Introduction To measurements

- Similarly other measures were invented

Physical quantity	*Standard unit*	*Definition*
Length	metre	The length of path travelled by light in an interval of 1/299 792 458 seconds
Mass	kilogram	The mass of a platinum–iridium cylinder kept in the International Bureau of Weights and Measures, Sèvres, Paris
Time	second	9.192631770×10^9 cycles of radiation from vaporized caesium-133 (an accuracy of 1 in 10^{12} or 1 second in 36 000 years)
Temperature	kelvin	The temperature difference between absolute zero and the triple point of water is defined as 273.16 kelvin
Current	ampere	One ampere is the current flowing through two infinitely long parallel conductors of negligible cross-section placed 1 metre apart in a vacuum and producing a force of 2×10^{-7} newtons per metre length of conductor
Luminous intensity	candela	One candela is the luminous intensity in a given direction from a source emitting monochromatic radiation at a frequency of 540 terahertz ($Hz \times 10^{12}$) and with a radiant density in that direction of 1.4641 mW/steradian. (1 steradian is the solid angle which, having its vertex at the centre of a sphere, cuts off an area of the sphere surface equal to that of a square with sides of length equal to the sphere radius)
Matter	mole	The number of atoms in a 0.012 kg mass of carbon-12

Table 1.2 Fundamental and derived SI units

(a) Fundamental units

Quantity	*Standard unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Matter	mole	mol

(c) Derived units

Quantity	*Standard unit*	*Symbol*
Area	square metre	m^2
Volume	cubic metre	m^3
Velocity	metre per second	m/s
Acceleration	metre per second squared	m/s^2
Angular velocity	radian per second	rad/s
Angular acceleration	radian per second squared	rad/s^2
Density	kilogram per cubic metre	kg/m^3
Specific volume	cubic metre per kilogram	m^3/kg
Mass flow rate	kilogram per second	kg/s
Volume flow rate	cubic metre per second	m^3/s
Force	newton	N
Pressure	newton per square metre	N/m^2
Torque	newton metre	N m
Momentum	kilogram metre per second	kg m/s
Moment of inertia	kilogram metre squared	$kg\,m^2$
Kinematic viscosity	square metre per second	m^2/s
Dynamic viscosity	newton second per square metre	$N\,s/m^2$
Work, energy, heat	joule	J
Specific energy	joule per cubic metre	J/m^3
Power	watt	W
Thermal conductivity	watt per metre kelvin	W/m K
Electric charge	coulomb	C
Voltage, e.m.f., pot. diff.	volt	V
Electric field strength	volt per metre	V/m
Electric resistance	ohm	Ω
Electric capacitance	farad	F
Electric inductance	henry	H
Electric conductance	siemen	S
Resistivity	ohm metre	Ω m
Permittivity	farad per metre	F/m
Permeability	henry per metre	H/m
Current density	ampere per square metre	A/m^2

- SI units (meter, kg, seconds,….etc) : systems international units
- Imperial system of units (miles, yards, inch, feet, slug,…etc)
- Still used : particularly in Britain and America.
- Trend to ban imperial system of units internationally.
- However: one can convert from one system to another.

Measurement system applications

- Applications can be classified into three major areas:

✓ regulating trade: measure physical quantities: length, volume, mass,..etc.

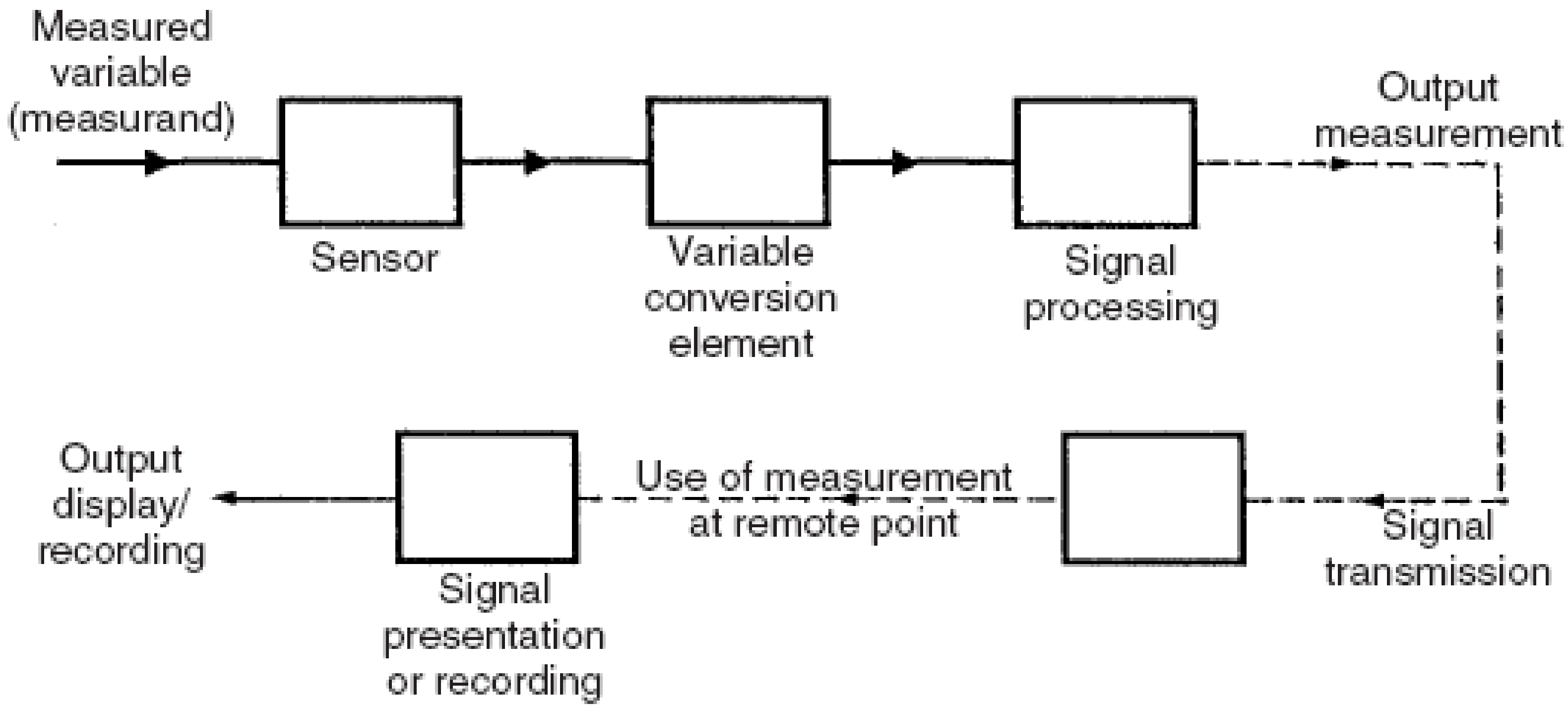
✓ Applications in monitoring functions

- to take actions (e.g. monitor Temp in greenhouses: windows on/off)
- in chemical process: reactions at certain temp & pressure.

✓ As a part of automatic feedback system

- e.g. Temperature control system

Elements of measurement system



Sensor: e.g. thermocouple, strain gauge / usually linear/ primary or complete (thermometer)

VCE: convenience, e.g. R>>V (strain gauge/bridge)
SPE: improve quality, e.g. op.amp for thermocouples (mV)

- ❑Transmission: for convenience or accessibility
- ❑Signal can be displayed or fedback to automatic control system

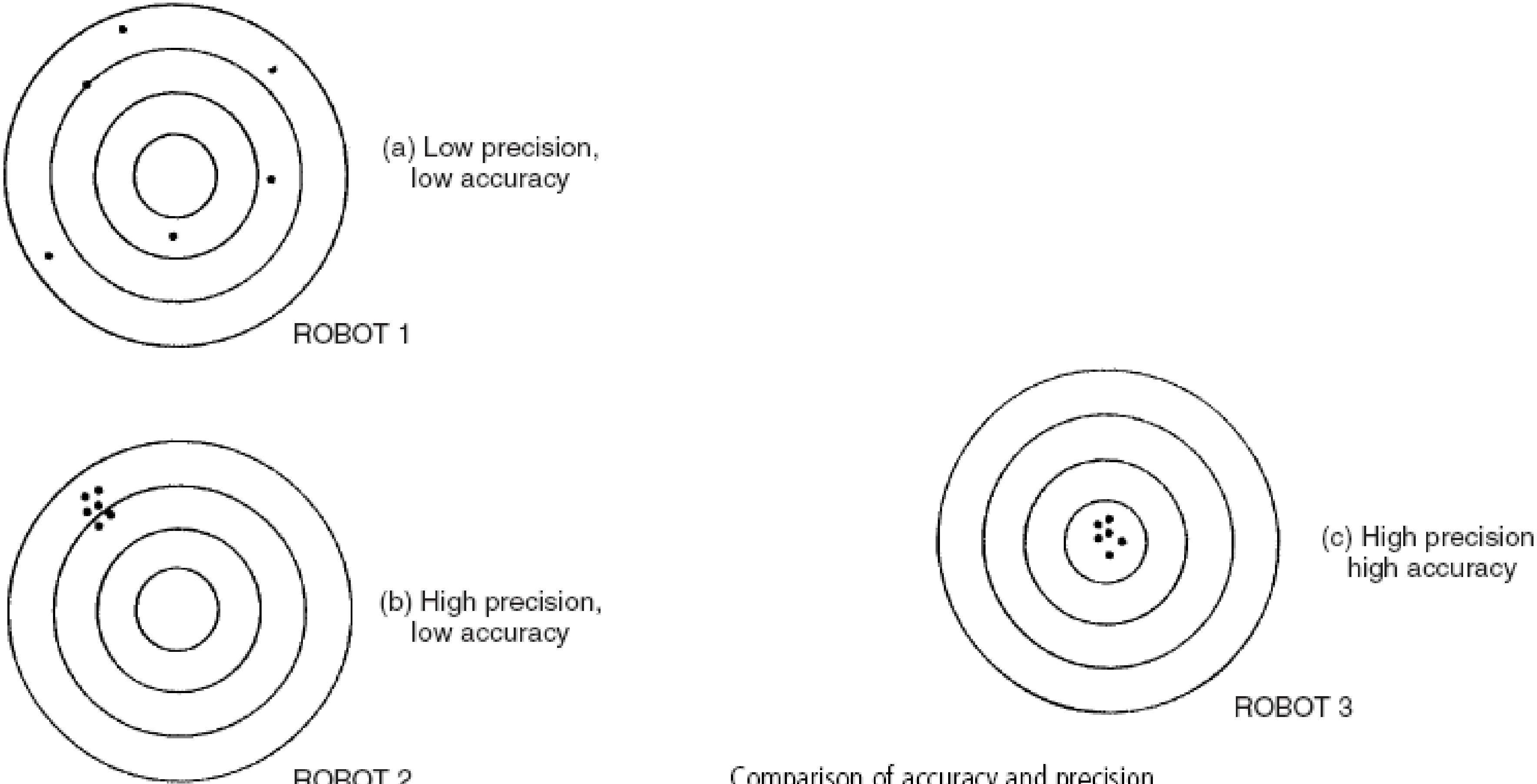
Selection of measuring instrument

- Specifications/characteristics: accuracy – resolution –sensitivity ..etc
- Environmental conditions: eliminate use OR protection (but might reduce dynamic response (e.g,measuring temperature) – might disturb the instrument (e,g, pressure sensor at high flow rate!)
- Cost

- Instrument Engineers: compromise/ select from list/ stay updated
- Better characteristics >>> higher the cost
- Consideration to: durability – maintainability – constancy of performance
- (Purchase cost + maintenance cost)/ projected life or period that instrument is expected to be used! = cost/year >>> unless instrument is reused

Static characteristics of instruments

- **Accuracy and inaccuracy (measurement uncertainty)**
  - measures how close the output reading to correct value
  - Inaccuracy: extend to which reading can be wrong – as percentage of full scale, e.g. ±1% >> can be crucial (thermometer in room vs factory) – match process and instrument range !
- **Precision/ repeatability/ reproducibility**
  - precision: degree of freedom from random errors (confused with accuracy!)
  - Repeatability: closeness to output when input is repeated (same conditions. e.g. instrument, observer, location)
  - Reproducibility : repeatability if conditions vary



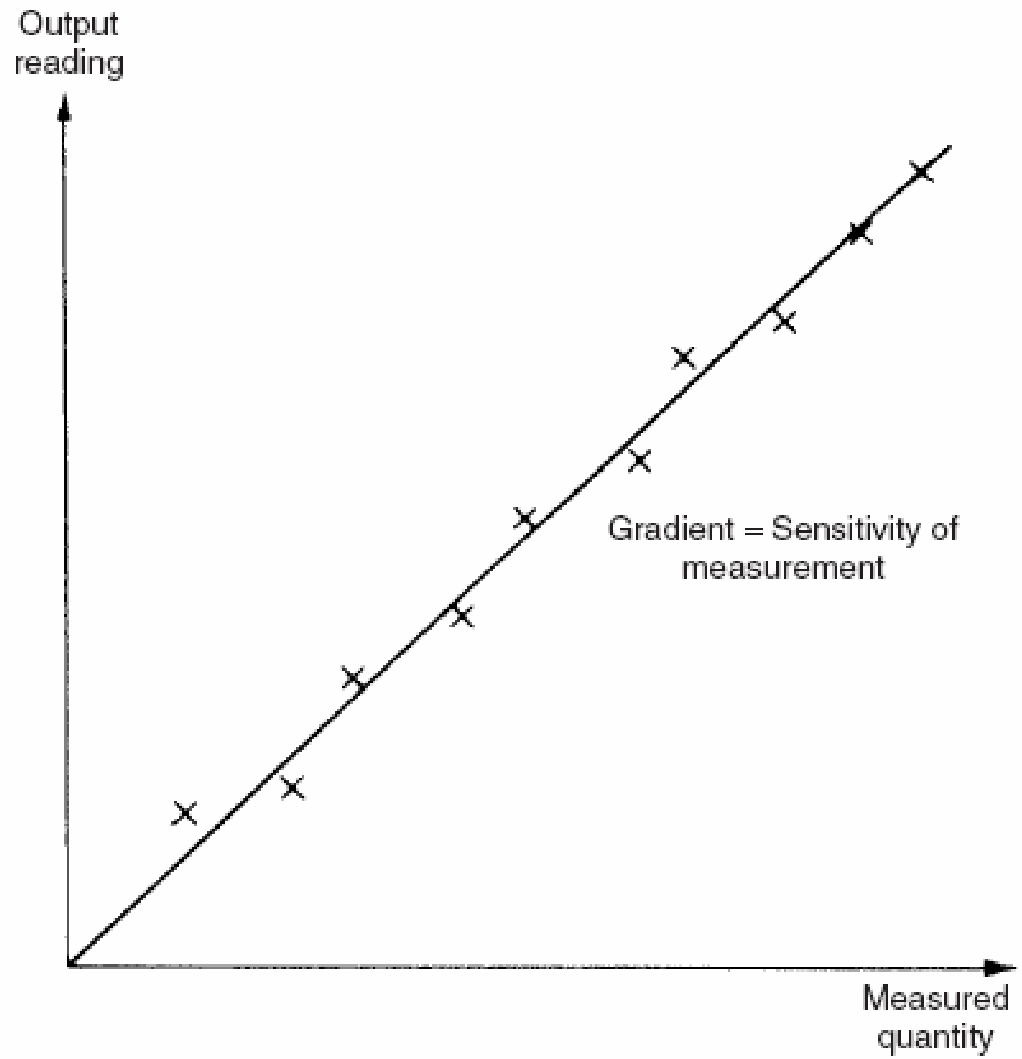
Comparison of accuracy and precision.

