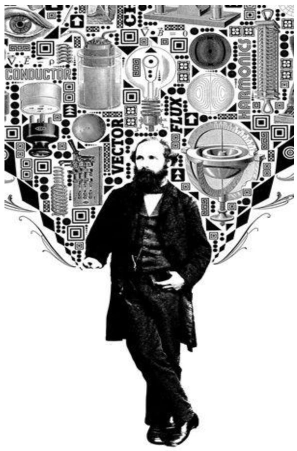

Electricity and Magnetism

Electrostatic Force and Coulomb's Law

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ELECTROMAGNETIC FIELDS By Ahmed W.

Lecture's Contents Part - II

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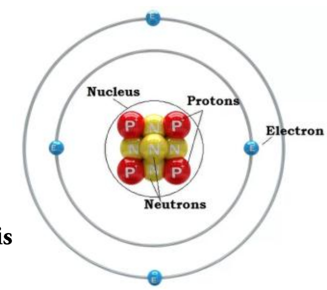
1. Electrostatic Force

– The Atomic Scale

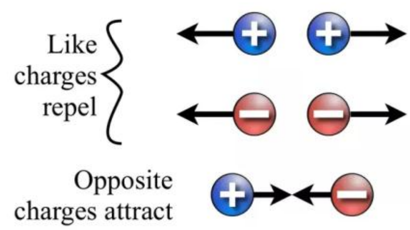
- The modern chemical picture of atom is consists of nucleus which have positive protons and neutron of no charge.
- The nucleus is extremely small compared to atom.
- Mathematically, $m_{p+} = m_n = 1.7 \times 10^{-27}\ kg$.
- Electrons are moves in cloud around the nucleus and hold a negative charge.
- Mathematically, $m_e = 9.1 \times 10^{-31}\ kg$. Extremely smaller than the mass of proton. That is why nucleus is heavy.
- If an atom lost an electron, it becomes a positive ion. If an atom gained electrons , it becomes a negative ion,
- Mathematically, $e^- = p^+ = 1.6 \times 10^{-19}\ C$.



1. Electrostatic Force

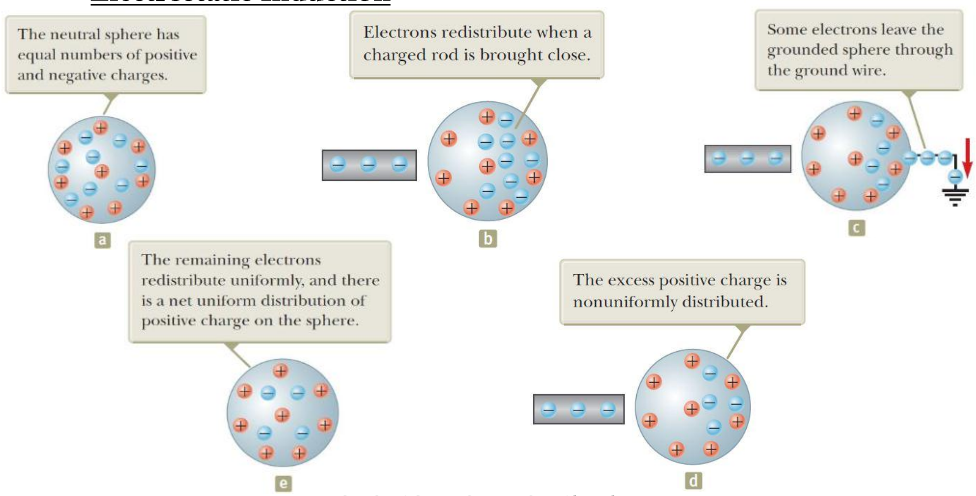
– The Electric Charge

- The discovery of two types of unlike charges backs to the 18th century, more specifically, 1700.
- Researchers found that there are positive charges and negative charges can be created by rubbing amber and/or silk with glass rod.
- All matter is held together due the attractive force between equal numbers of protons (positive charges) and electrons (negative charges).
- Like charges are repel each others, while unlike attracts.



