff's **voltage law twice**.



(a)

Fig. 1

For networks with three windows, as shown in Fig.1(b), **three** applications of Kirchhoff's voltage law are required, and so on.



(b)

Fig. 1 *Determining the number of independent closed loops.*

4. **Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.**

The **minimum number is one less than the number of independent nodes of the network**. For the purposes of this analysis, **a node is a junction of two or more branches**, where a branch is any combination of series elements. Figure 2 defines the number of applications of Kirchhoff's current law for each configuration of Fig. 1

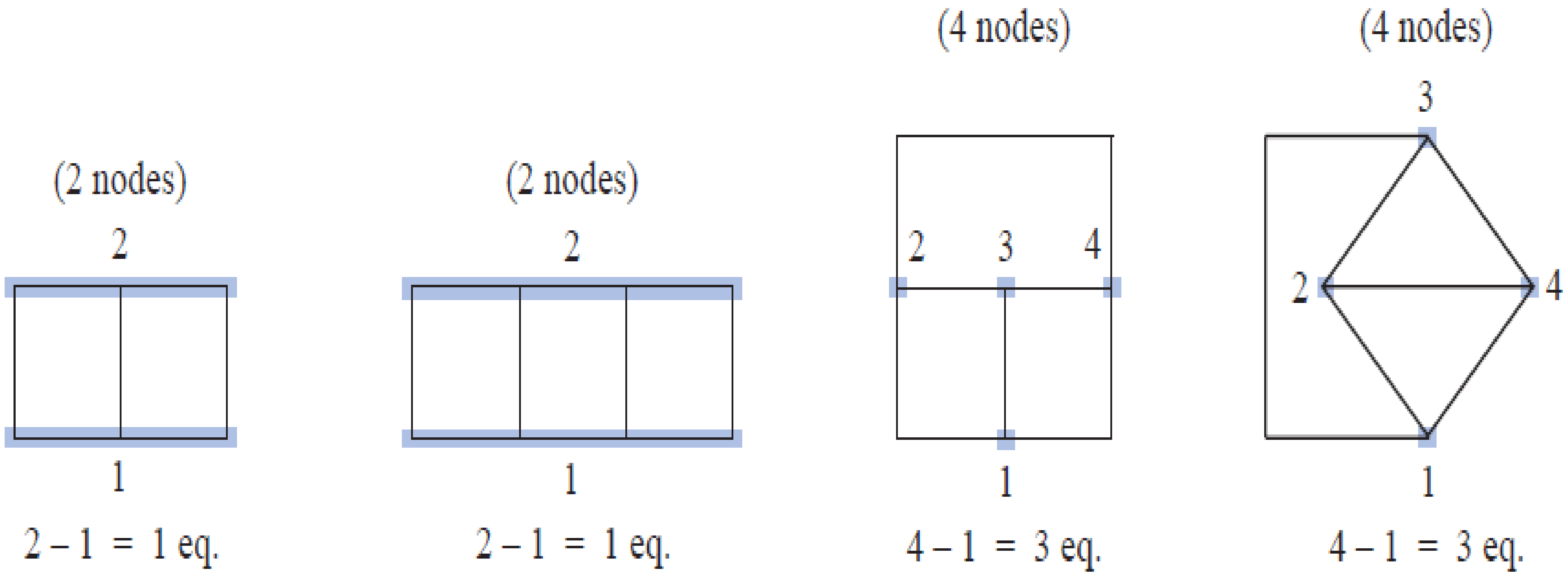


Figure 2 *Determining the number of applications of Kirchhoff's current law required.*

5. Solve the resulting simultaneous linear equations for assumed branch currents.

It is assumed that the use of the **determinants method** to solve for the currents I_1, I_2, and I_3 is understood and is a part of the student's mathematical background. If not, a detailed explanation of the procedure is provided in Appendix C.

DETERMINANTS

Determinants are employed to find the mathematical solutions for the variables in two or more simultaneous equations. Once the procedure is properly understood, solutions can be **obtained with a minimum of time and effort** and usually with **fewer errors** than when using other methods. Consider the following equations, where x and y are the unknown variables and a_1, a_2, b_1, b_2, c_1, and c_2 are constants:

Col. 1		Col. 2		Col. 3	
a_1x	$+$	b_1y	$=$	c_1	(C.1a)
a_2x	$+$	b_2y	$=$	c_2	(C.1b)

It is certainly possible to solve for one variable in Eq. (C.1a) and substitute into Eq. (C.1b). That is, solving for *x* in Eq. (C.1a),

$$x = \frac{c_1 - b_1y}{a_1}$$

and substituting the result in Eq. (C.1b),

$$a_2\left(\frac{c_1 - b_1 y}{a_1}\right) + b_2 y = c_2$$

It is now possible to **solve for *y***, since it is the only variable remaining, and then **substitute into either equation for *x*.** This is acceptable for two equations, but it becomes a **very tedious** and **lengthy process** for **three or more simultaneous equations**.

Using determinants to solve for x and y requires that the following formats be established for each variable:

$$\begin{array}{cc} & \text{Col. 1} \quad \text{Col. 2} \\ x = & \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \end{array} \qquad \begin{array}{cc} & \text{Col. 1} \quad \text{Col. 2} \\ y = & \dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \end{array} \tag{C.2}$$

First, note that only constants appear within the vertical brackets and that the **denominator** of each **is the same**. In fact, the denominator is simply the **coefficients of *x* and *y*** in the same arrangement as in Eqs. (C.1a) and (C.1b).

Second, when **solving for** x, replace the **coefficients of** x in the **numerator** by the constants to **the right of the equal sign in Eqs.** (C.1a) and (C.1b), and simply **repeat the coefficients of the** y **variable.**

Finally, when **solving for** y, replace the y **coefficients** in the **numerator** by the constants to the **right of the equal sign**, and **repeat the coefficients of** x.

Each configuration in the **numerator and denominator** of Eqs. (C.2) is referred to as a ***determinant*** **(*D*)**, which can be evaluated numerically in the following manner:

$$\begin{array}{c} \text{Col. Col.} \\ 1 \quad 2 \\ \hline \end{array}$$

$$\text{Determinant} = D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \qquad \text{(C.3)}$$

The **expanded value** is obtained by **first multiplying the top left element** by **the bottom right** and then **subtracting the product of the lower left and upper right elements**. This particular determinant is referred to as a ***second-order*** **determinant**, since it contains **two rows and two columns.**

EXAMPLE 1: Evaluate the following determinant:

$$\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix}$$

Solution: $\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = (2)(4) - (3)(2) = 8 - 6 = 2$

Expanding the entire expression for *x* and *y*, we have the following:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \quad \textbf{(C.4a)}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1} \quad \textbf{(C.4b)}$$

EXAMPLE 2: Solve for *x* and *y*:

$$2x + y = 3$$
$$3x + 4y = 2$$

Solution:

$$x = \frac{\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}} = \frac{(3)(4) - (2)(1)}{(2)(4) - (3)(1)} = \frac{12 - 2}{8 - 3} = \frac{10}{5} = \mathbf{2}$$

$$y = \frac{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}}{5} = \frac{(2)(2) - (3)(3)}{5} = \frac{4 - 9}{5} = \frac{-5}{5} = \mathbf{-1}$$

$$\begin{aligned} 2x + y &= (2)(2) + (-1) \\ &= 4 - 1 = 3 \qquad \text{(checks)} \\ 3x + 4y &= (3)(2) + (4)(-1) \\ &= 6 - 4 = 2 \qquad \text{(checks)} \end{aligned}$$

Any number of simultaneous linear equations:

The use of determinants is not limited to the solution of two simultaneous equations; determinants can be applied to any number of simultaneous linear equations.

Consider the three following simultaneous equations:

$$\begin{array}{cccccccc}
\text{Col. 1} & & \text{Col. 2} & & \text{Col. 3} & & \text{Col. 4} \\
\hline
a_1x & + & b_1y & + & c_1z & = & d_1 \\
a_2x & + & b_2y & + & c_2z & = & d_2 \\
a_3x & + & b_3y & + & c_3z & = & d_3
\end{array}$$

in which x, y, and z are the variables, and $a_{1,2,3}$, $b_{1,2,3}$, $c_{1,2,3}$, and $d_{1,2,3}$ are constants.

The determinant configuration for *x*, *y*, and *z* can be found in a manner similar to that for two simultaneous equations.

Where

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D}, \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

A shorthand method for evaluating the **third-order determinant** consists simply of **repeating the first two columns** of the **determinant** to the r**ight of the determinant** and then **summing the products along specific diagonals** as shown below:

$$\begin{array}{cccccc} & & & 4(-) & 5(-) & 6(-) \\ D = & \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} & & \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} & \\ & & & 1(+) & 2(+) & 3(+) \end{array}$$

The products of the **diagonals 1, 2, and 3** are **positive** and have the following magnitudes: $+a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3$

The products of the **diagonals 4, 5, and 6** are **negative** and have the following magnitudes: $-a_3b_2c_1 - b_3c_2a_1 - c_3a_2b_1$

The **total solution** is the **sum of the diagonals 1, 2, and 3 minus the sum of the diagonals 4, 5, and 6:**

$$\boxed{+(a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3) - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)} \quad (C.5)$$

Warning: **This method of expansion is good only for third-order determinants!** It cannot be applied to **fourth- and higher-order** systems.

EXAMPLE 3 Solve for *x*, *y*, and *z*:

$$\begin{aligned} 1x + 0y - 2z &= -1 \\ 0x + 3y + 1z &= +2 \\ 1x + 2y + 3z &= 0 \end{aligned}$$

Solution:

$$x = \frac{\left|\begin{matrix} -1 & 0 & -2 \\ 2 & 3 & 1 \\ 0 & 2 & 3 \end{matrix}\right| \begin{matrix} -1 & 0 \\ 2 & 3 \\ 0 & 2 \end{matrix}}{\left|\begin{matrix} 1 & 0 & -2 \\ 0 & 3 & 1 \\ 1 & 2 & 3 \end{matrix}\right| \begin{matrix} 1 & 0 \\ 0 & 3 \\ 1 & 2 \end{matrix}}$$

$$= \frac{[(-1)(3)(3) + (0)(1)(0) + (-2)(2)(2)] - [(0)(3)(-2) + (2)(1)(-1) + (3)(2)(0)]}{[(1)(3)(3) + (0)(1)(1) + (-2)(0)(2)] - [(1)(3)(-2) + (2)(1)(1) + (3)(0)(0)]}$$

$$= \frac{(-9 + 0 - 8) - (0 - 2 + 0)}{(9 + 0 + 0) - (-6 + 2 + 0)}$$

$$= \frac{-17 + 2}{9 + 4} = -\frac{15}{13}$$

$$y = \frac{\left|\begin{matrix} 1 & -1 & -2 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{matrix}\right| \begin{matrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{matrix}}{13}$$

$$= \frac{[(1)(2)(3) + (-1)(1)(1) + (-2)(0)(0)] - [(1)(2)(-2) + (0)(1)(1) + (3)(0)(-1)]}{13}$$

$$= \frac{(6 - 1 + 0) - (-4 + 0 + 0)}{13}$$

$$= \frac{5 + 4}{13} = \frac{9}{13}$$

$$z = \frac{\begin{vmatrix} 1 & 0 & -1 \\ 0 & 3 & 2 \\ 1 & 2 & 0 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 3 \\ 1 & 2 \end{matrix}}{13}$$

$$= \frac{[(1)(3)(0) + (0)(2)(1) + (-1)(0)(2)] - [(1)(3)(-1) + (2)(2)(1) + (0)(0)(0)]}{13}$$

$$= \frac{(0 + 0 + 0) - (-3 + 4 + 0)}{13}$$

$$= \frac{0 - 1}{13} = -\frac{1}{13}$$

or from $0x + 3y + 1z = +2,$

$$z = 2 - 3y = 2 - 3\left(\frac{9}{13}\right) = \frac{26}{13} - \frac{27}{13} = -\frac{1}{13}$$

The General Approach to Third-order or Higher determinants

The general approach to third-order or higher determinants requires that the determinant be expanded in the following form.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \underset{\substack{\uparrow \\ \text{Multiplying} \\ \text{factor}}}{a_1} \underbrace{\left(+ \underbrace{\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}}_{\text{Minor}} \right)}_{\text{Cofactor}} + \underset{\substack{\uparrow \\ \text{Multiplying} \\ \text{factor}}}{b_1} \underbrace{\left(- \underbrace{\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}}_{\text{Minor}} \right)}_{\text{Cofactor}} + \underset{\substack{\uparrow \\ \text{Multiplying} \\ \text{factor}}}{c_1} \underbrace{\left(+ \underbrace{\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}}_{\text{Minor}} \right)}_{\text{Cofactor}}$$

This expansion was obtained by **multiplying the elements of the first row of *D* by their corresponding cofactors.**

The **sign of each cofactor** is dictated by the position of the multiplying factors (a_1, b_1, and c_1 in this case) as in the following standard format:

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

For the determinant D, the elements would have the following signs:

$$\begin{vmatrix} a_1^{(+)} & b_1^{(-)} & c_1^{(+)} \\ a_2^{(-)} & b_2^{(+)} & c_2^{(-)} \\ a_3^{(+)} & b_3^{(-)} & c_3^{(+)} \end{vmatrix}$$

The **minors** associated with each multiplying factor are obtained by **covering up the row and column in which the multiplying factor is located** and **writing a second-order determinant to include the remaining elements in the same relative positions** that they have in the third order determinant. We can find the minors of a_1 and b_1 as follows:

$$a_{1(\text{minor})} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \qquad b_{1(\text{minor})} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix}$$

Using the **first column of *D***, we obtain the expansion

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\left(+\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}\right) + a_2\left(-\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}\right) + a_3\left(+\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}\right)$$

EXAMPLE 4 Expand the following third-order determinant:

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

Solution:

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 1\left(+ \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \right) + 3\left(- \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} \right) + 2 + \left(\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} \right)$$

$$\begin{aligned} &= 1[6 - 1] + 3[-(6 - 3)] + 2[2 - 6] \\ &= 5 + 3(-3) + 2(-4) \\ &= 5 - 9 - 8 \\ &= \mathbf{-12} \end{aligned}$$

EXAMPLE 5 Apply the branch-current method to the network Fig. 3.

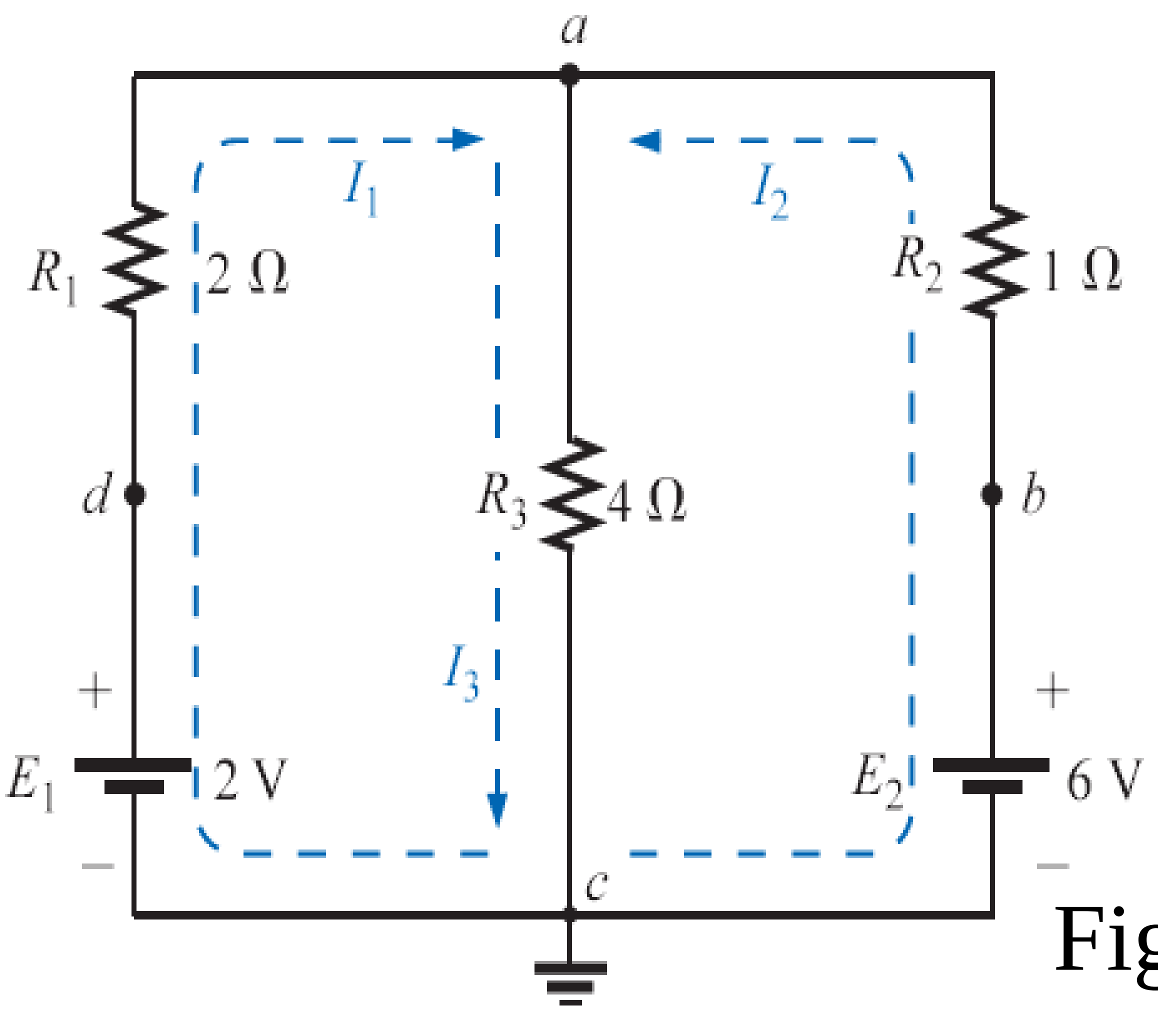


Fig. (3)

Solution:

Step 1: Since there are **three distinct branches (*cda*, *cba*, *ca*)**, three currents of arbitrary directions (I_1, I_2, I_3) are chosen, as indicated in Fig. 3. The current directions for I_1 and I_2 were chosen to **match the "pressure" applied by sources E_1 and E_2**, respectively. Since both I_1 and I_2 enter node *a*, I_3 is leaving.

Step 2: **Polarities for each resistor** are drawn to agree with **assumed current directions**, as indicated in Fig. 4.

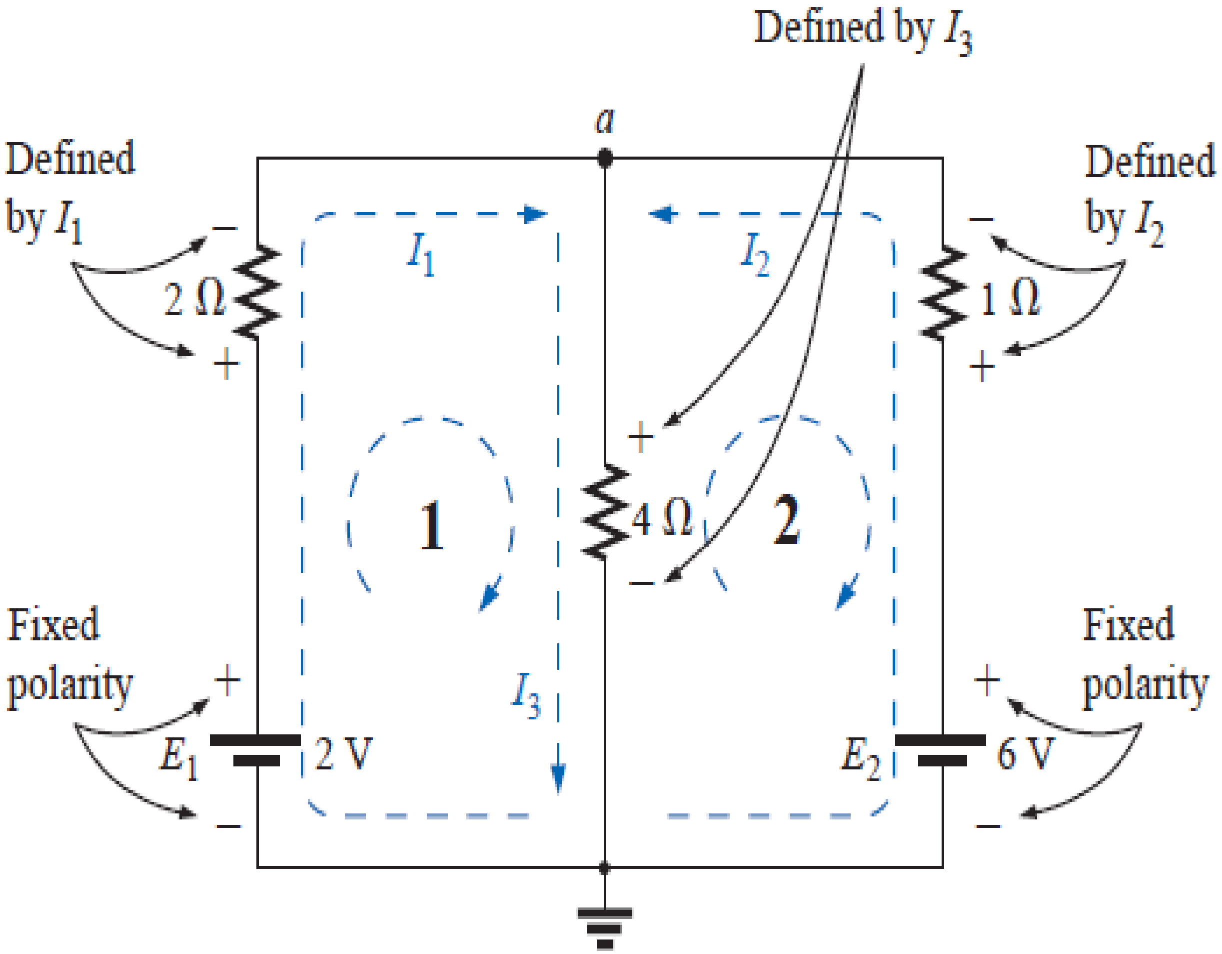


Figure 4: Inserting the polarities across the resistive elements as defined by the chosen branch currents.

Step 3: Kirchhoff’s voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

Rise in potential

$$\text{loop 1:} \quad \sum\nolimits_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Drop in potential

Rise in potential

$$\text{loop 2:} \quad \sum\nolimits_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

Drop in potential

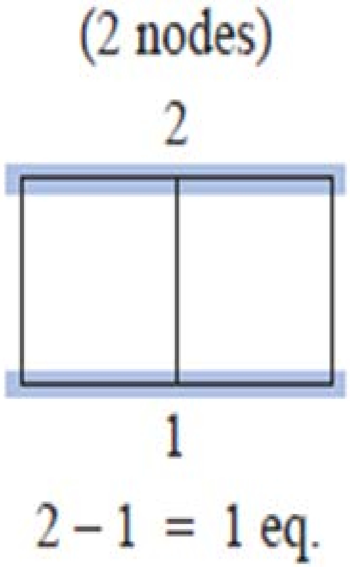
and

$$\text{loop 1:} \quad \sum\nolimits_{\circlearrowright} V = \underbrace{+2\text{ V}}_{\substack{\text{Battery}\\ \text{potential}}} - \underbrace{(2\ \Omega)I_1}_{\substack{\text{Voltage drop}\\ \text{across 2-}\Omega\\ \text{resistor}}} - \underbrace{(4\ \Omega)I_3}_{\substack{\text{Voltage drop}\\ \text{across 4-}\Omega\\ \text{resistor}}} = 0$$

$$\text{loop 2:} \quad \sum\nolimits_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: Applying Kirchhoff's current law at node *a* (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$



Step 5: There are three equations and three unknowns (units removed for clarity):

Solution 1

$$2 - 2I_1 - 4I_3 = 0 \qquad \text{Rewritten:} \quad 2I_1 + 0 + 4I_3 = 2$$

$$4I_3 + 1I_2 - 6 = 0 \qquad 0 + I_2 + 4I_3 = 6$$

$$I_1 + I_2 = I_3 \qquad I_1 + I_2 - I_3 = 0$$

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = -1\ \mathbf{A}$$

$$D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = \mathbf{2\ A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = \mathbf{1\ A}$$

Instead of using third-order determinants as in Solution 1, we could reduce the **three equations to two by substituting the third equation in the first and second equations:**

$$2 - 2I_1 - 4I_3 = 0$$

$$4I_3 + 1I_2 - 6 = 0$$

$$I_1 + I_2 = I_3$$

$$\left.\begin{array}{l} 2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} = 0 \\ \underline{4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 = 0} \end{array}\right\} \quad \begin{array}{l} 2 - 2I_1 - 4I_1 - 4I_2 = 0 \\ \underline{4I_1 + 4I_2 + I_2 - 6 = 0} \end{array}$$

or

$$\begin{aligned} -6I_1 - 4I_2 &= -2 \\ \underline{+4I_1 + 5I_2} &\underline{= +6} \end{aligned}$$

Multiplying through by -1 in the top equation yields

$$\begin{aligned} 6I_1 + 4I_2 &= +2 \\ \underline{4I_1 + 5I_2} &\underline{= +6} \end{aligned}$$

and using determinants,

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = \mathbf{-1\,A}$$

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ 4 & 6 \end{vmatrix}}{14} = \frac{36 - 8}{14} = \frac{28}{14} = \mathbf{2\ A}$$

$$I_3 = I_1 + I_2 = -1 + 2 = \mathbf{1\ A}$$

The **voltage across any resistor** can now be found using **Ohm's law**, and the **power delivered by either source or to any one of the three resistors** can be found using the appropriate **power equation.**

Applying Kirchhoff's voltage law around the loop 2. **Check**

$$\Sigma_{\circlearrowright} V = +(4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\ \text{V} = 0$$

or

$$(4\ \Omega)I_3 + (1\ \Omega)I_2 = 6\ \text{V}$$

and

$$(4\ \Omega)(1\ \text{A}) + (1\ \Omega)(2\ \text{A}) = 6\ \text{V}$$

$$4\ \text{V} + 2\ \text{V} = 6\ \text{V}$$

$$6\ \text{V} = 6\ \text{V} \quad \text{(checks)}$$

EXAMPLE 6 Apply branch-current analysis to the network of Fig. 5.