- **Tolerance**
  - maximum deviation of manufactured component from specified value. e.g. 1000 W resistors with tolerance 5% in power >> 950 to 1050 at random pick

- **Range or span**
  - minimum and maximum values of quantity the instrument is designed to measure

- **Linearity**
  - maximum deviation in output from fitted line (% full scale)

- **Sensitivity of measurement**
  - Change in output at a given input change :
  - scale deflection/value of measurand producing deflection = slope of fitted line


Output
reading
Gradient = Sensitivity of
measurement
Measured
quantity

- **Threshold**
  - Minimum detectable input (at start). E.g. car speedometer (15km/hr)

- **Resolution**
  - Minimum input produces detectable change in output. E.g. if car speedometer subdivision is 20 km/hr we can estimate changes upto 5km.hr roughly (5km/hr is the resolution)

- **Sensitivity to disturbance**
  - Standard ambient conditions are usually defined (e.g. temperature)
  - Measures the magnitude of change in characteristics of instrument due to condition change
  - Zero drift (bias): zero reading is modified. E.g. scale > remove bias. Also voltmeter due to change in temp >> Volts/ ^{0}C (zero drift coefficient /s > if other parameters !)
  - Sensitivity drift: varies as ambient condition varies

- **saturation**
  - Greater input than allowed

Errors during measurement

Introduction

- Errors during measurement > not associated with noise
- Aims at reducing errors or quantify them
- Problem arises from cumulative reading > overall magnitude of error
- Two types of errors : systematic & Random
- Systematic error: in output reading consistently on one side (all positive or all negative). Due to:

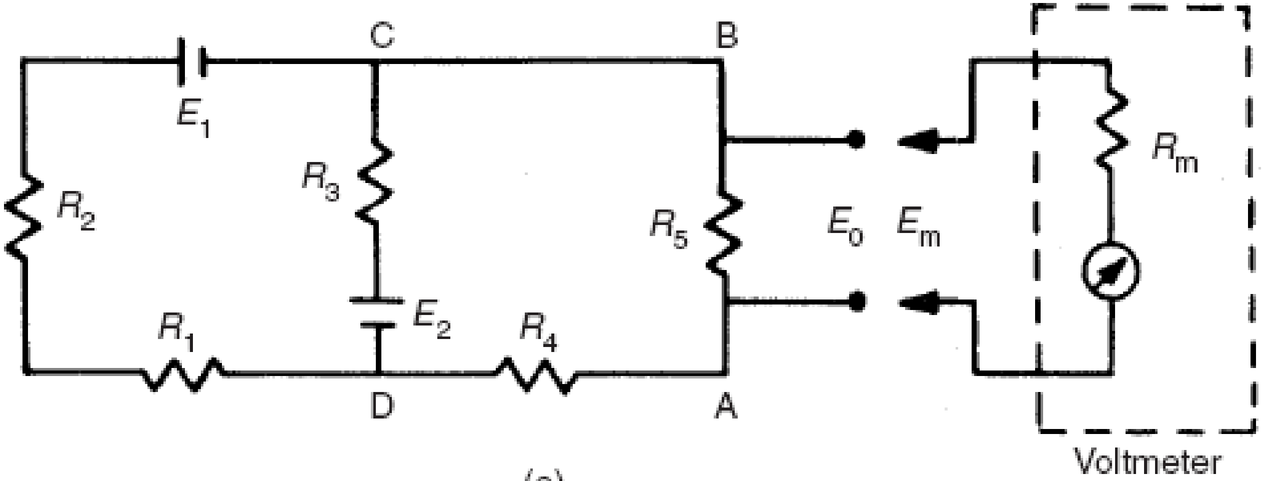
- Disturbance during measurement
- Environmental changes (modifying inputs)
- bent of needle
- Uncalibrated instrument , drift in instrument characteristics
- Cabling practice

Introduction

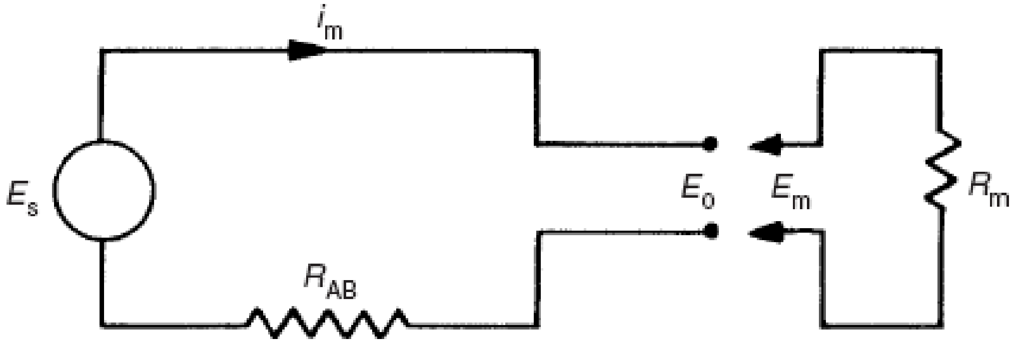
- Random error: perturbations on either sides by random and unpredictable effects(equal weights for positive and negative deviations). Due to:
- Wrong interpretation (e.g. interpolation)
- Electrical noise
- Statically quantified, and improved by averaging
- Quantification is based on probability of confidence (e.g. 99%)
- There is a chance of repeating the error! E.g. wrong reading

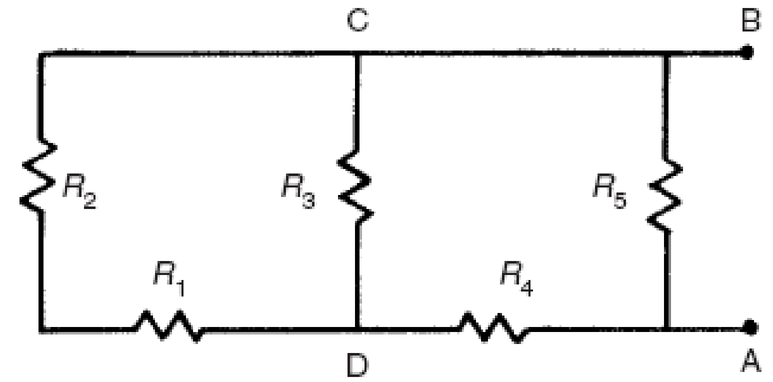
Sources of systematic error

- **System disturbance due to measurement:**
- By the act of measurement : e.g. thermometer in hot water or plate to measure pressure in a pipe
- Improved by reconsidering the design of the instrument
- **Measurement in electrical circuits:**
- Consider an example of voltage measured with voltmeter



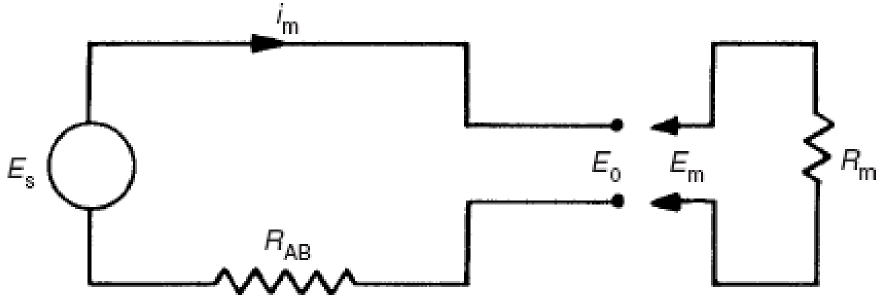
(a)





$$\frac{1}{R_{CD}} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} \quad \text{or} \quad R_{CD} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

$$\frac{1}{R_{AB}} = \frac{1}{R_{CD} + R_4} + \frac{1}{R_5} \quad \text{or} \quad R_{AB} = \frac{(R_4 + R_{CD})R_5}{R_4 + R_{CD} + R_5}$$



$$R_{AB} = \frac{\left[\dfrac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} + R_4\right] R_5}{\dfrac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} + R_4 + R_5}$$

$$I = \frac{E_0}{R_{AB} + R_m}, \qquad E_m = \frac{R_m E_0}{R_{AB} + R_m}.$$

R_{AB} should be large : ideally infinity…>>> but this presents other constrains !!

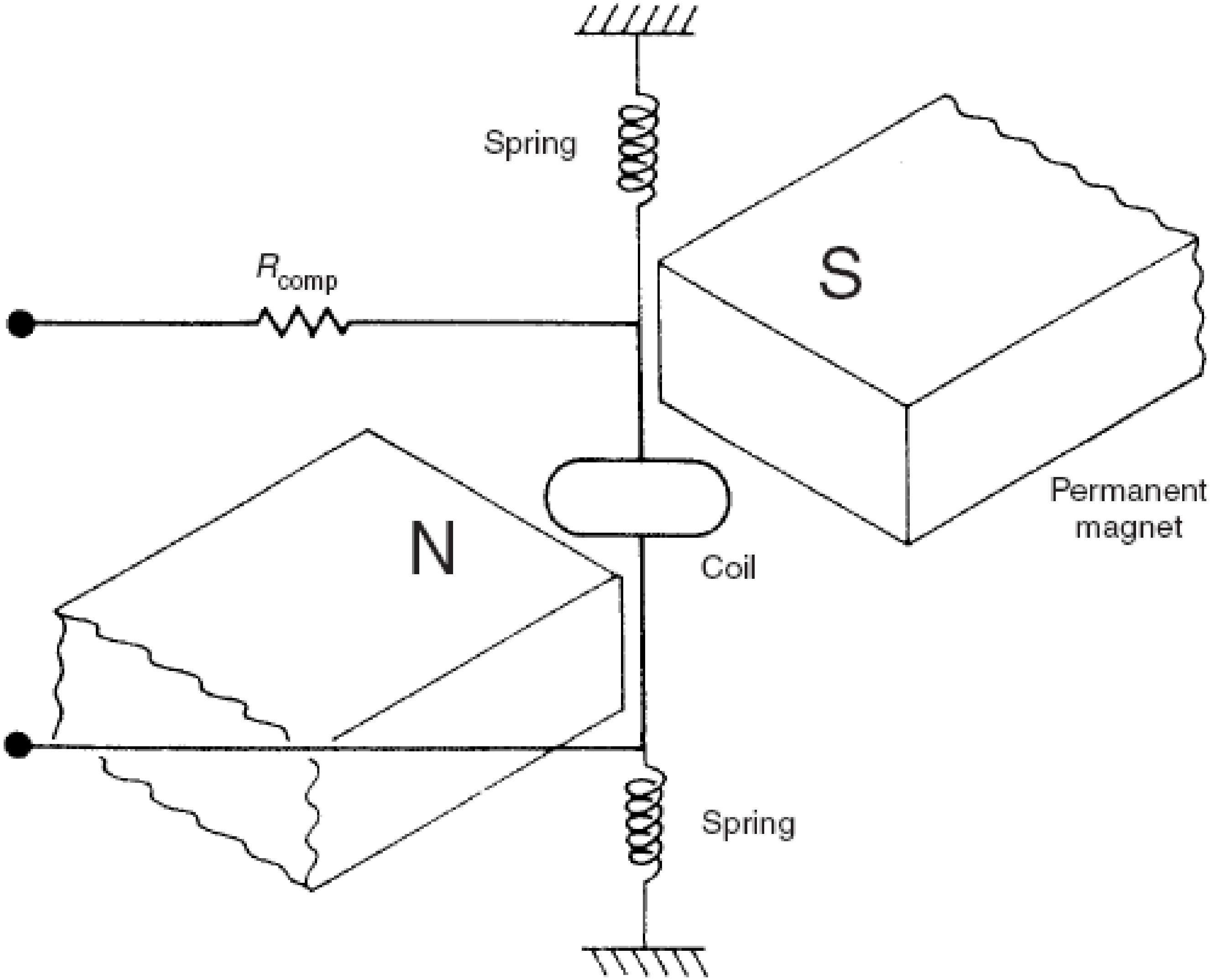
e.g. moving coil voltmeter

- **Errors due to environmental inputs:**
- Characteristics are specified initially
- E.g. closed box with something inside to know ! (real input, environmental input or a mixture of the two)
- Therefore environmental input must be measured first!

- **Errors due to connecting leads:**
- Taking account of the resistance of measuring leads
- E.g. resistance thermometer with copper leads (resistance + temperature coefficient)
- Also subjectivity to electrical or magnetic field > noise! > careful routing

❑ **Reduction of systematic errors**

- Careful instrument design might help……..E.g. strain gauge >> use material with very low temperature coefficient!!
- Method of opposing input > to cancel the effect >> e.g. compensating resistance with negative temperature coefficient to that of coil


Spring
R_{comp}
S
N
Permanent
magnet
Coil
Spring

Random errors

- Using averaging and statistical analysis
- Mean and Median values:

for any set of n measurements $x_1, x_2 \cdots x_n$ of a constant quantity

$$x_{\text{mean}} = \frac{x_1 + x_2 + \cdots x_n}{n}$$

$$x_{\text{median}} = x_{n+1}/2$$

398 420 394 416 404 408 400 420 396 413 430 (Measurement set A) mean = 409.0 and median = 408

409 406 402 407 405 404 407 404 407 407 408 (Measurement set B) mean = 406.0 and median = 407

34, whilst in set B, the spread is only 6 → Smaller spread > more confidence

409 406 402 407 405 404 407 404 407 407 408 406 410 406 405 408
406 409 406 405 409 406 407 (Measurement set C)

Now, mean = 406.5 and median = 406 → Mean approaches median as measurement increases

- Standard deviation and Variance:

- Better estimation of results distribution from mean (not smallest and highest)
- Deviation error of each measurement: d_i

$$d_i = x_i - x_{\text{mean}}$$

- Variance:

$$V = \frac{d_1^2 + d_2^2 \cdots d_n^2}{n-1}$$

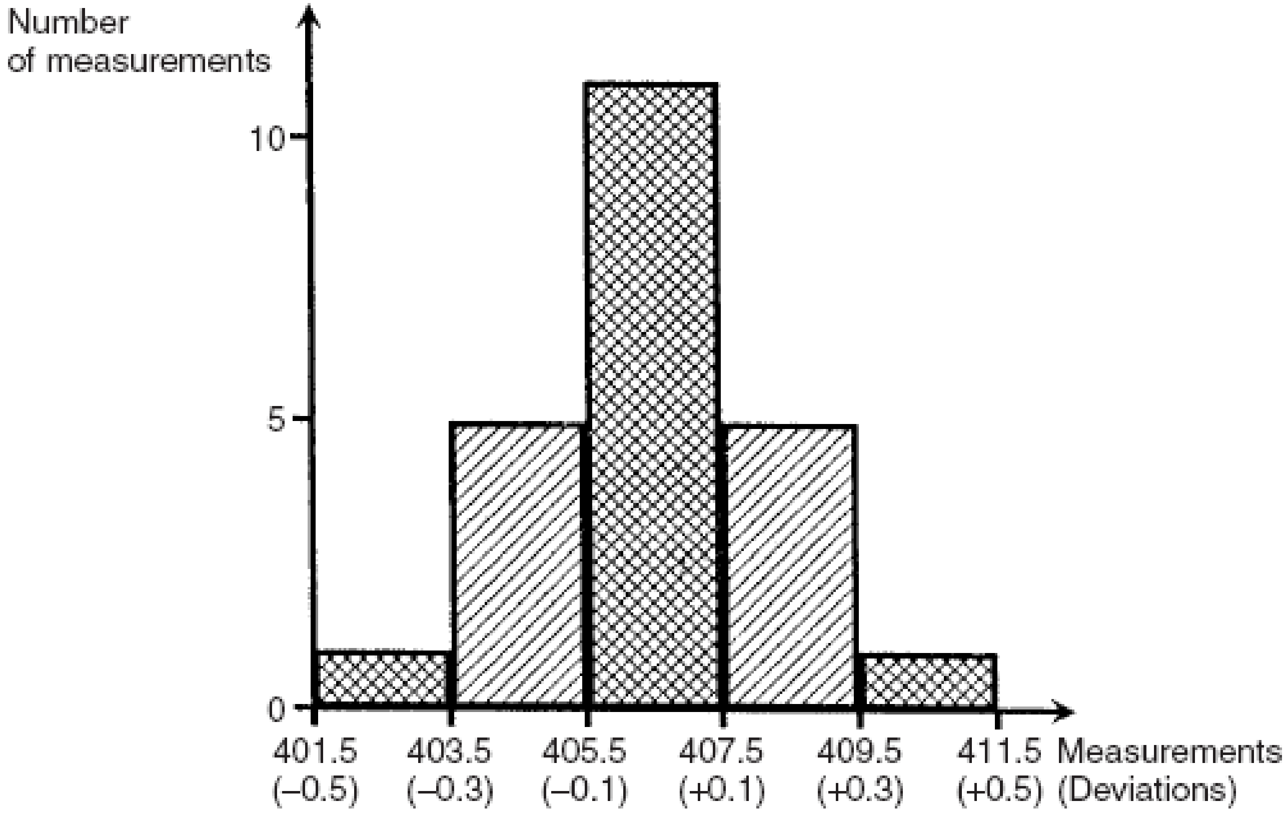
- Standard deviation :

$$\sigma = \sqrt{V} = \sqrt{\frac{d_1^2 + d_2^2 \cdots d_n^2}{n-1}}$$

- *Thus, as V and σ decrease for a measurement set, we are able to express greater confidence that the calculated mean or median value is close to the true value, i.e. that the averaging process has reduced the random error value close to zero.*
- *Comparing V and σ for measurement sets B and C, V and σ get smaller as the number of measurements increases, confirming that confidence in the mean value increases as the number of measurements increases.*

Graphical data analysis technique: frequency distribution

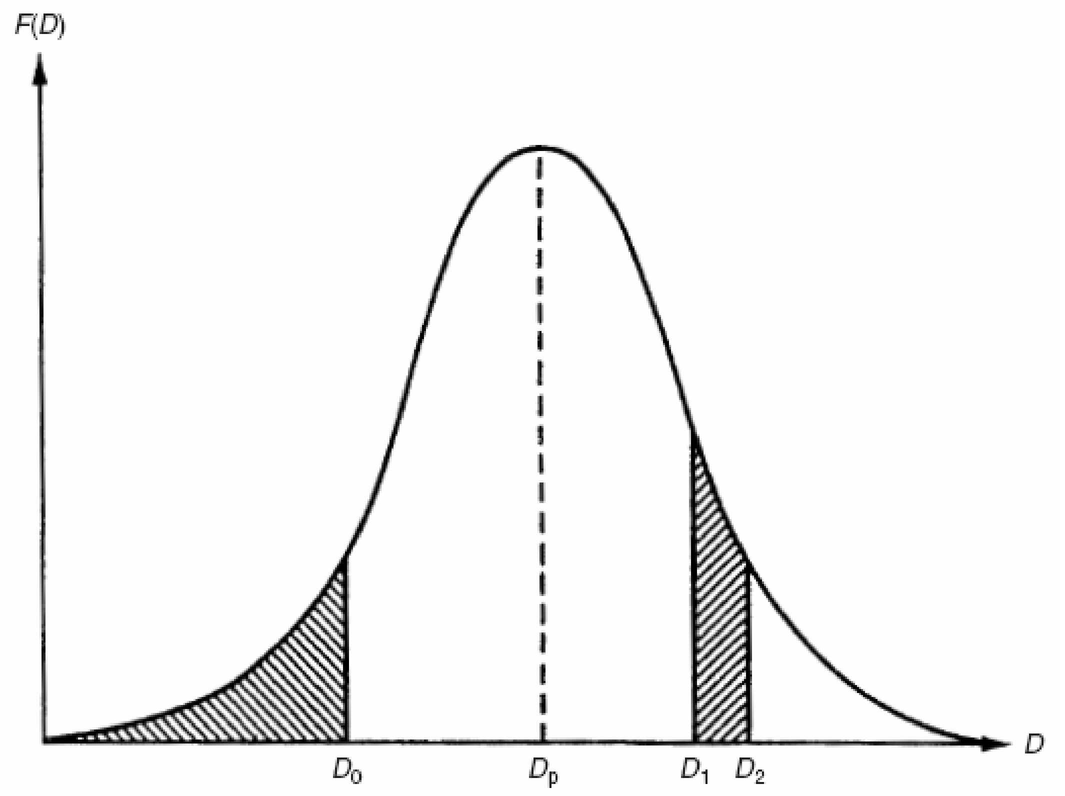
- Histogram and histogram of deviations



Graphical data analysis technique: frequency distribution

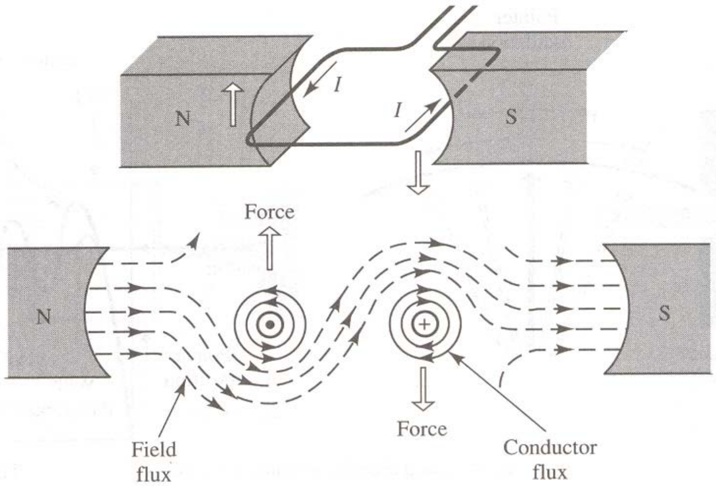
Frequency distribution curve of deviation:

- Frequency of occurrence of each deviation value Vs magnitude of deviation
- Asymmetry between curves at zero deviation
- Normalizing magnitude so the area under curve is unity >>>> probability curve
- D: probability density function

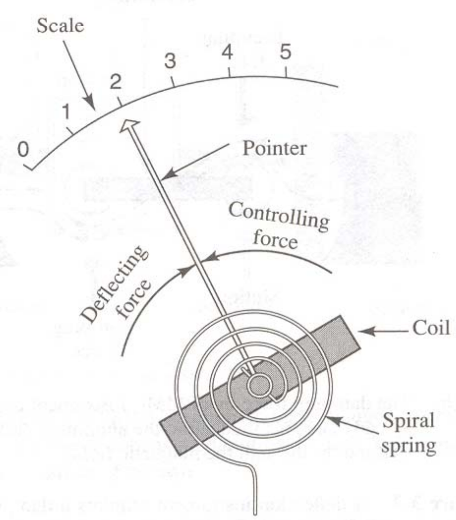


Electromechanical Instruments

- Permanent-Magnet Moving-Coil Instruments
  - Deflection Instrument Fundamentals
    - Deflecting force
      - causes the pointer to move from its zero position when a current flows
      - is magnetic force; the current sets up a magnetic field that interacts with the field of the permanent magnet (see Figure 3.1 (a))



(a) The deflecting force in a PMMC instrument is provided by a current-carrying coil pivoted in a magnetic field.

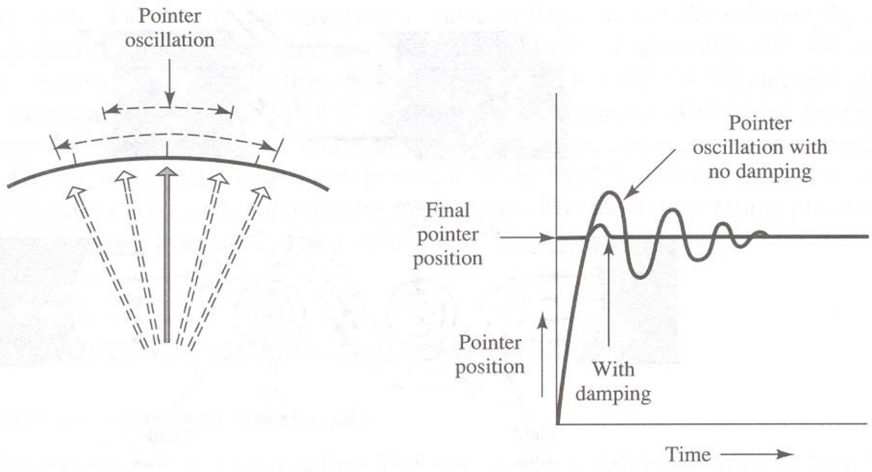


(b) The controlling force from the springs balances the deflecting force.

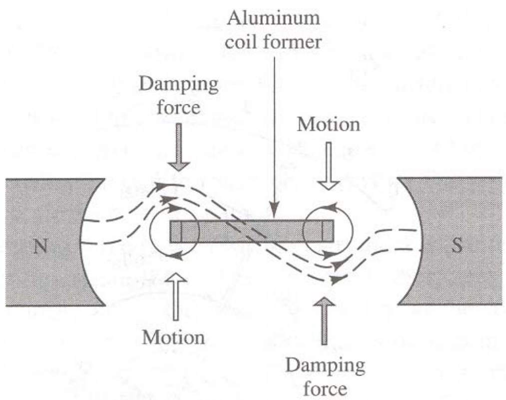
Figure 3-1 The deflecting force in a PMMC instrument is produced by the current in the moving coil. The controlling force is provided by spiral springs. The two forces are equal when the pointer is stationary.

- Controlling force
  - is provided by spiral springs (Figure 3.1 (b))
  - retain the coil and pointer at their zero position when no current is flowing
  - When current flows, the springs wind up as the coil rotates, and the force they exert on the coil increases
  - The coil and pointer stop rotating when the controlling force becomes equal to the deflecting force.
  - The spring material must be nonmagnetic to avoid any magnetic field influence on the controlling force.

  - Since the springs are used to make electrical connection to the coil, they must have a low resistance.
- Damping force
  - is required to minimize (or damp out) the oscillations
  - must be present only when the coil is in motion, thus it must be generated by the rotation of the coil
  - In PMMC instruments, the damping force is normally provided by eddy currents.



(a) Lack of damping causes the pointer to oscillate.



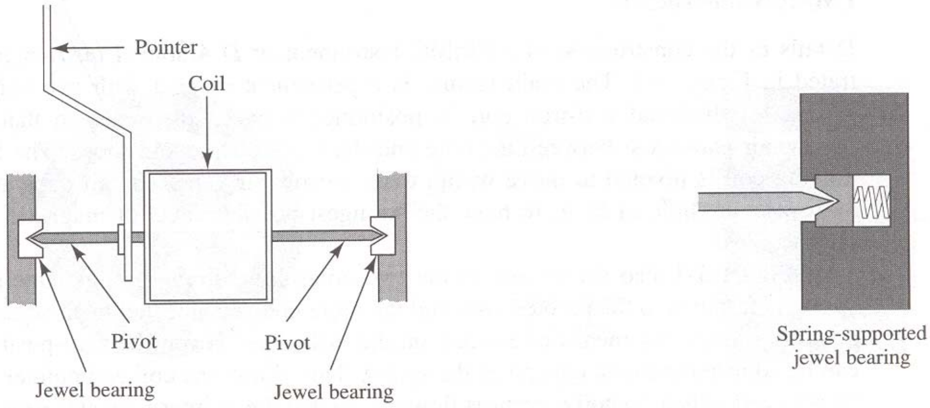
(b) The damping force in a PMMC instrument is provided by eddy currents induced in the aluminum coil former as it moves through the magnetic field.

Figure 3-2 A deflection instrument requires a damping force to stop the pointer oscillating about the indicated reading. The damping force is usually produced by eddy currents in a nonmagnetic coil former. These exist only when the coil is in motion.

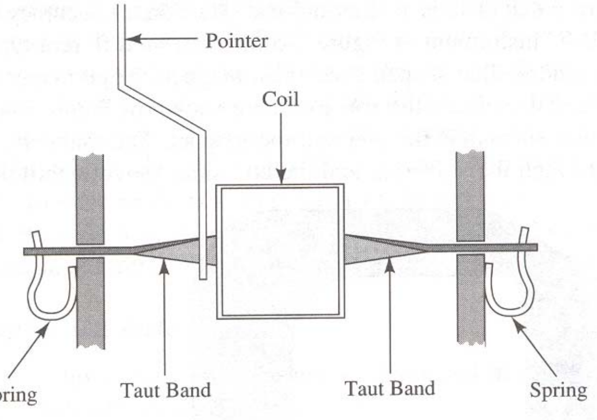
– Eddy currents induced in the coil former set up a magnetic flux that opposes the coil motion, thus damping the oscillations of the coil (see Figure 3.2 (b)).

- Two methods of supporting the moving system of a deflection instrument
  - Jeweled-bearing suspension
    - Cone-shaped cuts in jeweled ends of pivots
    - Least possible friction
    - Shock of an instrument spring $\Rightarrow$ supported to absorb such shocks
  - Taut-band method
    - Much tougher than jeweled-bearing
    - Two flat metal ribbons (phosphor bronze or platinum alloy) are held under tension by spring to support the coil

- Because of the spring, the metal ribbons behave like rubber under tension.
- The ribbons also exert a controlling force as they twist, and they can be used as electrical connections to the moving coil.
- Much more sensitive than the jeweled-bearing type because there is less friction
- Extremely rugged, not easily be shattered.



(a) Pivot and jewel-bearing suspension



(b) Taut-band suspension

Figure 3-3 The moving coil in a PMMC instrument may be supported by pivots in jeweled bearings, or by two flat metal ribbons held taut by springs. Taut-band suspension is the toughest and the most sensitive of the two.

