University of Technology
Laser & Optoelectronics Engineering Department
1st Year B.Sc.

Subject: Engineering Mechanics.

Syllabus (Part I)

Engineering Mechanics:

1. Static's
   - 1.1 Introduction.
   - 1.2 Scalar and vector-quantities.
   - 1.3 Resolution of a force into Components and resultant.
   - 1.4 Moment of a force a bout a point.
   - 1.5 Location of Resultant.
   - 1.6 Couples.
   - 1.7 Equilibrium.
   - 1.8 Free body diagram.
   - 1.9 Centroids & Centers of area.
   - 1.10. Center of gravity of a body.
   - 1.11. Moments of inertia.
   - 1.12 Friction.
2. Dynamics.
   - 2.1 Law of Linear Motion.
   - 2.2 Angular motion of a line.
   - 2.3 Motion of projectiles.

Books

* Engineering Mechanics – Higdon.

Assistant

* Engineering Mechanics – Singer
  Meryam.

Dr. Eng. Sudad Issam Youni

Engineering Mechanics

1. Introduction

1.1 Mechanics: may be defined as the science which describes and predicts the conditions of rest or motion of bodies under the action of forces.

- Mechanics
  - Rigid bodies
    - static
      - force systems
        - Concurrent
        - Parallel
        - Non - Concurrent
      - Applications
        - trusses
        - Centroids
        - friction
    - dynamics
      - Kinematics
        - Translation
        - Rotation
        - plane motion
      - Kinetics
        - Translation
        - Rotation
        - Plane motion
  - deformable bodies
  - fluids

1.2 Useful information

$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan(\theta)$

$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$

$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$, $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

Angle / function	15	30	45	60	75
Sin	0.259	0.5	0.707	0.866	0.966
Cos	0.966	0.866	0.707	0.5	0.259
tan	0.268	0.577	1	1.732	3.732

1.3 Principal SI Units used in Mechanics

Quantity	Symbol	Units	Formula
Acceleration	a		m/s^2
Angle	$\theta, \gamma, \varphi, \alpha$	rad	2π rad = 360°
Angular acceleration	α		rad/s^2
Angular velocity	Ω		rad/s
Area	A		m^2
Density	ρ		Kg/m^3
Energy	E	J	$N \cdot m$
Force	F	N	$kg \cdot m/s^2$
Frequency	f	Hz	s^{-1}
Impulse	I	$N \cdot s$	$Kg \cdot m/s$
Length	L		m
Mass	m		Kg
Moment of a force	M		$N \cdot m$
Power	P	W	J/s
Pressure	p	Pa	N/m^2
Stress	τ	Pa	N/m^2
time	t		sec
Velocity	v		m/s
Volume Solids Liquids	V		m^3 ℓ $(10^{-3} m^3)$
Work	W	J	$N \cdot m$

1.4 Newton's three fundamental laws

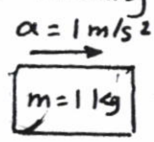
First law: if the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

Second law: if the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

$$F = m\,a$$

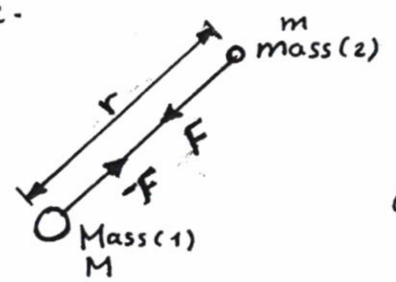
F, m & a represent, respectively, the resultant force acting on the particle, the mass of the particle and the acceleration of the particle.

Ex, Find the force of the 1 kg mass moving body shown in the figure.



$$F = m\,a$$
$$= (1\ \text{kg})(1\ \text{m/s}^2) = 1\ \text{N}.$$ in the direction of acceleration.

Third law, the forces of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense.

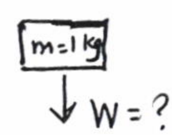


$$F = G\,\frac{Mm}{r^2}$$

G is universal constant called constant of gravitation.

$$g = \frac{GM}{R^2} = 9.81\ \text{m/s}^2 \qquad \& \quad W = mg$$

Ex Find the weight of the 1 kg mass shown



$$W = m\,g$$
$$= (1\ \text{kg})(9.81\ \text{m/s}^2)$$
$$= 9.81\ \text{N}.$$

1.5 Definitions

Rigid body : may be defined as that body that is not Change in its form or its Condition under the action of forces.