1. Electrostatic Force

– Electrostatic Induction



1. Electrostatic Force

– Electrostatic Force and Coulomb's Law

- **Coulomb (1736 – 1806) carried out an experiments to determine how the electrostatic force depends on the distance between "POINT" charges as well on the magnitude of their charges.**
- **Suppose "point" object (1) has (n_1) extra electrons, and "point" object (2) has (n_2) extra electrons. Each electron on object (1) will feel a force of (n_2F).**
- **Since there are (n_1) electrons on object (1), the magnitude of the net force between both objects is ($n_1 n_2$F) which is given by:**

$$|\vec{F}_e| \propto \frac{n_1 n_1}{r^2}$$

1. Electrostatic Force

– Electrostatic Force and Coulomb's Law

- **It is convenient to express the electrostatic force in term of Net Charge on the object instead of the number of electrons and/or protons.**

  - ❖ **The Net charge on Object (1):** $q_1 = \left(N(1)_e - N(1)_p\right)e$
  - ❖ **The Net charge on Object (2):** $q_2 = \left(N(2)_e - N(2)_p\right)e$

- N_e **and** N_p**:** **Number of electrons and protons, respectively.**
- **Coulomb's Law:**

$$\left|\vec{\mathbf{F}}_{\mathbf{e}}\right| = k\frac{|q_1||q_2|}{r^2}$$

- **The absolute values are used since the charges can be positive or negative and the equation is an expression for the magnitude of electrostatic force.**

1. Electrostatic Force

– Electrostatic Force and Coulomb's Law

- **It is convenient to express Coulomb's force in term of direction:**

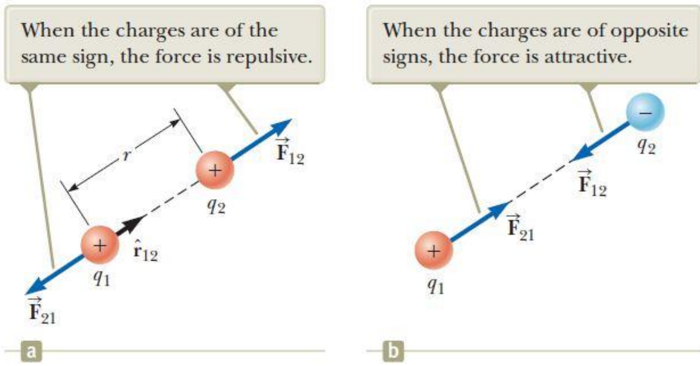
$$\overrightarrow{\mathbf{F}_{\mathbf{e}}} = k\frac{q_1 q_2}{r^2}\hat{\mathbf{n}}$$

- $\hat{\mathbf{n}}$ **is the unit vector pointing from the charge (1) to charge (2).**
- k **is Coulomb's constant** 8.987×10^9 N. m^2/C^2

$$k = \frac{1}{4\pi\epsilon_0}$$

- ϵ_0 **is the permittivity of free space** 8.85×10^{-12} C^2/(N. m^2)

1. Electrostatic Force



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1. Electrostatic Force

– **Notes:**
  - **The formula is only valid for point particles. How small is small enough to be considered a "point charge"? If the separation distance between particles is very large compared to the dimension of charges.**
  - **The mass of the charge does not effect the electrical force. Charge does not add to the inertia of an object, nor does it affect the gravitational force.**
  - **The force is called electrostatic because the force does not depend on the velocity of the particles.**
  - **The force magnitude falls** $(1/r^2)$.
  - **Coulomb's law holds at the atomic length scales.**
  - **The electrostatic force is much larger compared to the gravitational force.**

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1. Electrostatic Force

– How Large the Electrostatic Force:

- The electron and proton of a hydrogen atom are separated by a distance of approximately 5.3×10^{-11}m. Find the magnitude of the electric force and the gravitational force between the two particles.

$$|\vec{F}_e| = k\frac{|e||-e|}{r^2} = \left(8.99\times10^9\ \mathrm{N.}\frac{m^2}{C^2}\right)\frac{(1.6\times10^{-19}\ C)^2}{(5.3\times10^{-11}\ m)^2} = 8.2\times10^{-8}\ N$$

$$|\vec{F}_g| = G\frac{m_e m_p}{r^2} = \left(6.679\times10^{-11}\ \mathrm{N.}\frac{m^2}{kg^2}\right)\frac{(9.11\times10^{-31}kg)(1.67\times10^{-27}kg)}{(5.3\times10^{-11}\ m)^2} = 3.6\times10^{-47}\ N$$

$$\frac{F_e}{F_g} = 2\times10^{39}$$

1. Electrostatic Force

– Superposition Principle

- When more than two charges are present, the force between any pair of them is given by the vector sum of the electrostatic force due to all other charges:

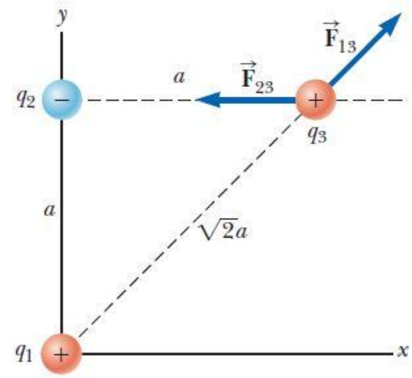
$$\vec{\mathbf{F}}_1(\mathrm{NET}) = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31}$$

$$\vec{\mathbf{F}}_1 = k\left(\frac{q_1q_2}{r_{21}^2}\hat{\mathbf{n}}_{21} + \frac{q_1q_3}{r_{31}^2}\hat{\mathbf{n}}_{31}\right)$$

1. Electrostatic Force

– Three Charges Example

Consider three point charges located at the corners of a right triangle as shown in the figure beside, where $q_1 = q_3 = 5\mu C$, $q_2 = -2\mu C$, and $a = 0.1$ m. Find the resultant force exerted on q_3.



2. Electric Field and Electric Field Lines

– The Electric Field

- **The electric field vector $\vec{E}$ at a point in space is defined as the electric force acting on a positive test charge (q_0) placed at the point divided by the test charge:**

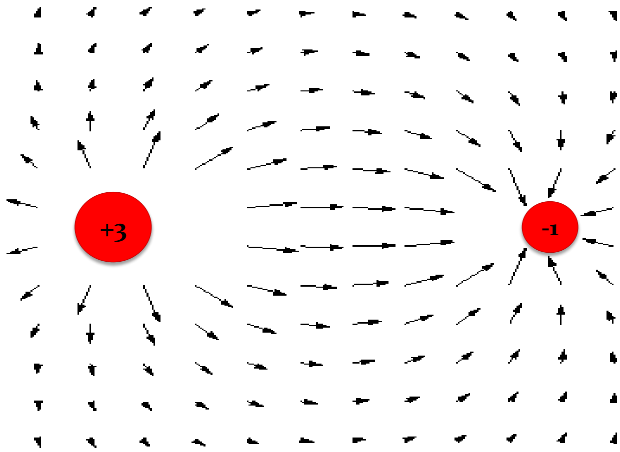
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F_e}}}{q_0} = k\frac{Q}{r^2}\hat{\mathbf{n}}$$

- **For n-numbers of charges:**

$$\vec{\mathbf{E}} = k\left(\frac{q_1}{r_1^2}\mathbf{n_1} + \frac{q_2}{r_2^2}\hat{\mathbf{n}}_2 + \frac{q_3}{r_3^2}\hat{\mathbf{n}}_3 + \cdots + \frac{q_n}{r_n^2}\hat{\mathbf{n}}_n\right)$$

2. Electric Field and Electric Field Lines

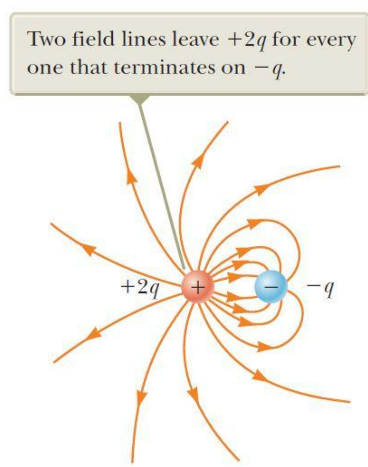
– **Electric Field of +3 and -1 Charges**



1. Electrostatic Force

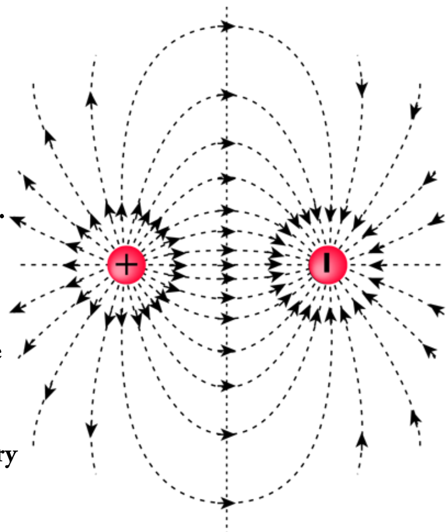
– **Electric Field Lines**

- **The electric field vector is tangent to the electric field lines at each point.**
- **The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.**
- **The lines must begin on a positive charge and terminate in a negative charge.**
- **No two field lines can cross**