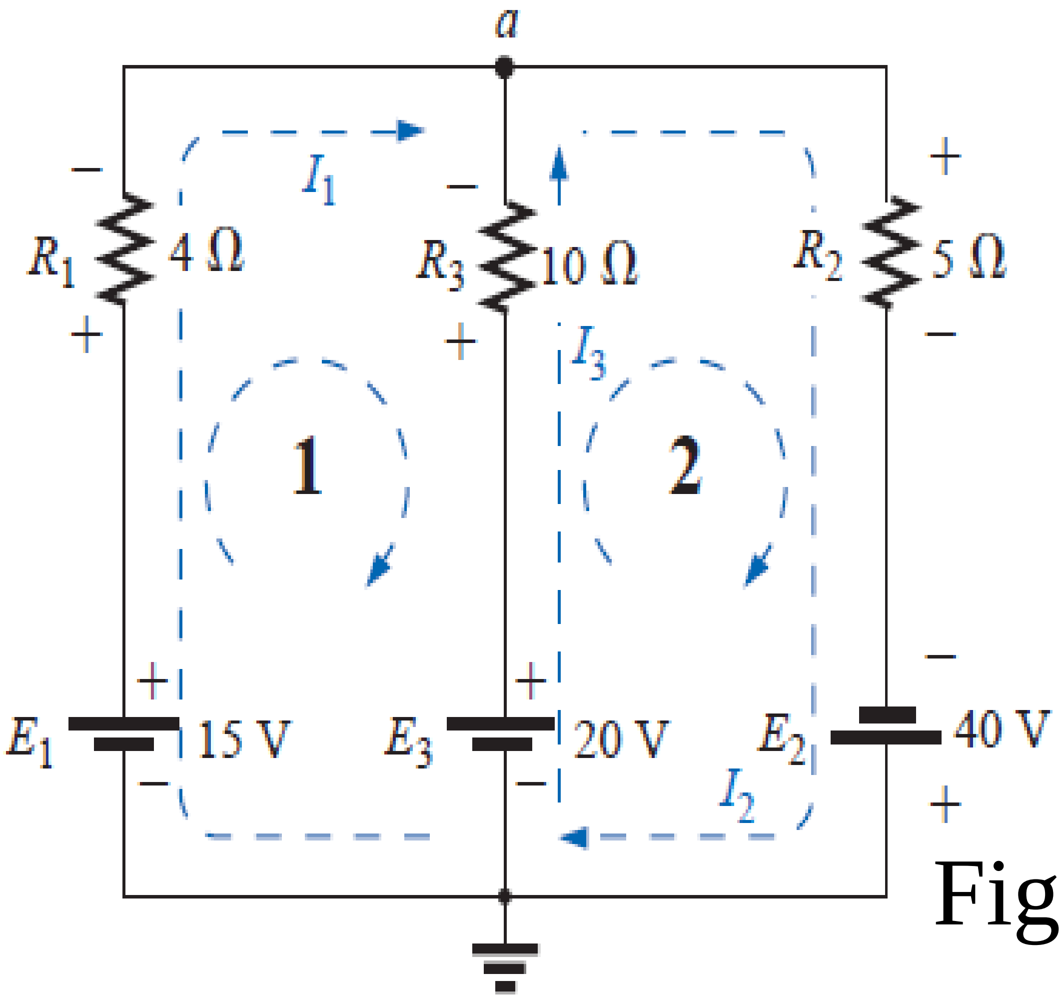


Fig. (5)

Solution:

Again, the current directions were chosen to match the "pressure" of each battery.

The polarities are then added and Kirchhoff's voltage law is applied around each closed loop in the clockwise direction. The result is as follows:

$$\text{loop 1:} \quad +15\text{ V} - (4\ \Omega)I_1 + (10\ \Omega)I_3 - 20\text{ V} = 0$$
$$\text{loop 2:} \quad +20\text{ V} - (10\ \Omega)I_3 - (5\ \Omega)I_2 + 40\text{ V} = 0$$

Applying Kirchhoff's current law at node *a*,

$$I_1 + I_3 = I_2$$

Substituting the third equation into the other two yields (with units removed for clarity)

$$\left.\begin{aligned} 15 - 4I_1 + 10I_3 - 20 &= 0 \\ 20 - 10I_3 - 5(I_1 + I_3) + 40 &= 0 \end{aligned}\right\}$$

Substituting for I_2 (since it occurs only once in the two equations)

or

$$\begin{aligned} -4I_1 + 10I_3 &= 5 \\ -5I_1 - 15I_3 &= -60 \end{aligned}$$

Multiplying the lower equation by -1, we have

$$-4I_1 + 10I_3 = 5$$
$$5I_1 + 15I_3 = 60$$

$$I_1 = \frac{\begin{vmatrix} 5 & 10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 10 \\ 5 & 15 \end{vmatrix}} = \frac{75 - 600}{-60 - 50} = \frac{-525}{-110} = \mathbf{4.773\ A}$$

$$I_3 = \frac{\begin{vmatrix} -4 & 5 \\ 5 & 60 \end{vmatrix}}{-110} = \frac{-240 - 25}{-110} = \frac{-265}{-110} = \mathbf{2.409\ A}$$

$$I_2 = I_1 + I_3 = 4.773 + 2.409 = \mathbf{7.182\ A}$$

revealing that the assumed directions were the actual directions, with I_2 equal to the sum of I_1 and I_3.

2- MESH ANALYSIS (GENERAL APPROACH)

The second method of analysis to be described is called **mesh analysis.** The term *mesh* is derived from the similarities in appearance between the closed loops of a network and a wire mesh fence.

The systematic approach outlined below should be followed when applying this method.

This is the **crux** of the terminology: ***independent***. No matter how you choose your loop currents,

the number of loop currents required is always equal to the number of windows of a planar (no-crossovers) network.

1. **Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current.**

For the network of Fig. 6, the loop current I_1 is the branch current of the branch containing the 2-Ω resistor and 2-V battery.

The current through the 4-Ω resistor is not I_1, however, since there is also a loop current I_2 through it.

Since they have opposite directions, $I_{4\Omega}$ equals the difference between the two, $I_1 - I_2$ or $I_2 - I_1$, depending on which you choose to be the defining direction.

In other words, *a loop current is a branch current only when it is the only loop current assigned to that branch.*

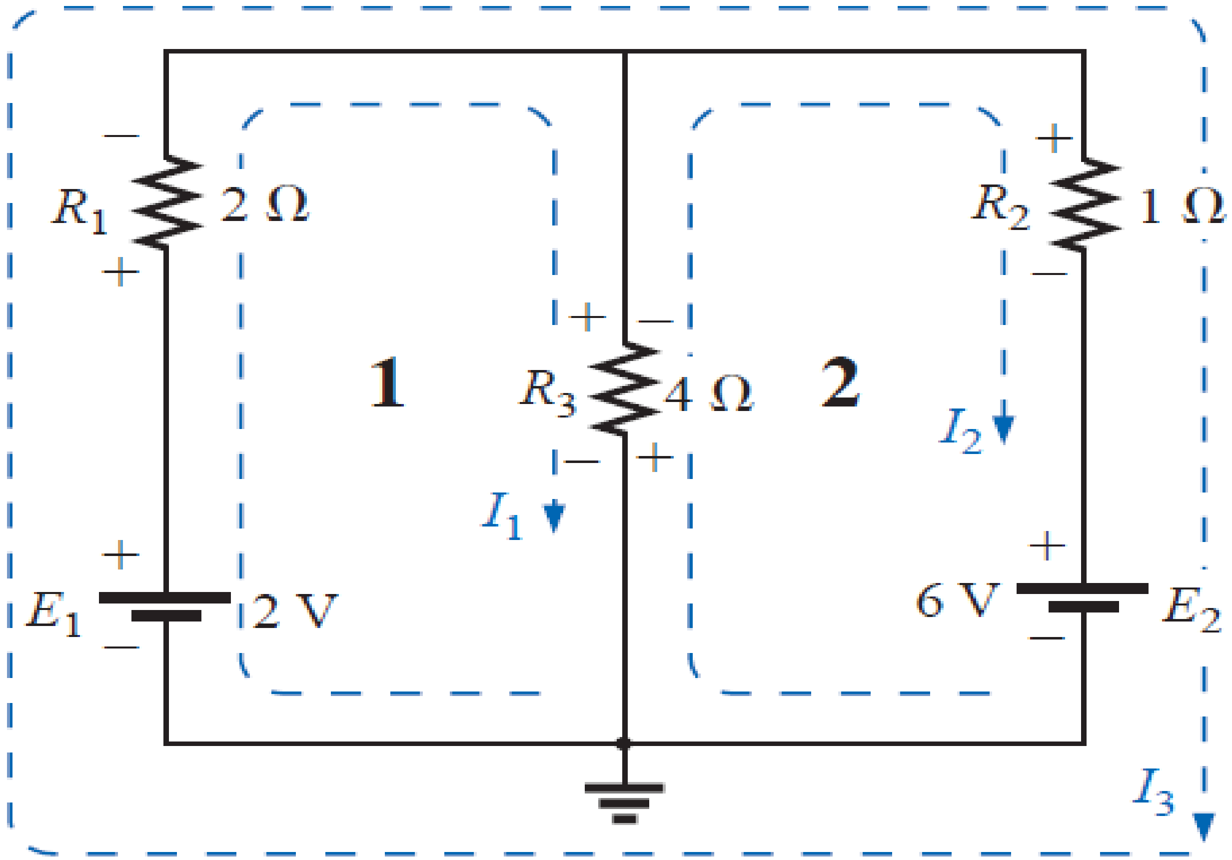


Fig. 6 *Defining the mesh currents for a "two window" network.*

2. Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop.

This requires, as shown in Fig. 6, that the 4Ω resistor have two sets of polarities across it.

3. Apply Kirchhoff's voltage law around each closed loop in the clockwise direction.

a. If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.

b. The polarity of a voltage source is unaffected by the direction of the assigned loop currents.

4. Solve the resulting simultaneous linear equations for the assumed loop currents.

EXAMPLE 7 Find the current through each branch of the network of the Fig. 7.

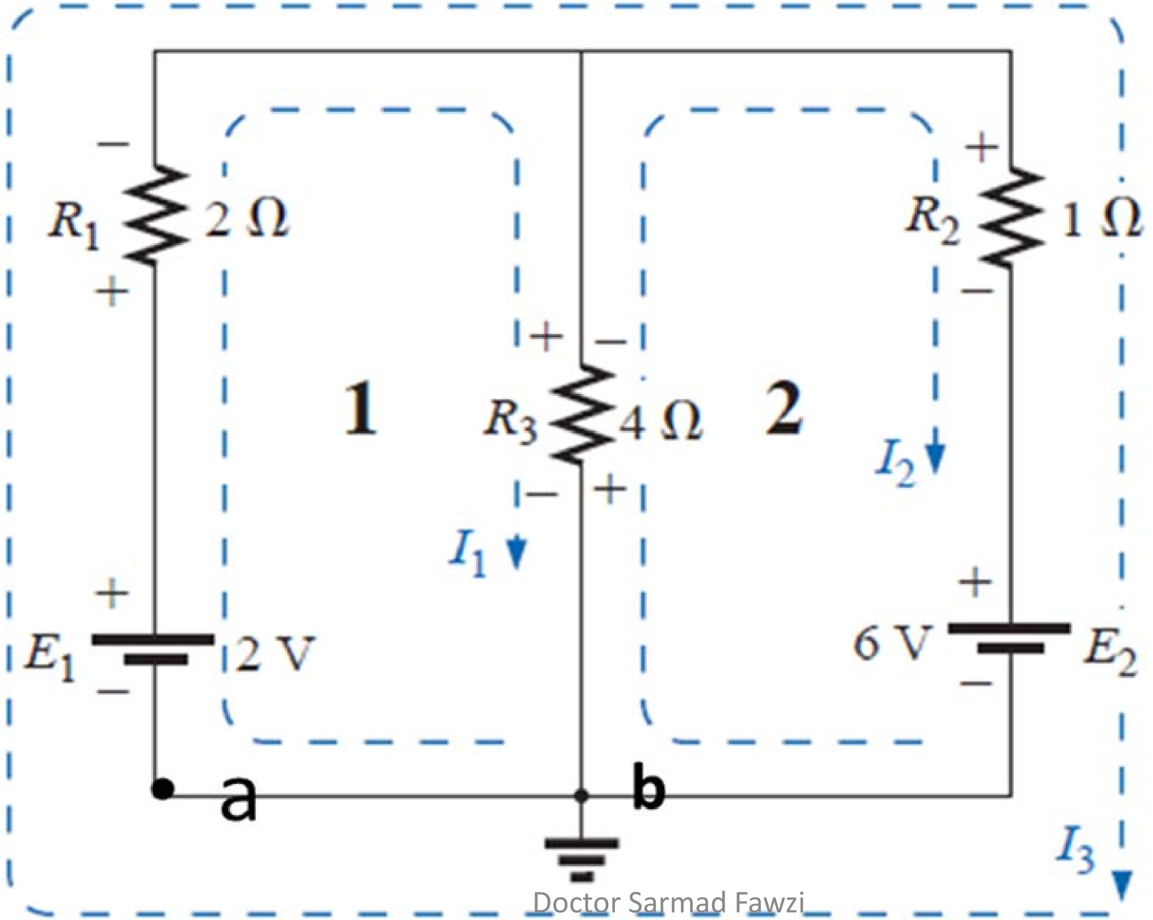


Fig. 7 *Defining the mesh currents for a “two window” network.*

Solution: *Step 1:* Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network. A third loop (I_3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4-Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff’s voltage law is applied around each loop in the clockwise direction.

Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element.

The voltage across each resistor is determined by ***V= IR***, and for a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions.

If **clockwise applications of Kirchhoff's voltage law** are always chosen, the other **loop currents** will always be **subtracted from the loop current of the loop being analyzed**.

loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\,\Omega)\,I_1 - \overbrace{(4\,\Omega)\underbrace{(I_1 - I_2)}_{\substack{\text{Total current}\\ \text{through}\\ 4\text{-}\Omega\text{ resistor}}}}^{\substack{\text{Voltage drop across}\\ 4\text{-}\Omega\text{ resistor}}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\,\Omega)(I_2 - I_1) - (1\,\Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for clarity):

$$\text{loop 1:}\quad +2 - 2I_1 - 4I_1 + 4I_2 = 0$$
$$\text{loop 2:}\quad -4I_2 + 4I_1 - 1I_2 - 6 = 0$$

and

$$\text{loop 1:}\quad +2 - 6I_1 + 4I_2 = 0$$
$$\text{loop 2:}\quad -5I_2 + 4I_1 - 6 = 0$$

or

$$\text{loop 1:}\quad -6I_1 + 4I_2 = -2$$
$$\text{loop 2:}\quad +4I_1 - 5I_2 = +6$$

Applying determinants will result in

$$I_1 = \mathbf{-1\,A} \quad \text{and} \quad I_2 = \mathbf{-2\,A}$$

The **minus signs** indicate that the currents have **a direction opposite** to that indicated by the **assumed loop current**.

The actual current through the **2-V source** and **2-Ω resistor** is therefore **1 A** in the **other direction**, and the **current** through the **6-V source and 1-Ω resistor is 2 A** in the **opposite direction** indicated on the circuit.

The current through the 4-Ω resistor is determined by the following equation from the original network:

loop 1: $I_{4\Omega} = I_1 - I_2 = -1\text{ A} - (-2\text{ A}) = -1\text{ A} + 2\text{ A}$
$= \mathbf{1\ A}$ (in the direction of I_1)

The **outer loop (*I_3*) and *one* inner loop (either *I_1* or *I_2*)** would also have produced the correct results. This approach, however, will often **lead to errors since the loop equations may be more difficult to write.** The **best method** of **picking these loop currents** is **to use the window approach.**

EXAMPLE 8 Find the branch currents of the network of Fig. 8

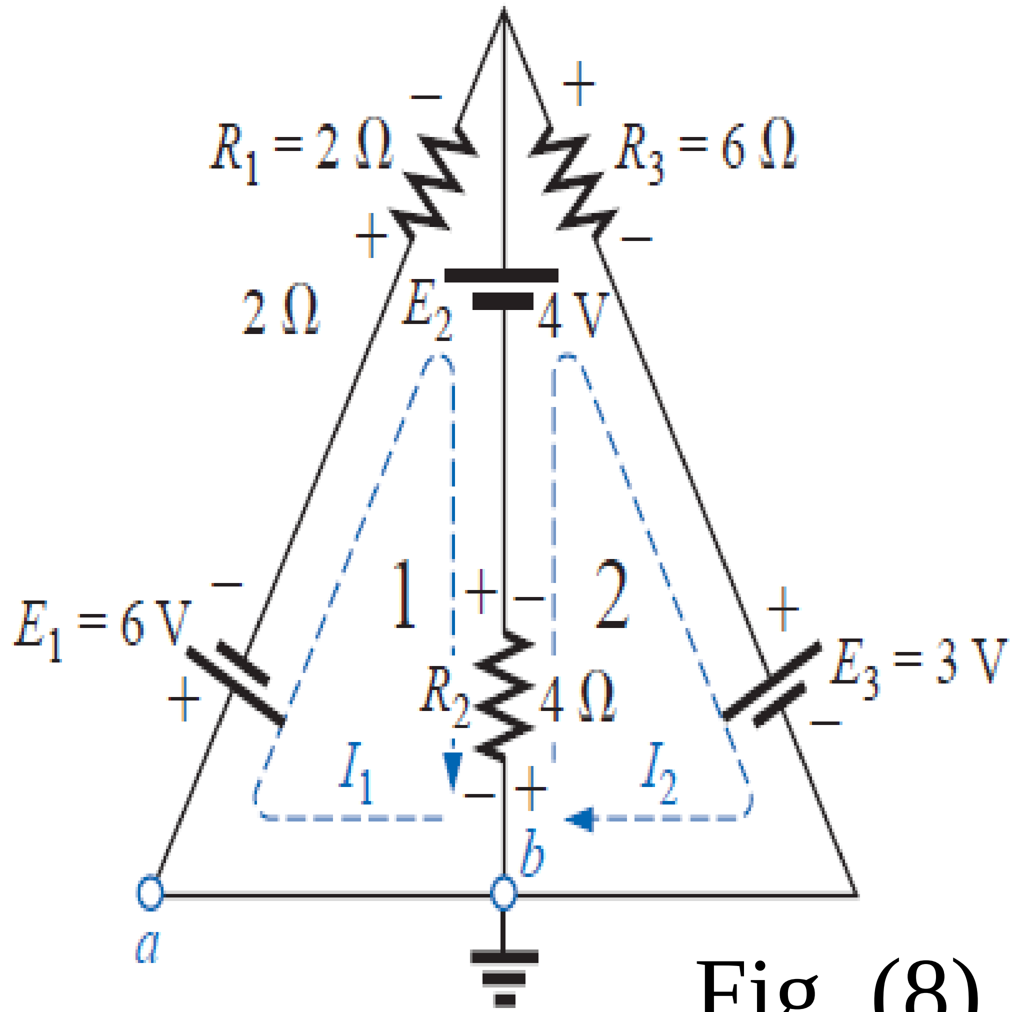


Fig. (8)

Solution:

Steps 1 and 2 are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

$$\text{loop 1:} \quad -E_1 - I_1R_1 - E_2 - V_2 = 0 \quad \text{(clockwise from point } a\text{)}$$

$$-6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$$

$$\text{loop 2:} \quad -V_2 + E_2 - V_3 - E_3 = 0 \quad \text{(clockwise from point } b\text{)}$$

$$-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0$$

which are rewritten as

$$\left.\begin{aligned} -10 - 4I_1 - 2I_1 + 4I_2 &= 0 \\ +1 + 4I_1 - 4I_2 - 6I_2 &= 0 \end{aligned}\right\} \begin{aligned} -6I_1 + 4I_2 &= +10 \\ +4I_1 - 10I_2 &= -1 \end{aligned}$$

or, by multiplying the top equation by -1, we obtain

$$6I_1 - 4I_2 = -10$$

$$4I_1 - 10I_2 = -1$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.182\ A}$$

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.773\ A}$$

The current in the 4-Ω resistor and 4-V source for loop 1 is

$$\begin{aligned} I_1 - I_2 &= -2.182\ \mathrm{A} - (-0.773\ \mathrm{A}) \\ &= -2.182\ \mathrm{A} + 0.773\ \mathrm{A} \\ &= \mathbf{-1.409\ A} \end{aligned}$$

revealing that it is 1.409 A in a direction opposite (due to the minus sign) to I_1 in loop 1.

Supermesh Currents

On occasion there **will be current sources** in the network to which mesh analysis is to be applied. **In such cases one can convert the current source to a voltage source (if a parallel resistor is present)** and proceed as before or utilize a *supermesh* current and proceed as follows.

- **Start as before** and **assign a mesh current** to each **independent loop**, **including the current sources**, as if they were resistors or voltage sources.
- Then mentally (redraw the network if necessary) **remove the current sources (replace with open-circuit equivalents)**, and
- **Apply Kirchhoff's voltage law** to all the **remaining independent paths** of the network using the **mesh currents** just defined.

Any resulting path, **including two or more mesh currents**, is said to be the path of **a supermesh current.**

- Then relate the chosen mesh currents of the network to the independent current sources of the network, and solve for the mesh currents.

EXAMPLE 9 Using mesh analysis, determine the currents for the network of Fig. 9.

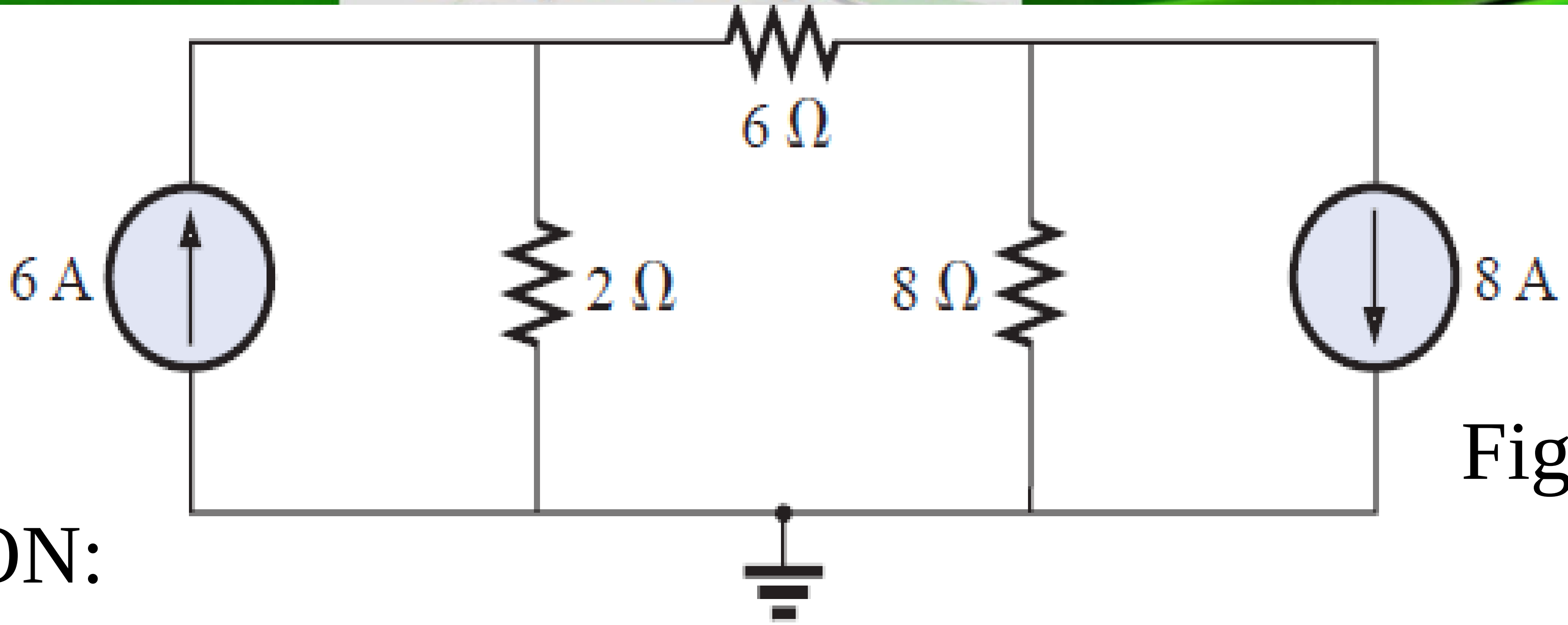


Fig. 9

SOLUTION:

The mesh currents are defined in Fig. 10.

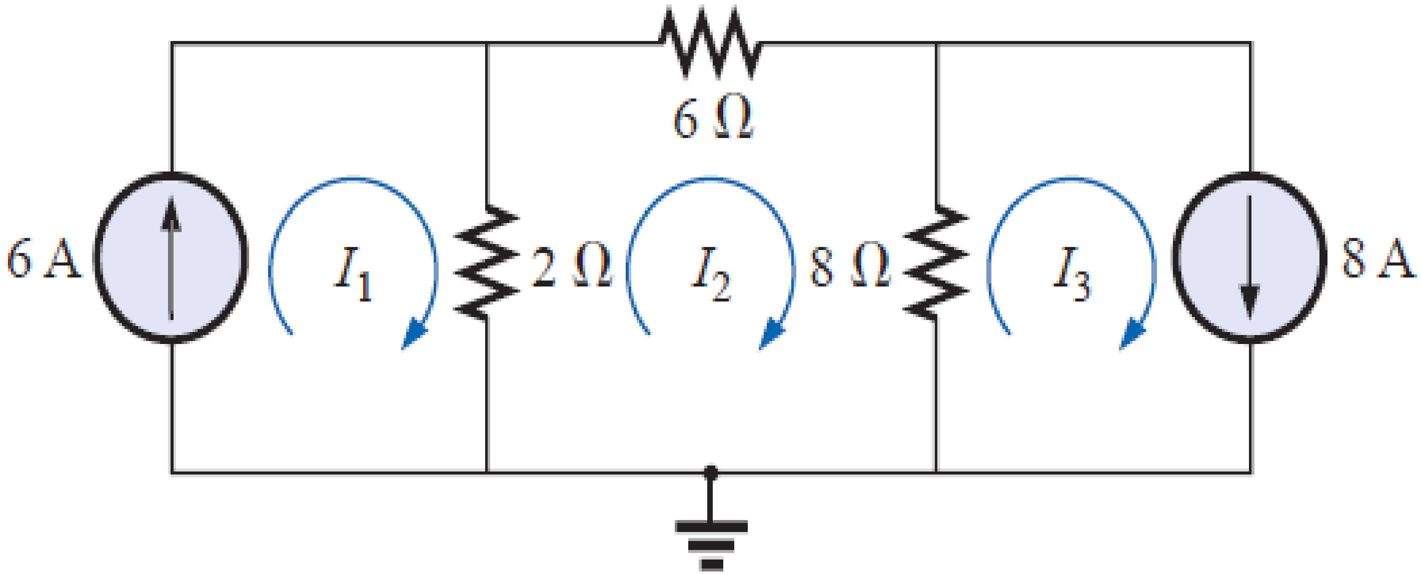


Fig. 10 Defining the mesh currents for the network

The current sources are removed, and the single supermesh path is defined in Fig. 11.