Force : represents the action of one body on another. it may be expressed by actual Contact or at distance, as in the case of gravitational force and magnetic force.

※ The mechanics of rigid body is subdivided into statics and dynamics. The former dealing with bodies at rest, the latter with body in motion, when the force system (body) in equilibrium.

※ The force is Characterized by its point of application, its magnitude and its direction, a force is represent by a vector.

2. Scalar & vector quantities

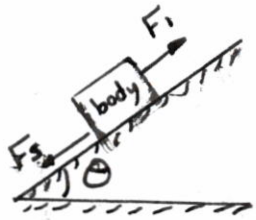
2.1 Scalar quantities; these quantities have magnitude only, as mass, volume, area --- etc.

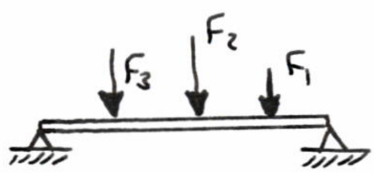
2.2 Vector quantities; these quantities have magnitude, direction and position as force, velocity, acceleration --- etc.

The vectors can be divided into two kinds:-

a. Free vectors, this is a quantity that has unlimited slope, and position, as particles in the body, wind.

b. localized vector, this quantity has limited slope, sence and position, as forces that affected upon a bar or body, friction force.



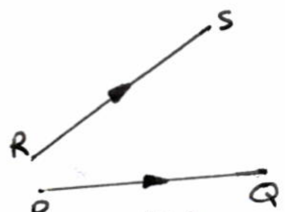


2.3 Representation of a vector.

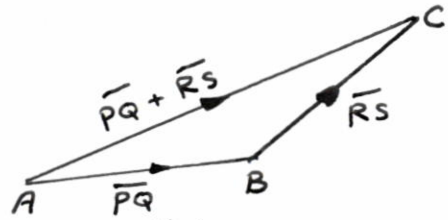
A vector is represented by a directed line as shown below. It may be noted that the length of OA represents the magnitude of the vector $\vec{OA}$ (or $\overline{OA}$). The direction of vector $\vec{OA}$ is from O (starting point) to A (ending point). It is also known as vector P.

2.4 Addition of vectors

Consider two vectors $\overline{PQ}$ and $\overline{RS}$ which are required to be added as shown in figure (a).



(a)

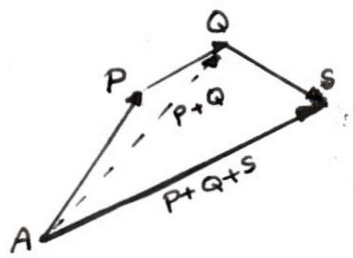


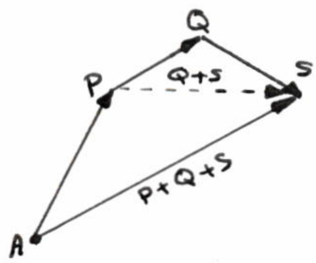
(b)

Take point A and draw line (AB) parallel and equal in magnitude to the vector $\overrightarrow{PQ}$ to some convenient scale.

Through B draw (BC) parallel and equal to vector $\overline{RS}$ to the same scale. Join AC which will give the required sum of the vectors $\overline{PQ}$ and $\overline{RS}$ as shown in the figure (b) above. This method of adding the two vectors is called the (Triangle law) of addition of vectors.

Similarly, if more than two vectors are to be added, the same may be done, first two vectors, and then by adding the third vector to the resultant of the first two and soon. This method of adding more than two vectors is called (Polygon law) of addition of vectors.





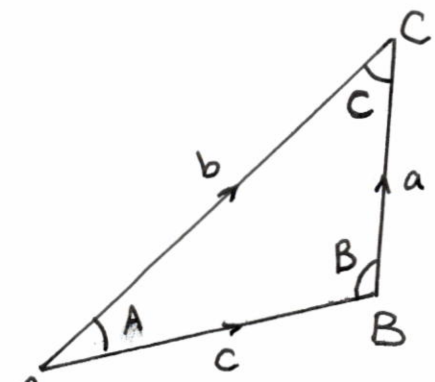
notes

Sine law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

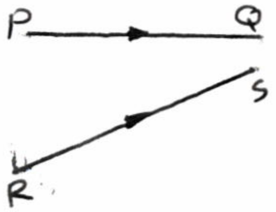
parallogram law

$$R = \sqrt{a^2 + c^2 - 2ac\cos B}$$

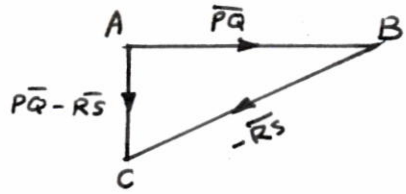


2.5 Subtraction of vectors

Consider two vectors $\overline{PQ}$ and $\overline{RS}$ whose difference required to be found out as shown in figure (a)



