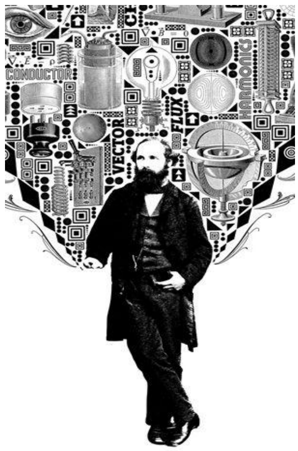

Electricity and Magnetism

An Introduction to Vectors and Calculus

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Lecture's Contents Part - I

1. **Basic Vector Algebra**
   – Scalars and Vectors
   – Position and Distance Vector
   – Vector Addition and Subtraction
   – Vector Multiplication
   – Triple Product of Scalar and Vector
2. **Coordinate Systems**
   – Cartesian Coordinate System
   – Cylindrical Coordinate System
   – Spherical Coordinate System
3. **Systems Transformation**
   – Cartesian to Cylindrical
   – Cartesian to Spherical
   – Spherical to Cylindrical

Why Studying Vectors

- The Subject of electromagnetism is very fundamental and lots of new concepts are in contents.
- Forces between charges, Electric Field and Electric Field Lines, Energy and Potentials Magnetic Fields become clearly understandable in term of its magnitude and their associated direction in any medium.
- Vector tells us the magnitude and where the direction is. OUR TOOL.
- In this lecture, vectors will be covered in details. Some useful examples will be separately presented during this lecture.

1. Basic Vector Algebra

– Scalars
- A single real number (positive or negative) is referred as a scalar value.
- Many Examples represent scalars such as Temperature and Pressure.
- Example:\\ $T = 290^oK$

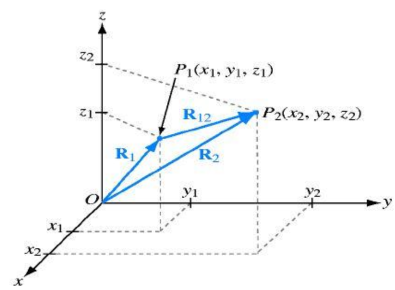
– Vectors
- Many quantities are not complete without specifying their directions. Such quantities are well-known as vector quantities.
- An example of vector quantities is the velocity, where its complete figure can be understandable in term of its velocity and the direction that the object derived.
- Example:\\ v = 30 kmph - North

1. Basic Vector Algebra

– Scalar and Vector Fields

- Most of the work in this module is concerned about vector and scalar fields.
- The vector or scalar field can be defined as a function that connects any arbitrary point to a position in space.

- R_1 and R_2 are defined as vectors position which they can be determined by its three position in the coordinate system.
- R_{12} is the distance vector between two the defined points which is given by $R_{12} = R_2 - R_1$



1. Basic Vector Algebra

– Vector structure

- Any vector can be written as follow:

$$\vec{\mathbf{A}} = \mathrm{A_x}\hat{\mathbf{x}} + \mathrm{A_y}\hat{\mathbf{y}} + \mathrm{A_z}\hat{\mathbf{z}}$$

- $\mathrm{A_x}, \mathrm{A_y}, \mathrm{A_z}$ are defined as the vector components.
- $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are known as the unit vectors of the vector $\vec{\mathrm{A}}$.
- Unit vectors are vectors of unit lengths in the directions of x, y, z, respectively.
- Example:

$$\vec{\mathbf{A}} = 3\hat{\mathbf{x}} + 7\hat{\mathbf{y}} - 5\hat{\mathbf{z}}$$

1. Basic Vector Algebra

– Vector Magnitude and Direction

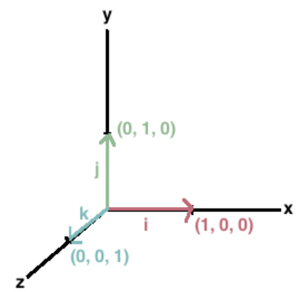
- **Vector magnitude:**

$$\vec{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$$

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

- **The direction of the vector $\vec{A}$:**

$$\hat{\mathbf{a}} = \frac{\vec{A}}{A} = \frac{A_x\hat{\mathbf{x}}+A_y\hat{\mathbf{y}}+A_z\hat{\mathbf{z}}}{\sqrt{A_x^2+A_y^2+A_z^2}}$$