2. Electric Field and Electric Field Lines

– **Electrostatic Field of an Electric Dipole**

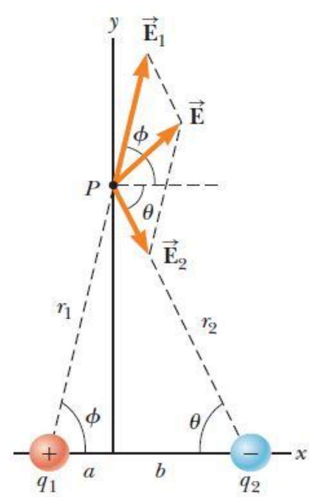


- Because the charges are of the equal magnitudes, the number of lines that begins at positive charges is equal of that lines end up at the negative lines.
- At points very near the charges, the field lines are radial, as for single isolated charges.
- The high density of lines between the charges indicates a region of strong electric field.
- How to see electric field at a point very far away than the separation between charges.?

2. Electric Field and Electric Field Lines

– **Example**

- Charges (q_1) and (q_2) are located on the x – axis, at distance (a) and (b), respectively, from the origin as shown in the figure. Find the component of the net electric field at the point (P), which is at position (0, y).



3. E-Field of Continuous Charge Distribution

– Charge Distribution

- **When performing calculation of charge distribution, it is convenient to use the concept of a CHARGE DENSITY along with the following notations:**
  - **If a charge Q is uniformly distributed throughout a volume (V), the volume charge density (ρ) is defined as:**

$$\rho = \frac{Q}{V} \quad \left[\frac{C}{m^3}\right]$$

  - **If a charge Q is uniformly distributed on a surface of area (S), the surface charge density (σ) is defined by:**

$$\sigma = \frac{Q}{S} \quad \left[\frac{C}{m^2}\right]$$

  - **If a charge Q is uniformly distributed along a surface of area (l), the line charge density (λ) is defined by:**

$$\lambda = \frac{Q}{l} \quad \left[\frac{C}{m}\right]$$

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3. E-Field of Continuous Charge Distribution

– E-Field of Charge Distribution

- **The *ELECTRIC FIELD* due to differential charge element (dq) at any point in space can be performed by:**

$$\overrightarrow{d\mathbf{E}} = k\frac{dq}{r_{QP}^2}\,\hat{\mathbf{n}}_{\mathbf{QP}}$$

- **The total electric field due to line, surface, or a volume charge distribution is:**

$$\vec{\mathbf{E}} = k\int_x \frac{dq}{r_{QP}^2}\,\hat{\mathbf{n}}_{\mathbf{QP}}$$

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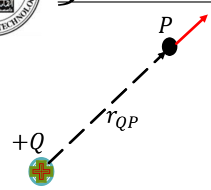
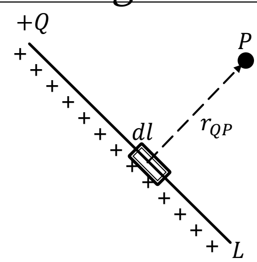
3. E-Field of Continuous Charge Distribution