– PMMC Construction

- D’Arsonval or horseshoe magnet
- Core-magnet

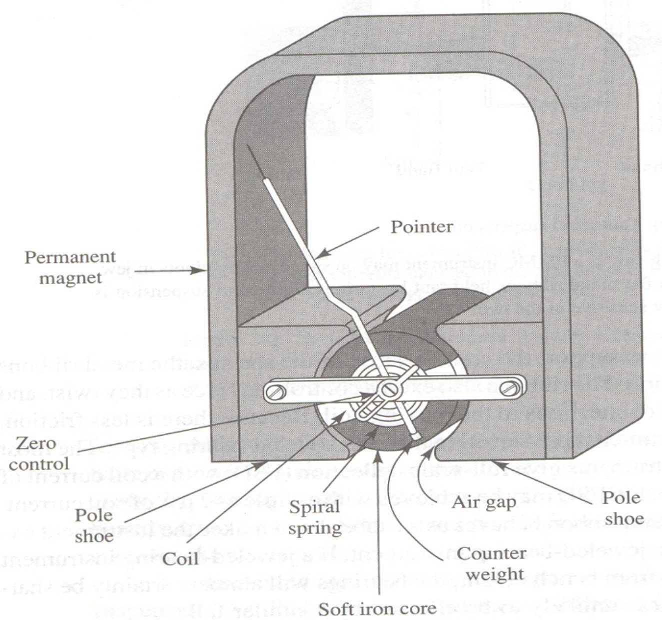


Figure 3-4 A typical PMMC instrument is constructed of a *horseshoe* magnet, soft-iron pole shoes, a soft-iron core, and a suspended coil that moves in the air gap between the core and the pole shoes.

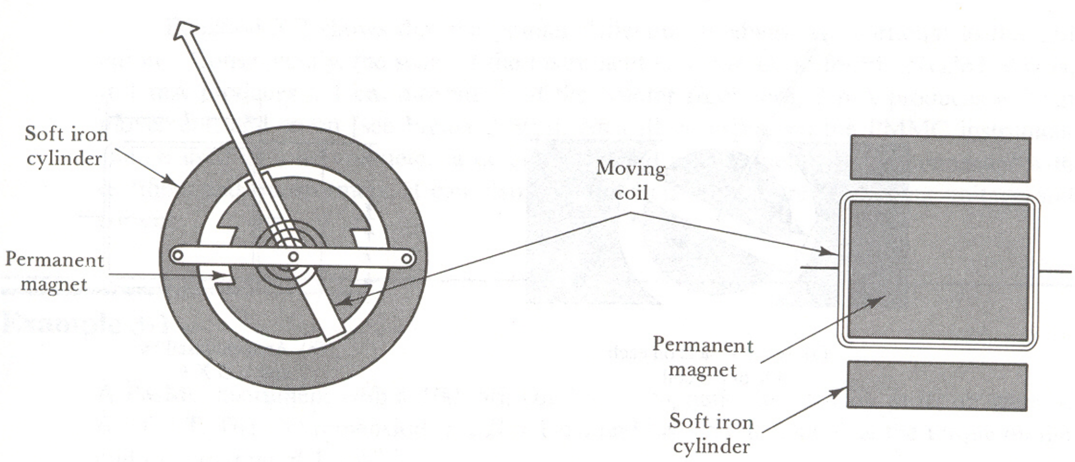


Figure 3-5 In a core-magnet PMMC instrument, the permanent magnet is located inside the moving coil, and the coil and magnet are positioned inside a soft-iron cylinder.

– Torque Equation and Scale

- When a current I flows through a one-turn coil situated in a magnetic field, a force F is exerted on each side of the coil

$$F = BIl \quad newtons$$

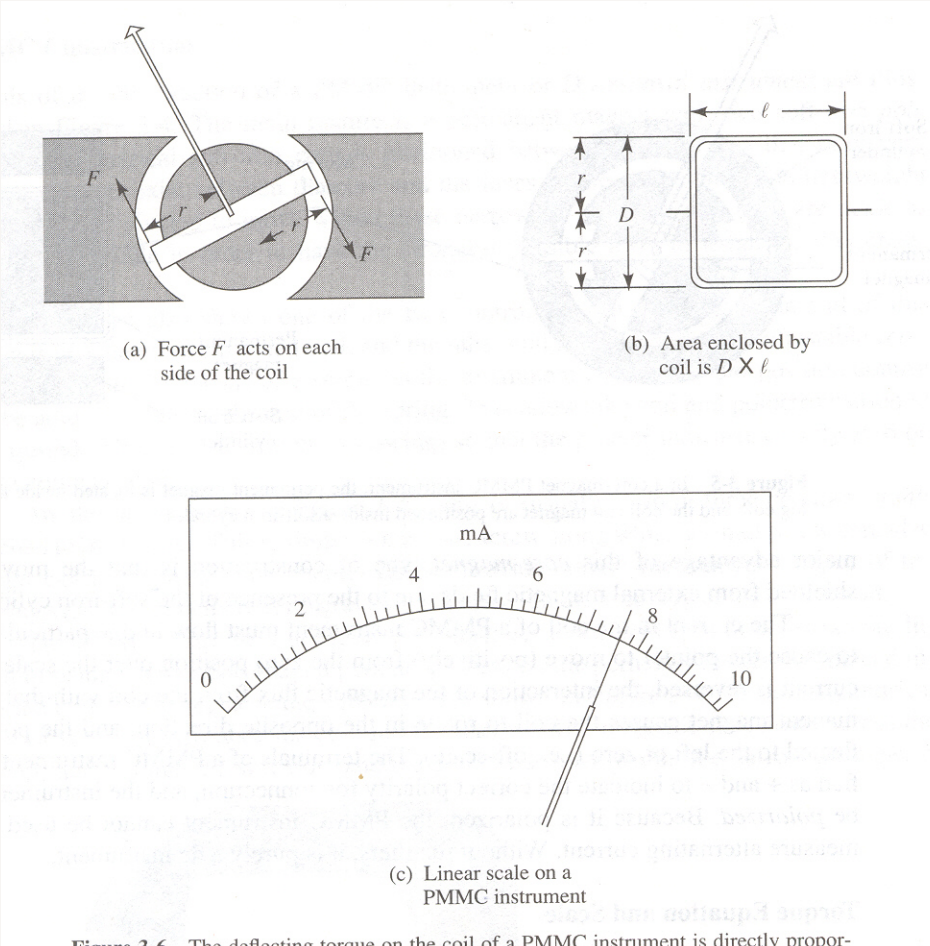


Figure 3-6 The deflecting torque on the coil of a PMMC instrument is directly proportional to the magnetic flux density, the coil dimensions, and the coil current. This gives the instrument a linear scale.

- Since the force acts on each side of the coil, the total force for a coil of N turns is

$$F = 2BIlN$$

- The force on each side acts at a radius r, producing a deflecting torque:

$$T_D = 2BlINr = BlIN(2r)$$
$$= BlIND$$
$$= BAIN$$

- The controlling torque exerted by the spiral springs is directly proportional to the deformation or windup of the springs. Thus, the controlling torque is proportional to the actual angle of deflection of the pointer.

$$T_C = K\theta \quad \text{where } K \text{ is a constant}$$

- For a given deflection, the controlling and deflecting torque are equal:

$$K\theta = BlIND$$

$$\theta = CI \qquad \textit{where C is a constant}$$

Example 3.1 A PMMC instrument with a 100-turn coil has a magnetic flux density in its air gaps of B = 0.2 T. The coil dimension are D = 1 cm and l = 1.5 cm. Calculate the torque on the coil for a current of 1 mA.

Solution

$$T_d = BlIND = (0.2T)(1.5\times10^{-2})(1\times10^{-3})(100)(1\times10^{-2})$$

$$= 3\times10^{-6}\ Nm$$

Resistor Types

Importance parameters

- ❖Value
- ❖Tolerance
- ❖Power rating
- ❖Temperature coefficient

Type	Values (Ω)	Power rating (W)	Tolerance (%)	Temperature coefficient (ppm/°C)	picture
Wire wound (power)	10m~3k	3~1k	±1~±10	±30~±300	
Wire wound (precision)	10m~1M	0.1~1	±0.005~±1	±3~±30	
Carbon film	1~1M	0.1~3	±2~±10	±100~±200	
Metal film	100m~1M	0.1~3	±0.5~±5	±10~±200	
Metal film (precision)	10m~100k	0.1~1	±0.05~±5	±0.4~±10	
Metal oxide film	100m~100k	1~10	±2~±10	±200~±500	

Data: Transistor technology (10/2000)

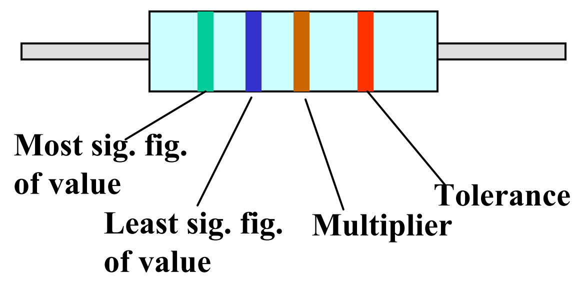
Resistor Values

- Color codes
- Alphanumeric

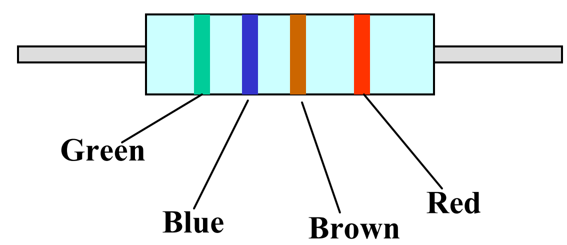
Color		Digit	Multiplier	Tolerance (%)		Temperature coefficient (ppm/°C)	
	Silver	-	10^{-2}	±10	K	-	-
	Gold		10^{-1}	±5	J		
	Black	0	10^{0}	-	-	±250	K
	Brown	1	10^{1}	±1	F	±100	H
	Red	2	10^{2}	±2	G	±50	G
	Orange	3	10^{3}	-	-	±15	D
	Yellow	4	10^{4}		-	±25	F
	Green	5	10^{5}	±0.5	D	±20	E
	Blue	6	10^{6}	±0.25	C	±10	C
	Violet	7	10^{7}	±0.1	B	±5	B
	Gray	8	10^{8}	-	-	±1	A
	White	9	10^{9}		-	-	-
			-	±20	M		

Data: Transistor technology (10/2000)

4 band color codes



Ex.



***R* = 560 Ω ± 2%**

Alphanumeric

R, K, M, G, and T = $x10^{0}$, $x10^{3}$, $x10^{6}$, $x10^{9}$, and $x10^{12}$

Ex. 6M8 = 6.8 x 10^{6} Ω
59P04 = 59.04 Ω

Resistor Values

$$R = x \pm \%\Delta x$$

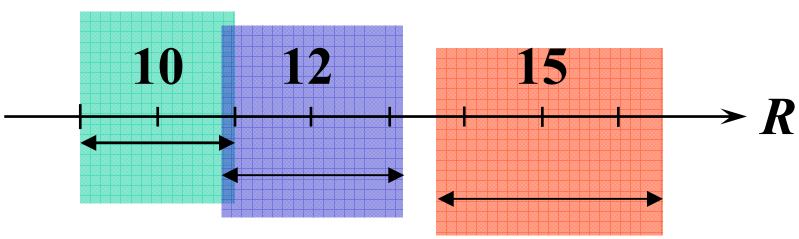
Tolerance

Nominal value

Ex. 1 kΩ ± 10% ≡ 900-1100 Ω

For 10% resistor

10, 12, 15, 18, …



$$R \approx \sqrt[E]{10^n}$$

where E = 6, 12, 24, 96

for 20, 10, 5, 1% tolerance

n = 0, 1, 2, 3, …

For 10% resistor E = 12

$n = 0$; $R = 1.00000\ldots$

$n = 1$; $R = 1.21152\ldots$

$n = 2$; $R = 1.46779\ldots$

$n = 3$; $R = 1.77827\ldots$

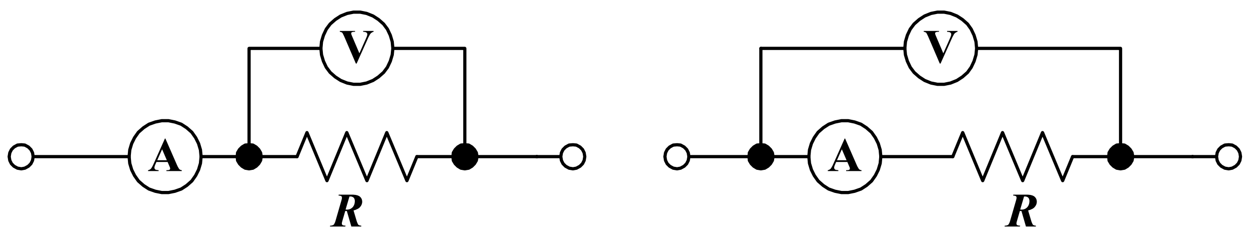
Commonly available resistance for a fixed resistor

±1%	±2%	±5%	±10%	±1%	±2%	±5%	±10%
100	100	10	10	316	316		
102				324			
105	105			332	332	33	33
107				340			
110	110	11		348	348		
113				357			
115	115			365	365	36	
118				374			
121	121	12	12	383	383		
124				392		39	39
127	127			407	407		
130		13		412			
133	133			422	422		
137				432		43	
140	140			442	442		
143				453			
147	147			464	464		
150		15	15	475		47	47
154	154			487	487		
158				499			
162	162	16		511	511	51	
165				523			
169	169			536	536		
174				549			
178	178			562	562	56	56
182		18	18	576			
187	187			590	590		
191				604			
196	196			619	619	62	
200		20		634			
205	205			649	649		
210				665			
215	215			681	681	68	68
221		22	22	698			
226	226			715	715		
232				732			
237	237			750	750	75	
243		24		765			
249	249			787	787		
255				806			
261	261			825	825	82	82
267				845			
274	274	27	27	866	866		
280				887			
287	287			909	909	91	
294				931			
301	301	30		953	953		
309				976			