1. Basic Vector Algebra

– Vector Addition:

Lets *A* and *B* are two vectors, where:

$$\vec{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$$

$$\vec{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\mathbf{x}} + (A_y+B_y)\hat{\mathbf{y}} + (A_z+B_z)\hat{\mathbf{z}}$$

– Vector Subtraction:

$$\vec{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$$

$$\vec{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{\mathbf{x}} + (A_y-B_y)\hat{\mathbf{y}} + (A_z-B_z)\hat{\mathbf{z}}$$

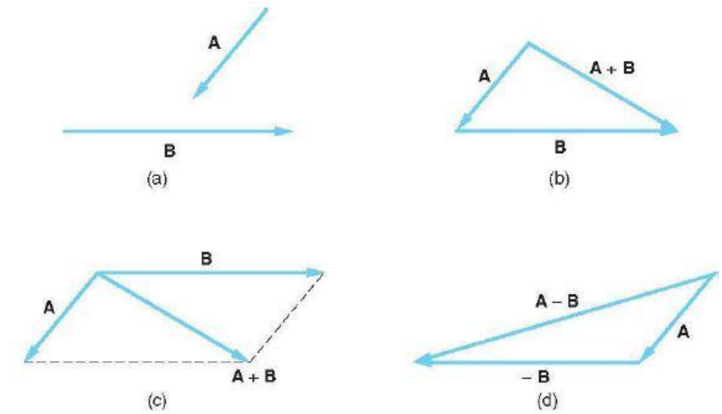
– Notes:

- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- $\vec{A} - \vec{B} \neq \vec{B} + \vec{A}$

1. Basic Vector Algebra

– A pair of vectors $\vec{A}$ and $\vec{B}$ shown in (a) are added by head-to-tail method (b) and by completing the trapezoid (c).

– In (d) the vector $\vec{B}$ is subtracted from vector $\vec{A}$.

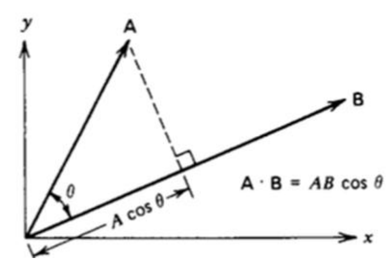


1. Basic Vector Algebra

– **Dot (Scalar) Multiplication**

- Lets *A* and *B* are two vectors:
  $\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$
  $\vec{\mathbf{B}} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$

- The dot product of two vectors:
  $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$



θ is the smallest angle between the two vectors.

- The result of dot product is a SCALAR.
- The dot product has its maximum magnitude at $\theta = 0$.
- The dot product it has ZERO magnitude at $\theta = \pi/2$.
- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$

1. Basic Vector Algebra

– Dot (Scalar) Multiplication

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{AB}$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

- **The dot product of two identical unit vectors is always ONE.**
- **The dot product of two non-identical unit vectors is always ZERO.**
- **Example:**

$$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \mathbf{1}$$
$$\hat{\mathbf{x}} \cdot \hat{\mathbf{z}} = \mathbf{0}$$

1. Basic Vector Algebra

– Cross Product

- **Lets $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are two vectors:**

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$$
$$\vec{\mathbf{B}} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$$

- **The cross product of two vectors:**

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\sin\theta$$

$$= \begin{bmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} = (A_y B_z - A_z B_y)\hat{\mathbf{x}} + (A_z B_x - A_x B_z)\hat{\mathbf{y}} + (A_x B_y - A_y B_x)\hat{\mathbf{z}}$$

- **The cross product is ZERO for collinear vectors ($\theta = 0$).**
- **The cross product is MAX for perpendicular vectors ($\theta = \frac{\pi}{2}$).**

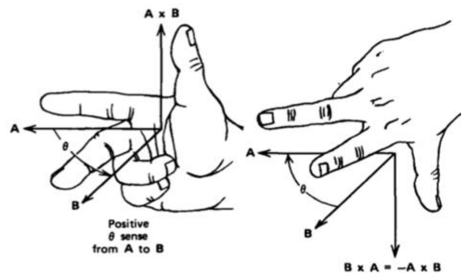
1. Basic Vector Algebra

– Cross Product

- The resultant of cross product of two vectors is a new vector perpendicular to the direction of both.
- The cross product of two identical unit vectors is ZERO.
- The cross product of two non-identical unit vectors is new vector perpendicular to both of them.
- $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$
- Example:\\