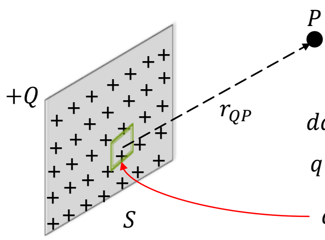
Point Charge



$$dq = \lambda\, dl$$

$$q = \int_L \lambda\, dl$$

Line of Charge

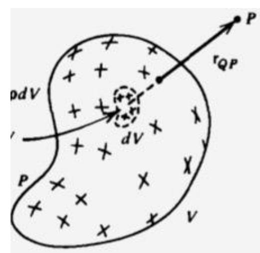


$$dq = \sigma\, dS$$

$$q = \int_S \sigma\, dS$$

dS

Surface of Charge



$$dq = \rho\, dv$$

$$q = \int_v \rho\, dv$$

Volume of Charge

3. E-Field of Continuous Charge Distribution

– **E-Field of Line of Charge**

$$\mathrm{dE = dE_1 + dE_2}$$

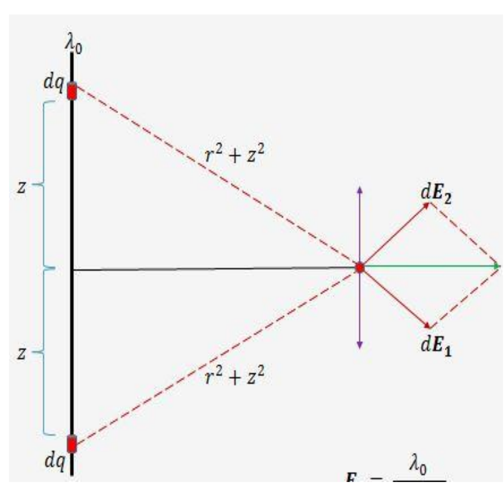
$$\mathrm{dE_r} = \frac{\lambda_0 \mathrm{dz}}{4\pi\varepsilon_0(\mathrm{z}^2 + \mathrm{r}^2)} \cos\theta$$

$$\mathrm{dE_r} = \frac{\lambda_0\, \mathrm{r\, dz}}{4\pi\varepsilon_0(\mathrm{z}^2 + \mathrm{r}^2)^{3/2}}$$

$$\mathrm{E_r} = \int_{-\infty}^{+\infty} \frac{\lambda_0\, \mathrm{r\, dz}}{4\pi\varepsilon_0(\mathrm{z}^2 + \mathrm{r}^2)^{3/2}}$$

$$\mathrm{E_r} = \frac{\lambda_0\, \mathrm{r}}{4\pi\varepsilon_0} \frac{\mathrm{z}}{\mathrm{r}^2(\mathrm{z}^2 + \mathrm{r}^2)^{1/2}} \Big|_{-\infty}^{+\infty}$$

$$\mathrm{E_r} = \frac{\lambda_0}{2\pi\varepsilon_0\, \mathrm{r}} = \frac{\lambda_0}{2\pi\varepsilon_0\, \mathrm{r}}\hat{\mathbf{n}}$$



3. E-Field of Continuous Charge Distribution

– **E-Field of Surface of Charge**

$$dE_y = \frac{\sigma_S\, dx}{2\pi\varepsilon_0 (x^2+y^2)^{1/2}} \cos\theta = \frac{\sigma_S\, y\, dx}{2\pi\varepsilon_0 (x^2+y^2)}$$

$$E_y = \frac{\sigma_0\, y}{2\pi\varepsilon_0} \int_{-\infty}^{+\infty} \frac{dx}{x^2+y^2} = \frac{\sigma_S\, y}{2\pi\varepsilon_0 y} \frac{1}{y} \tan^{-1}\frac{x}{y} \Big|_{-\infty}^{+\infty}$$

$$E_y = \begin{cases} \dfrac{\sigma_S}{2\varepsilon_0}, & y > 0 \\ -\dfrac{\sigma_S}{2\varepsilon_0}, & y < 0 \end{cases}$$

$$\vec{\mathbf{E}} = \frac{\sigma_S}{2\epsilon_0}\hat{\mathbf{n}}$$

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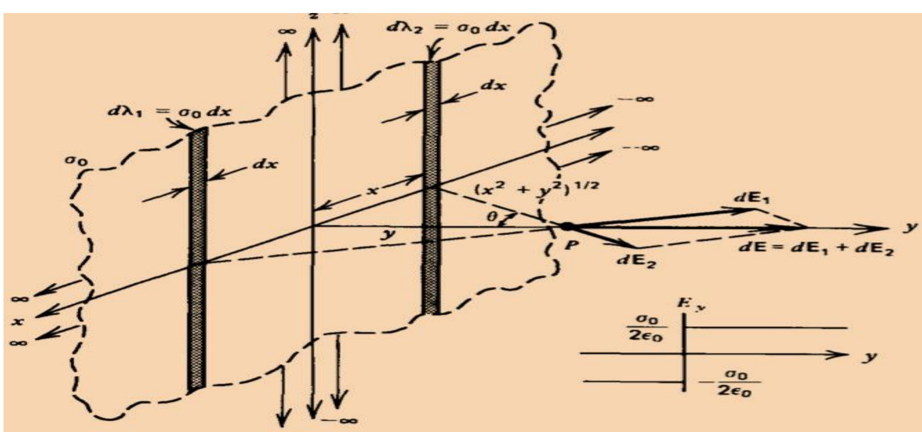
3. E-Field of Continuous Charge Distribution

– **E-Field of Surface of Charge**



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