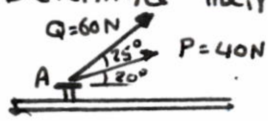
(a)



(b)

Take a point A, and draw line AB parallel and equal in magnitude to the vector $\overline{PQ}$ to some convenient scale. Through B draw BC parallel and equal to $\overline{RS}$ but in opposite direction to that of the vector $\overline{RS}$ to the same scale. Join AC, which will give the required difference of the vectors $\overline{PQ}$ and $\overline{RS}$ as shown in figure (b) above.

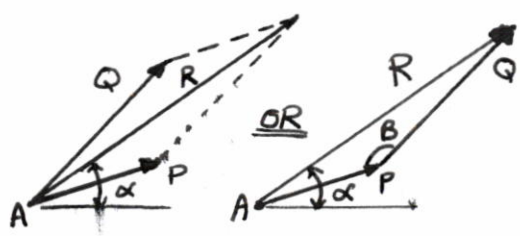
Ex The two forces P and Q act on a bolt A. Determine their resultant.



Graphical solution A parallelogram with sides equal to P and Q is drawn to scale. The magnitude and direction of the resultant are measured and found to be

$R = 98\ N \quad \alpha = 35°$ $\quad R = 98\ N \angle 35°$

The same results obtained if the triangle rule be used.



Trigonometric solution (by applying the law of Cosines

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$
$$= (40)^2 + (60)^2 - 2(40)(60) \cos 155°$$
$$R = 97.7\ N$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R}, \quad \frac{\sin A}{60} = \frac{\sin 155}{97.7}$$

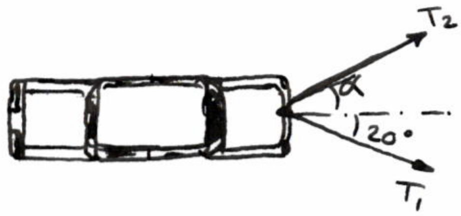
$$\sin A = \frac{60 \sin 155}{97.7} \Rightarrow A = 15°$$

$$\alpha = 20 + A = 35°.$$

Example A disabled automobile is pulled by means of two ropes as shown. If the resultant of the two forces exerted by the ropes is a 300 N parallel to the axis of the automobile, find

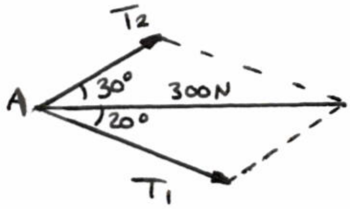
(a) the tension in each of the ropes, knowing that $\alpha = 30°$.

(b) the value of α such that the tension in rope 2 is minimum.

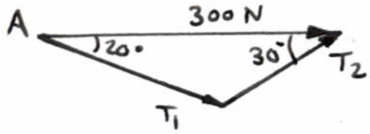


Solution

a. Tension for $\alpha = 30°$.



Graphically $T_1 = 196$ N $T_2 = 134$ N.

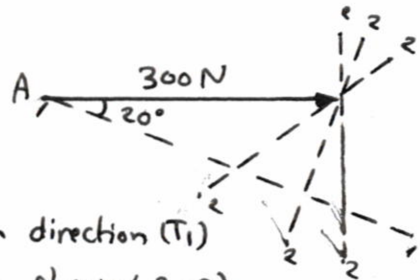


Trigonometric solution

$$\frac{T_1}{\sin 30} = \frac{T_2}{\sin 20} = \frac{300}{\sin 130}$$

$$T_1 = \frac{300 \sin 30}{\sin 130} = 195.8 \text{ N}.$$

$$T_2 = \frac{300 \sin 20}{\sin 130} = 133.9 \text{ N}.$$



b. Value of α for minimum T_2

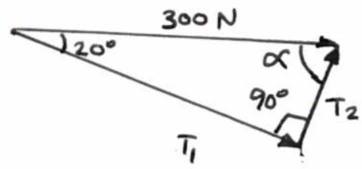
by using the triangle rule, plotting the 300 N (resultant force) and the 20° tension direction (T_1) There are several possible direction of T_2 as shown (2-2) we note that the minimum value of T_2 occurs when T_1 & T_2 are perpendicular.