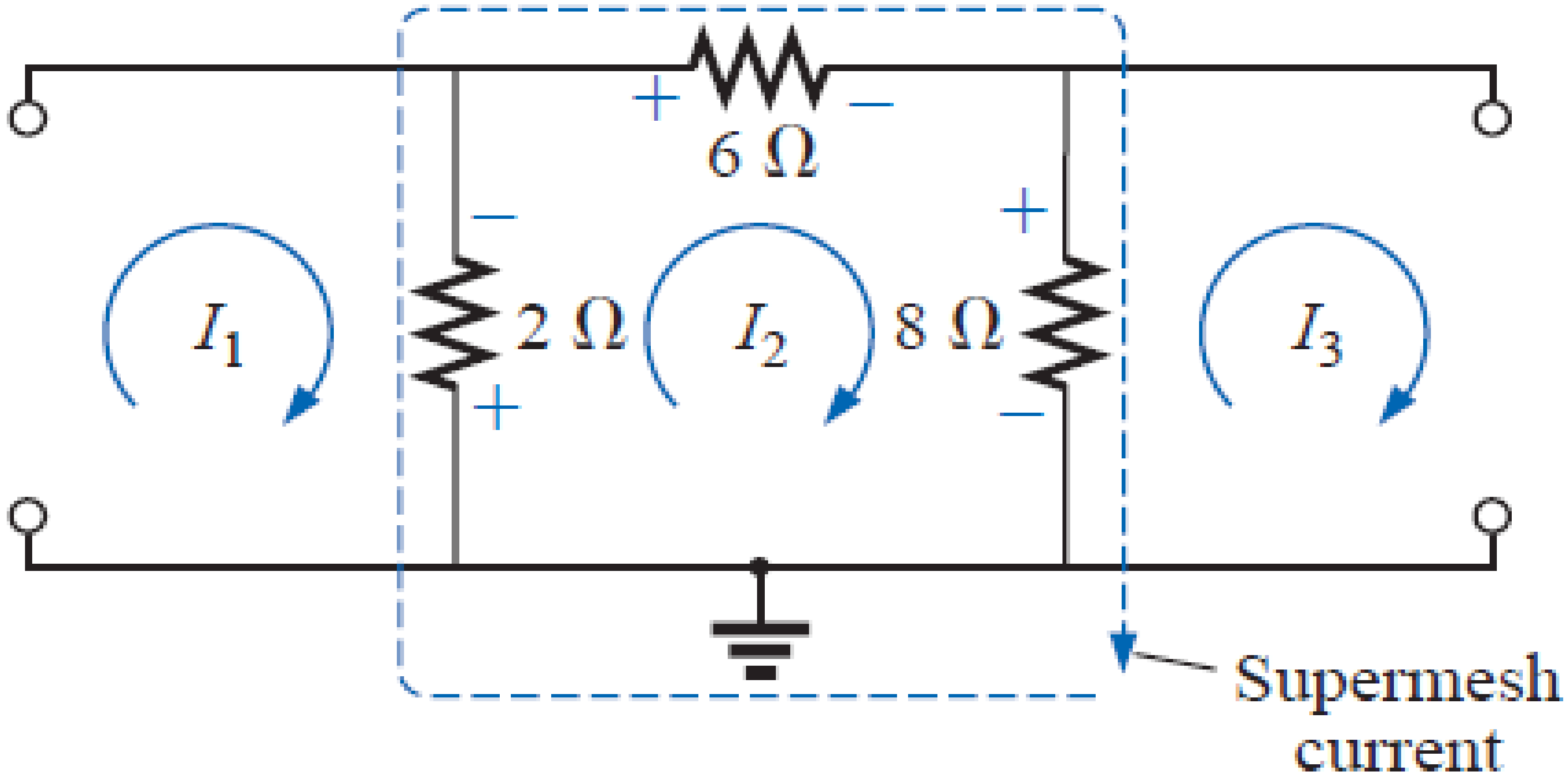


Fig. 11 *Defining the supermesh current for the Network*

Applying Kirchhoff's voltage law around the supermesh path:

$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$

$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$

$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$

$$2I_1 - 16I_2 + 8I_3 = 0$$

Introducing the relationship between the mesh currents and the current sources:

$$I_1 = 6\text{ A}$$

$$I_3 = 8\text{ A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$

$$2(6\text{ A}) - 16I_2 + 8(8\text{ A}) = 0$$

And

$$I_2 = \frac{76\text{ A}}{16} = \mathbf{4.75\text{ A}}$$

Then

$$I_{2\Omega}\downarrow = I_1 - I_2 = 6\text{ A} - 4.75\text{ A} = \mathbf{1.25\text{ A}}$$

And $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\ \text{A} - 4.75\ \text{A} = \mathbf{3.25\ A}$$

Again, note that you must stick with your **original definitions** of the various mesh currents when a**pplying Kirchhoff's voltage law** around the resulting **supermesh paths**.

MESH ANALYSIS (FORMAT APPROACH)

The **format approach** can be applied **only** to networks in **which all current sources have been converted to their equivalent voltage source.**

The below statements can be extended to develop the following *format approach* to mesh analysis:

1. Assign a loop current to each independent, closed loop in a clockwise direction.

2. The number of required equations is equal to the number of chosen independent, closed loops. Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.

3. We must now consider the mutual terms, which, as noted in the examples below, are always subtracted from the first column.

Mutual term **is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current.**

This will be demonstrated in an example to follow. Each term is the product of the mutual resistor and the other loop current passing through the same element.

4. **The column to the right of the equality sign is the *algebraic sum* of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal.**

A negative sign is assigned to those potentials for which the reverse is true.

5. **Solve the resulting simultaneous equations for the desired loop currents.**

EXAMPLE 10 Write the mesh equations for the network of Fig. (12) and find the current through the 7Ω resistor.

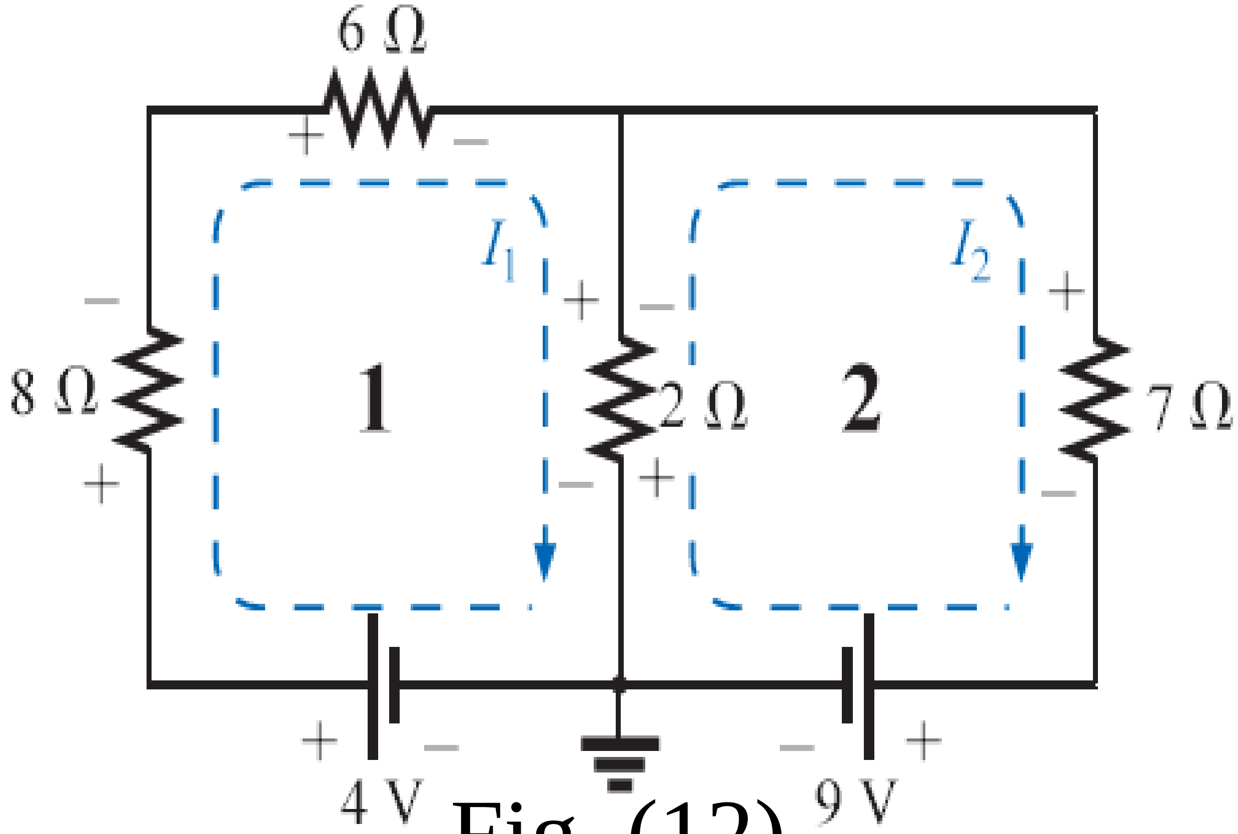


Fig. (12)

Solution:

Step 1: As indicated in Fig. 12, each assigned loop current has a clockwise direction.

Steps 2 to 4:

$$I_1: \quad (8\ \Omega + 6\ \Omega + 2\ \Omega)I_1 - (2\ \Omega)I_2 = 4\ \text{V}$$
$$I_2: \quad (7\ \Omega + 2\ \Omega)I_2 - (2\ \Omega)I_1 = -9\ \text{V}$$

$$16I_1 - 2I_2 = 4$$
$$9I_2 - 2I_1 = -9$$

which, for determinants, are

$$\begin{aligned} 16I_1 - 2I_2 &= 4 \\ -2I_1 + 9I_2 &= -9 \end{aligned}$$

and
$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}$$
$$= \mathbf{-0.971\ A}$$

<u>EXAMPLE 11</u> Find the current through the 10-Ω resistor of the network of Fig. 13.

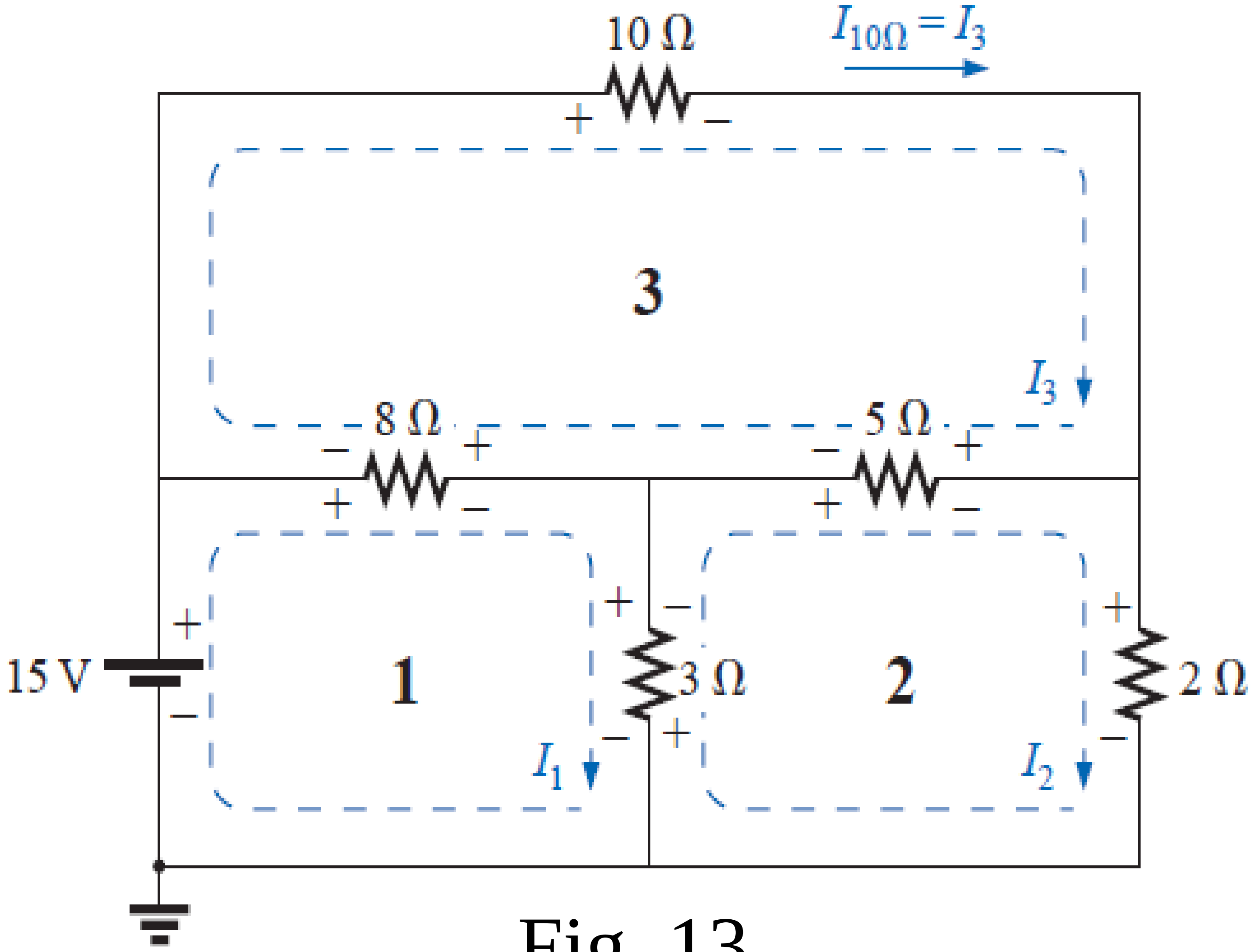


Fig. 13

Solution:

$$I_1: \quad (8\,\Omega + 3\,\Omega)I_1 - (8\,\Omega)I_3 - (3\,\Omega)I_2 = 15\text{ V}$$

$$I_2: \quad (3\,\Omega + 5\,\Omega + 2\,\Omega)I_2 - (3\,\Omega)I_1 - (5\,\Omega)I_3 = 0$$

$$I_3: \quad (8\,\Omega + 10\,\Omega + 5\,\Omega)I_3 - (8\,\Omega)I_1 - (5\,\Omega)I_2 = 0$$

$$11I_1 - 8I_3 - 3I_2 = 15$$

$$10I_2 - 3I_1 - 5I_3 = 0$$

$$23I_3 - 8I_1 - 5I_2 = 0$$

Or

$$
\begin{array}{cccc}
 & C & A & B \\
A & 11I_1 - & 3I_2 - & 8I_3 = 15 \\
 & -3I_1 + & 10I_2 - & 5I_3 = 0 \\
B & -8I_1 - & 5I_2 + & 23I_3 = 0
\end{array}
$$

Note that the **coefficients of the *A* and *B* diagonals are equal**. This ***symmetry*** about the ***C*-axis** will always **be true for equations written using the format approach**. It is **a check** on whether the equations were obtained **correctly.**

$$I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \mathbf{1.220\ A}$$

3- NODAL ANALYSIS (GENERAL APPROACH)

We will now employ **Kirchhoff's current law** to develop a method referred to as **nodal analysis.**

A **node** is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, **a point of zero potential or ground**), the remaining nodes of the network will all have a fixed potential relative to this reference.

For a network of N nodes, therefore, there will exist **(N -1) nodes** with a fixed potential relative to the assigned reference node. **Equations** relating these nodal voltages can be written by applying **Kirchhoff's current law** at each of the (N -1) nodes.

The nodal analysis method is applied as follows:

1. Determine the number of nodes within the network.

2. Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.

3. Apply Kirchhoff's current law at each node except the reference.

Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.

4. Solve the resulting equations for the nodal voltages.

EXAMPLE 12 Apply nodal analysis to the network of Fig. 14.

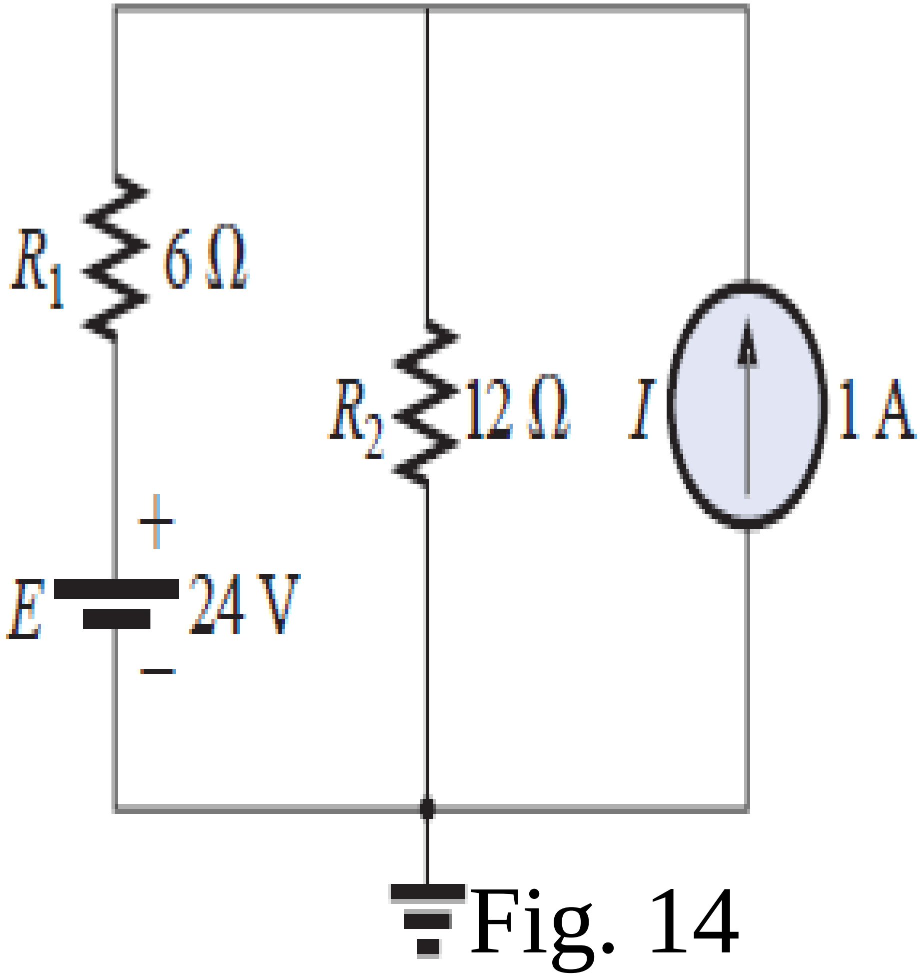


Fig. 14

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 15.

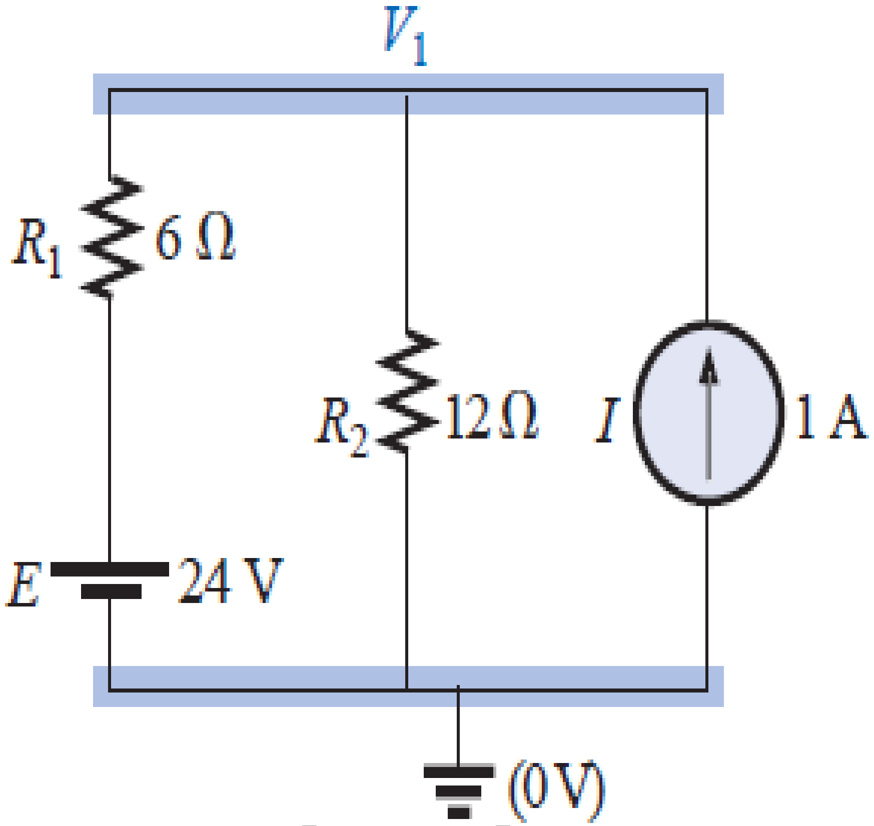


Fig. 15 Network with assigned nodes.

The lower node is defined as the reference node at **ground potential (zero volts)**, and the **other node as** V_1, the voltage from node 1 to ground.

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 16, and **Kirchhoff's current law** is applied as follows:

$$I = I_1 + I_2$$

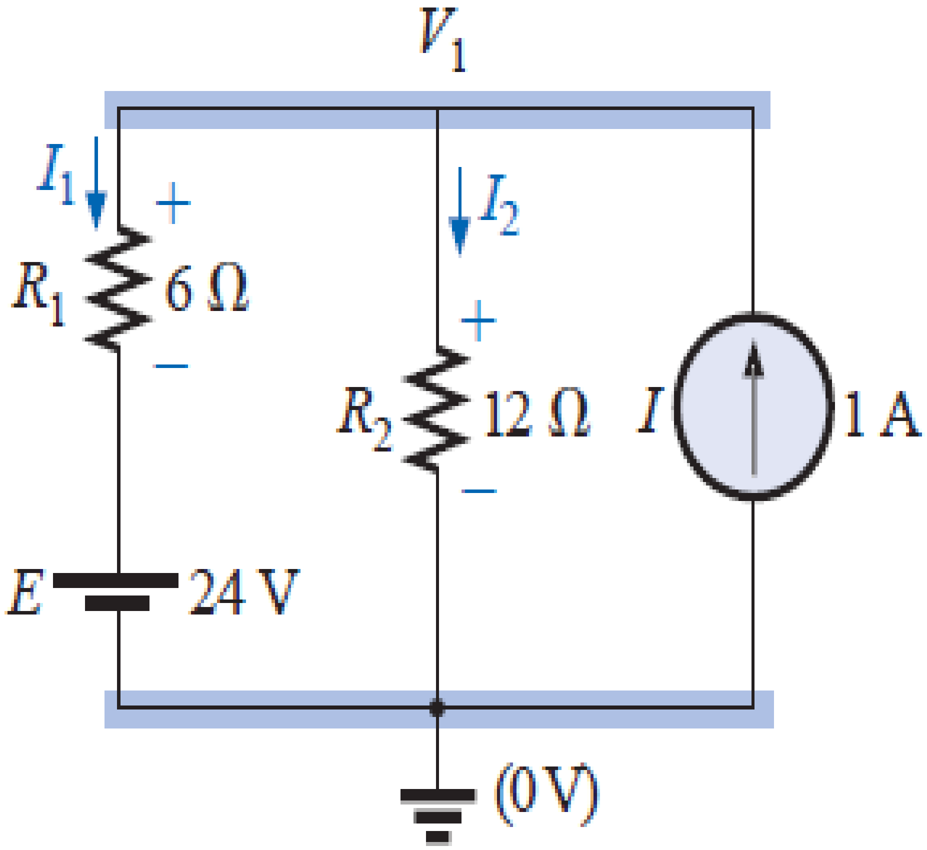


Fig. 16 *Applying Kirchhoff's current law to the node* V_1.

The current I_2 is related to the nodal voltage V_1 by **Ohm's law**:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by **Ohm's law** as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

With $V_{R_1} = V_1 - E$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

Or $V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + I$

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega}\right) = \frac{24\ \text{V}}{6\ \Omega} + 1\ \text{A} = 4\ \text{A} + 1\ \text{A}$$

$$V_1\left(\frac{1}{4\ \Omega}\right) = 5\ \text{A}$$

$$V_1 = \mathbf{20\ V}$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\ \text{V} - 24\ \text{V}}{6\ \Omega} = \frac{-4\ \text{V}}{6\ \Omega}$$

$$= \mathbf{-0.667\ A}$$

The **minus sign** indicates simply that the current I_1 has a direction opposite to that appearing in Fig. 16.

$$I_2 = \frac{V_1}{R_2} = \frac{20\ \text{V}}{12\ \Omega} = \mathbf{1.667\ A}$$

EXAMPLE 13 Determine the nodal voltages for the network of Fig. 17.

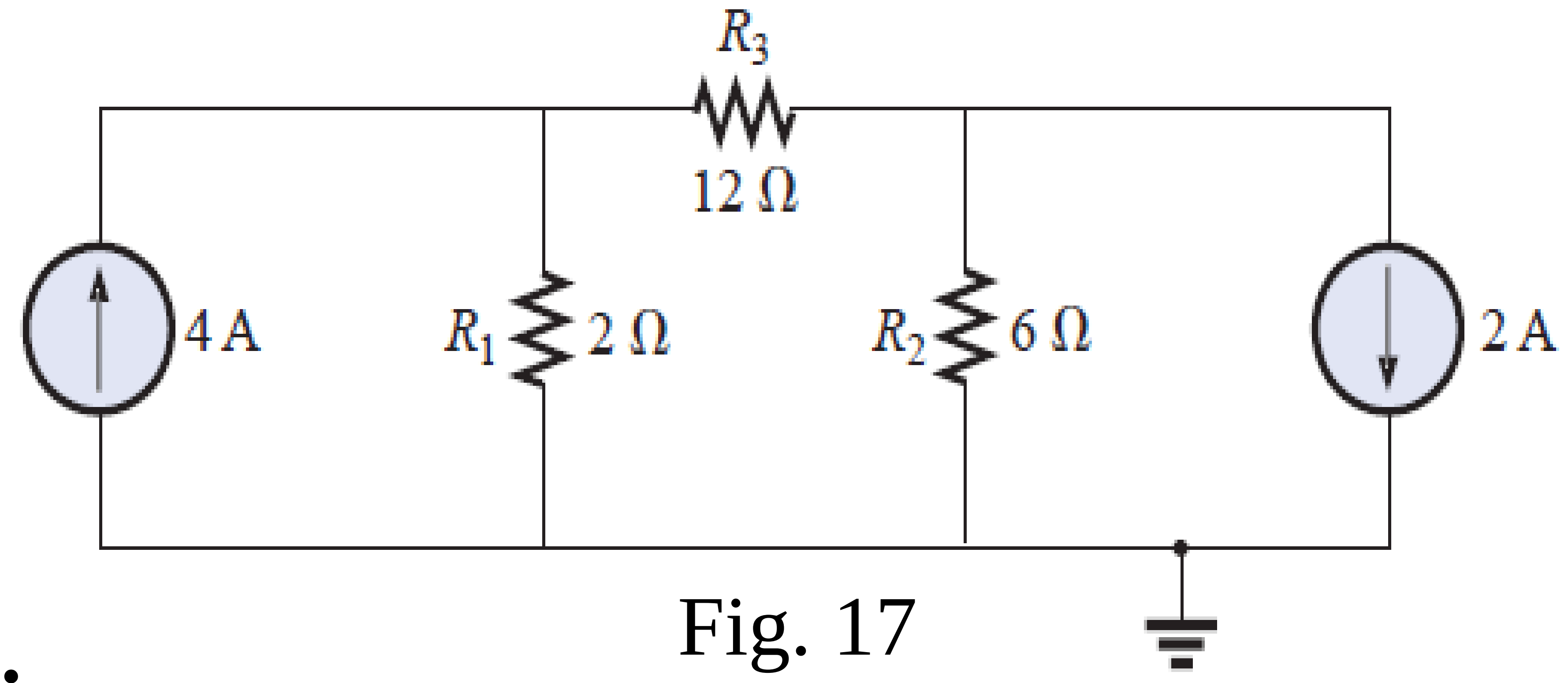


Fig. 17

Solution:

Steps 1 and 2: As indicated in Fig. 18.

