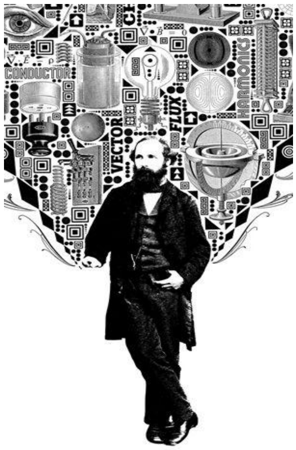

Electricity and Magnetism

Electric Flux and Gauss's Law

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2017 - 2018

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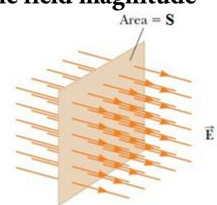
Lecture's Contents Part - II

1. Electric Flux

– The concept of Electric Flux

- **The total number of lines (electric field line density) penetrating a surface of area (S) is proportional to the product of that electric field time the area.**
- **The *Electric Flux* (∅) is defined as the product of electric field magnitude ($\vec{\mathbf{E}}$) time the surface area (A)**



$$\emptyset = |\vec{\mathbf{E}}|S$$

- **In SI units, the units of electric flux is:** $\left[\frac{\mathrm{N\,m^2}}{\mathrm{C}}\right]$
- **Electric flux is proportional to the number of electric field lines penetrating the surface.**
- **The electric flux is a SCALAR value takes any value (zero, positive or negative)**

1. Electric Flux

– Electric Flux Density

- **The *Electric Flux Density* ($\vec{\mathbf{D}}$) is a vector field defined as the density of electric field lines crossing a squared meter of surface area.**
- **The electric flux density relates to the electric field (in free space) by:**

$$\vec{\mathbf{D}} = \epsilon_0 \vec{\mathbf{E}}$$

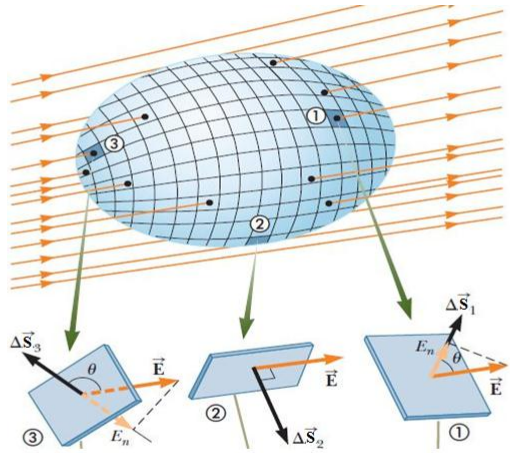
- **A general form of electric flux density for charge distribution**

$$\vec{\mathbf{D}} = \int \frac{dq}{4\pi r^2}\,\hat{\mathbf{n}}$$

1.Electric Flux

– Closed Surface Electric Flux

- We are often interested in evaluating the flux through a *closed surface*, defined as a surface that divides space into an *inside* and *outside* region so that one cannot move from one region to the other without *crossing* the surface.
- In the figure, the vector $(\Delta\vec{S})$ is always normal to the surface elements and the electric field vector is always pointing from inside to outside.



1. Electric Flux

– Closed Surface Electric Flux

- At the element labeled (1), the field lines are crossing the surface from the inside to the outside and $\theta < 90^o$, the flux is positive.
- At the element labeled (2), the field lines are graze the surface from the inside to the outside and $\theta = 90^o$, the flux is zero.
- At the element labeled (1), the field lines are crossing the surface from the inside to the outside and $180° > \theta > 90^o$, the flux is negative.
- In other words: If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.

- The flux of closed surface is:

$$\emptyset = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \oint E\, dS$$

2. Gauss's Law

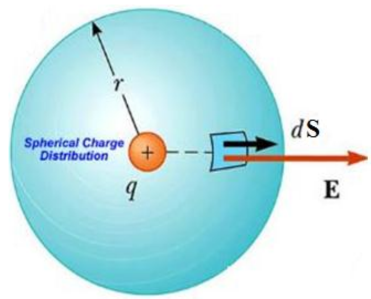
– **The Gauss's Law**

- **Consider a positive point charge (+q) located at the center of a sphere of radius (r) as shown in figure.**
- **The electric field lines are radially pointing outward of the sphere and perpendicular to the sphere's surface element everywhere.**
- **The electric Flux is:**