The minimum value of T_2 is

$$T_2 = 300 \sin 20 = 102.6 \text{ N}$$

$$T_1 = 300 \cos 20 = 282 \text{ N}$$

& $\alpha = 90 - 20 = 70°$



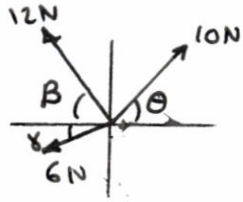
2.6 Definitions *

* **Force system** - when several forces act in a given situation they are called a system of force. They can be classified according to the arrangement of the lines of action of the forces of the system as follows.

- **Collinear** All forces have a common line of action.

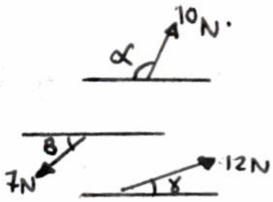
- **Concurrent Coplanar**

The action lines of all forces of the system are in the same plane and intersect at a common point.



- **parallel Coplanar**

The action line of all the forces of the system are parallel and lie in the same plane.



- **Non Concurrent, non parallel coplanar** The action lines of all forces of the system are in the same plane, but they are not all parallel and they are not intersect at a common point.

- **Concurrent, non Coplanar** The action lines of all forces of the system intersect at a common point, but they are not all in one plane.

?! parallel, non Coplanar & Non Concurrent, Non parallel, non Coplanar

* **Unit vector** A vector, whose magnitude is unity is known as unit vector.

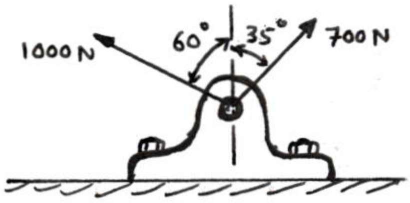
* **Equal vectors** the vectors which are parallel to each other, have the same direction and equal magnitude are known as equal vectors.

* **Like vectors** The vectors which are parallel to each other and have same direction but unequal magnitude are known as like vectors.

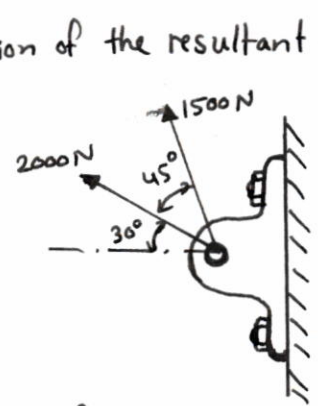
* **negative vectors** - The vectors which have the same magnitude but in opposite direction are known as negative vectors.

Problems

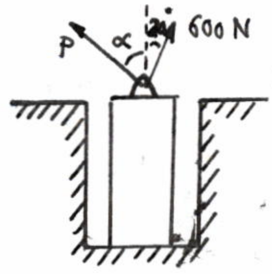
1. Determine graphically the magnitude and direction of the resultant of the two forces shown, using the parallelogram law.



2. Determine graphically the magnitude and direction of the resultant of the two forces shown, using the triangle rule.



3. Knowing that $\alpha = 30°$, determine the magnitude of the force P so that the resultant force exerted on the cylinder is vertical. What is the corresponding magnitude of the resultant?

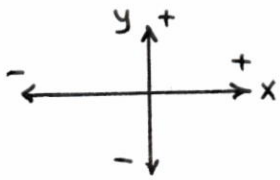


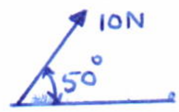
4. A cylinder is to be lifted by two cables. Knowing that the tension in one cable is 600 N, determine the magnitude and direction of force P so that the resultant is vertical force of 900 N.

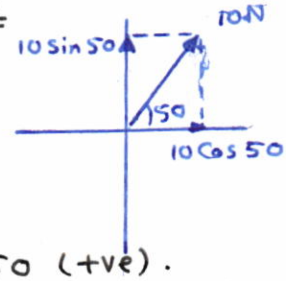
5. If the resultant of the two forces acting on the cylinder above is to be vertical, find
(a) the value of α for which the magnitude of P is minimum.
(b) the corresponding magnitude of P.