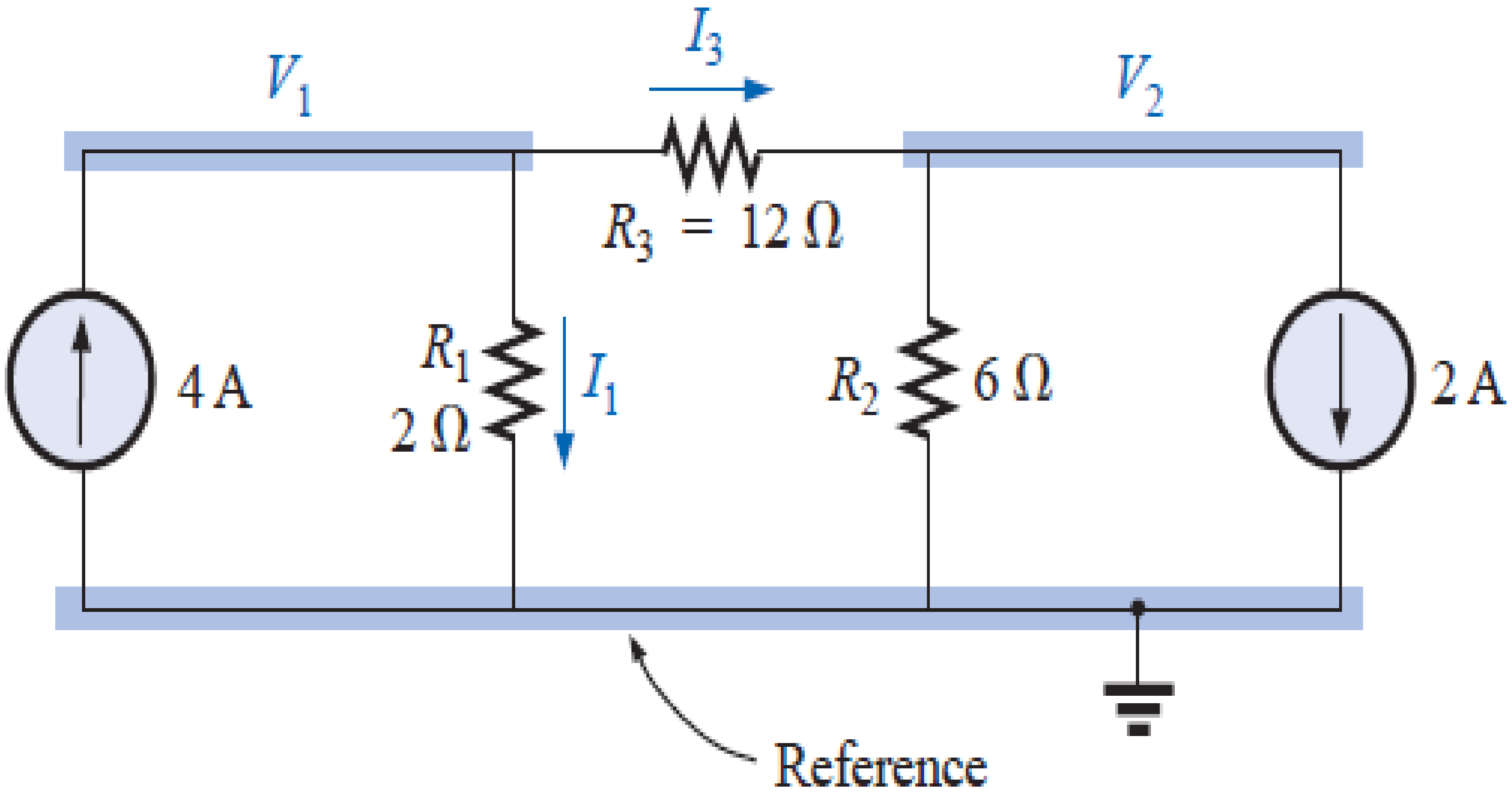


Fig. 18 *Defining the nodes and applying Kirchhoff's current law to the node* V_1.

Step 3: Included in Fig. 18 for the node V_1. Applying Kirchhoff's current law:

$$4\text{ A} = I_1 + I_3$$

And
$$4\text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2\,\Omega} + \frac{V_1 - V_2}{12\,\Omega}$$

Expanding and rearranging:

$$V_1\left(\frac{1}{2\,\Omega} + \frac{1}{12\,\Omega}\right) - V_2\left(\frac{1}{12\,\Omega}\right) = 4\text{ A}$$

For node V_2 the currents are defined as in Fig. 19.

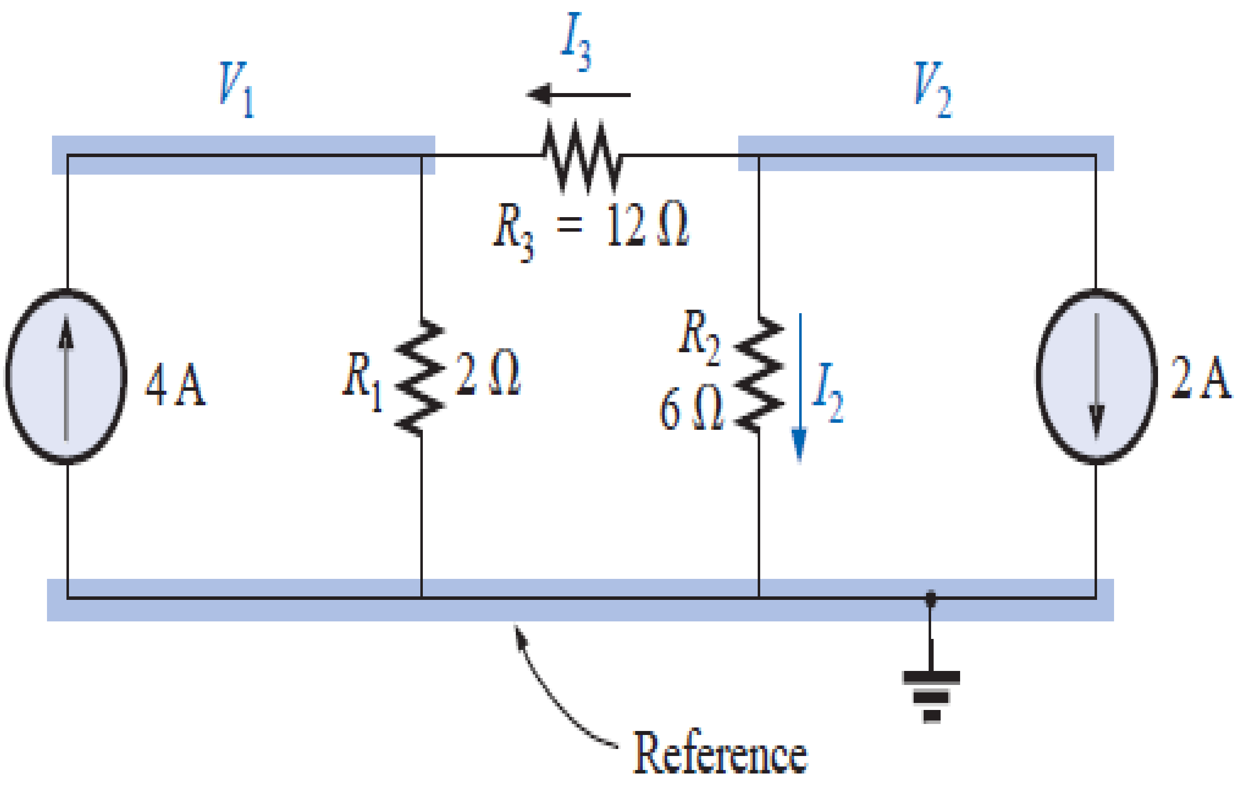


Fig. 19 *Applying Kirchhoff's current law to the node* V_2.

Applying Kirchhoff's current law:

$$0 = I_3 + I_2 + 2\,\text{A}$$

And $$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2\,\text{A} = 0 \rightarrow \frac{V_2 - V_1}{12\,\Omega} + \frac{V_2}{6\,\Omega} + 2\,\text{A} = 0$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) = -2\,\text{A}$$

resulting in two equations and two unknowns (numbered for later reference):

$$V_1\left(\frac{1}{2\,\Omega}+\frac{1}{12\,\Omega}\right)-V_2\left(\frac{1}{12\,\Omega}\right)=+4\text{ A}$$

$$V_2\left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}\right)-V_1\left(\frac{1}{12\,\Omega}\right)=-2\text{ A}$$

Producing

$$\frac{7}{12}V_1-\frac{1}{12}V_2=+4$$

$$-\frac{1}{12}V_1+\frac{3}{12}V_2=-2$$

Then, $7V_1 - V_2 = 48$

$$-1V_1 + 3V_2 = -24$$

And

$$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = \mathbf{+6\ V}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = \mathbf{-6\ V}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\,\text{V} - (-6\,\text{V})}{12\,\Omega} = \frac{12\,\text{V}}{12\,\Omega} = \mathbf{1\,A}$$

The fact that V_1 is positive results in a current I_{R1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\,\text{V}}{2\,\Omega} = \mathbf{3\,A}$$

Finally, since V_2 is negative, the current I_{R2} flows from ground to V_2 and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\ \mathrm{V}}{6\ \Omega} = \mathbf{1\ A}$$

Supernode

On occasion there will be **independent voltage sources** in the network to which **nodal analysis** is to be applied. In such cases we can **convert the voltage source to a current source (if a series resistor is present)** and **proceed as before**, **or** we can introduce the concept of a ***supernode*** and proceed as follows.

1. **Start as before and assign a nodal voltage to each independent node of the network, including each independent voltage source as if it were a resistor or current source.**
2. **Then mentally replace the independent voltage sources with short-circuit equivalents, and**

3. **Apply Kirchhoff's current law to the defined nodes of the network.** Any node including the effect of elements tied only to other nodes is referred to as a supernode (since it has an additional number of terms).

4. **Finally, relate the defined nodes to the independent voltage sources of the network, and solve for the nodal voltages.**

EXAMPLE 14 Determine the nodal voltages V_1 and V_2 of Fig. 20 using the concept of a supernode.

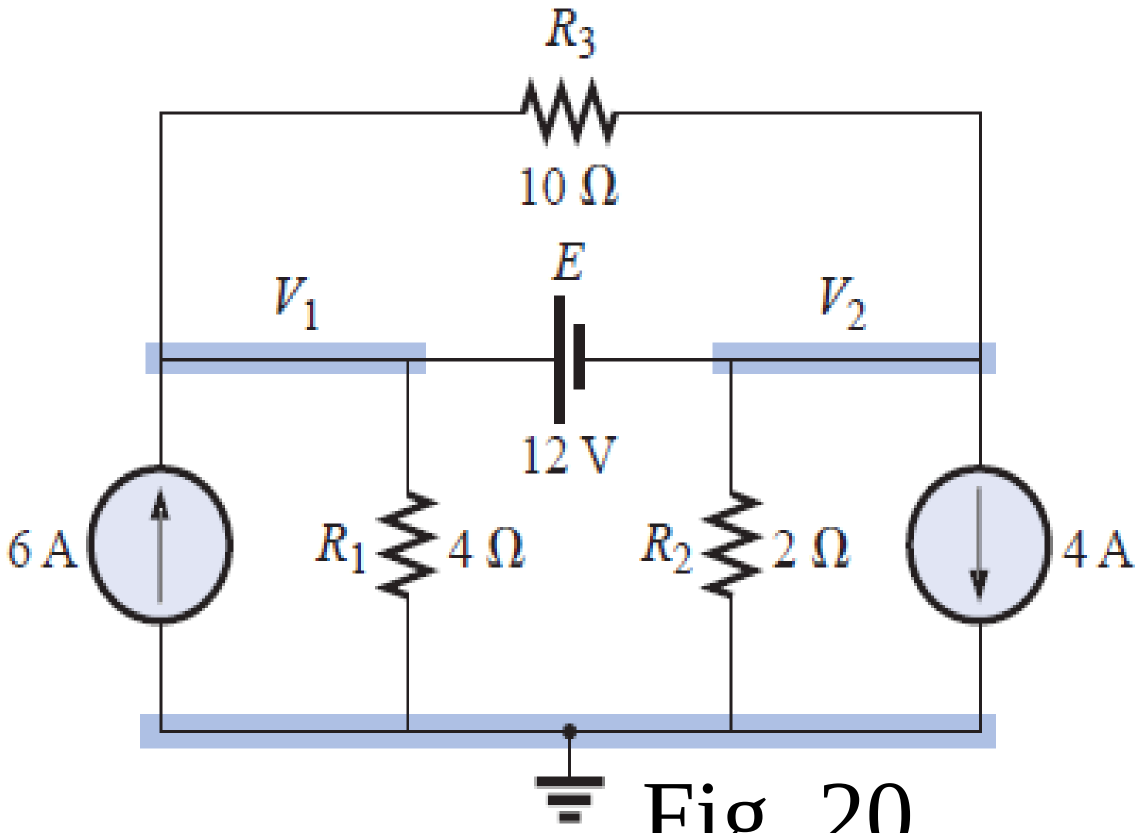


Fig. 20

Solution: Replacing the independent voltage source of 12 V with a short-circuit equivalent will result in the network of Fig. 21.

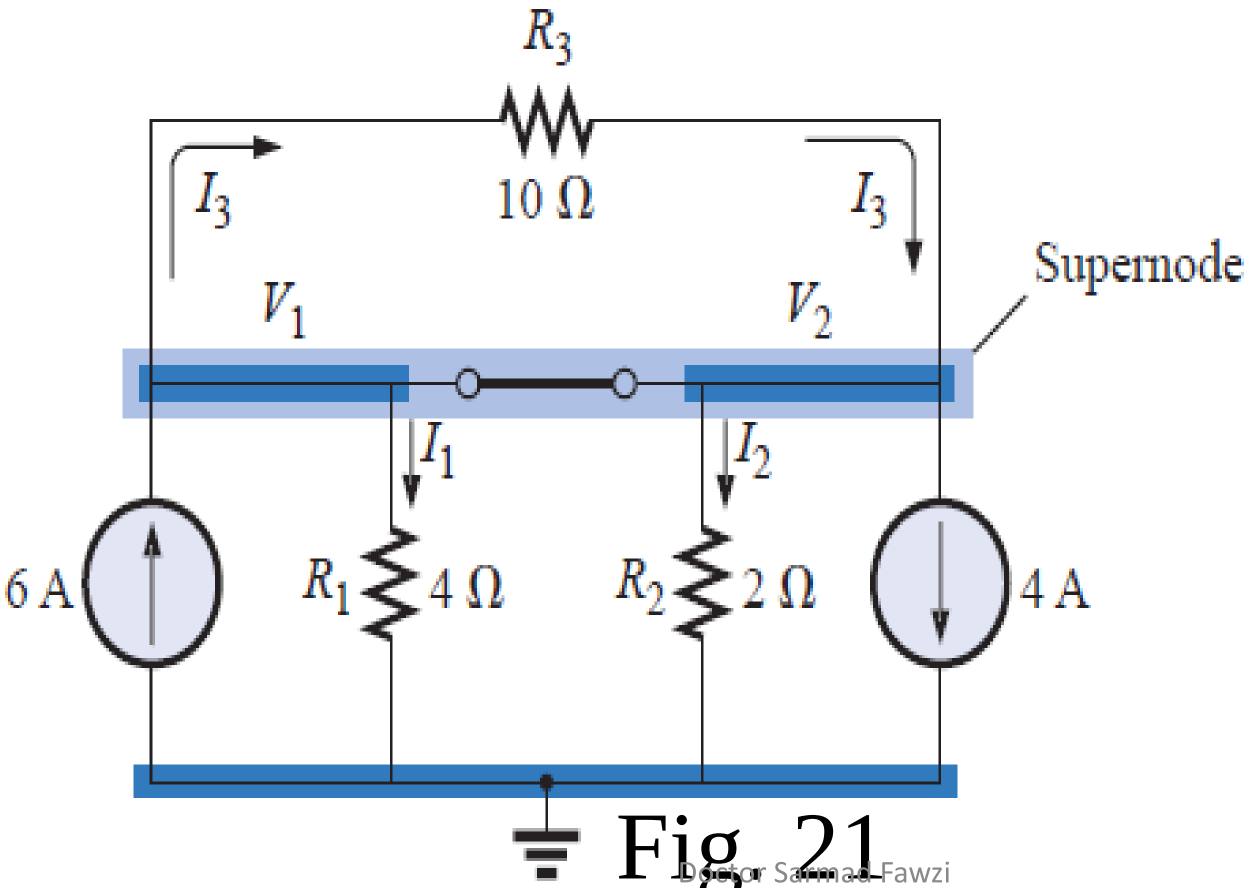


Fig. 21

The result is **a single supernode** for which Kirchhoff's current law must be applied.

Be sure to leave the other defined nodes in place and use them to define the currents from that region of the network.

In particular (in detail), note that the **current I_3** will **leave the supernode at V_1** and then **enter the same supernode at V_2**. It must therefore **appear twice** when applying **Kirchhoff's current law**, as shown below:

$$\sum I_i = \sum I_o$$

$$6\,\text{A} + I_3 = I_1 + I_2 + 4\,\text{A} + I_3$$

Or $I_1 + I_2 = 6\,\text{A} - 4\,\text{A} = 2\,\text{A}$

Then $\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} = 2\,\text{A}$

And $\dfrac{V_1}{4\,\Omega} + \dfrac{V_2}{2\,\Omega} = 2\,\text{A}$

Relating the defined nodal voltages to the independent voltage source, ربط الجهد العقدي المحدد بمصدر الجهد المستقل

we have $V_1 - V_2 = E = 12\text{ V}$

which results in two equations and two unknowns:

$$\begin{aligned} 0.25V_1 + 0.5V_2 &= 2 \\ V_1 - \quad 1V_2 &= 12 \end{aligned}$$

Substituting: حل محل آخر

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

And $0.75V_2 = 2 - 3 = -1$

so that.لهذا السبب

$$V_2 = \frac{-1}{0.75} = \mathbf{-1.333\ V}$$

And $V_1 = V_2 + 12\text{ V} = -1.333\text{ V} + 12\text{ V} = \mathbf{+10.667\ V}$

The current of the network can then be determined as follows:
يمكن بعد ذلك تحديد تيار الشبكة على النحو التالي:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.667\ \text{V}}{4\ \Omega} = \mathbf{2.667\ A}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.333\ \text{V}}{2\ \Omega} = \mathbf{0.667\ A}$$

$$\underset{\rightarrow}{I_3} = \frac{V_1 - V_2}{10\ \Omega} = \frac{10.667\ \text{V} - (-1.333\ \text{V})}{10\ \Omega} = \frac{12\ \text{V}}{10\ \Omega} = \mathbf{1.2\ A}$$

NODAL ANALYSIS (FORMAT APPROACH)

A **major requirement,** however, is that ***all voltage sources must first be converted to current sources before the procedure is applied.*** Note the parallelism تماثلbetween the following four steps of application and those required for mesh analysis.

1. Choose a reference node and assign a subscripted voltage label to the (N _ 1) remaining nodes of the network.

2. The number of equations required for a complete solution is equal to the number of subscripted voltages (N_1). Column 1 of each equation is formed by summing the conductances tied(connected) to the node of interest and multiplying the result by that subscripted nodal voltage

3. We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term* مصطلح مشترك *if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This will be demonstrated in an example to follow* سيتم توضيح ذلك في مثال يتبعه*. Each mutual term is the product of the mutual conductance* سلوك متبادل *and the other nodal voltage tied to that conductance.

4. The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.

5. Solve the resulting simultaneous equations for the desired voltages.

Let us now consider a few examples. .دعونا ننظر الآن في بعض الأمثلة

EXAMPLE 15 Write the nodal equations for the network of Fig. 22.

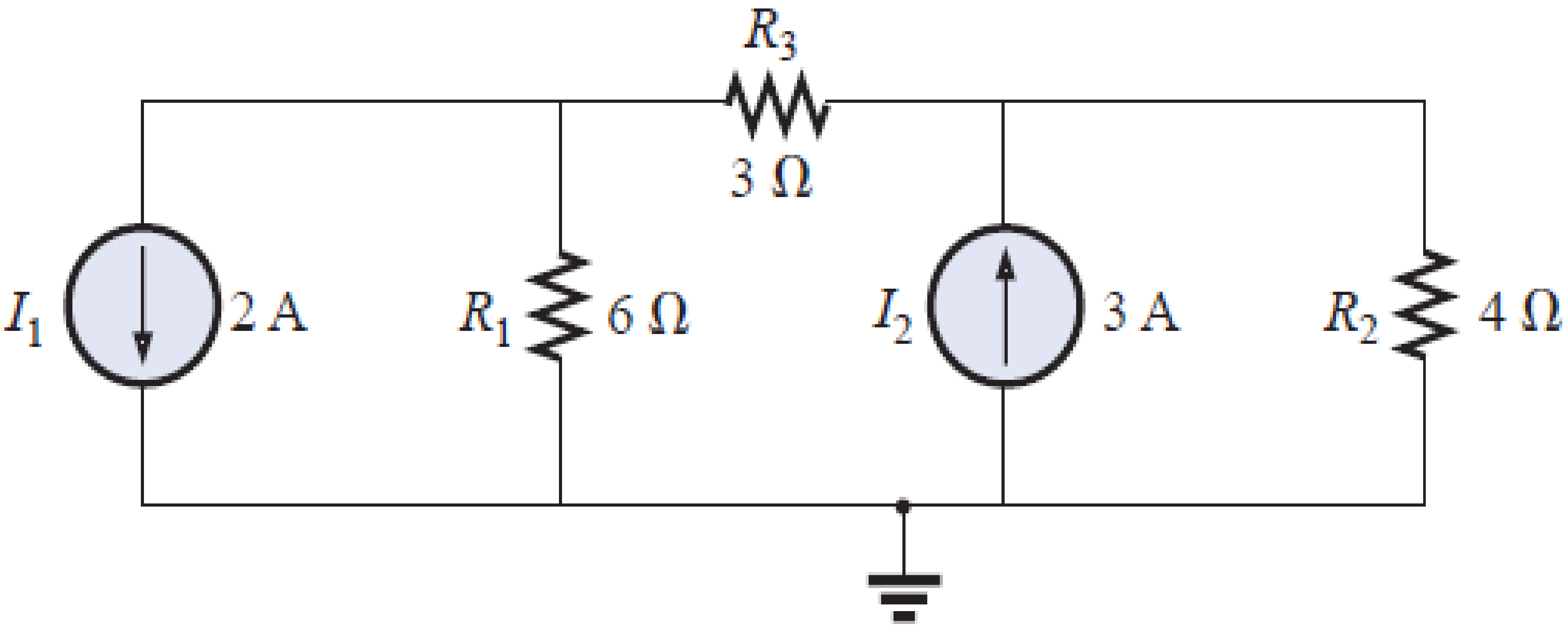


Fig. 22

Solution:

Step 1: The figure is redrawn with assigned subscripted voltages in Fig. 23.

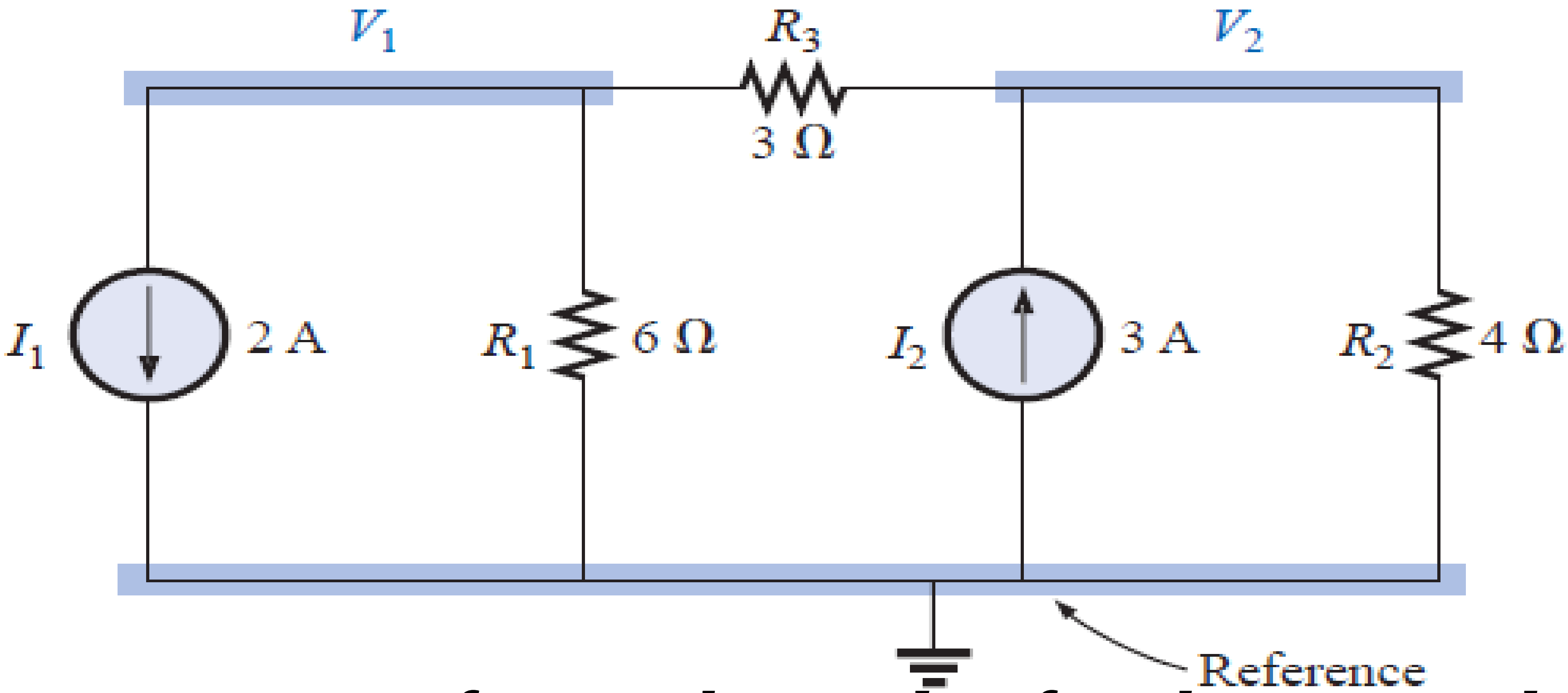


Fig. 23 *Defining the nodes for the network.*

Steps 2 to 4:

$$V_1:\quad \underbrace{\left(\frac{1}{6\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}} V_1-\underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_2=\overset{\substack{\text{Drawing current}\\ \text{from node 1}\\ \downarrow}}{-2\text{ A}}$$

$$V_2:\quad \underbrace{\left(\frac{1}{4\,\Omega}+\frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}}V_2-\underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}}V_1=\overset{\substack{\text{Supplying current}\\ \text{to node 2}\\ \downarrow}}{+3\text{ A}}$$

And

$$\frac{1}{2}V_1-\frac{1}{3}V_2=-2$$

$$-\frac{1}{3}V_1+\frac{7}{12}V_2=3$$

We can find V_1 and V_2 by using the determinants.