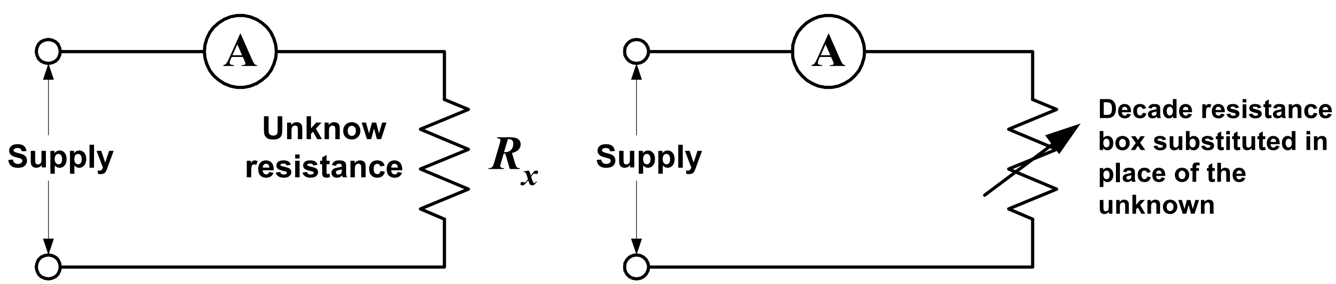
Resistance Measurement Techniques

- **Bridge circuit**
- **Voltmeter-ammeter**
- **Substitution**
- **Ohmmeter**

Voltmeter-ammeter



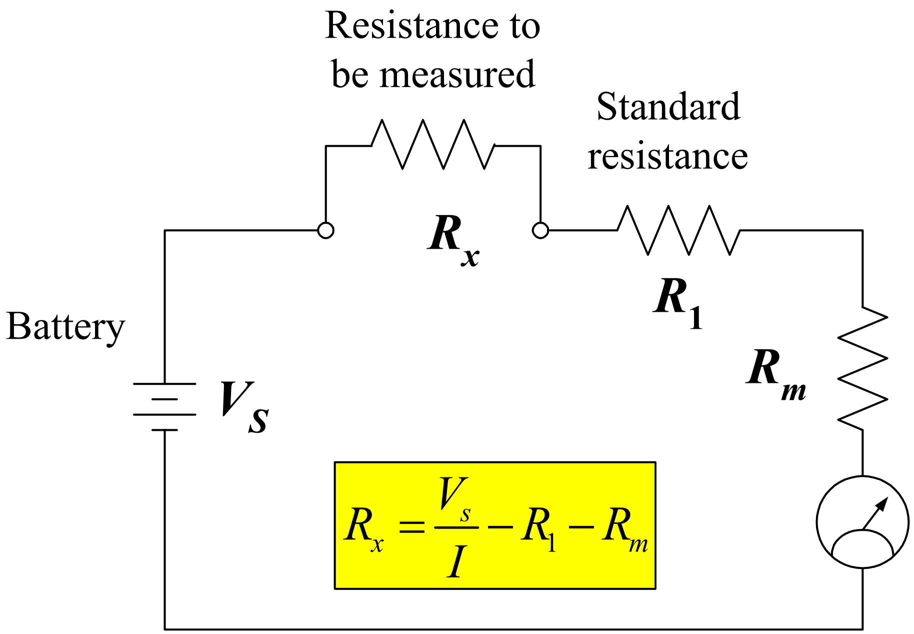
Substitution



Ohmmeter

- Voltmeter-ammeter method is rarely used in practical applications (mostly used in Laboratory)
- Ohmmeter uses only one meter by keeping one parameter constant

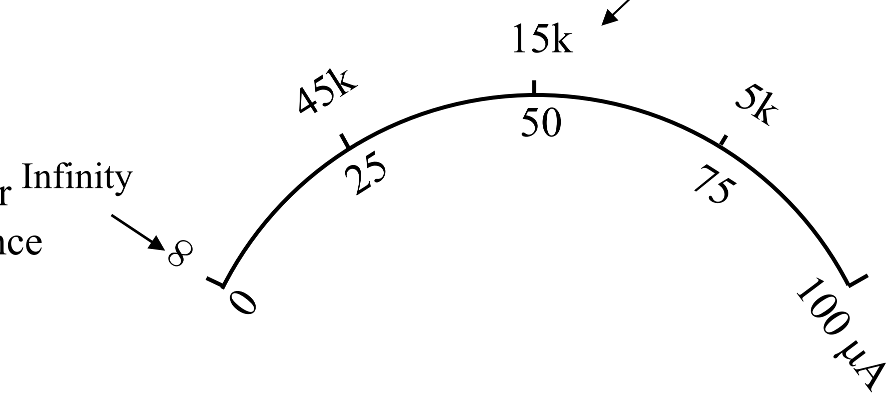
Example: series ohmmeter



$$R_x = \frac{V_s}{I} - R_1 - R_m$$

Basic series ohmmeter

Nonlinear scale



Ohmmeter scale

Basic series ohmmeter consisting of a PMMC and a series-connected standard resistor (R_1). When the ohmmeter terminals are shorted ($R_x = 0$) meter full scale defection occurs. At half scale defection $R_x = R_1 + R_m$, and at zero defection the terminals are open-circuited.

Bridge Circuit

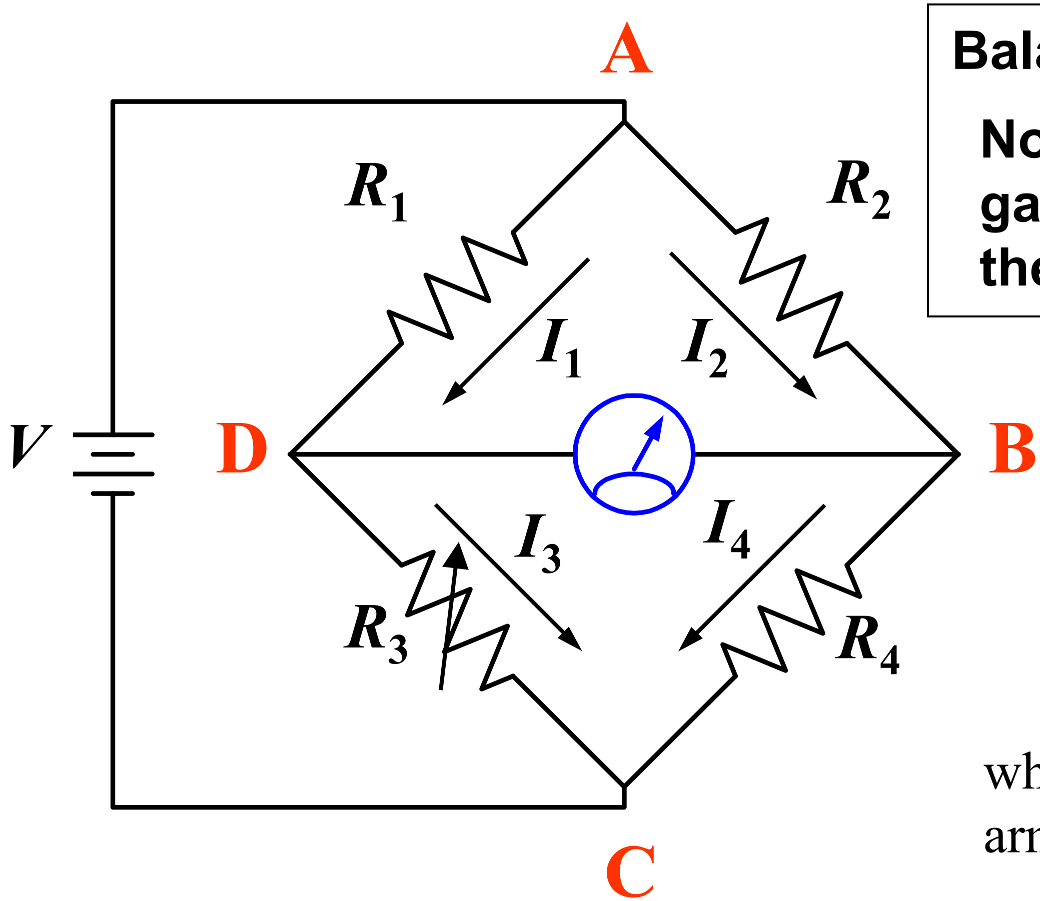
Bridge Circuit is a null method, operates on the principle of comparison. That is a known (standard) value is adjusted until it is equal to the unknown value.

Bridge Circuit

- **DC Bridge (Resistance)**
  - Wheatstone Bridge
  - Kelvin Bridge
  - Megaohm Bridge
- **AC Bridge**
  - **Inductance**
    - Maxwell Bridge
    - Hay Bridge
    - Owen Bridge
    - Etc.
  - **Capacitance**
    - Schering Bridge
  - **Frequency**
    - Wien Bridge

Wheatstone Bridge and Balance Condition

Suitable for moderate resistance values: 1 Ω to 10 MΩ



Balance condition:

No potential difference across the galvanometer (there is no current through the galvanometer)

Under this condition: $V_{AD} = V_{AB}$

$$I_1 R_1 = I_2 R_2$$

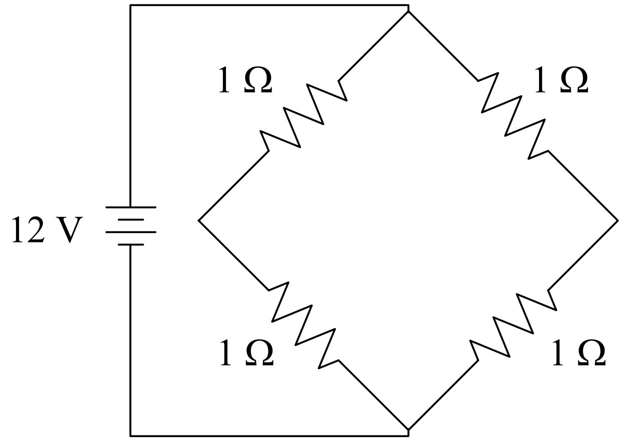
And also $V_{DC} = V_{BC}$

$$I_3 R_3 = I_4 R_4$$

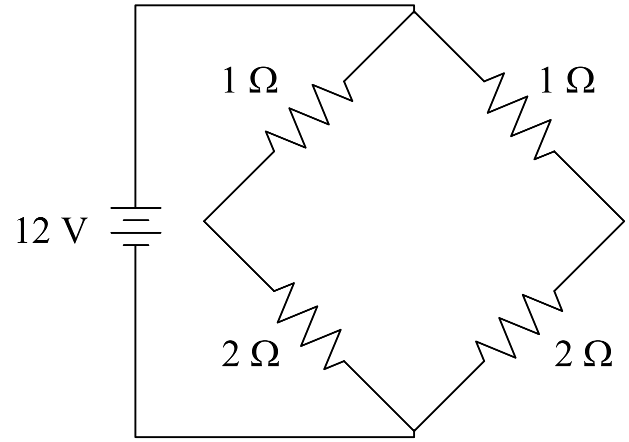
where I_1, I_2, I_3, and I_4 are current in resistance arms respectively, since $I_1 = I_3$ and $I_2 = I_4$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \quad \text{or} \quad R_x = R_4 = R_3 \frac{R_2}{R_1}$$

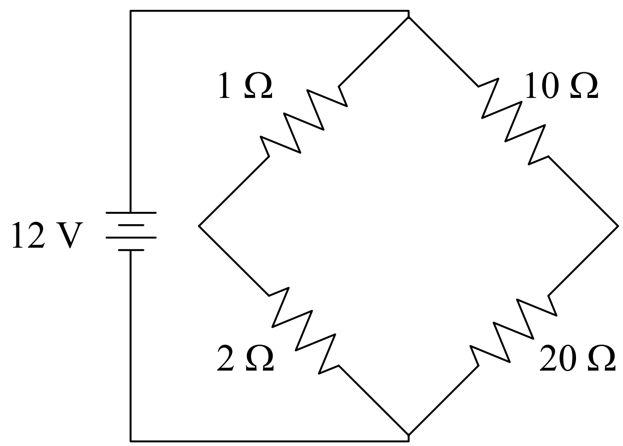
Example



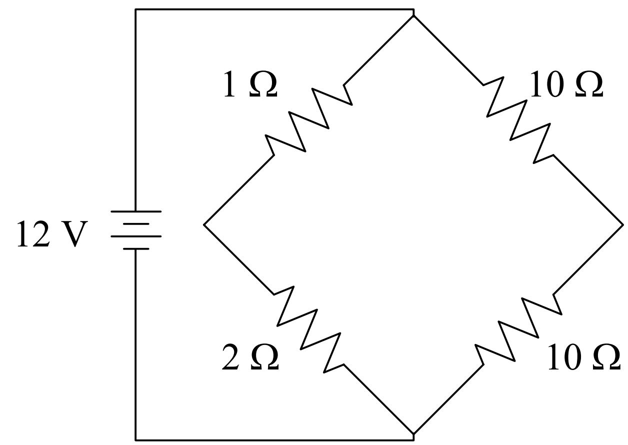
(a) Equal resistance



(b) Proportional resistance



(c) Proportional resistance



(d) 2-Volt unbalance

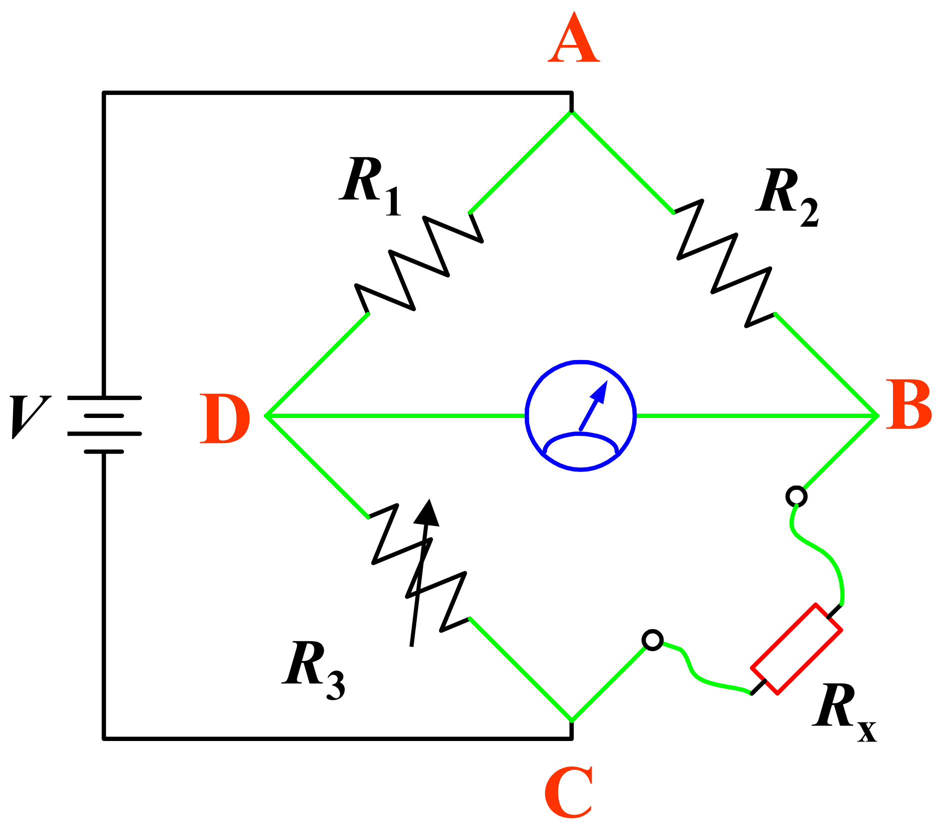
Measurement Errors

1. Limiting error of the known resistors

$$R_x = \left(R_3 \pm \Delta R_3\right)\left(\frac{R_2 \pm \Delta R_2}{R_1 \pm \Delta R_1}\right)$$

Using 1st order approximation:

$$R_x = R_3 \frac{R_2}{R_1}\left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3}\right)$$



2. Insufficient sensitivity of Detector

3. Changes in resistance of the bridge arms due to the heating effect (I^2R) or temperatures

4. Thermal emf or contact potential in the bridge circuit

5. Error due to the lead connection

3, 4 and 5 play the important role in the measurement of low value resistance

Example In the Wheatstone bridge circuit, R_3 is a decade resistance with a specified in accuracy ±0.2% and R_1 and R_2 = 500 Ω ± 0.1%. If the value of R_3 at the null position is 520.4 Ω, determine the possible minimum and maximum value of R_X

SOLUTION Apply the error equation

$$R_x = R_3 \frac{R_2}{R_1}\left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3}\right)$$

$$R_x = \frac{520.4 \times 500}{500}\left(1 \pm \frac{0.1}{100} \pm \frac{0.1}{100} \pm \frac{0.2}{100}\right) = 520.4(\ 1 \pm 0.004) = 520.4 \pm 0.4\%$$

Therefore the possible values of R_3 are 518.32 to 522.48 Ω

Example A Wheatstone bridge has a ratio arm of 1/100 (R_2/R_1). At first balance, R_3 is adjusted to 1000.3 Ω. The value of R_x is then changed by the temperature change, the new value of R_3 to achieve the balance condition again is 1002.1 Ω. Find the change of R_x due to the temperature change.