$$\emptyset = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E \oint dS = ES$$

$$S = 4\pi r^2, \qquad E = \frac{q}{4\pi\epsilon_0 r^2} \quad , \qquad \emptyset = E(4\pi r^2)$$

$$\emptyset = \frac{q}{4\pi\epsilon_0 r^2}(4\pi r^2) = \frac{q}{\epsilon_0}$$



2. Gauss's Law

– **The Gauss's Law – The 1st equation of J. C. Maxwell**

- **The integral form of Gauss's law:**

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{\sum q_{in}}{\epsilon_0} \quad \text{or} \quad \oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} = q_{in}$$

- **The Differential For od Gauss law:**

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$

- **This equation shows that the net flux through the spherical surface is proportional to the charge inside the surface.**
- **The flux is independent of the radius (r) because the area of the spherical surface is proportional to (r^2), whereas the electric field is proportional to $\left(\frac{1}{r^2}\right)$. Therefore, in the product of area and electric field, the dependence on (r) cancels.**

2. Gauss's Law

– Gauss's Law for Charge Distribution

- **For several point charges, Gauss's law:**

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{\sum q_{in}}{\epsilon_0} \quad \text{or} \quad \oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} = \sum q_{in}$$

- **Line Charge Distribution:**

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{\int \lambda \, dl}{\epsilon_0} \quad \text{or} \quad \oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} = \int \lambda \, dl$$

- **Surface Charge Distribution (not necessarily closed surface):**

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{\int \sigma \, dS}{\epsilon_0} \quad \text{or} \quad \oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} = \int \sigma \, dS$$

- **Volume Charge Distribution:**

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{\int \rho \, dv}{\epsilon_0} \quad \text{or} \quad \oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} = \int \rho \, dv$$

3. Applications of Gauss's Law

– Symmetrical Condition

- **The solution using Gauss's law is hold only if:**
  - ❖ **The symmetry argument should be exist. If it is not existed, then we cannot use Gauss's law**
  - ❖ **The electric field vector is either normal or tangential to the closed surface.**
  - ❖ **On that portion of the closed surface for which the dot product of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} \neq 0$, then $E =$ constant.**
  - ❖ **The electric field vector should be either pointing away from positive charge or pointing toward a negative charge.**

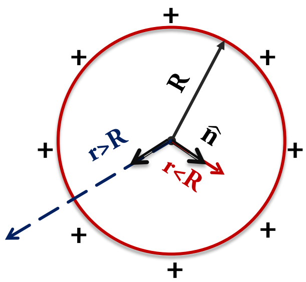
- **Three Gaussian surfaces are used to perform symmetry argument:**

Surface Symmetry | **Cylindrical Symmetry** | **Spherical Symmetry**

3. Applications of Gauss's Law

– A spherical Symmetric Charge Distribution

A conducting spherical shell of radius (R) carries a total positive charge $+Q$ uniformly distributed over its surface. Calculate the magnitude of the electric field at a point (r) inside and outside the sphere.



- **Since the charges are uniformly distributed over the sphere surface in a symmetric manner, we apply Gauss law.**
- **Choosing the Gaussian surface is very crucial to satisfy that the dot product of $\vec{E}\cdot d\vec{S}$ has an angle of either 0 or 180. (The Gaussian surface is sphere).**
- **Since the shell has a charge of $(+q)$, the normal to the surface area of the shell and the electric field are pointing in the same direction.** $\vec{E}\cdot d\vec{S} = ES$

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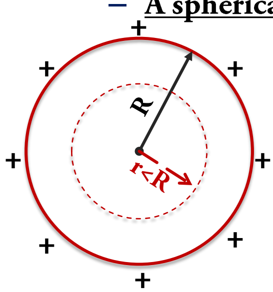
3. Applications of Gauss's Law

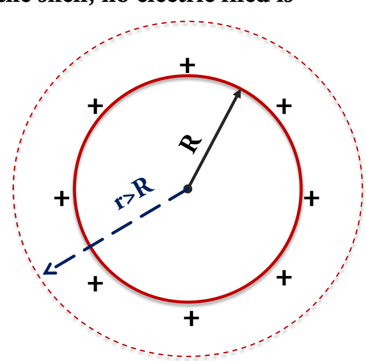
– A spherical Symmetric Charge Distribution