EXAMPLE 16 Find the voltage across the 3-Ω resistor of Fig. 24 by nodal analysis.

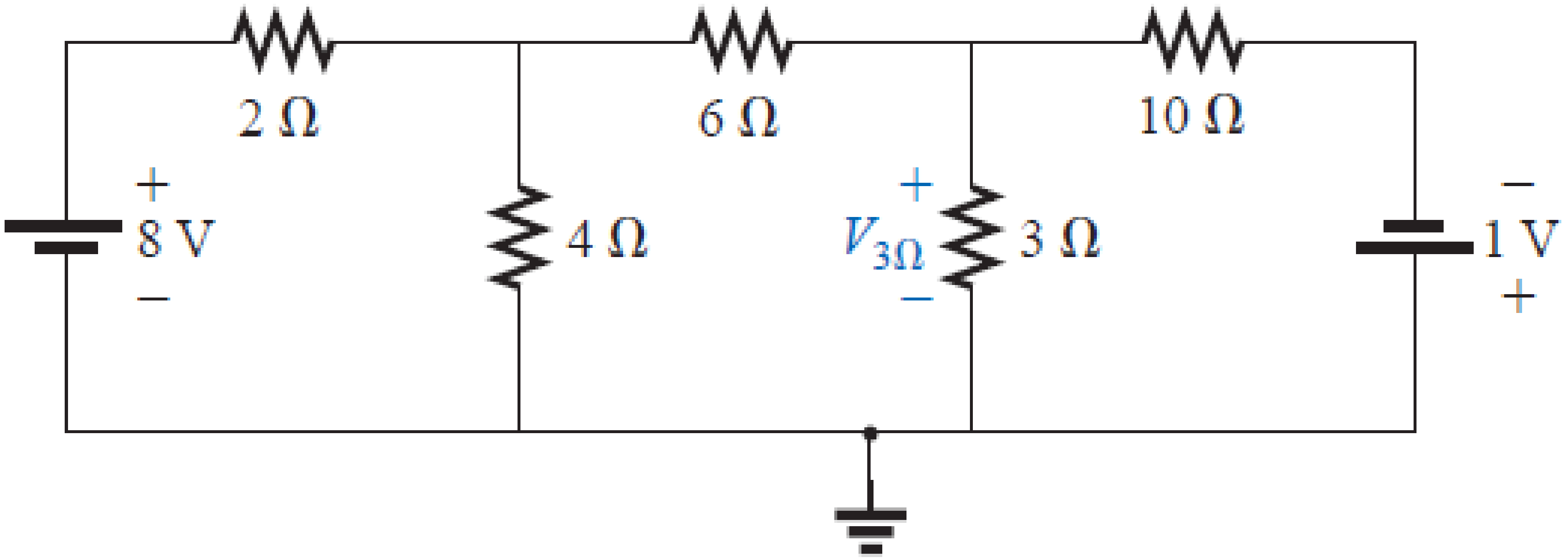


Fig. 24

Solution: Converting sources (fig. 25)

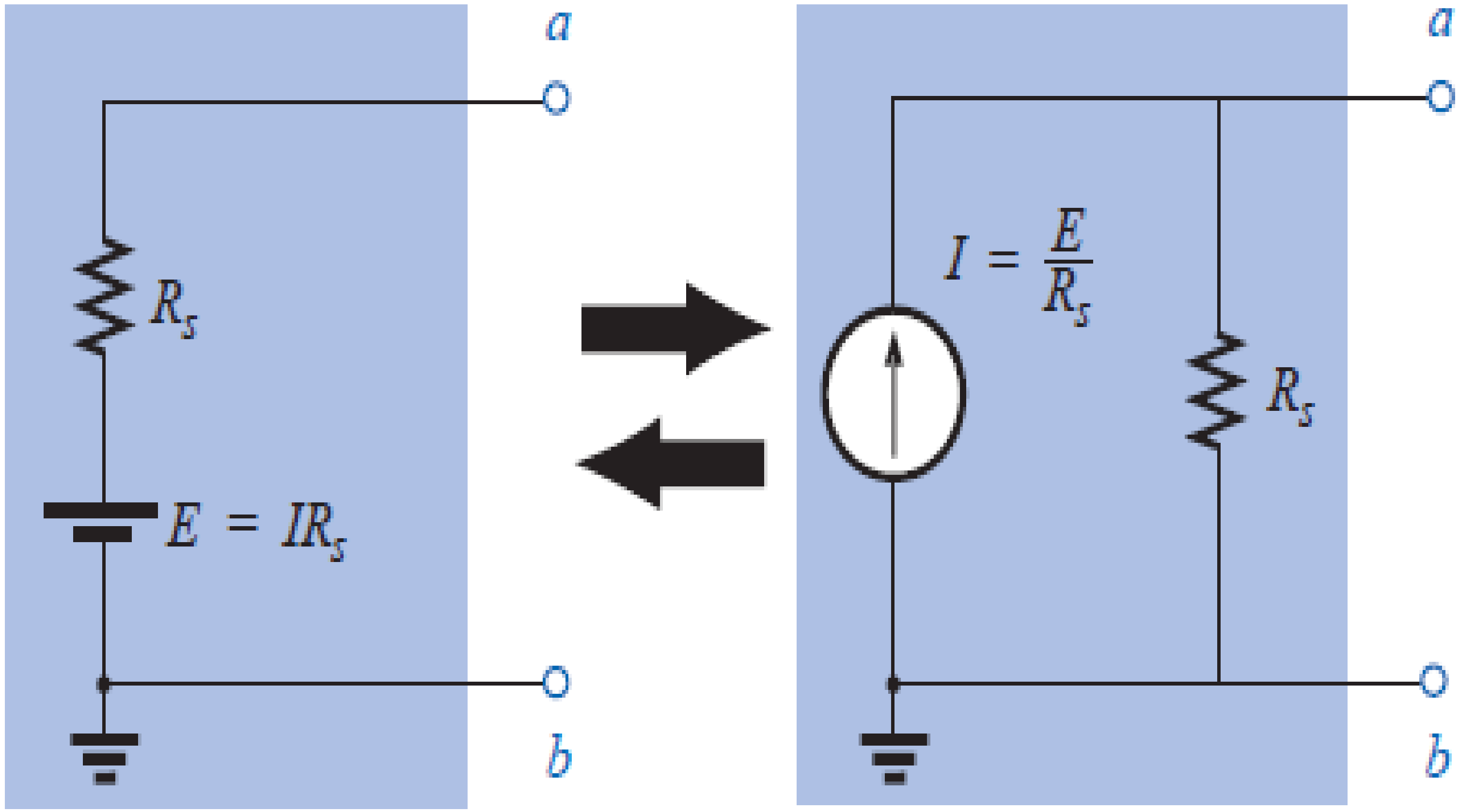


fig. 25 Source conversion.

and choosing nodes (Fig. 26), we have

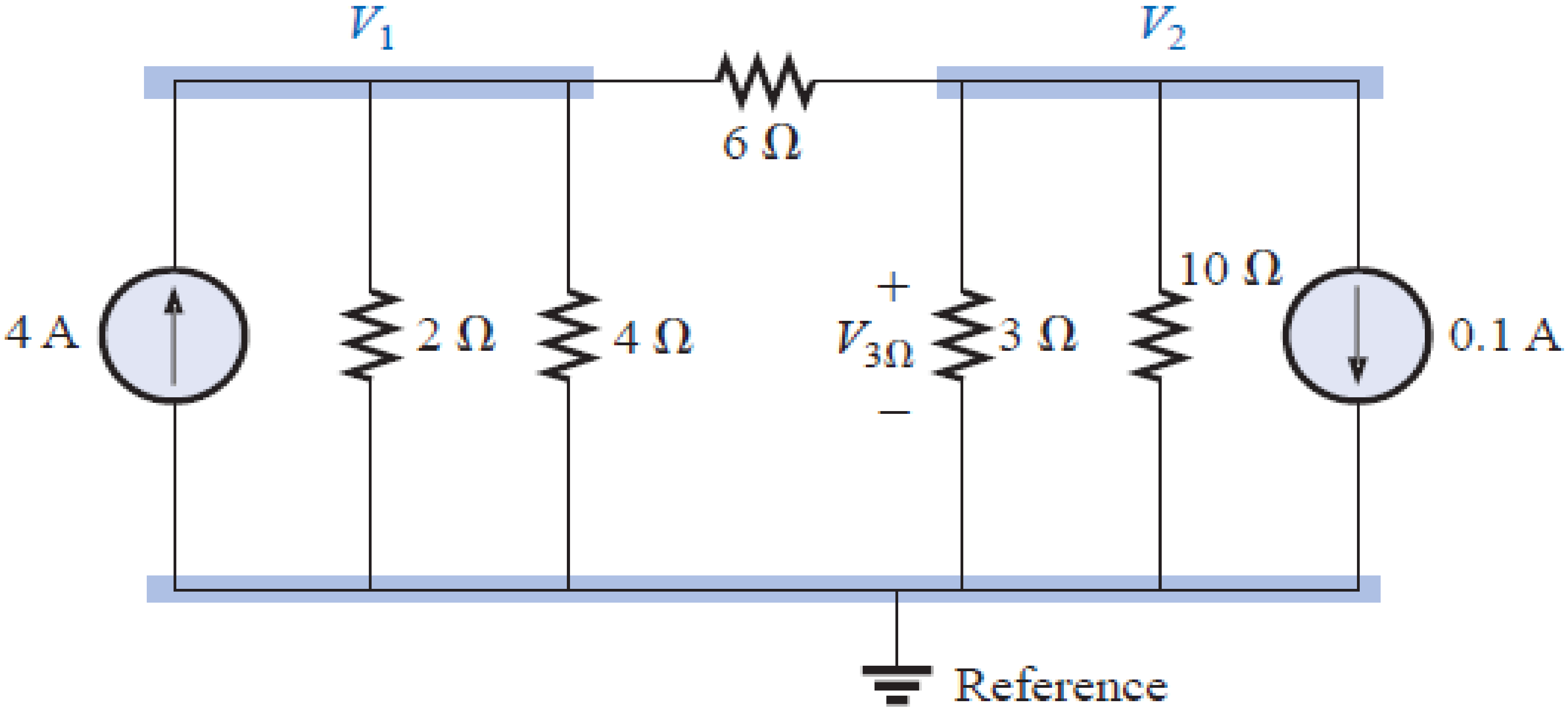


Fig. 26 *Defining the nodes for the network.*

$$\left.\begin{aligned}\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}\right)V_1-\left(\frac{1}{6\,\Omega}\right)V_2=+4\text{ A}\\ \left(\frac{1}{10\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}\right)V_2-\left(\frac{1}{6\,\Omega}\right)V_1=-0.1\text{ A}\end{aligned}\right\}$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

resulting in مما يؤدي الى

by multiplying the top equation by 30 and the bottom equation by 30, we obtain

$$11V_1 - \ 2V_2 = +48$$
$$-5V_1 + 18V_2 = -3$$

And

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \mathbf{1.101\ V}$$

As demonstrated for كما هو موضح ل **mesh analysis, nodal analysis** التحليل العقدي can also be a very useful technique for solving networks with **only one source**.

EXAMPLE 17 Using nodal analysis, determine the potential across the 4-Ω resistor in Fig. 27.

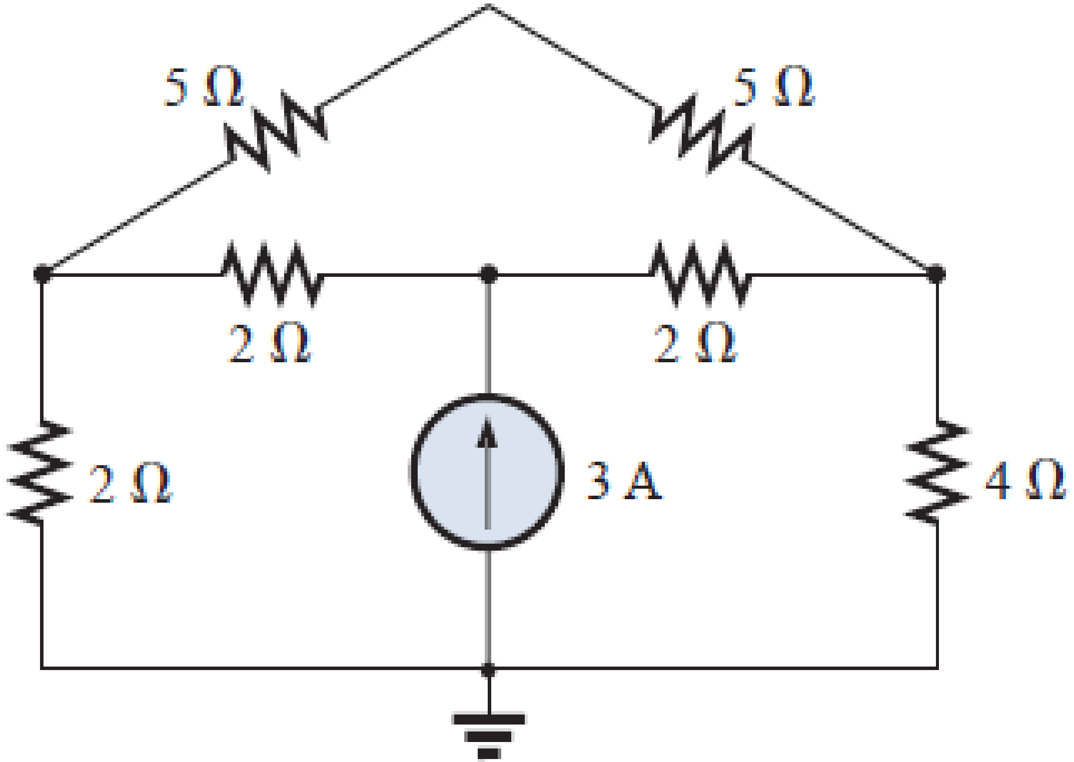


Fig. 27

Solution: The **reference** and **four subscripted voltage levels** were chosen as shown in Fig. 28.

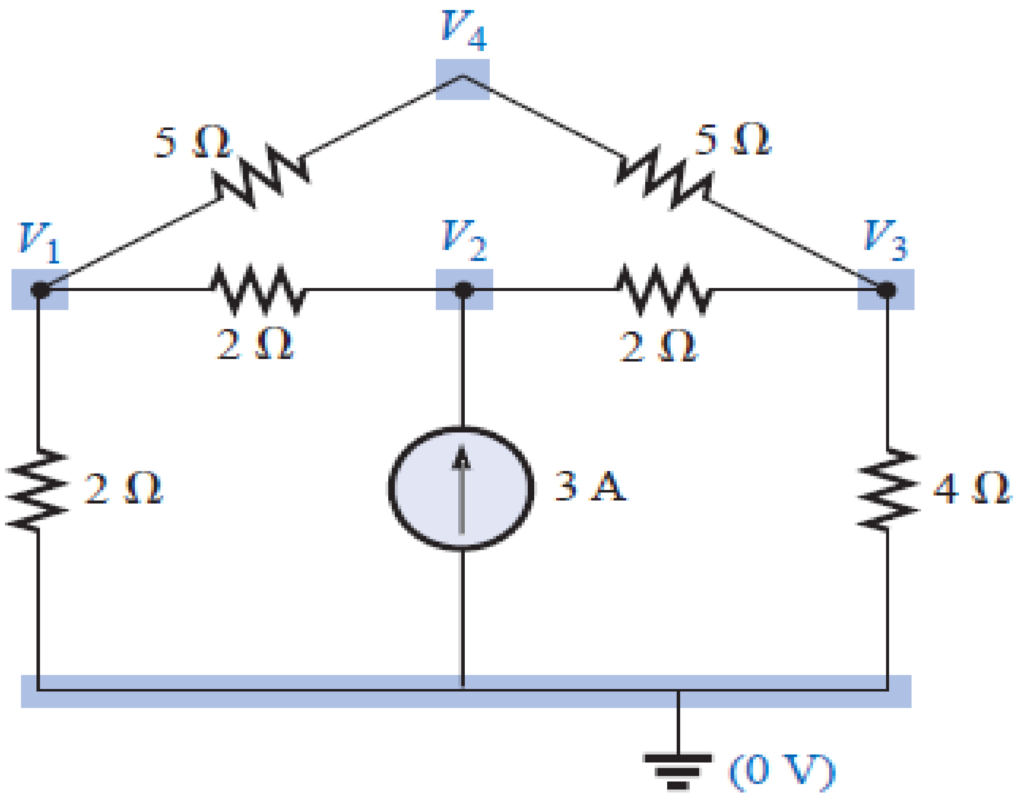


Fig. 28 *Defining the nodes for the network.*

A moment of reflection should reveal يجب أن تكشف لحظة من التأمل that for any difference in potential between $\boldsymbol{V_1}$ **and** $\boldsymbol{V_3}$, the **current through** and the **potential drop across each 5-Ω resistor** will be the **same**.

Therefore, V_4 is simply a **midvoltage** level between V_1 and V_3 and is known if V_1 and V_3 are available.

We will therefore **not include** it in **a nodal voltage** and will redraw the network as shown in Fig. 29. Understand, **however, that V_4 can be included if desired**, although **four nodal voltages** will result rather than بدلا من the **three** to be obtained in the solution of **this problem**.

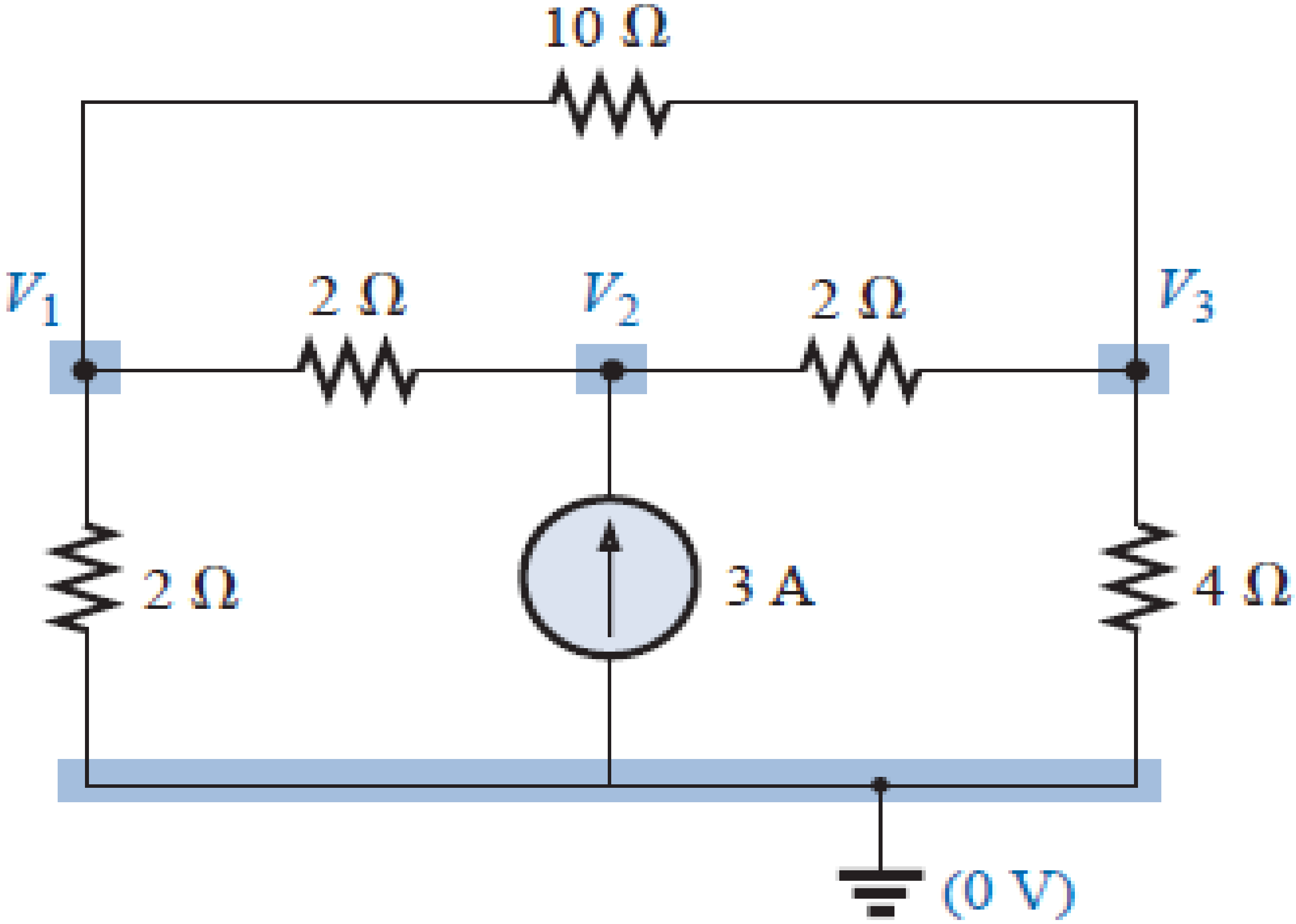


Fig. 29 *Reducing the number of nodes for the network by combining the two 5-Ω resistors.*

$$V_1: \quad \left(\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{10\,\Omega}\right)V_1-\left(\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{10\,\Omega}\right)V_3=0$$

$$V_2: \quad \left(\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{2\,\Omega}\right)V_1-\left(\frac{1}{2\,\Omega}\right)V_3=3\text{ A}$$

$$V_3: \quad \left(\frac{1}{10\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}\right)V_3-\left(\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{10\,\Omega}\right)V_1=0$$

which are rewritten as والتي يتم إعادة كتابتها ب

$$1.1V_1 - 0.5V_2 - 0.1V_3 = 0$$

$$V_2 - 0.5V_1 - 0.5V_3 = 3$$

$$0.85V_3 - 0.5V_2 - 0.1V_1 = 0$$

For determinants,

$$\begin{array}{llll}
c & b & a & \\
1.1V_1 & -\ 0.5V_2 & -\ 0.1V_3 & = 0 \\
b & & & \\
-0.5V_1 & +\ 1V_2 & -\ 0.5V_3 & = 3 \\
a & & & \\
-0.1V_1 & -\ 0.5V_2 & +\ 0.85V_3 & = 0
\end{array}$$

Before continuing, note the **symmetry** about the **major diagonal** in the equation above. Recall a **similar result for mesh analysis**.

Keep this thought in mind as a check on future applications of nodal analysis.

ضع في اعتبارك هذا الفكر كتحقق من التطبيقات المستقبلية للتحليل العقدي.

$$V_3 = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = \mathbf{4.645\ V}$$

The next example has only one source applied to a **ladder network** شبكة سلم.

EXAMPLE 18 Write the nodal equations and find the voltage across the 2-Ω resistor for the network of Fig. 30.

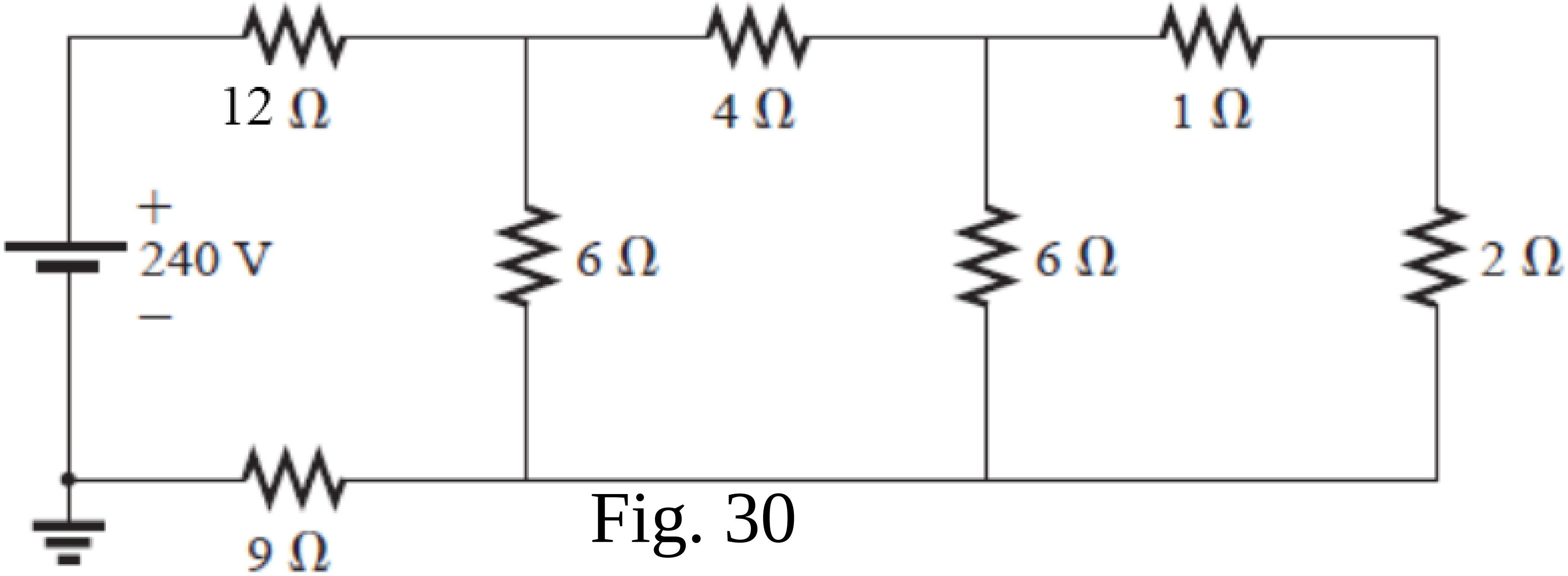


Fig. 30

Solution: The nodal voltages are chosen as shown in Fig. 31.