  $\hat{\mathbf{x}} \times \hat{\mathbf{x}} = \mathbf{0}$

  $\hat{\mathbf{x}} \times \hat{\mathbf{z}} = \hat{\mathbf{y}}$



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1. Basic Vector Algebra

- $\vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{B}} \cdot (\vec{\mathbf{C}} \times \vec{\mathbf{A}}) = \vec{\mathbf{C}} \cdot (\vec{\mathbf{A}} \times \vec{\boldsymbol{B}})$
- $\vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$
- $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{B}}(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}) - \vec{\mathbf{C}}(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})$
- $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) \neq (\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \times \vec{\mathbf{C}}$

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2. Coordinate Systems

– Introduction

- **Our reference point is usually the origin**
- **The location of any object or point in space can be defined by intersecting three perpendicular surfaces.**
- **We do study THREE coordinate systems:**

  - ❖ **Cartesian (Rectangular) Coordinates.**
  - ❖ **Circular (Cylindrical) Coordinates.**
  - ❖ **Spherical Coordinates.**

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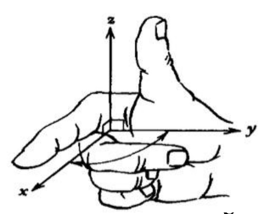
2. Coordinate Systems

– Cartesian Coordinates

- **With respect to the origin (0,0,0), we define any object in space by intersection of *THREE PERPENDICULAR* planes parallel to (x,y,z) axis:**
  - ❖ **x – plane perpendicular to x – axis**
  - ❖ **y – plane perpendicular to y – axis**
  - ❖ **z – plane perpendicular to z – axis**



- **A right hand coordinate system can specify the directions of Cartesian coordinate:**

  "Curls of the fingers in the direction from x-axis to y s in the direction of +ve x-axis and the middle finger in the direction of +ve y-axis and the thump is referring to the +ve z-axis.

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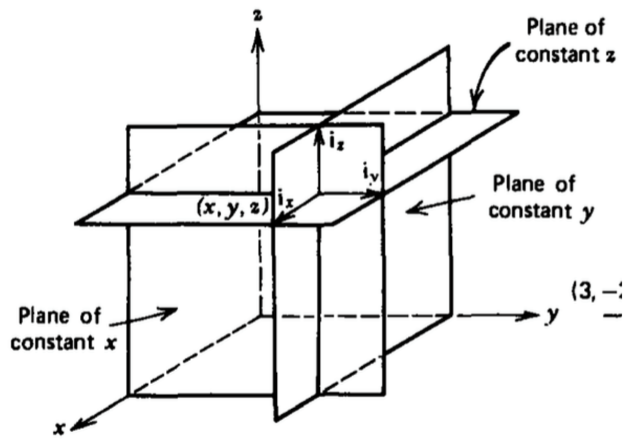
2. Coordinate Systems

– **Cartesian Coordinates**



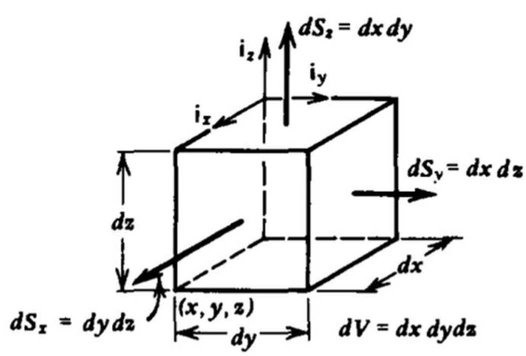
2. Coordinate Systems

– **Surface and Volume deferential**

- **The differential length:**
  $d\hat{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$

- **Each surface has an area:**
  $d\hat{\mathbf{S}}_x = d_y d_z$
  $d\hat{\mathbf{S}}_y = d_x d_z$
  $d\hat{\mathbf{S}}_z = d_x d_y$