- **At** $(r < R)$:

$$\oint \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0} \Rightarrow E\oint R^2 \sin\theta\, d\theta d\phi = \frac{q}{\epsilon_0}$$

Since there is no net charge inside the shell, no electric filed is zero $(E_{in} = 0)$



- **At** $(r > R)$:

$$\oint \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0} \Rightarrow E\oint R^2 \sin\theta\, d\theta d\phi = \frac{q}{\epsilon_0}$$

$$4\pi r^2 E = \frac{q}{\epsilon_0} \quad \Rightarrow \quad E = \frac{q}{4\pi\epsilon_0 r^2}$$

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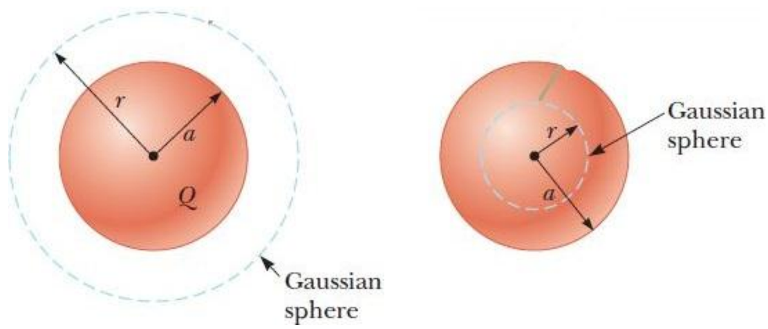
3. Applications of Gauss's Law

– A spherical Symmetric Charge Distribution

An insulating solid sphere of radius (a) **has a uniform volume charge density** (ρ) **and carrier a total positive charge** (+Q). **Calculate the magnitude of** ($\vec{\mathbf{E}}$) **at a point outside and inside the sphere.**



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3. Applications of Gauss's Law

– A spherical Symmetric Charge Distribution

- **For** (r > a):

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E \oint dS = E \oint r^2 \sin(\theta)\, d\theta d\emptyset = \frac{Q}{\epsilon_0}$$

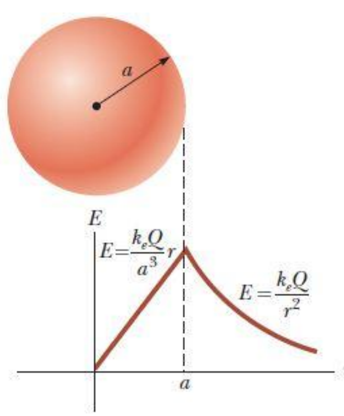
$$4\pi r^2 E = \frac{Q}{\epsilon_0} \Longrightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

- **For** (r < a)**: we calculate the electric field in this case for smaller sphere of volume** v′.

$$Q_{in} = \rho v' = \rho\left(\frac{4}{3}\pi r^3\right)$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E \oint dS = \frac{Q_{in}}{\epsilon_0} \Longrightarrow 4\pi r^2 E = \frac{4\rho\pi r^3}{3\epsilon_0}$$

$$E = \frac{\rho r}{3\epsilon_0}$$



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3. Applications of Gauss's Law

– **Conceptualization**

- **This result for (E) differs from the one obtained in part $(r > a)$. It shows that $(E \rightarrow 0)$ as $(r \rightarrow 0)$. Therefore, the result eliminates the problem that would exist at $(r = 0)$ if (E) varied as $\left(\frac{1}{r^2}\right)$ inside the sphere as it does outside the sphere. That is, if $\left(E \propto \frac{1}{r^2}\right)$ for $(r < a)$, the field would be infinite at $(r = 0)$, which is physically impossible.**

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3. Applications of Gauss's Law