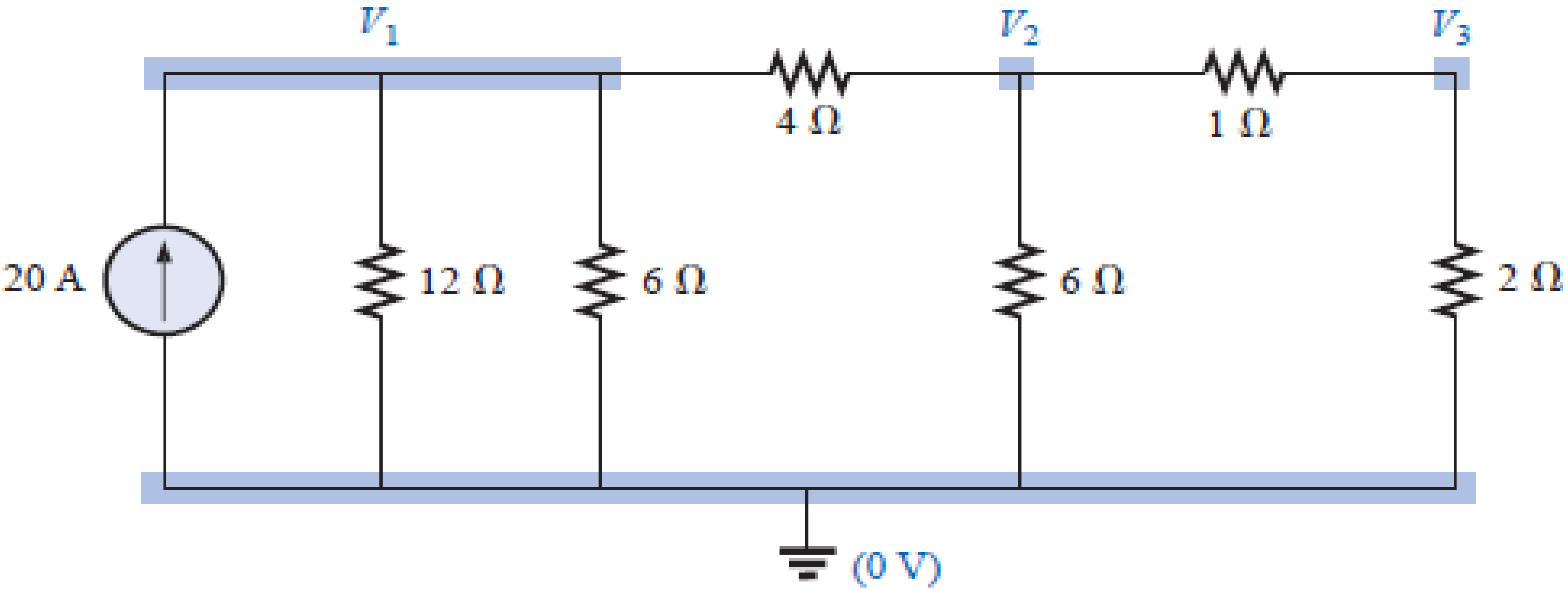


Fig. 31 *Converting the voltage source to a current source and defining the nodes for the network.*

$$V_1:\quad \left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{4\,\Omega}\right)V_2+\quad 0 \quad = 20\text{ V}$$

$$V_2:\quad \left(\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{1\,\Omega}\right)V_2-\left(\frac{1}{4\,\Omega}\right)V_1-\left(\frac{1}{1\,\Omega}\right)V_3=0$$

$$V_3:\quad \left(\frac{1}{1\,\Omega}+\frac{1}{2\,\Omega}\right)V_3-\left(\frac{1}{1\,\Omega}\right)V_2+\quad 0 \quad =0$$

And

$$\begin{aligned}0.5V_1-0.25V_2+0 \quad &= 20\\ -0.25V_1+\frac{17}{12}V_2 \quad -1V_3 &= 0\\ 0 \quad -1V_2 \quad +1.5V_3 &= 0\end{aligned}$$

Note the symmetry present about the major axis. Application of determinants reveals that

$$V_3 = V_{2\Omega} = 10.667\ \text{V}$$

BRIDGE NETWORKS

This section introduces the **bridge network,** a configuration that has a multitude of applications. In the chapters to follow, it will be employed in both **dc and ac meters**.

The bridge network may appear in one of the three forms as indicated in Fig. 32.

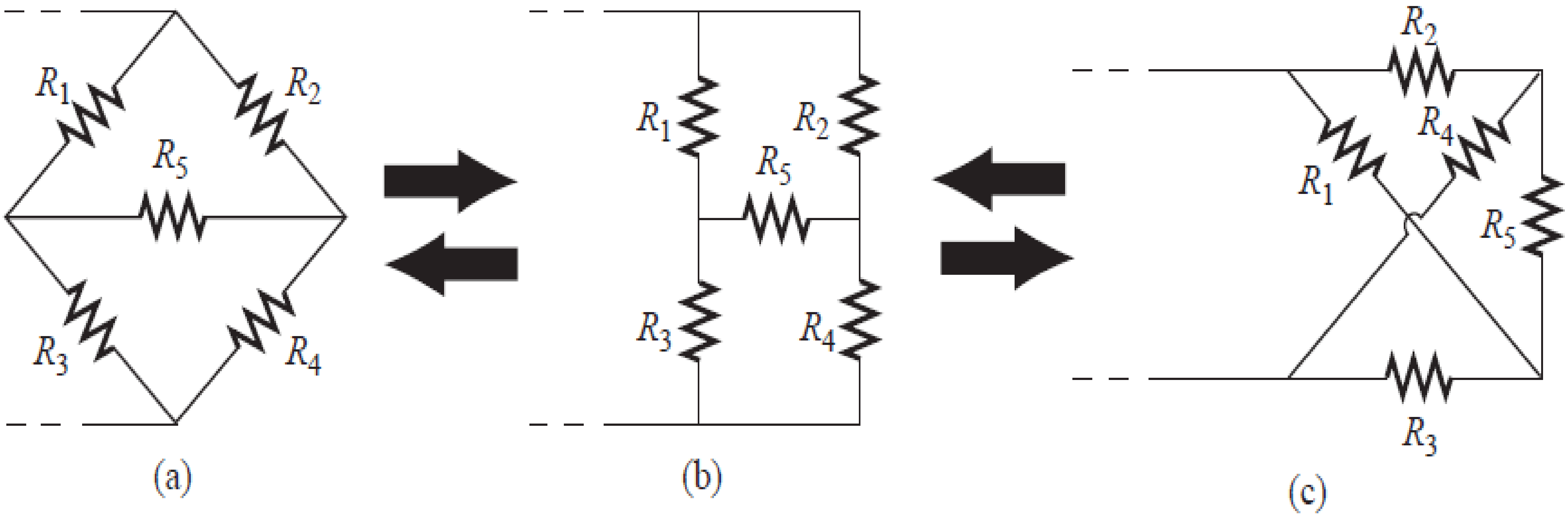


Fig. 32 *Various formats for a bridge network.*

Let us examine the network of Fig. 33 using mesh and nodal analysis.

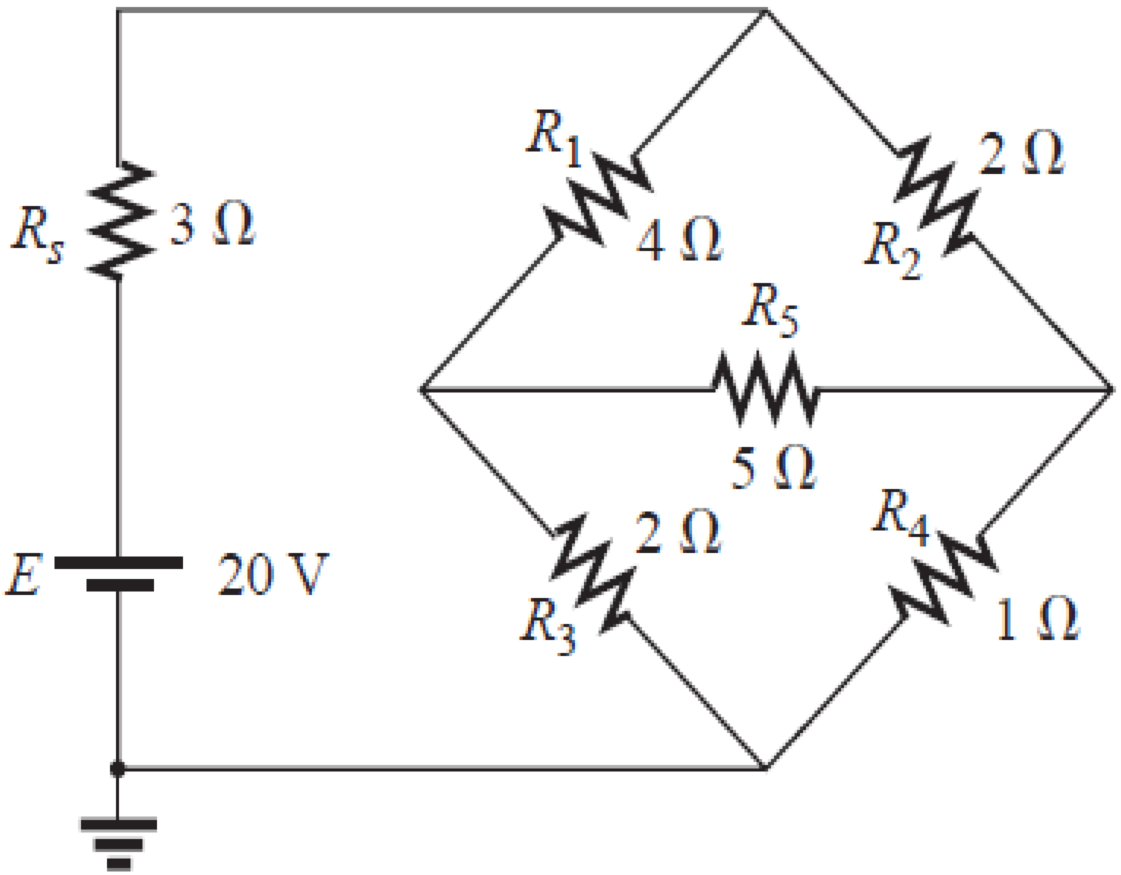


Fig. 33 *Standard bridge configuration.*

Mesh analysis (Fig. 34) yields

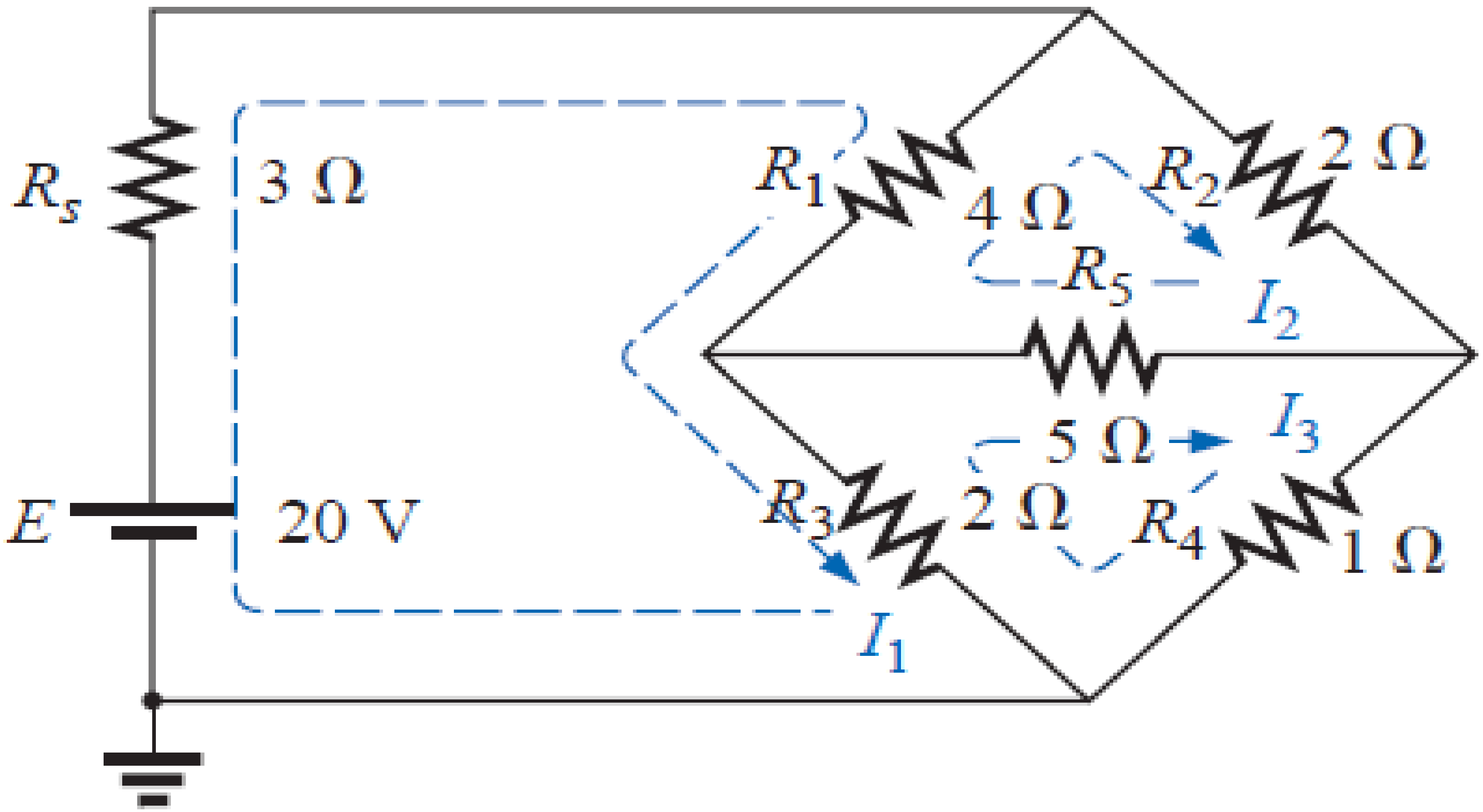


Fig. 34 *Assigning the mesh currents to the network.*

$$(3\ \Omega + 4\ \Omega + 2\ \Omega)I_1 - (4\ \Omega)I_2 - (2\ \Omega)I_3 = 20\ \text{V}$$

$$(4\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (4\ \Omega)I_1 - (5\ \Omega)I_3 = 0$$

$$(2\ \Omega + 5\ \Omega + 1\ \Omega)I_3 - (2\ \Omega)I_1 - (5\ \Omega)I_2 = 0$$

And

$$9I_1 - 4I_2 - 2I_3 = 20$$

$$-4I_1 + 11I_2 - 5I_3 = 0$$

$$-2I_1 - 5I_2 + 8I_3 = 0$$

with the result that

$$I_1 = 4\ \text{A}$$

$$I_2 = 2.667\ \text{A}$$

$$I_3 = 2.667\ \text{A}$$

The net current through the 5-Ω resistor is

$$I_{5\Omega} = I_2 - I_3 = 2.667\ \text{A} - 2.667\ \text{A} = 0\ \text{A}$$

Nodal analysis (Fig. 35) yields

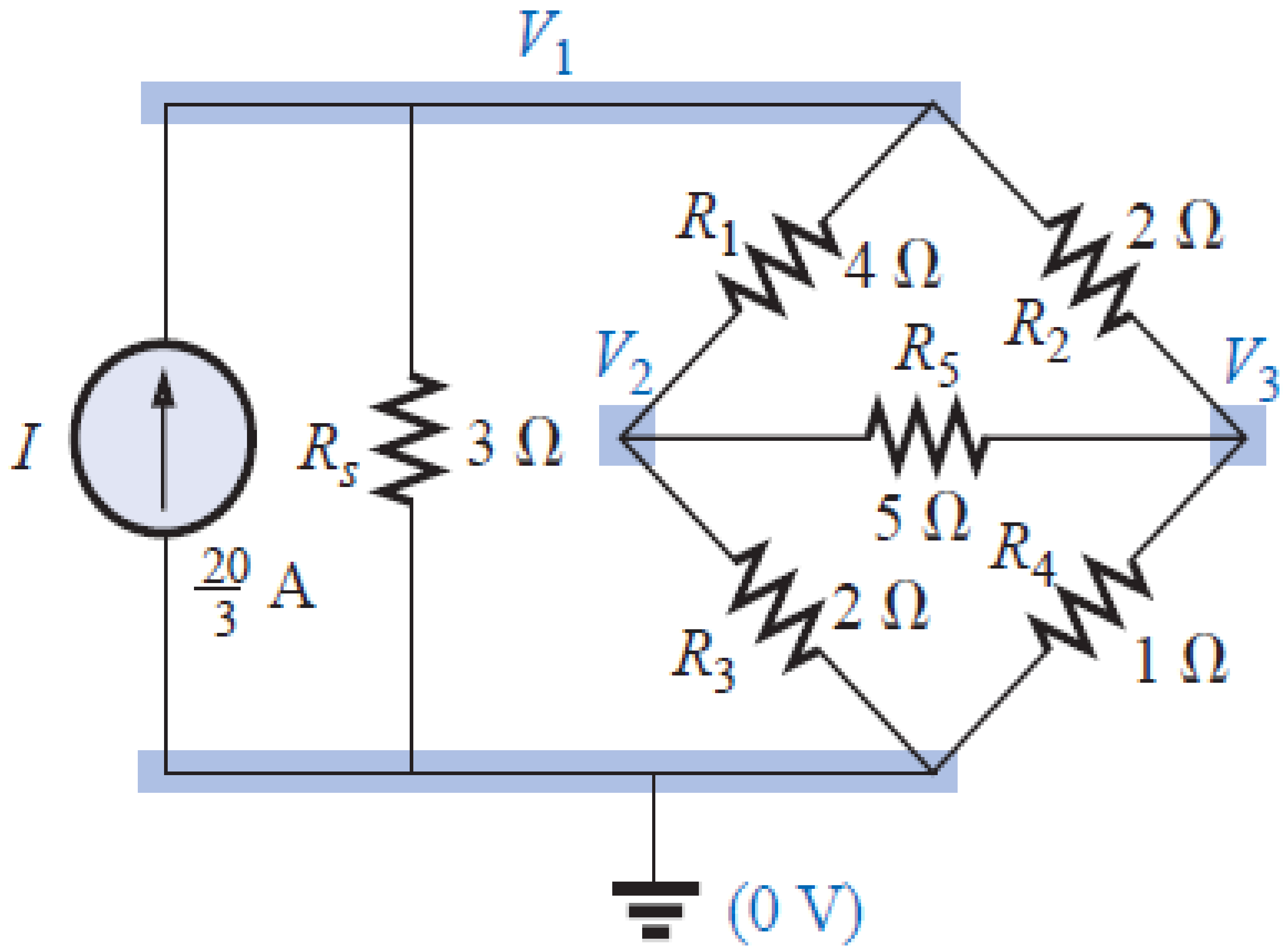


Fig. 35 *Defining the nodal voltages for the network.*

$$\left(\frac{1}{3\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{4\ \Omega}\right)V_2-\left(\frac{1}{2\ \Omega}\right)V_3=\frac{20}{3}\ \mathrm{A}$$

$$\left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2-\left(\frac{1}{4\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_3=0$$

$$\left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3-\left(\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_2=0$$

And

$$\left(\frac{1}{3\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{4\ \Omega}\right)V_2-\left(\frac{1}{2\ \Omega}\right)V_3=\frac{20}{3}\ \text{A}$$

$$-\left(\frac{1}{4\ \Omega}\right)V_1+\left(\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{5\ \Omega}\right)V_2-\left(\frac{1}{5\ \Omega}\right)V_3=0$$

$$-\left(\frac{1}{2\ \Omega}\right)V_1-\left(\frac{1}{5\ \Omega}\right)V_2+\left(\frac{1}{5\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}\right)V_3=0$$

Finally,

and V_1=**8 V**

Similarly, V_2 = **2.667 V** and V_3 = **2.667 V**

and the voltage across the 5-Ω resistor is

$$V_{5\Omega} = V_2 - V_3 = 2.667\text{ V} - 2.667\text{ V} = 0\text{ V}$$

Since $V_{5\Omega}$ = 0V, we can insert a short in place of the bridge arm without affecting the network behavior. (Certainly $V = IR = I\cdot(0) = 0V$.)

Since $V_{5\Omega}$ = 0V, we can insert a short in place of the bridge arm without affecting the network behavior. (Certainly $V = IR = I\cdot(0) = 0V$.)

In Fig. 36, a short circuit has replaced the resistor R_5, and the voltage across R_4 is to be determined.

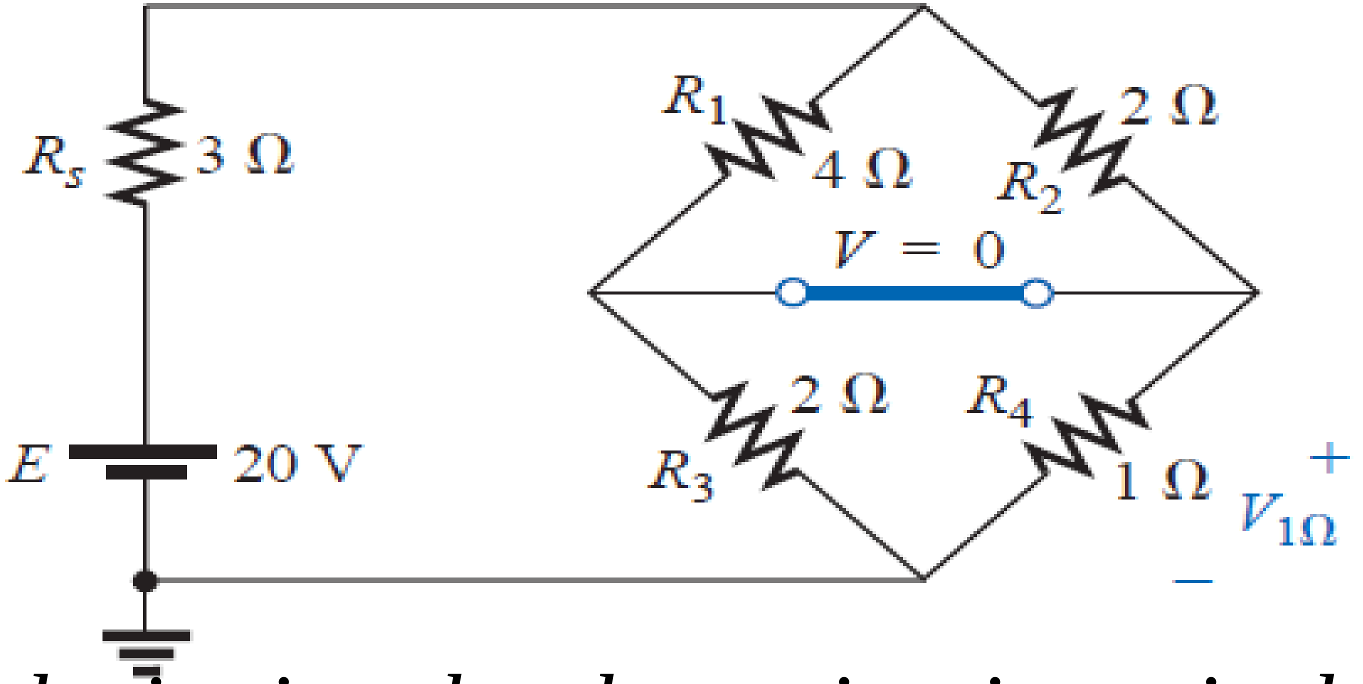


Fig. 36 *Substituting the short-circuit equivalent for the balance arm of a balanced bridge.*

The network is redrawn in Fig. 37, and

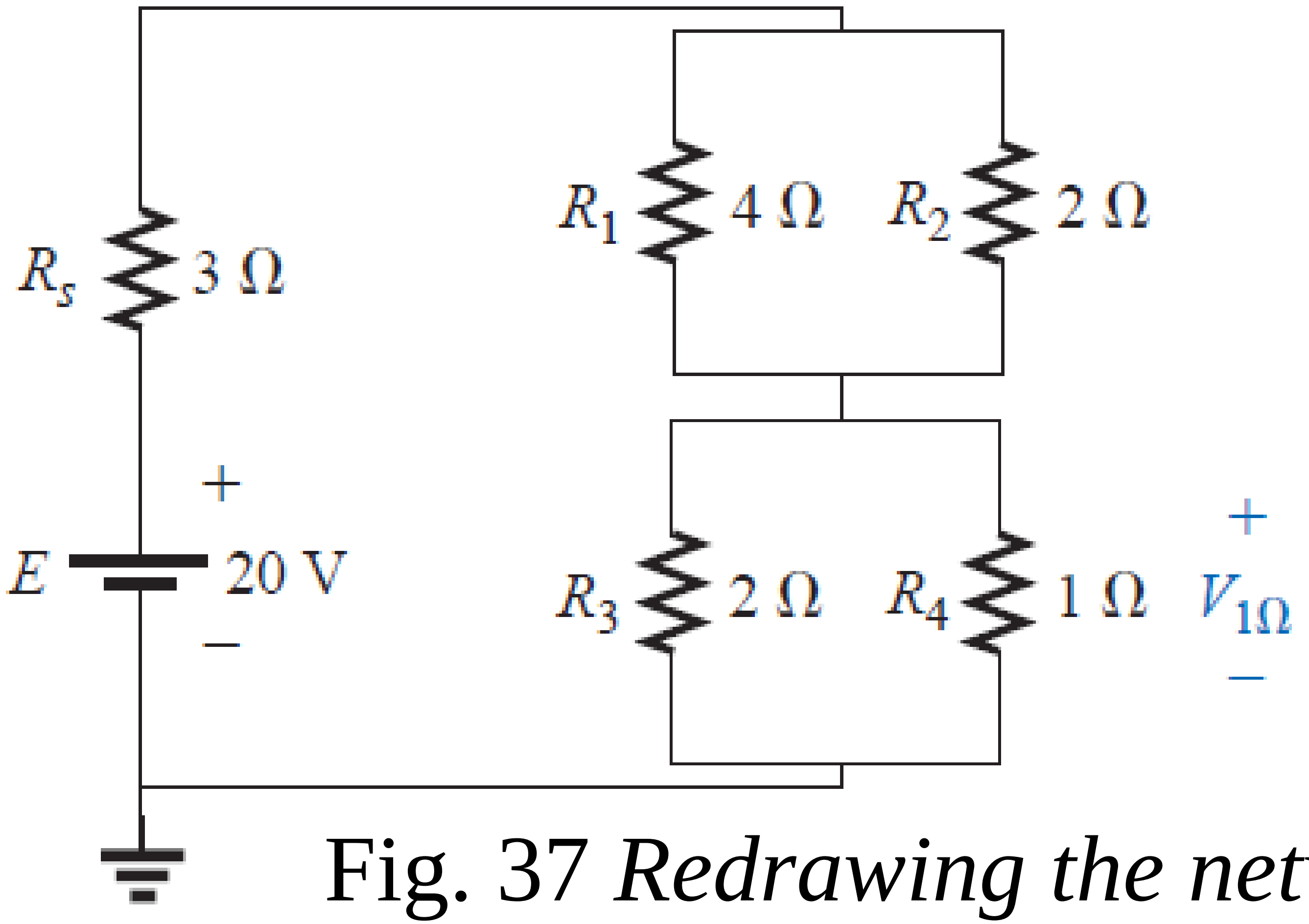


Fig. 37 *Redrawing the network.*

$$V_{1\Omega} = \frac{(2\ \Omega \parallel 1\ \Omega)20\ \text{V}}{(2\ \Omega \parallel 1\ \Omega) + (4\ \Omega \parallel 2\ \Omega) + 3\ \Omega}$$

$$= \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{8}{6} + 3} = \frac{\frac{2}{3}(20\ \text{V})}{\frac{2}{3} + \frac{4}{3} + \frac{9}{3}} \quad \text{(voltage divider rule)}$$

$$= \frac{2(20\ \text{V})}{2 + 4 + 9} = \frac{40\ \text{V}}{15} = \mathbf{2.667\ V}$$

as obtained earlier.