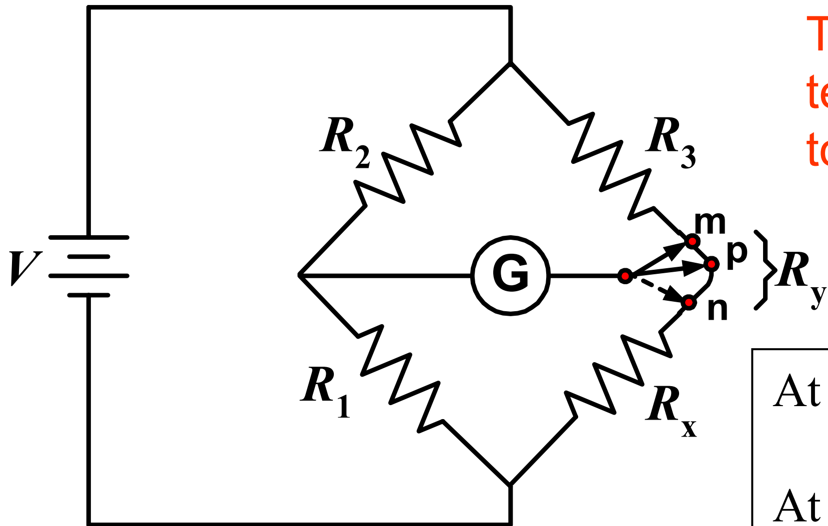
SOLUTION At first balance: $$R_x\text{old} = R_3 \frac{R_2}{R_1} = 1000.3 \times \frac{1}{100} = 10.003\ \Omega$$

After the temperature change: $$R_x\text{new} = R_3 \frac{R_2}{R_1} = 1002.1 \times \frac{1}{100} = 10.021\ \Omega$$

Therefore, the change of R_x due to the temperature change is 0.018 Ω

Low resistance Bridge: $R_x < 1\ \Omega$

Effect of connecting lead



The effects of the connecting lead and the connecting terminals are prominent when the value of R_x decreases to a few Ohms

R_y = the resistance of the connecting lead from R_3 to R_x

At point m: R_y is added to the unknown R_x, resulting in too high and indication of R_x

At point n: R_y is added to R_3, therefore the measurement of R_x will be lower than it should be.

At point p: $$R_x + R_{np} = \left(R_3 + R_{mp}\right)\frac{R_1}{R_2}$$

rearrange $$R_x = R_3\frac{R_1}{R_2} + R_{mp}\frac{R_1}{R_2} - R_{np}$$

Where R_{mp} and R_{np} are the lead resistance from m to p and n to p, respectively.

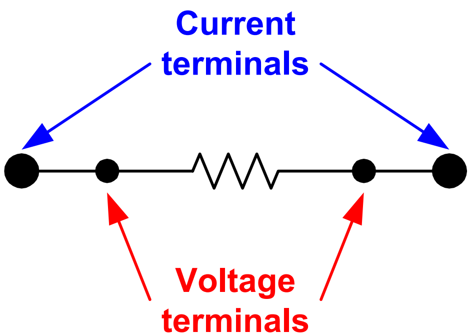
The effect of the connecting lead will be canceled out, if the sum of 2nd and 3rd term is zero.

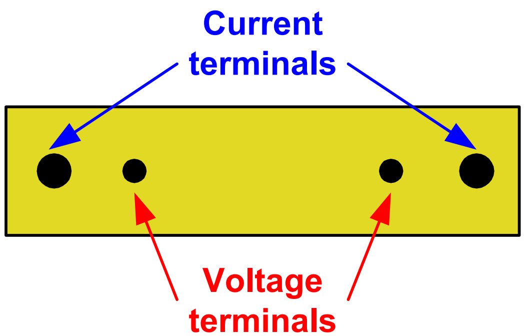
$$R_{mp}\frac{R_1}{R_2} - R_{np} = 0 \text{ or } \frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2}$$

$$R_x = R_3\frac{R_1}{R_2}$$

Kelvin Double Bridge: 1 to 0.00001 Ω

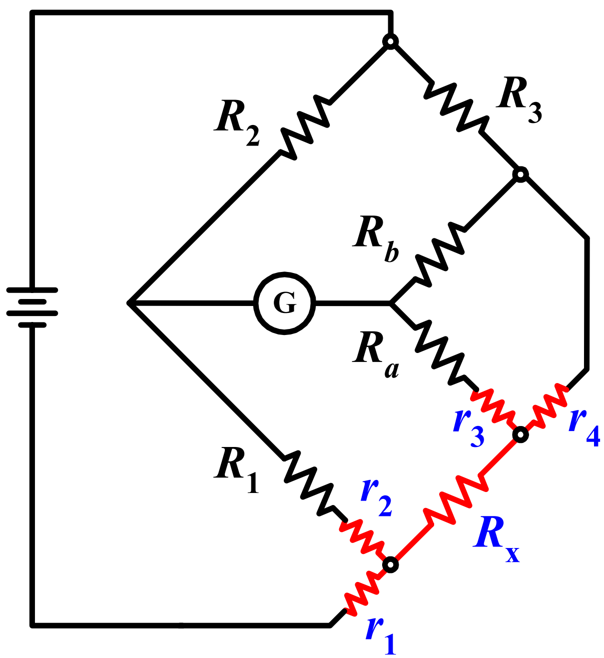
Four-Terminal Resistor





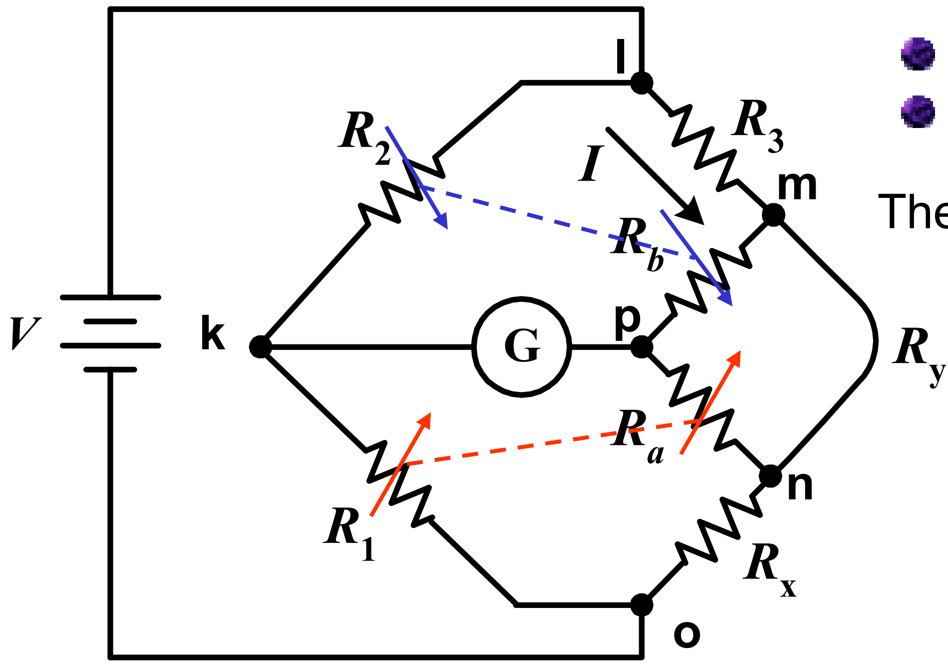
Four-terminal resistors have current terminals and potential terminals. The resistance is defined as that between the potential terminals, so that contact voltage drops at the current terminals do not introduce errors.

Four-Terminal Resistor and Kelvin Double Bridge



- r_1 causes no effect on the balance condition.
- The effects of r_2 and r_3 could be minimized, if $R_1 >> r_2$ and $R_a >> r_3$.
- The main error comes from r_4, even though this value is very small.

Kelvin Double Bridge: 1 to 0.00001 Ω



- 2 ratio arms: R_1-R_2 and R_a-R_b
- the connecting lead between m and n: yoke

The balance conditions: $V_{lk} = V_{lmp}$ or $V_{ok} = V_{onp}$

$$V_{lk} = \frac{R_2}{R_1 + R_2} V \qquad (1)$$

here $V = IR_{lo} = I[R_3 + R_x + (R_a + R_b) // R_y]$

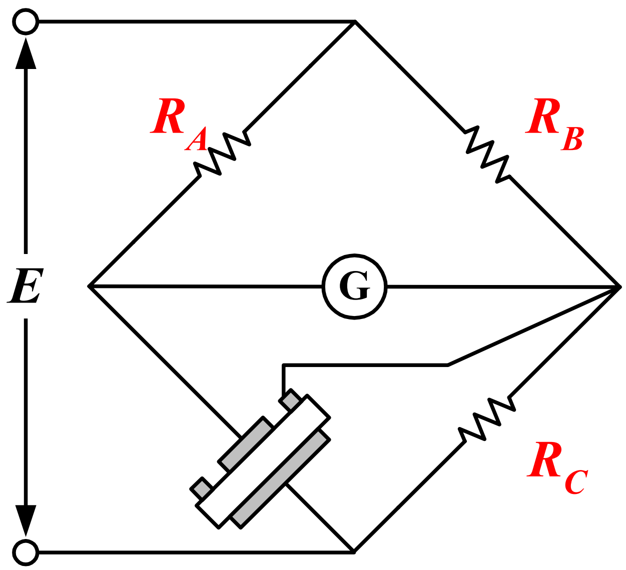
$$V_{lmp} = I\left[R_3 + \frac{R_y}{R_a + R_b + R_y} R_b\right] \qquad (2)$$

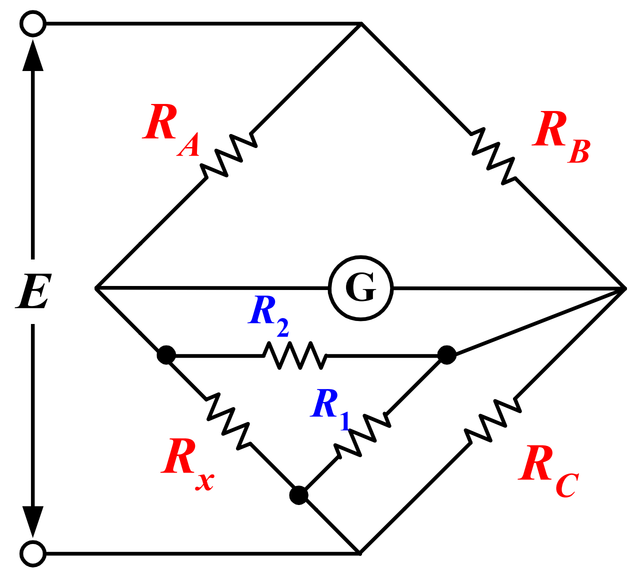
Eq. (1) = (2) and rearrange: $R_x = R_3 \frac{R_1}{R_2} + \frac{R_b R_y}{R_a + R_b + R_y}\left(\frac{R_1}{R_2} - \frac{R_a}{R_b}\right)$ ➡ $R_x = R_3 \frac{R_1}{R_2}$

If we set $R_1/R_2 = R_a/R_b$, the second term of the right hand side will be zero, the relation reduce to the well known relation. In summary, The resistance of the yoke has no effect on the measurement, if the two sets of ratio arms have equal resistance ratios.

MegaOhm Bridge

- Just as low-resistance measurements are affected by series lead impedance, high-resistance measurements are affected by shunt-leakage resistance.





- the guard terminal is connect to a bridge corner such that the leakage resistances are placed across bridge arm with low resistances

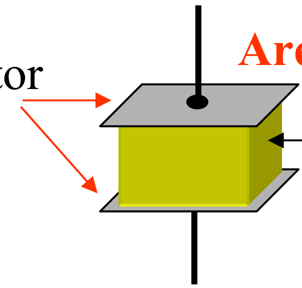
$R_1 // R_C \approx R_C$ since $R_1 >> R_C$

$R_2 // R_g \approx R_g$ since $R_2 >> R_g$

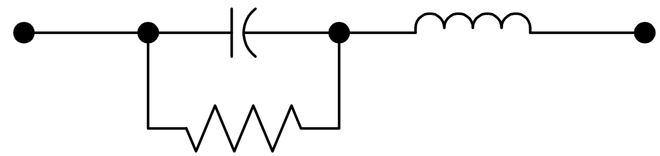
$$R_x \approx R_A \frac{R_C}{R_B}$$

Capacitor

Capacitance – the ability of a dielectric to store electrical charge per unit voltage



$$C = \frac{A\,\varepsilon_0 \varepsilon_r}{d}$$

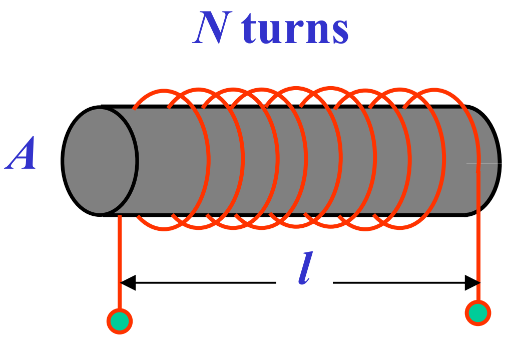


Typical values pF, nF or μF

Dielectric	Construction	Capacitance	Breakdown,V
Air	Meshed plates	10-400 pF	100 (0.02-in air gap)
Ceramic	Tubular	0.5-1600 pF	500-20,000
	Disk	1pF to 1 μF	
Electrolytic	Aluminum	1-6800 μF	10-450
	Tantalum	0.047 to 330 μF	6-50
Mica	Stacked sheets	10-5000 pF	500-20,000
Paper	Rolled foil	0.001-1 μF	200-1,600
Plastic film	Foil or Metallized	100 pF to 100 μF	50-600

Inductor

Inductance – the ability of a conductor to produce induced voltage when the current varies.