– **A cylindrically symmetric charge distribution**

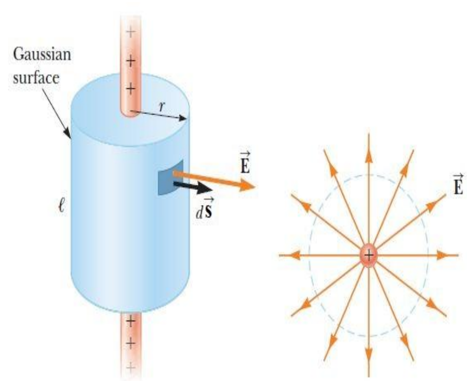
find the electric field a distance (r) from a line of positive charge of infinite length and constant charge per unit length.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{q}{\epsilon_0} \Longrightarrow q = \epsilon_0 E \int_{sides} dS$$

$$q = \epsilon_0 E \int_{z=0}^{\ell} \int_{\emptyset=0}^{2\pi} r d\emptyset dz = \epsilon_0 E 2\pi r \ell$$

$$E = \frac{q}{2\pi\epsilon_0 r \ell} \qquad \text{but} \quad q = \lambda\ell$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$



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3. Applications of Gauss's Law

– **Analysis**

This result shows that the electric field due to a cylindrically symmetric charge distribution varies as $\left(\frac{1}{r}\right)$, whereas the field external to a spherically symmetric charge distribution varies as $\left(\frac{1}{r^2}\right)$. Equation found can also be derived by direct integration over the charge distribution.

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3. Applications of Gauss's Law

– **A Plane of Charge**

Find the electric field due to an infinite plane of positive charge with uniform surface charge density (σ)

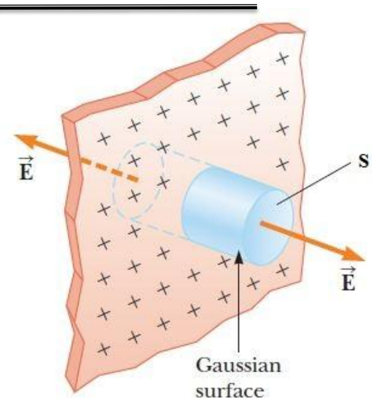
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{q}{\epsilon_0} \Longrightarrow q = \epsilon_0 E \int_{sides} dS$$

$$q = \epsilon_0 E\, S$$

$$\text{But } q = \sigma S$$

$$\sigma S = \epsilon_0 E\, S$$

$$E = \frac{\sigma}{\epsilon_0}$$



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4. Conductors in Electrostatic Equilibrium

– **Conductors in Electrostatic Equilibrium**

- **A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium.**
- **A conductor in electrostatic equilibrium has the following properties:**
  - ❖ **The electric field is zero everywhere inside the conductor, whether the conductor is solid or hollow.**
  - ❖ **If the conductor is isolated and carries a charge, the charge resides on its surface.**
  - ❖ **The electric field at a point just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude $\left(\frac{\sigma}{\epsilon_0}\right)$, where s is the surface charge density at that point.**
  - ❖ **On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.**

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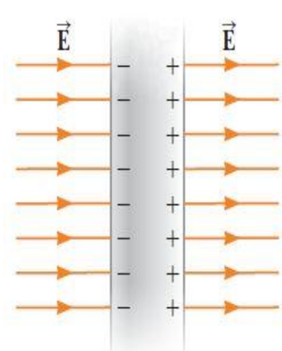
4. Conductors in Electrostatic Equilibrium

– **Conductors in Electrostatic Equilibrium**

We can understand the first property by considering a conducting slab placed in an external field $(\vec{E})$. The electric field inside the conductor *must* be zero, assuming electrostatic equilibrium exists. If the field were not zero, free electrons in the conductor would experience an electric force $(\vec{F} = q\vec{E})$ and would accelerate due to this force. This motion of electrons, however, would mean that the conductor is not in electrostatic equilibrium. Therefore, the existence of electrostatic equilibrium is consistent only with a zero field in the conductor.



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4. Conductors in Electrostatic Equilibrium

– **Conductors in Electrostatic Equilibrium**

- **Before the external field is applied, free electrons are uniformly distributed throughout the conductor.**
- **When the external field is applied, the free electrons accelerate in one side, causing a plane of negative charge to accumulate to one side of the surface.**
- **The movement of electrons to the opposite side results in a plane of positive charge on the right surface.**
- **These planes of charge create an additional electric field inside the conductor that opposes the external field.**
- **As the electrons move, the surface charge densities on the left and right surfaces increase until the magnitude of the internal field equals that of the external field, resulting in a net field of zero inside the conductor.**
- **The time it takes a good conductor to reach equilibrium is on the order of 10216 s, which for most purposes can be considered instantaneous.**