We found through mesh analysis that $I_{5\Omega} = 0$ A, which has as its equivalent an open circuit as shown in Fig. 38 (a).

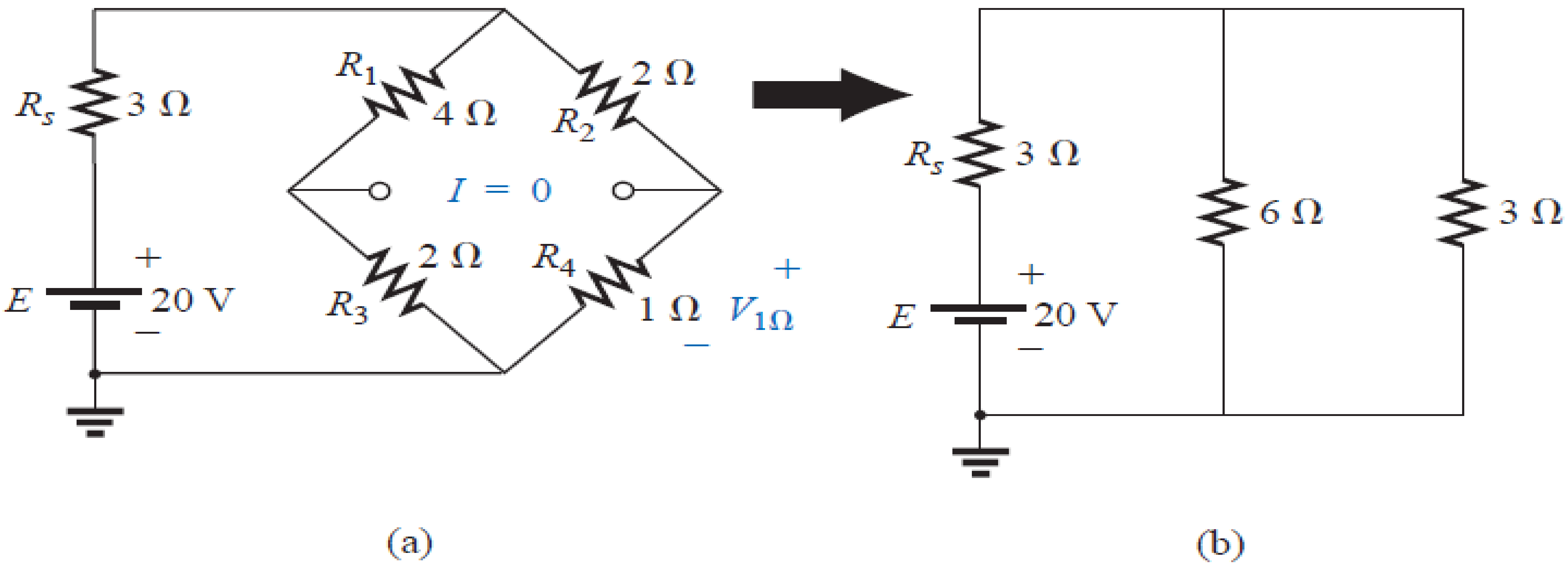


Fig. 38 Substituting the open-circuit equivalent for the balance arm of a balanced bridge.

(Certainly $I = V/R = 0/(\infty\ \Omega) = 0$ A.) The voltage across the resistor R_4 will again be determined and compared with the result above.

The network is redrawn after combining series elements, as shown in Fig. 38 (b), and

$$V_{3\Omega} = \frac{(6\ \Omega \parallel 3\ \Omega)(20\ \text{V})}{6\ \Omega \parallel 3\ \Omega + 3\ \Omega} = \frac{2\ \Omega(20\ \text{V})}{2\ \Omega + 3\ \Omega} = 8\ \text{V}$$

And $$V_{1\Omega} = \frac{1\ \Omega(8\ \text{V})}{1\ \Omega + 2\ \Omega} = \frac{8\ \text{V}}{3} = \mathbf{2.667\ V}$$

Y-Δ (T-π) AND Δ-Y (π-T) CONVERSIONS

Circuit configurations are often encountered in which the resistors do **not appear to be in series or parallel**. Under these conditions, it may be necessary to **convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied.**

Two circuit configurations that often account for these difficulties are the **wye (Y)** and **delta (Δ) configurations,** depicted مصور(drawn) in Fig. 39(a).

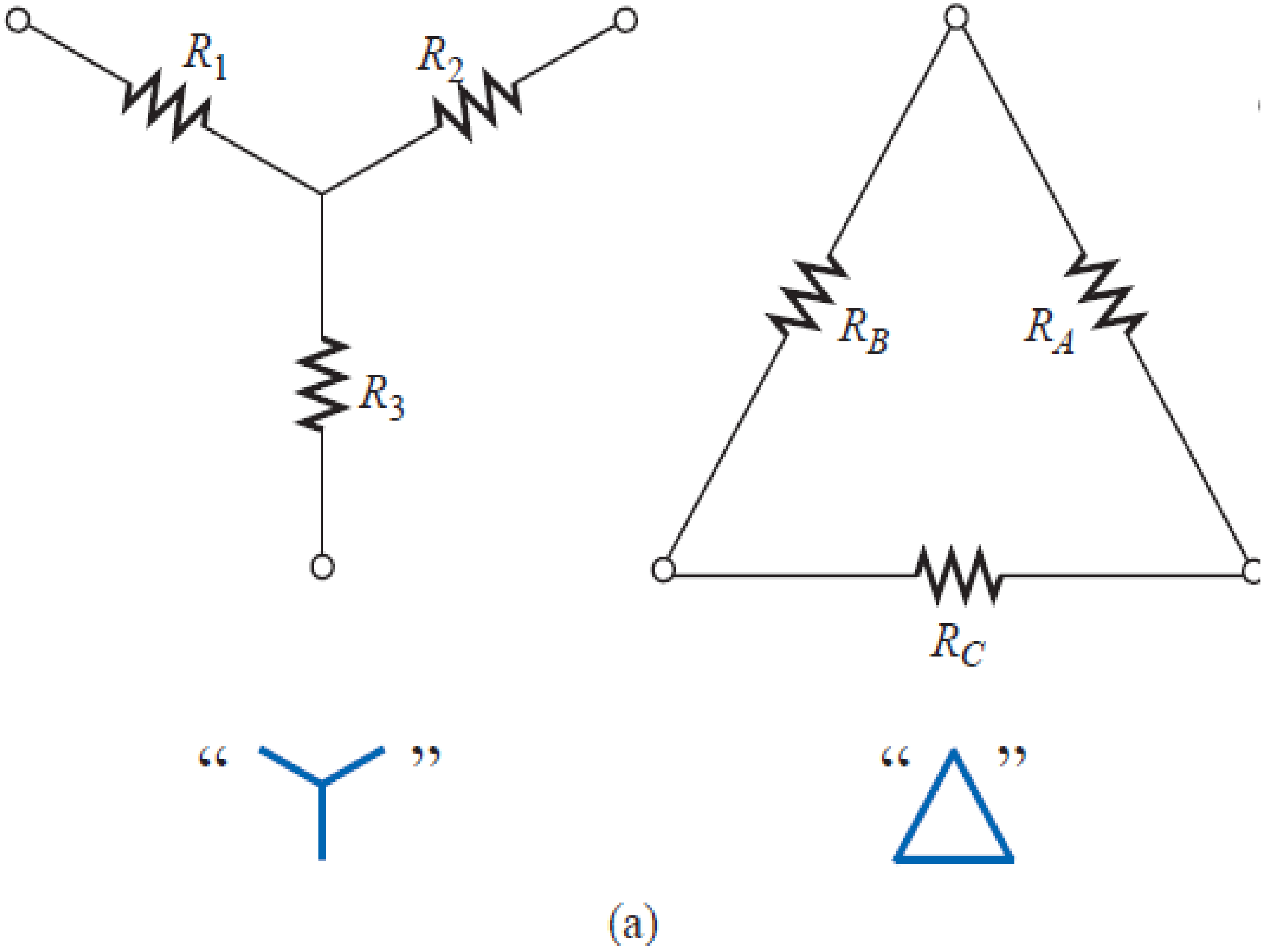


Fig. 39 *The Y (T) and Δ (π) configurations.*

They are also referred to as the **tee (T)** and **pi (π),** respectively, as indicated in Fig. 39 (b). Note that the **pi** is actually an inverted **delta**.

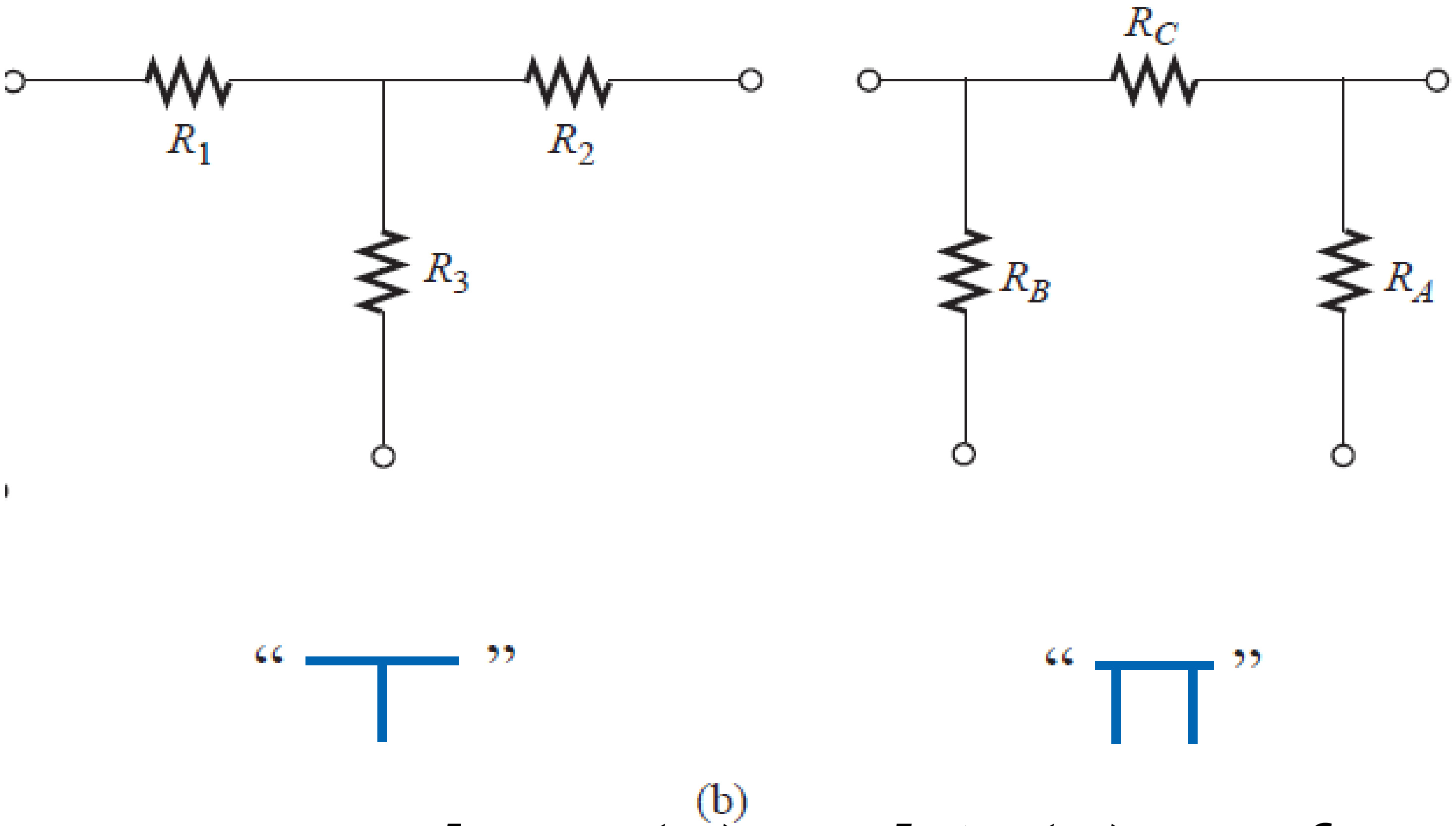
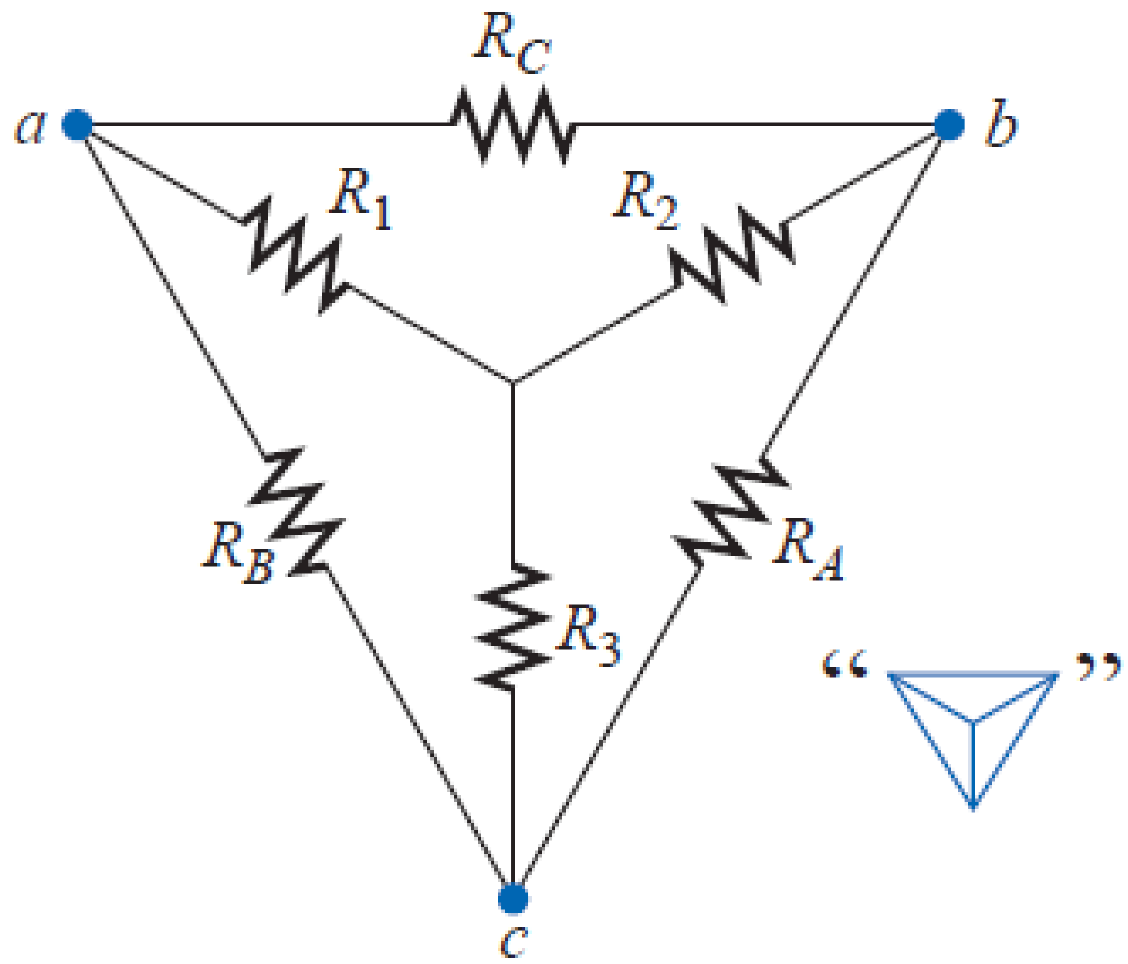


Fig. 39 The Y (T) and Δ (π) configurations.

The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa والعكس صحيح

In other words, in Fig.40, with terminals *a*, *b*, and *c* held fast, if the wye (Y) configuration were desired *instead of* the inverted delta (Δ) configuration, all that would be necessary is a direct application of the equations



$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

Fig.40 Introducing the concept of Δ-Y or Y-Δ conversions.

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest الأبعد from the resistor to be determined.

Let us consider what would occur if all the values of a Δ or Y were the same. If $R_A = R_B = R_C$, Equation

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

would become (using R_A only) the following:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{R_A R_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \qquad R_2 = \frac{R_A}{3}$$

In general, therefore,

$$\boxed{R_Y = \frac{R_\Delta}{3}}$$

Or

$$\boxed{R_\Delta = 3R_Y}$$

which indicates that *for a* Y *of three equal resistors, the value of each resistor of the* Δ *is equal to three times the value of any resistor of the* Y.

The Y and the Δ will often appear as shown in Fig. 41. They are then referred to as a **tee (T)** and a **pi (π)** network, respectively. The equations used to convert from one form to the other are exactly **the same as** those developed for the **Y and Δ transformation تحويل**.

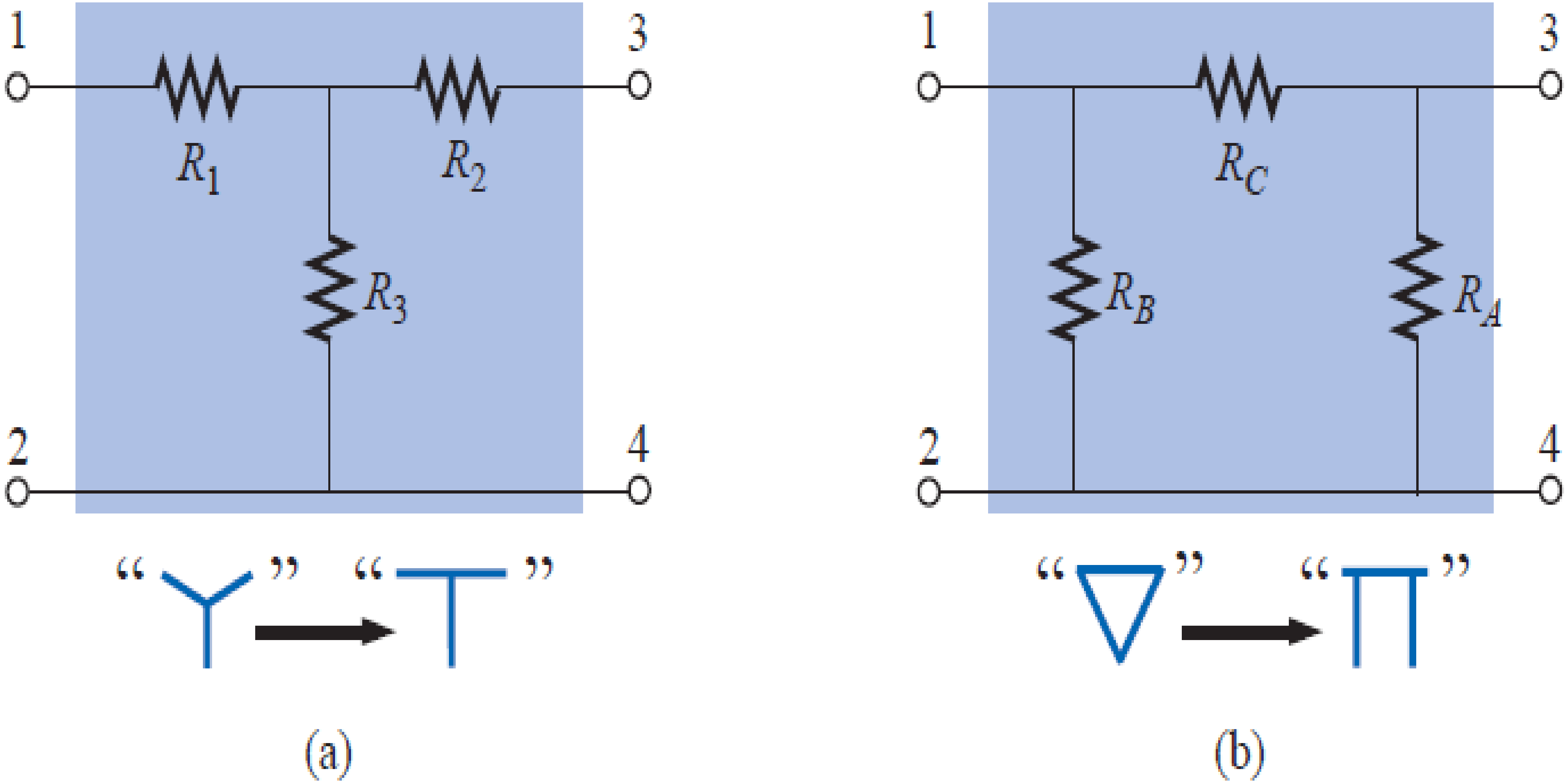


Fig. 41 *The relationship between the Y and T configurations and the Δ and π Configurations.*

EXAMPLE 19 Convert the Δ of Fig. 42 to a Y.

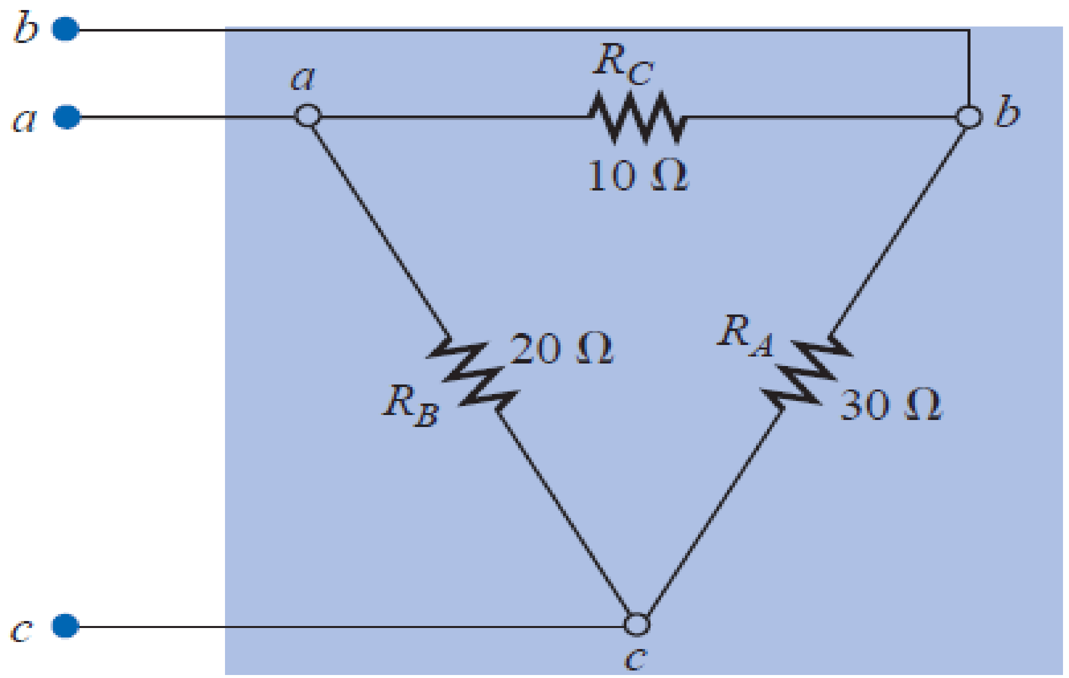


Fig. 42

Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = 3\tfrac{1}{3}\ \Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. 43

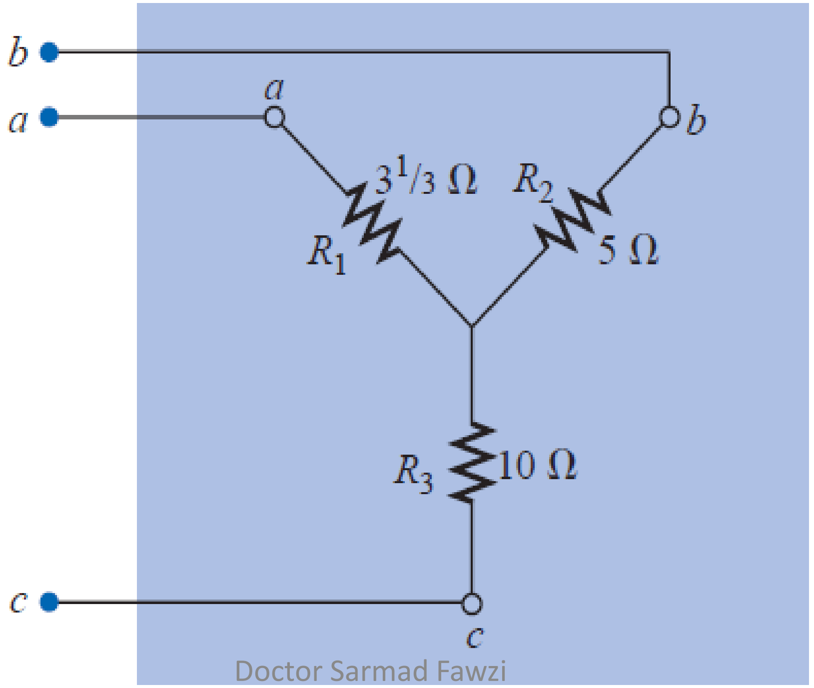


Fig. 43 *The Y equivalent for the* Δ.

EXAMPLE 20 Convert the Y of Fig. 44 to a Δ.