- **The volume of the cube:**
  $d\hat{\mathbf{V}} = d_x d_y d_z$



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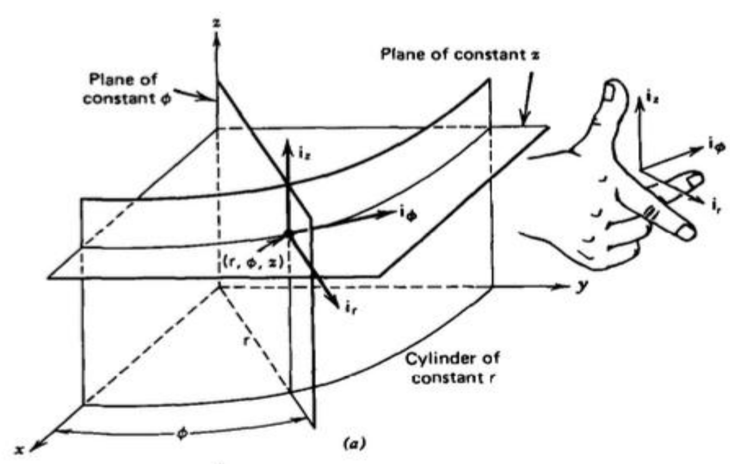
2. Coordinate Systems

– **Cylindrical Coordinates**

- **The location of any object in space can be defined by intersecting of three perpendicular surface of circular or cylindrical coordinated parameters ($\hat{\mathbf{r}}$, $\hat{\mathbf{\Phi}}$, $\hat{\mathbf{z}}$).**
- **The unit vectors are:** $\hat{\mathbf{r}}$, $\hat{\Phi}$, $\hat{\mathbf{z}}$.
- **The direction of** $(\hat{\mathbf{k}})$ **is independent of position.**
- $\hat{\mathbf{r}}, \hat{\emptyset}$ **are changing in direction.**
- **NOTE:**
  - ➢ **For** $\emptyset = 0$; $\hat{\mathbf{r}} = \hat{\mathbf{x}}$ $\hat{\mathbf{\Phi}} = \hat{\mathbf{y}}$
  - ➢ **For** $\emptyset = \frac{\pi}{2}$: $\hat{\mathbf{r}} = \hat{\mathbf{y}}$ $\hat{\mathbf{\Phi}} = -\hat{\mathbf{x}}$

2. Coordinate Systems

– **Cylindrical Coordinates**



2. Coordinate Systems

– Cylindrical Coordinates

- **The differential length is given by:**

  $d\hat{\mathbf{l}} = dr\,\hat{\mathbf{r}} + rd\emptyset\,\widehat{\Phi} + dz\,\hat{\mathbf{z}}$

- **Each surface has an area:**

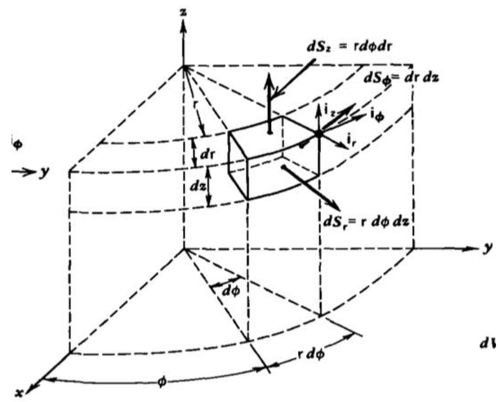
  $d\widehat{\mathbf{S}_{\mathbf{r}}} = \; r\,d_{\emptyset}d_{z}$

  $d\widehat{\mathbf{S}_{\emptyset}} = \; d_{r}d_{z}$

  $d\widehat{\mathbf{S}_{\mathbf{z}}} = r\,d_{r}d_{\emptyset}$

- **The volume is given by:**

  $d\mathbf{V} = rd_{r}d_{\emptyset}d_{z}$



2. Coordinate Systems

– Spherical Coordinates

- **The differential length is given by:**

  $$d\hat{\mathbf{l}} = dR\,\hat{\mathbf{r}} + rd\theta\,\widehat{\Theta} + r\sin\theta\,d\emptyset\,\widehat{\Phi}$$

- **Each surface has an area given by:**

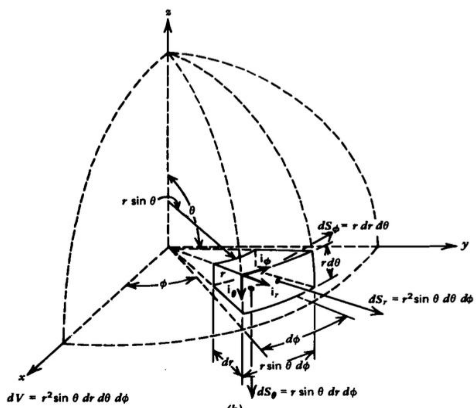
  $$d\widehat{\mathbf{S}_{\mathbf{R}}} = \; R^{2}\sin\theta\,d_{\theta}d_{\emptyset}$$
  $$d\widehat{\mathbf{S}_{\boldsymbol{\theta}}} = R\sin\theta\,d_{r}d_{\emptyset}$$
  $$d\widehat{\mathbf{S}_{\emptyset}} = Rd_{r}d_{\theta}$$