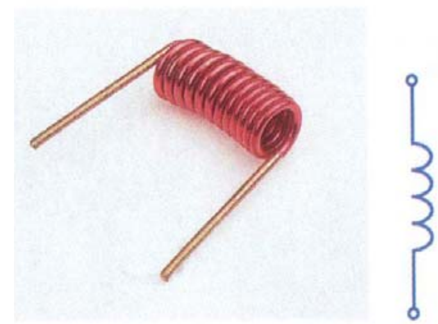
$$L = \frac{\mu_o \mu_r N^2 A}{l}$$

$\mu_o = 4\pi \times 10^{-7}$ H/m

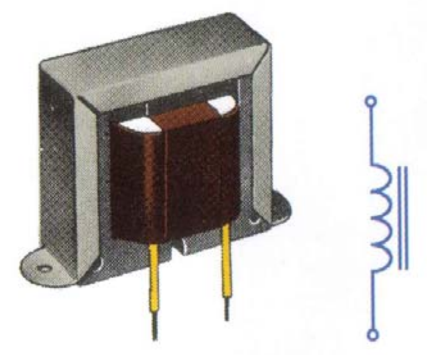
μ_r – relative permeability of core material

Ni ferrite: $\mu_r > 200$

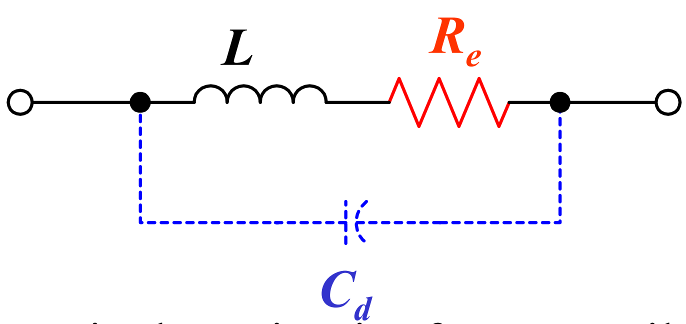
Mn ferrite: $\mu_r > 2{,}000$



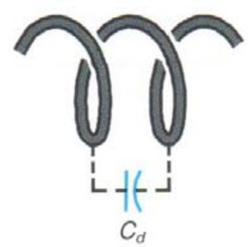
(a) Air core inductor



(b) Iron core inductor



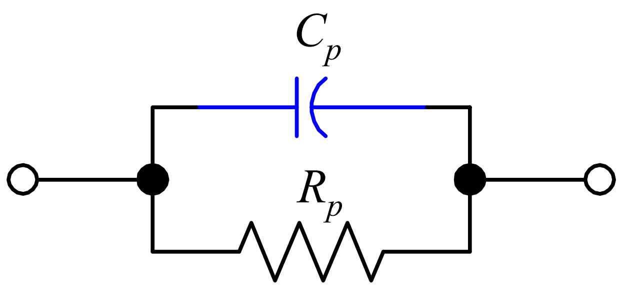
Equivalent circuit of an RF coil



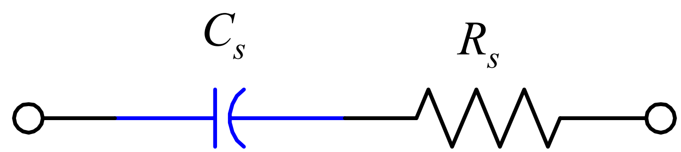
Distributed capacitance C_d between turns

Quality Factor of Inductor and Capacitor

Equivalent circuit of capacitance

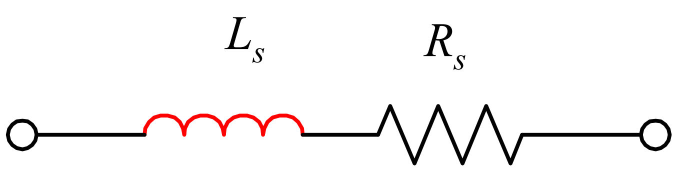


Parallel equivalent circuit

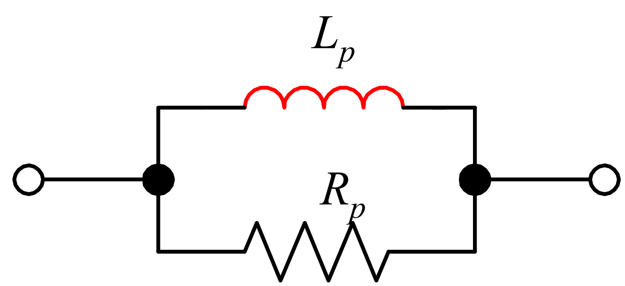


Series equivalent circuit

Equivalent circuit of Inductance



Series equivalent circuit



Parallel equivalent circuit

$$R_p = \frac{R_s^2 + X_s^2}{R_s}$$

$$X_p = \frac{R_s^2 + X_s^2}{X_s}$$

$$R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2}$$

$$X_s = \frac{X_p R_p^2}{R_p^2 + X_p^2}$$

Quality Factor of Inductor and Capacitor

Quality factor of a coil: the ratio of reactance to resistance (frequency dependent and circuit configuration)

Inductance series circuit: $Q = \frac{X_s}{R_s} = \frac{\omega L_s}{R_s}$

Typical $Q \sim 5 - 1000$

Inductance parallel circuit: $Q = \frac{R_p}{X_p} = \frac{R_p}{\omega L_p}$

Dissipation factor of a capacitor: the ratio of reactance to resistance (frequency dependent and circuit configuration)

Capacitance parallel circuit: $D = \frac{X_p}{R_p} = \frac{1}{\omega C_p R_p}$

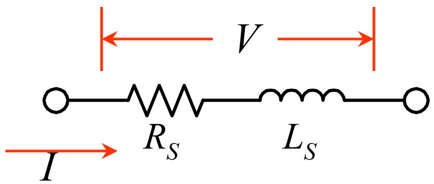
Typical $D \sim 10^{-4} - 0.1$

Capacitance series circuit: $D = \frac{R_s}{X_s} = \omega C_s R_s$

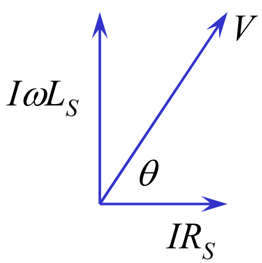
Inductor and Capacitor

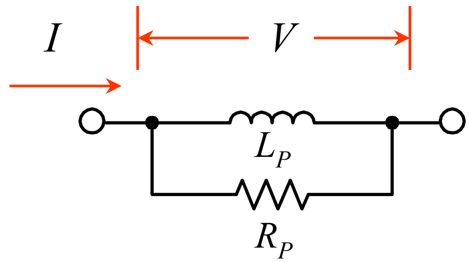
$$L_S = \frac{R_P^2}{R_P^2 + \omega^2 L_P^2} \cdot L_P$$

$$R_S = \frac{\omega^2 L_P^2}{R_P^2 + \omega^2 L_P^2} \cdot R_P$$



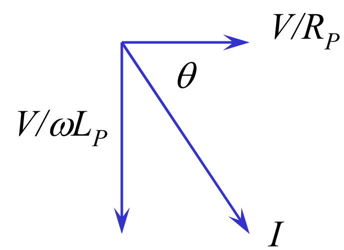
$$Q = \frac{\omega L_S}{R_S}$$





$$L_P = \frac{R_S^2 + \omega^2 L_S^2}{\omega^2 L_S^2} \cdot L_S$$

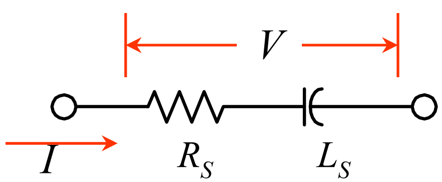
$$R_P = \frac{R_S^2 + \omega^2 L_S^2}{R_S^2} \cdot R_S$$



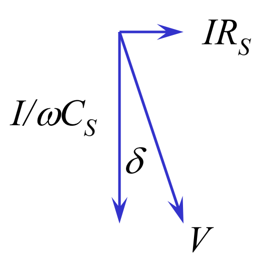
$$Q = \frac{R_P}{\omega L_P}$$

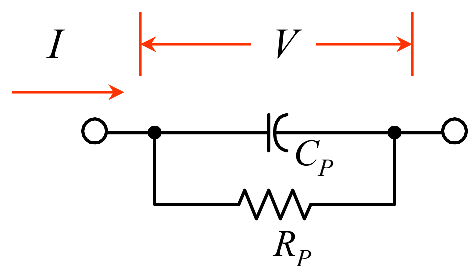
$$C_S = \frac{1 + \omega^2 C_P^2 R_P^2}{\omega^2 C_P^2 R_P^2} \cdot C_P$$

$$R_S = \frac{1}{1 + \omega^2 C_P^2 R_P^2} \cdot R_P$$



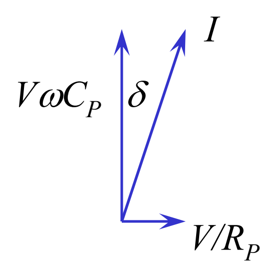
$$D = \omega C_S R_S$$





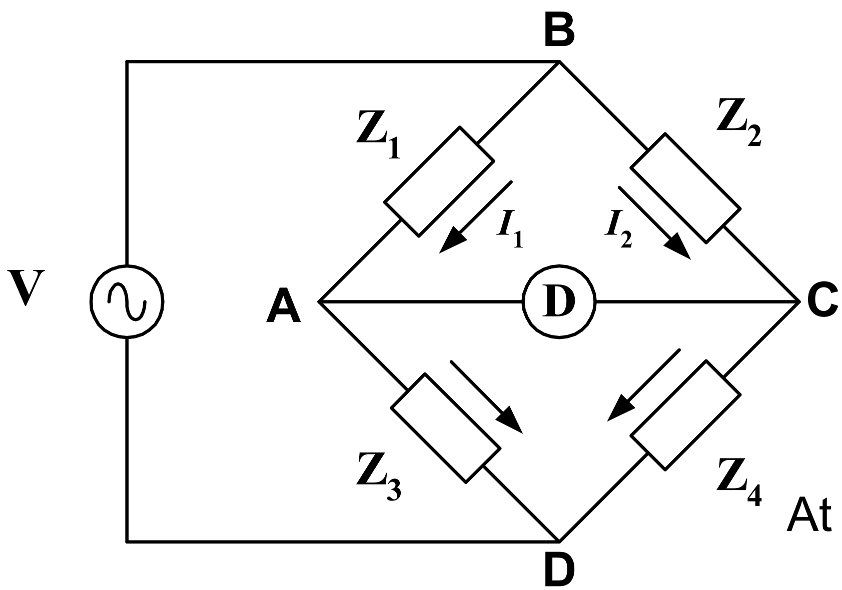
$$C_P = \frac{1}{1 + \omega^2 C_S^2 R_S^2} \cdot C_S$$

$$R_P = \frac{1 + \omega^2 C_S^2 R_S^2}{\omega^2 C_S^2 R_S^2} \cdot R_S$$



$$D = \frac{1}{\omega C_P R_P}$$

AC Bridge: Balance Condition



- all four arms are considered as impedance (frequency dependent components)
- The detector is an ac responding device: headphone, ac meter
- Source: an ac voltage at desired frequency

$\mathbf{Z_1}$, $\mathbf{Z_2}$, $\mathbf{Z_3}$ and $\mathbf{Z_4}$ are the impedance of bridge arms

At balance point: $\mathbf{E_{BA}} = \mathbf{E_{BC}}$ or $\mathbf{I_1 Z_1} = \mathbf{I_2 Z_2}$

$$\mathbf{I_1} = \frac{\mathbf{V}}{\mathbf{Z_1} + \mathbf{Z_3}} \text{ and } \mathbf{I_2} = \frac{\mathbf{V}}{\mathbf{Z_2} + \mathbf{Z_4}}$$

General Form of the ac Bridge

Complex Form: $\mathbf{Z_1 Z_4} = \mathbf{Z_2 Z_3}$

Polar Form:

$$Z_1 Z_4 (\angle\theta_1 + \angle\theta_4) = Z_2 Z_3 (\angle\theta_2 + \angle\theta_3)$$

Magnitude balance: $Z_1 Z_4 = Z_2 Z_3$

Phase balance: $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

Example The impedance of the basic ac bridge are given as follows:

$\mathbf{Z}_1 = 100\ \Omega\ \angle 80^{\circ}$ (inductive impedance)

$\mathbf{Z}_2 = 250\ \Omega$ (pure resistance)

$\mathbf{Z}_3 = 400\ \angle 30^{\circ}\Omega$ (inductive impedance)

$\mathbf{Z}_4 = \text{unknown}$

Determine the constants of the unknown arm.

SOLUTION The first condition for bridge balance requires that

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1,000\ \Omega$$

The second condition for bridge balance requires that the sum of the phase angles of opposite arms be equal, therefore

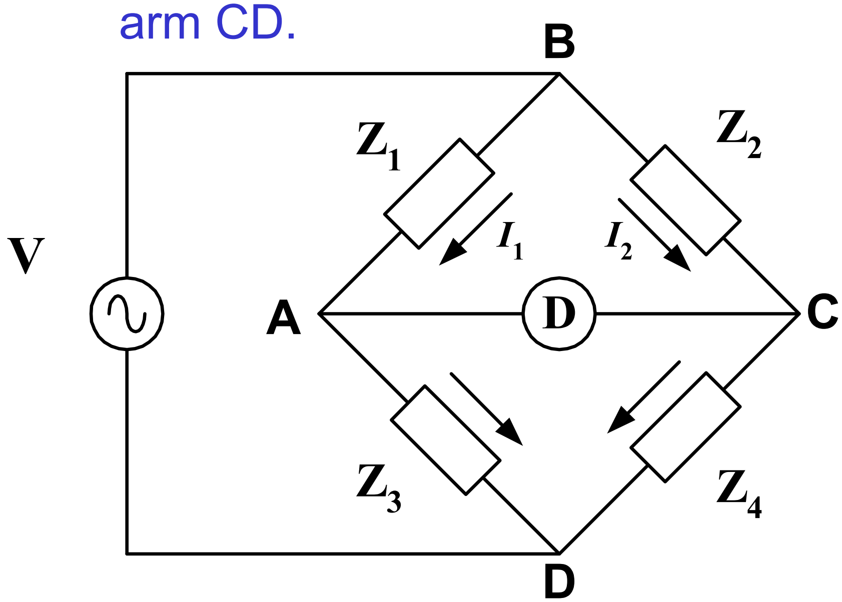
$$\angle\theta_4 = \angle\theta_2 + \angle\theta_3 - \angle\theta_1 = 0 + 30 - 80 = -50^{\circ}$$

Hence the unknown impedance $\mathbf{Z}_4$ can be written in polar form as

$$\mathbf{Z}_4 = 1,000\ \Omega\ \angle -50^{\circ}$$

Indicating that we are dealing with a capacitive element, possibly consisting of a series combination of at resistor and a capacitor.

Example an ac bridge is in balance with the following constants: arm AB, R = 200 Ω in series with L = 15.9 mH R; arm BC, R = 300 Ω in series with C = 0.265 μF; arm CD, unknown; arm DA, = 450 Ω. The oscillator frequency is 1 kHz. Find the constants of arm CD.