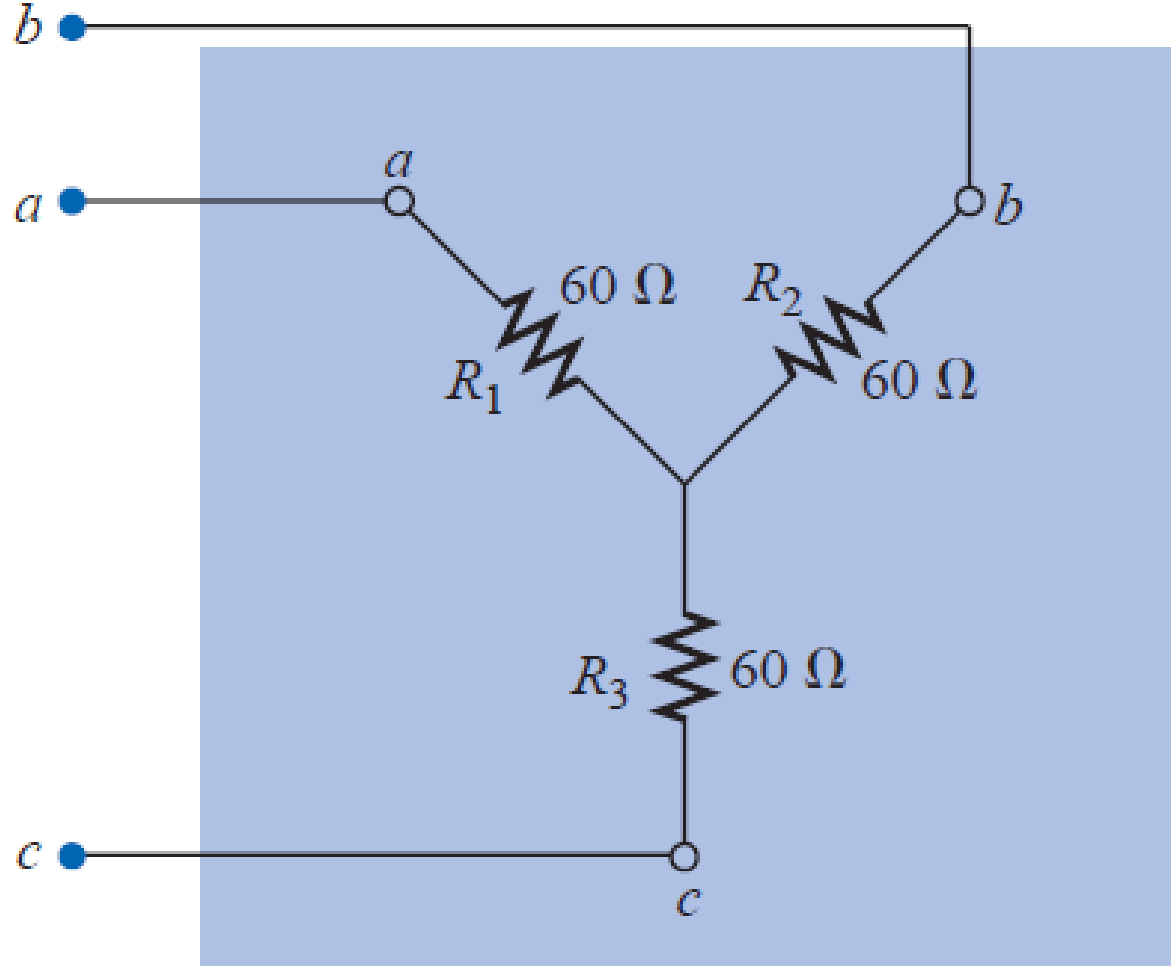


Fig. 44

Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal, permitting the use of Eq. $R_\Delta = 3R_Y$ and yielding

$$= 3(60\ \Omega) = 180\ \Omega$$

And $R_B = R_C = \mathbf{180\ \Omega}$

The equivalent network is shown in Fig. 45.

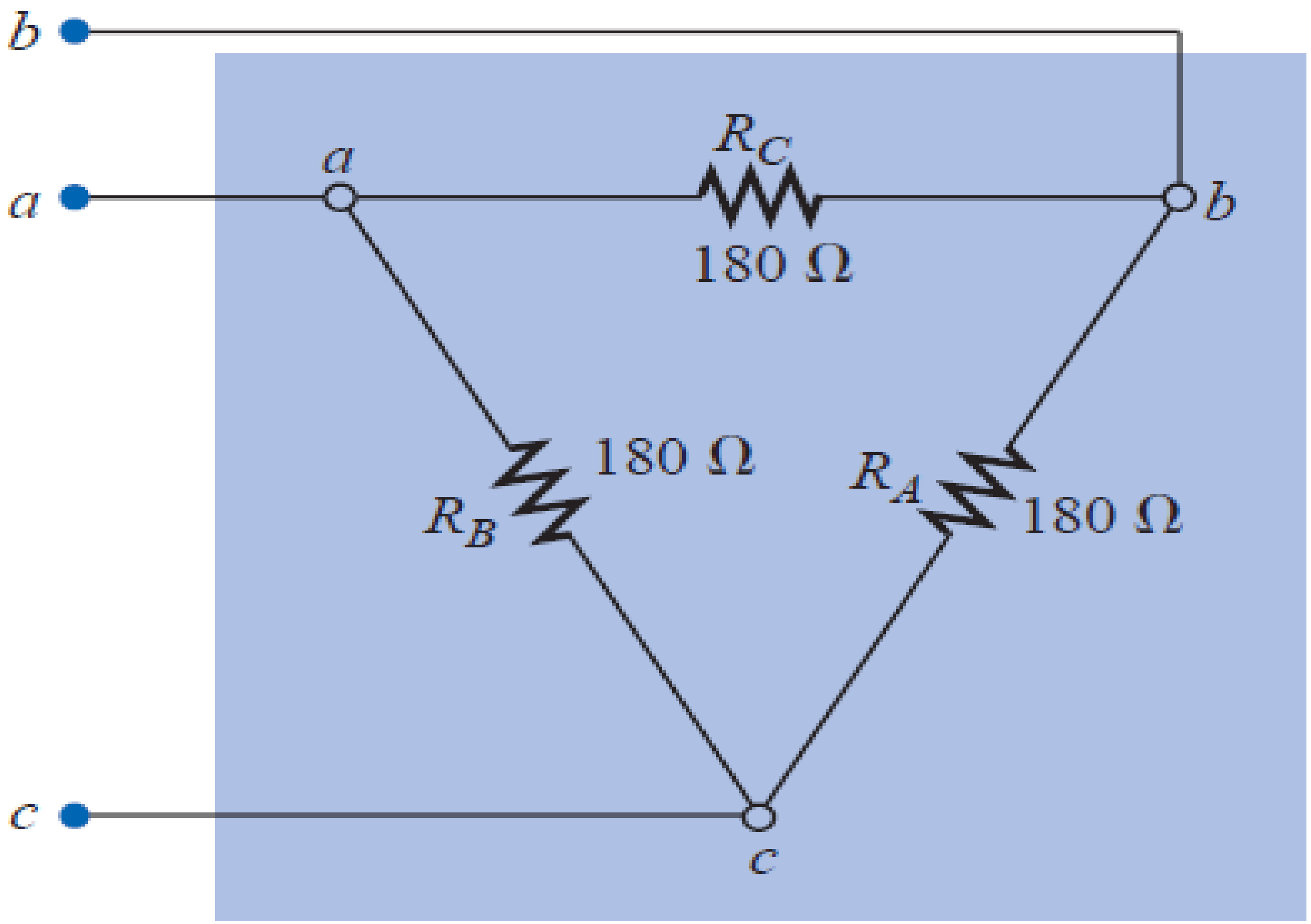


Fig. 45 *The* Δ *equivalent for the Y.*

EXAMPLE 21 Find the total resistance of the network of Fig. 46, where $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.

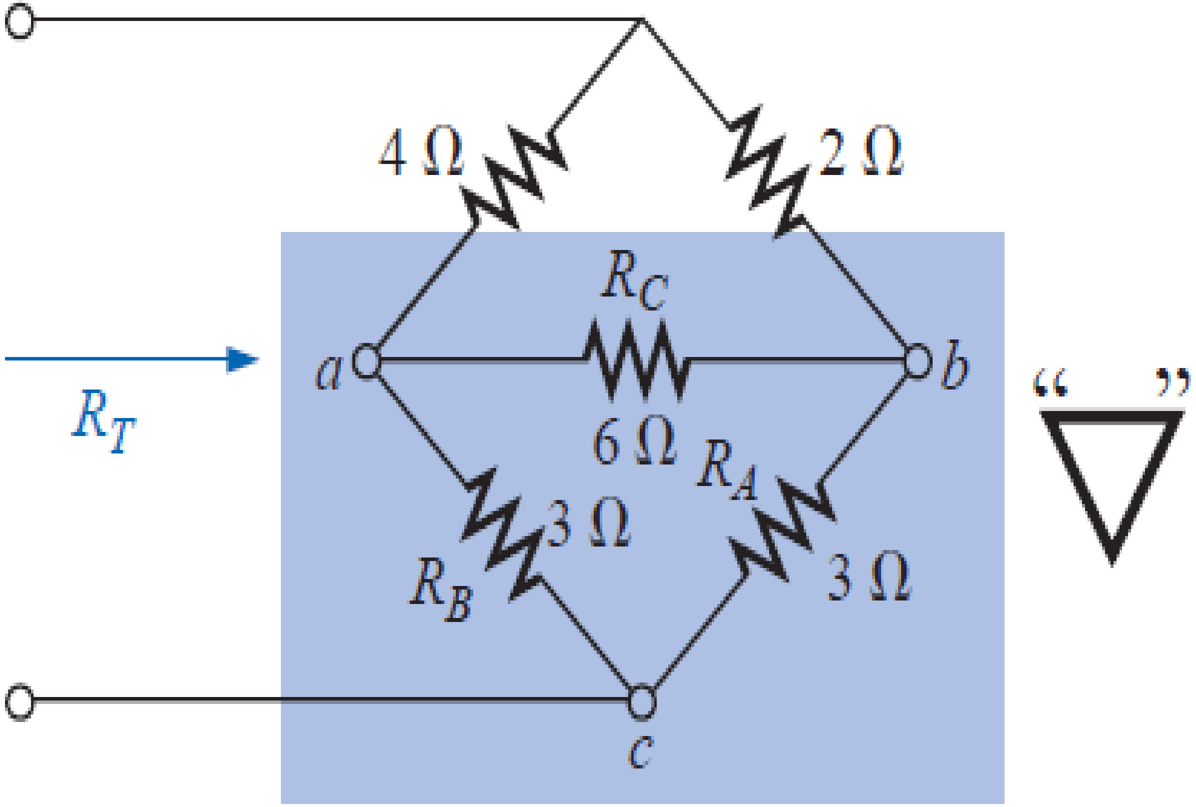


Fig. 46

Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig. 47, yields

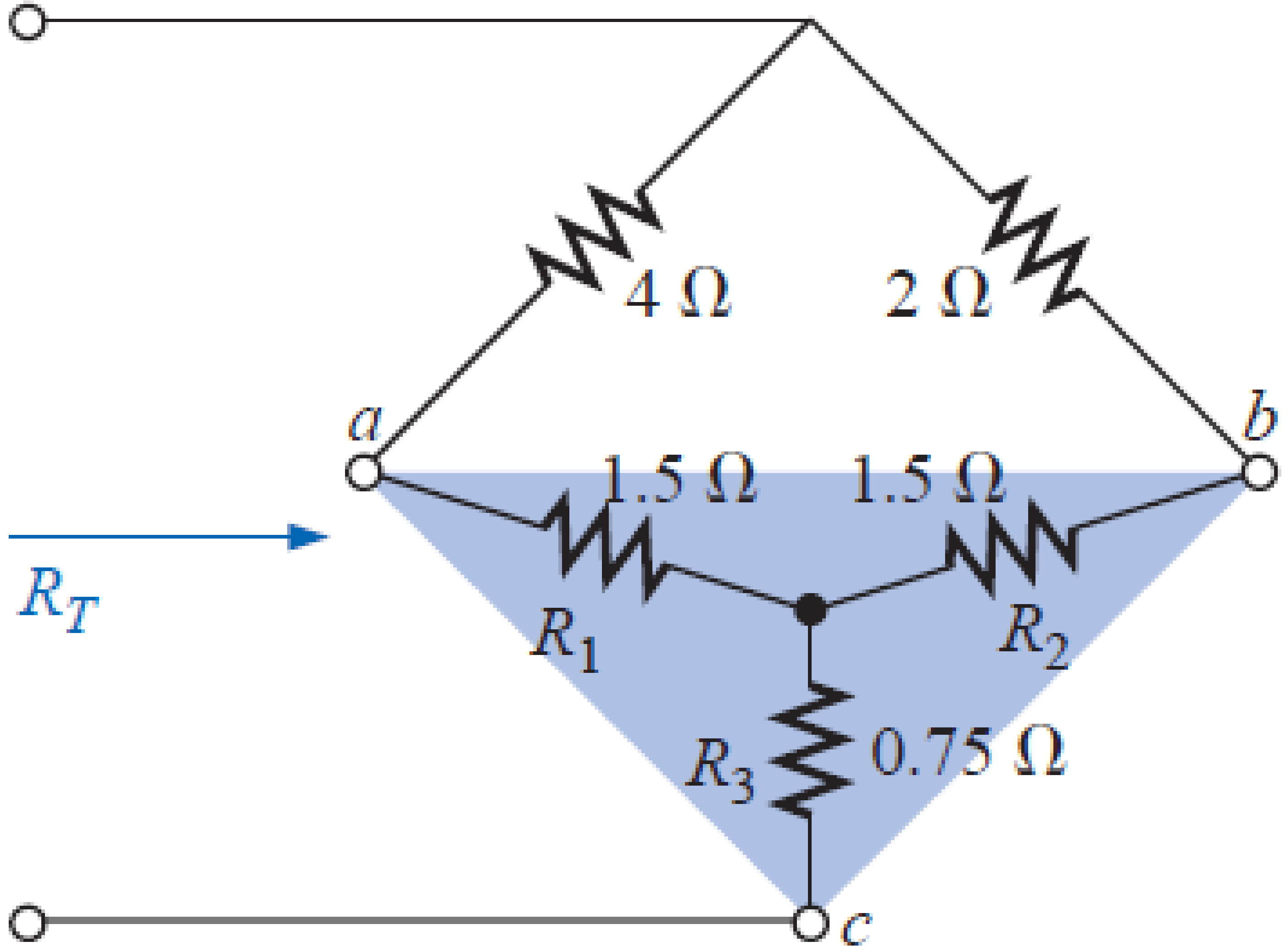


Fig. 47 *Substituting the Y equivalent for the bottom Δ.*

EXAMPLE 22 Find the total resistance of the network of Fig. 48.

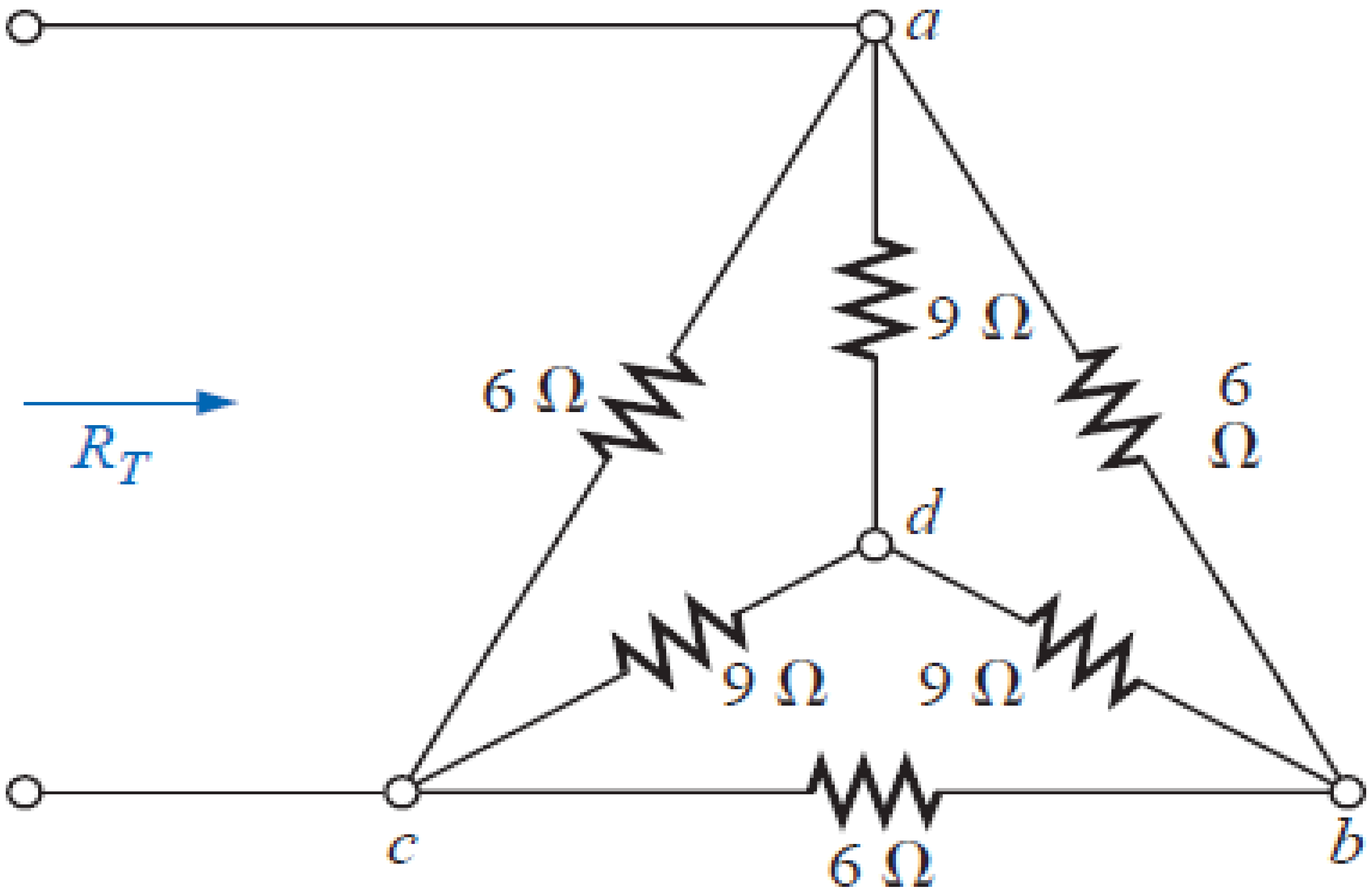


Fig. 48

Solutions: Since all the resistors of the Δ or Y are the same,

a. ***Converting the* Δ *to a Y***. Note: When this is done, the resulting *d* ′ of the new Y will be the same as the point *d* shown in the original figure, only because both systems are “balanced.” That is, the resistance in each branch of each system has the same value:

$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega$$

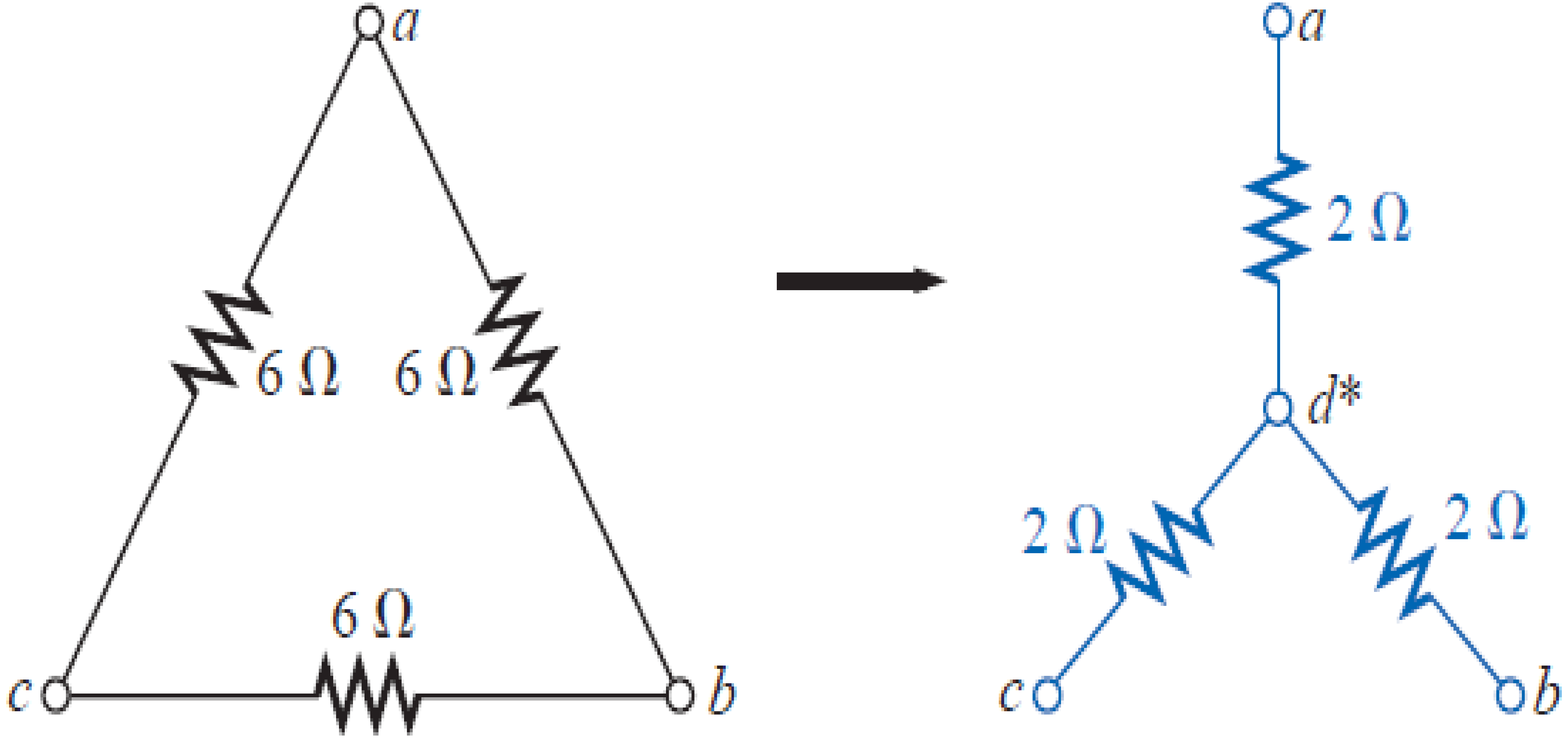


Fig. 49 Converting the Δ configuration to a Y configuration.

The network then appears as shown in Fig. 50.

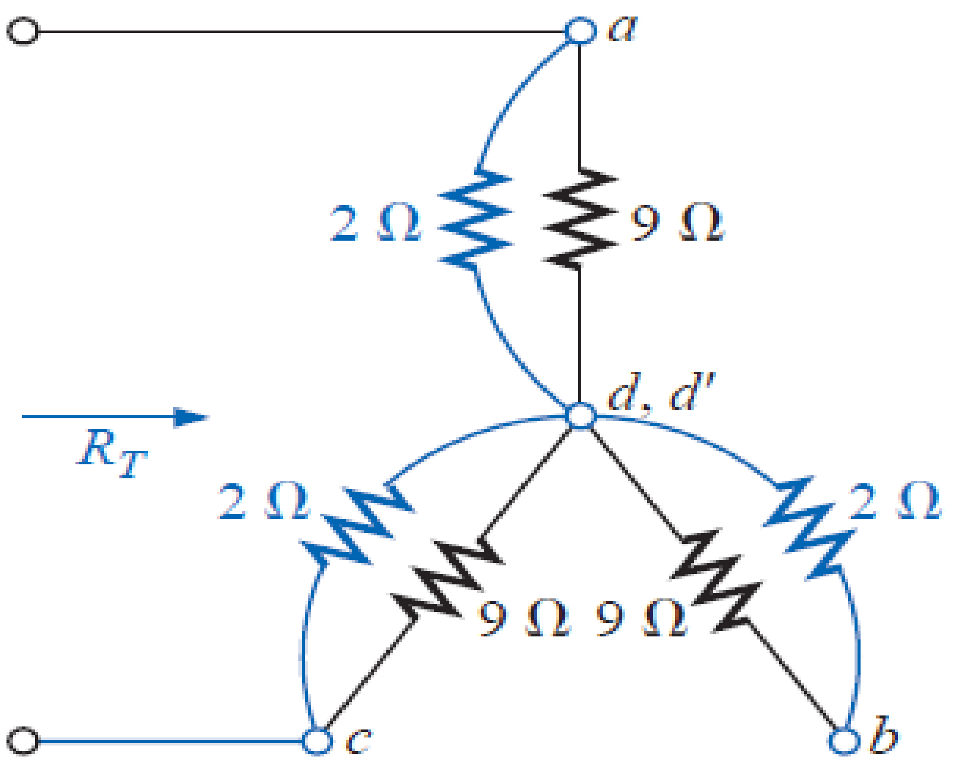


Fig. 50 *Substituting the Y configuration for the converted Δ into the network.*

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.2727\ \Omega}$$

b. ***Converting the Y to a Δ:*** (Fig. 51)

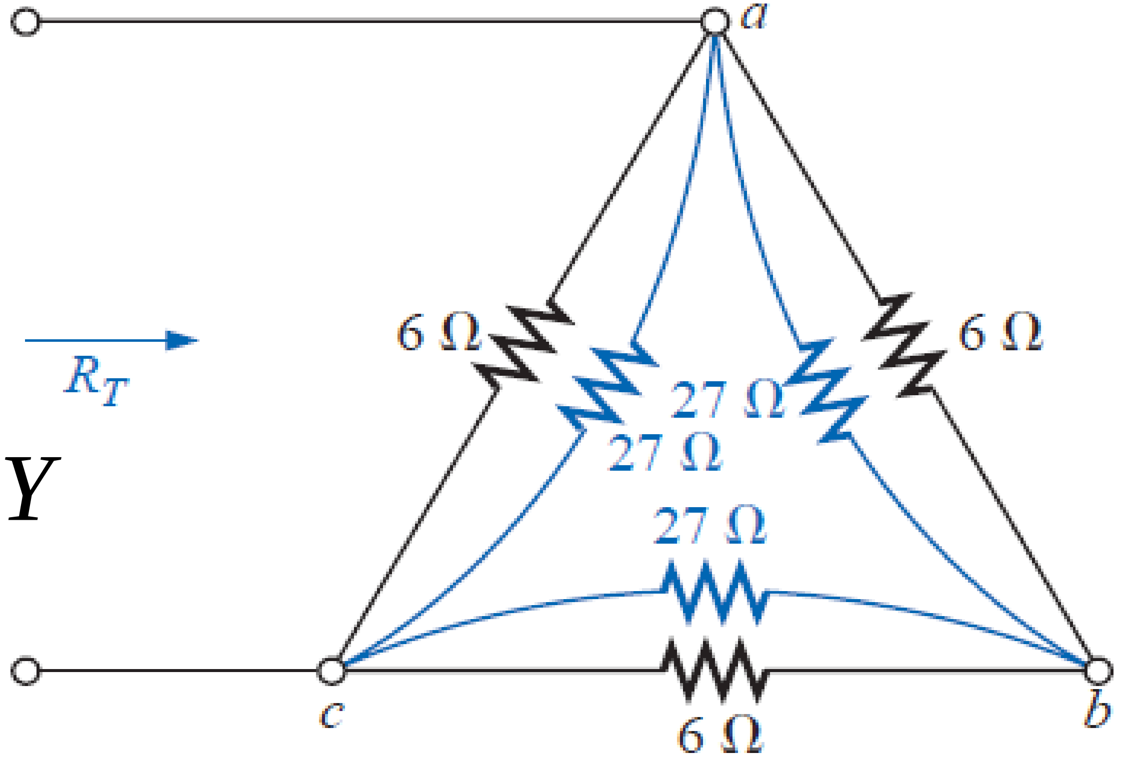


Fig. 51 *Substituting the converted Y configuration into the network.*

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.9091\ \Omega$$

$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

$$= \frac{2(4.9091\ \Omega)}{3} = \mathbf{3.2727\ \Omega}$$

which checks with the previous solution.

Please read and try understand in the first reference (Chapter 8).

References:

1- Introductory Circuits Analysis, By Robert L. Boylestad, Tenth (10th) Edition.

2- Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.

3- Any reference that has a Direct Current Circuits Analysis (DCCA).

Thank you for listening