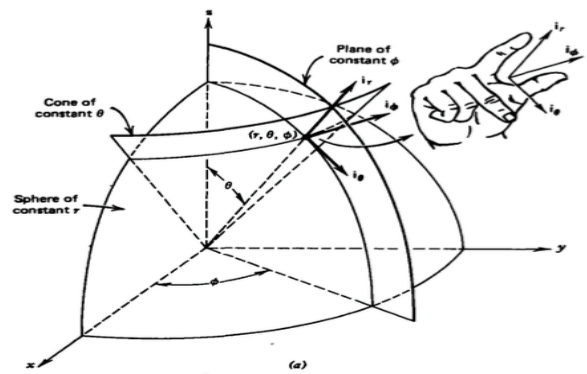
– **The volume is given by:**

  $$d\widehat{\mathbf{V}} = R^{2}\sin\theta\,d_{r}d_{\theta}d_{\emptyset}$$

2. Coordinate Systems

– **Spherical Coordinates**





2. Coordinates Systems - Summary

Summary

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Base vectors properties	$\hat{x}\cdot\hat{x}=\hat{y}\cdot\hat{y}=\hat{z}\cdot\hat{z}=1$ $\hat{x}\cdot\hat{y}=\hat{y}\cdot\hat{z}=\hat{z}\cdot\hat{x}=0$ $\hat{x}\times\hat{y}=\hat{z}$ $\hat{y}\times\hat{z}=\hat{x}$ $\hat{z}\times\hat{x}=\hat{y}$	$\hat{r}\cdot\hat{r}=\hat{\phi}\cdot\hat{\phi}=\hat{z}\cdot\hat{z}=1$ $\hat{r}\cdot\hat{\phi}=\hat{\phi}\cdot\hat{z}=\hat{z}\cdot\hat{r}=0$ $\hat{r}\times\hat{\phi}=\hat{z}$ $\hat{\phi}\times\hat{z}=\hat{r}$ $\hat{z}\times\hat{r}=\hat{\phi}$	$\hat{R}\cdot\hat{R}=\hat{\theta}\cdot\hat{\theta}=\hat{\phi}\cdot\hat{\phi}=1$ $\hat{R}\cdot\hat{\theta}=\hat{\theta}\cdot\hat{\phi}=\hat{\phi}\cdot\hat{R}=0$ $\hat{R}\times\hat{\theta}=\hat{\phi}$ $\hat{\theta}\times\hat{\phi}=\hat{R}$ $\hat{\phi}\times\hat{R}=\hat{\theta}$
Dot product, $\mathbf{A}\cdot\mathbf{B}=$	$A_xB_x+A_yB_y+A_zB_z$	$A_rB_r+A_\phi B_\phi+A_zB_z$	$A_RB_R+A_\theta B_\theta+A_\phi B_\phi$
Cross product, $\mathbf{A}\times\mathbf{B}=$	$\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$	$\begin{vmatrix}\hat{r} & \hat{\phi} & \hat{z}\\ A_r & A_\phi & A_z\\ B_r & B_\phi & B_z\end{vmatrix}$	$\begin{vmatrix}\hat{R} & \hat{\theta} & \hat{\phi}\\ A_R & A_\theta & A_\phi\\ B_R & B_\theta & B_\phi\end{vmatrix}$
Differential length, $d\mathbf{l}=$	$\hat{x}\,dx+\hat{y}\,dy+\hat{z}\,dz$	$\hat{r}\,dr+\hat{\phi}r\,d\phi+\hat{z}\,dz$	$\hat{R}\,dR+\hat{\theta}R\,d\theta+\hat{\phi}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x=\hat{x}\,dy\,dz$ $d\mathbf{s}_y=\hat{y}\,dx\,dz$ $d\mathbf{s}_z=\hat{z}\,dx\,dy$	$d\mathbf{s}_r=\hat{r}r\,d\phi\,dz$ $d\mathbf{s}_\phi=\hat{\phi}\,dr\,dz$ $d\mathbf{s}_z=\hat{z}r\,dr\,d\phi$	$d\mathbf{s}_R=\hat{R}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta=\hat{\theta}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi=\hat{\phi}R\,dR\,d\theta$
Differential volume, $d\mathcal{V}=$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