SOLUTION

$$\mathbf{Z}_1 = R + j\omega L = 200 + j100\ \Omega$$
$$\mathbf{Z}_2 = R + 1/j\omega C = 300 - j600\ \Omega$$
$$\mathbf{Z}_3 = R = 450\ \Omega$$
$$\mathbf{Z}_4 = \text{unknown}$$

The general equation for bridge balance states that $\mathbf{Z_1Z_4 = Z_2Z_3}$

$$\mathbf{Z}_4 = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1} = \frac{450 \times (200 + j100)}{(300 - j600)} = j150\ \Omega$$

This result indicates that $\mathbf{Z}_4$ is a pure inductance with an inductive reactance of 150 Ω at at frequency of 1kHz. Since the inductive reactance X_L = 2πfL, we solve for L and obtain L = 23.9 mH

Comparison Bridge: Capacitance

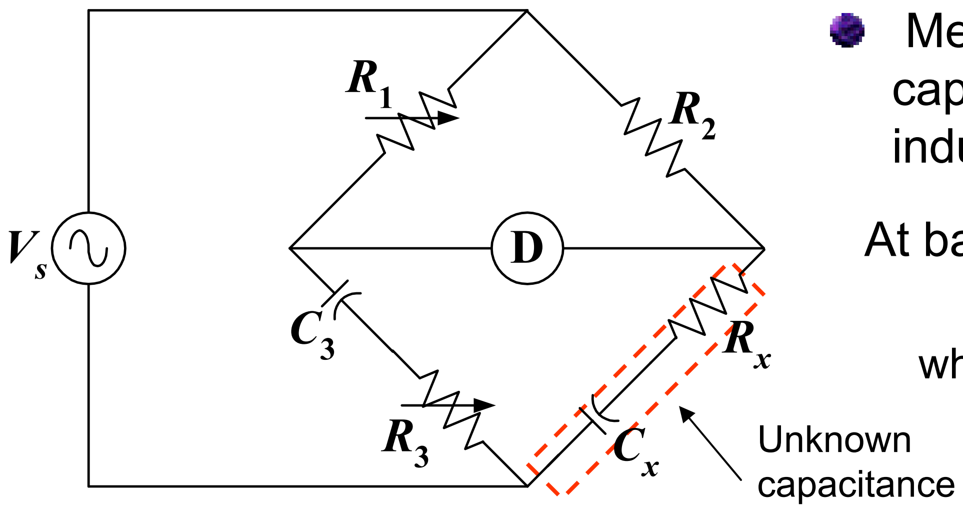


Diagram of Capacitance Comparison Bridge

- Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: $\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$

where $\mathbf{Z}_1 = R_1; \mathbf{Z}_2 = R_2;$ and $\mathbf{Z}_3 = R_3 + \frac{1}{j\omega C_3}$

$$R_1\left(R_x + \frac{1}{j\omega C_x}\right) = R_2\left(R_3 + \frac{1}{j\omega C_3}\right)$$

Separation of the real and imaginary terms yields: $R_x = \frac{R_2 R_3}{R_1}$ and $C_x = C_3 \frac{R_1}{R_2}$

- Frequency independent
- To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.

Comparison Bridge: Inductance

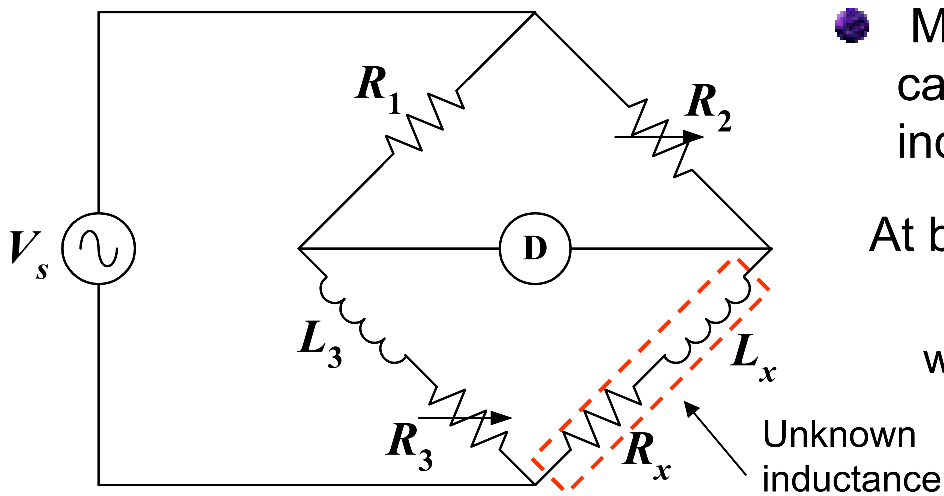


Diagram of Inductance Comparison Bridge

- Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: $\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$

where $\mathbf{Z}_1 = R_1; \mathbf{Z}_2 = R_2;$ and $\mathbf{Z}_3 = R_3 + j\omega L_3$

$$R_1\left(R_x + j\omega L_x\right) = R_2\left(R_S + j\omega L_S\right)$$

Separation of the real and imaginary terms yields: $R_x = \dfrac{R_2 R_3}{R_1}$ and $L_x = L_3 \dfrac{R_2}{R_1}$

- Frequency independent
- To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.

Maxwell Bridge

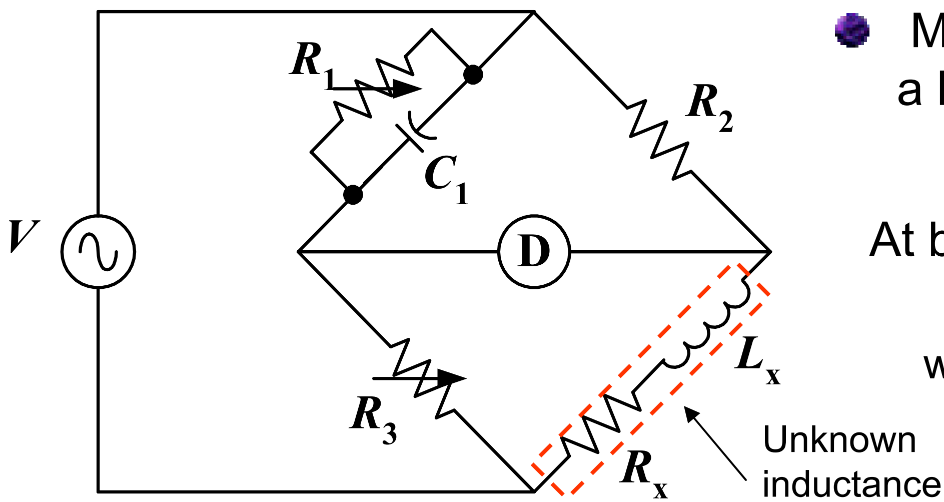


Diagram of Maxwell Bridge

- Measure an unknown inductance in terms of a known capacitance

At balance point: $\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3\mathbf{Y}_1$

where $\mathbf{Z}_2 = R_2;\ \mathbf{Z}_3 = R_3;\ \text{and}\ \mathbf{Y}_1 = \frac{1}{R_1} + j\omega C_1$

$$\mathbf{Z}_x = R_x + j\omega L_x = R_2 R_3\left(\frac{1}{R_1} + j\omega C_1\right)$$

Separation of the real and imaginary terms yields: $R_x = \frac{R_2 R_3}{R_1}$ and $L_x = R_2 R_3 C_1$

- Frequency independent
- Suitable for Medium Q coil (1-10), impractical for high Q coil: since R_1 will be very large.

Hay Bridge

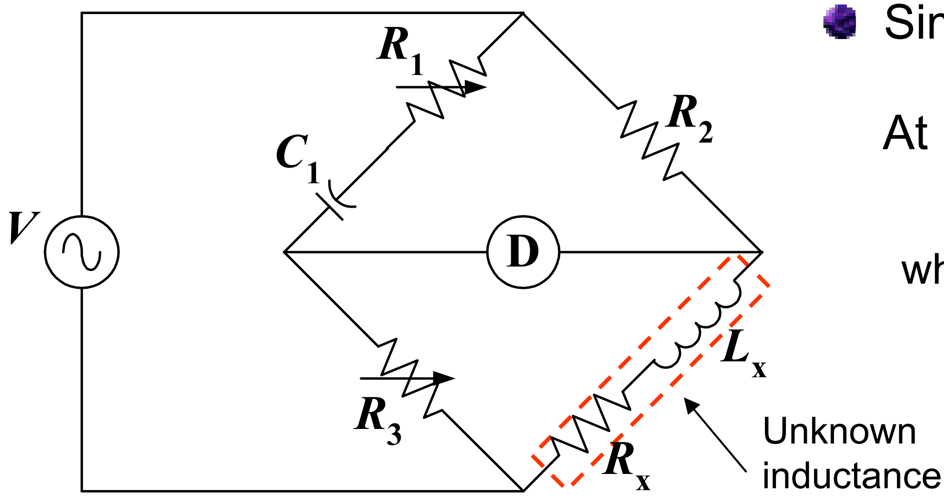


Diagram of Hay Bridge

- Similar to Maxwell bridge: but R_1 series with C_1

At balance point: $$\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$$

where $\mathbf{Z}_1 = R_1 - \dfrac{j}{\omega C_1}$; $\mathbf{Z}_2 = R_2$; and $\mathbf{Z}_3 = R_3$

$$\left(R_1 + \frac{1}{j\omega C_1}\right)\left(R_x + j\omega L_x\right) = R_2 R_3$$

which expands to $$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$$

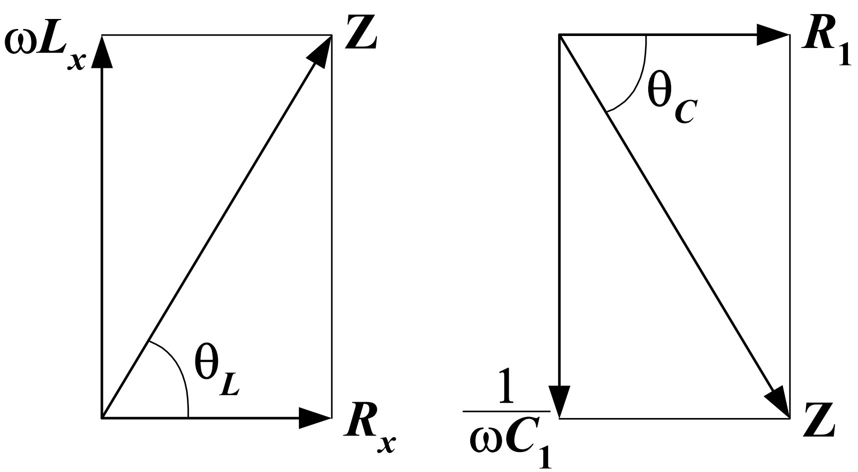
$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{..........(1)}$$

$$\frac{R_x}{\omega C_1} = \omega L_x R_1 \quad \text{..........(2)}$$

Solve the above equations simultaneously

Hay Bridge: continues

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 C_1^2 R_1^2}$$ and $$L_x = \frac{R_2 R_3 C_1}{1+\omega^2 C_1^2 R_1^2}$$



Phasor diagram of arm 4 and 1

$$\tan\theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q$$

$$\tan\theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1}$$

$$\tan\theta_L = \tan\theta_C \text{ or } Q = \frac{1}{\omega C_1 R_1}$$

Thus, L_x can be rewritten as

$$L_x = \frac{R_2 R_3 C_1}{1+(1/Q^2)}$$

For high Q coil (> 10), the term $(1/Q)^2$ can be neglected

$$L_x \approx R_2 R_3 C_1$$

Schering Bridge

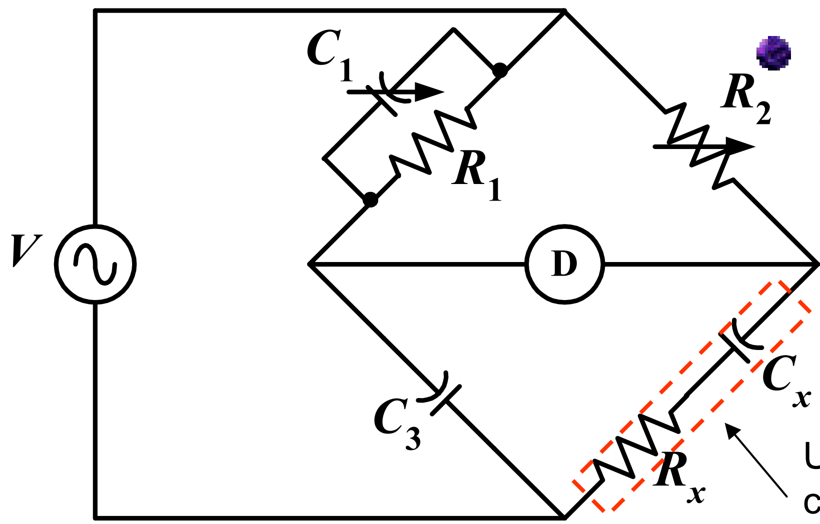


Diagram of Schering Bridge

- Used extensively for the measurement of capacitance and the quality of capacitor in term of D

At balance point: $\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3\mathbf{Y}_1$

where $\mathbf{Z}_2 = R_2;\ \mathbf{Z}_3 = \dfrac{1}{j\omega C_3};$ and $\mathbf{Y}_1 = \dfrac{1}{R_1} + j\omega C$

$$R_x - \frac{j}{\omega C_x} = R_2\left(\frac{-j}{\omega C_x}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

which expands to $$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_3} - \frac{jR_2}{\omega C_3 R_1}$$

Separation of the real and imaginary terms yields: $R_x = R_2\dfrac{C_1}{C_3}$ and $C_x = C_3\dfrac{R_1}{R_2}$

Schering Bridge: continues

Dissipation factor of a series RC circuit: $$D = \frac{R_x}{X_x} = \omega R_x C_x$$

Dissipation factor tells us about the quality of a capacitor, how close the phase angle of the capacitor is to the ideal value of 90°

For Schering Bridge: $$D = \omega R_x C_x = \omega R_1 C_1$$

For Schering Bridge, R_1 is a fixed value, the dial of C_1 can be calibrated directly in D at one particular frequency

Wien Bridge

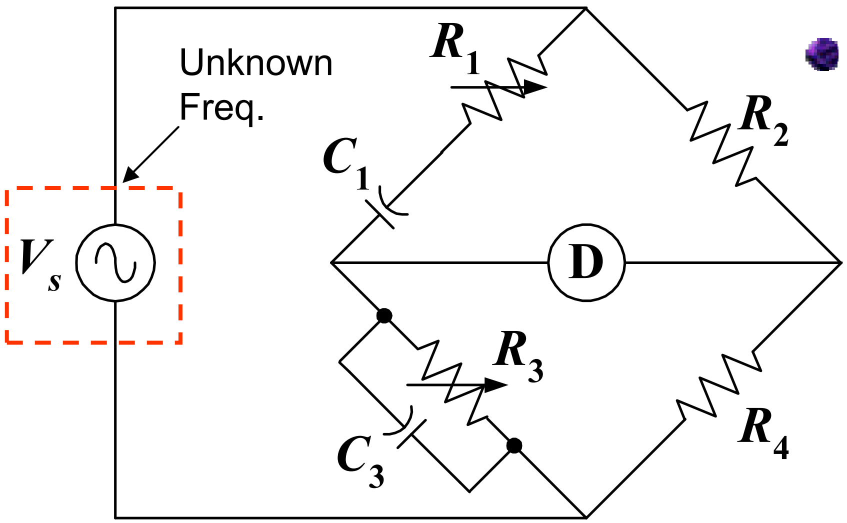


Diagram of Wien Bridge

- Measure frequency of the voltage source using series RC in one arm and parallel RC in the adjoining arm

At balance point: $\mathbf{Z}_2 = \mathbf{Z}_1\mathbf{Z}_4\mathbf{Y}_3$

$$\mathbf{Z}_1 = R_1 + \frac{1}{j\omega C_1};\ \mathbf{Z}_2 = R_2;\ \mathbf{Y}_3 = \frac{1}{R_3} + j\omega C_3;\ \text{and } \mathbf{Z}_4 = R_4$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

which expands to $R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1}$

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \text{............ (1)}$$

$$\omega C_3 R_1 = \frac{1}{\omega C_1 R_3} \quad \text{............ (2)}$$

Rearrange Eq. (2) gives $f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}$

In most, Wien Bridge, $R_1 = R_3$ and $C_1 = C_3$

(1) → $R_2 = 2R_4$ (2) → $f = \frac{1}{2\pi RC}$