2. Coordinates Systems - Summary

Summary

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}}=\hat{\mathbf{y}}\cdot\hat{\mathbf{y}}=\hat{\mathbf{z}}\cdot\hat{\mathbf{z}}=1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}}=\hat{\mathbf{y}}\cdot\hat{\mathbf{z}}=\hat{\mathbf{z}}\cdot\hat{\mathbf{x}}=0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}}=\hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}}=\hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}}=\hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}=\hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}}=\hat{\mathbf{z}}\cdot\hat{\mathbf{z}}=1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}}=\hat{\mathbf{z}}\cdot\hat{\mathbf{r}}=0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}}=\hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}}=\hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}}=\hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}}=\hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}}=\hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}}=1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}}=\hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}}=0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}}=\hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}}=\hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}}=\hat{\boldsymbol{\theta}}$
Dot product, $\mathbf{A}\cdot\mathbf{B}=$	$A_xB_x+A_yB_y+A_zB_z$	$A_rB_r+A_\phi B_\phi+A_zB_z$	$A_RB_R+A_\theta B_\theta+A_\phi B_\phi$
Cross product, $\mathbf{A}\times\mathbf{B}=$	$\begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$	$\begin{vmatrix}\hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}}\\ A_r & A_\phi & A_z\\ B_r & B_\phi & B_z\end{vmatrix}$	$\begin{vmatrix}\hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}}\\ A_R & A_\theta & A_\phi\\ B_R & B_\theta & B_\phi\end{vmatrix}$
Differential length, $d\mathbf{l}=$	$\hat{\mathbf{x}}\,dx+\hat{\mathbf{y}}\,dy+\hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr+\hat{\boldsymbol{\phi}}r\,d\phi+\hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR+\hat{\boldsymbol{\theta}}R\,d\theta+\hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x=\hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y=\hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z=\hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r=\hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi=\hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z=\hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R=\hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta=\hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi=\hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume, $d\mathcal{V}=$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

4. Systems Transformation

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r=\sqrt[+]{x^2+y^2}$ $\phi=\tan^{-1}(y/x)$ $z=z$	$\hat{\mathbf{r}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_r=A_x\cos\phi+A_y\sin\phi$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$ $A_z=A_z$
Cylindrical to Cartesian	$x=r\cos\phi$ $y=r\sin\phi$ $z=z$	$\hat{\mathbf{x}}=\hat{\mathbf{r}}\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{r}}\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{z}}$	$A_x=A_r\cos\phi-A_\phi\sin\phi$ $A_y=A_r\sin\phi+A_\phi\cos\phi$ $A_z=A_z$
Cartesian to spherical	$R=\sqrt[+]{x^2+y^2+z^2}$ $\theta=\tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi=\tan^{-1}(y/x)$	$\hat{\mathbf{R}}=\hat{\mathbf{x}}\sin\theta\cos\phi$ $+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi$ $+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=-\hat{\mathbf{x}}\sin\phi+\hat{\mathbf{y}}\cos\phi$	$A_R=A_x\sin\theta\cos\phi$ $+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi$ $+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
Spherical to Cartesian	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{\mathbf{x}}=\hat{\mathbf{R}}\sin\theta\cos\phi$ $+\hat{\boldsymbol{\theta}}\cos\theta\cos\phi-\hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}}=\hat{\mathbf{R}}\sin\theta\sin\phi$ $+\hat{\boldsymbol{\theta}}\cos\theta\sin\phi+\hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_x=A_R\sin\theta\cos\phi$ $+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi$ $+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical to spherical	$R=\sqrt[+]{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{\mathbf{R}}=\hat{\mathbf{r}}\sin\theta+\hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}}=\hat{\mathbf{r}}\cos\theta-\hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
Spherical to cylindrical	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{\mathbf{r}}=\hat{\mathbf{R}}\sin\theta+\hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}}=\hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}}=\hat{\mathbf{R}}\cos\theta-\hat{\boldsymbol{\theta}}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$