3. Resolution of forces into Components and resultant.

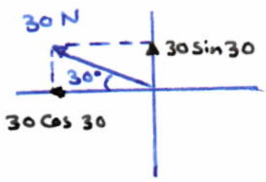
3.1 Resolve the force into its Components



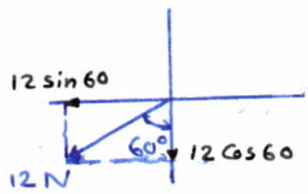




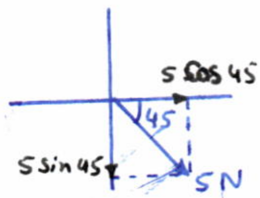
X Component of the force 10N = 10 Cos 50 (+ve).
Y Component of the force 10N = 10 Sin 50 (+ve).



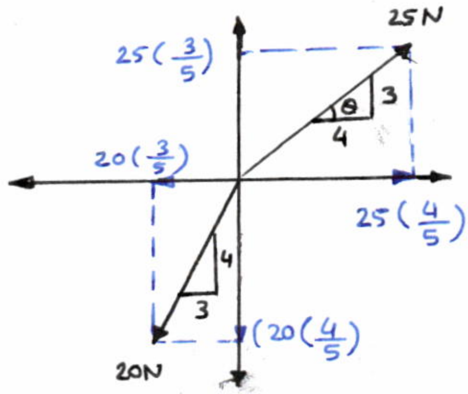
F_x = 30 Cos 30 (−ve)
F_y = 30 Sin 30 (+ve)



F_x = 12 sin 60 (−ve)
F_y = 12 Cos 60 (−ve)



F_x = 5 Cos 45 (+ve)
F_y = 5 Sin 45 (−ve)



※ في المسائل التي تحتوي على الطول اضلاع لمثلث الميل

الطريقة الاولى ١. نستخرج الوتر (فيثاغورس)

الوتر = $\sqrt{3^2+4^2}$ = 5 (وحدة)

٢. نضرب مركبة القوة بالضلع الموازي لها مقسوماً على الوتر.

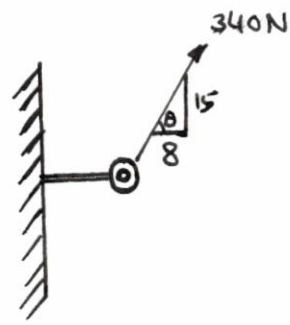
الطريقة الثانية

نستخرج الزاوية θ ونحل كما في الامثلة السابقة

$$\tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\frac{3}{4} \approx 37^\circ.$$

Ex Find the Components of the force shown.



$$\sqrt{8^2+15^2} = 17$$

$$F_x = 340 * \frac{8}{17} = 160 \text{ N}$$

$$F_y = 340 * \frac{15}{17} = 300 \text{ N}$$

OR

$$\theta = \tan^{-1}\left(\frac{15}{8}\right) = 61.928$$

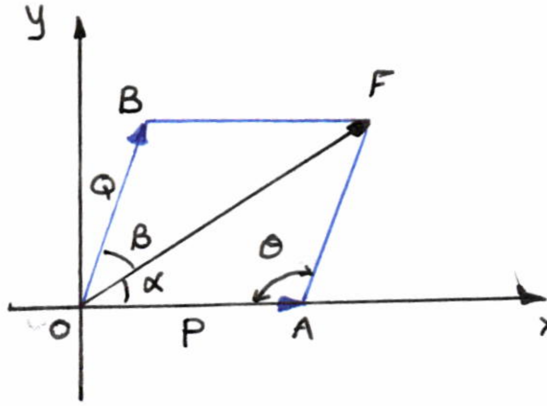
$$F_x = F\cos\theta = 340 * \cos(61.928) = 160 \text{ N}$$

$$F_y = F\sin\theta = 340 * \sin(61.928) = 300 \text{ N}$$

Ex

In figure, the resultant force F is 300 N and the angles α, B are 25° and 45° respectively.

Resolve the force F into a pair of Components P along line OA and Q along OB.



Solution

$F_x = F \cos\alpha = 300 \cos 25 = 271.9 \text{ N}$

$F_y = F \sin\alpha = 300 \sin 25 = 126.8 \text{ N}$

$$\frac{F}{\sin\theta} = \frac{P}{\sin\beta} = \frac{Q}{\sin\alpha}$$

$$\frac{300}{\sin 110} = \frac{Q}{\sin 25} = \frac{P}{\sin 45}$$

$$Q = \frac{300 \sin 25}{\sin 110} = 134.9 \text{ N}$$

$$P = \frac{300 \sin 45}{\sin 110} = 225.75 \text{ N}$$

Another method

$F_y = Q \cos(90 - (\alpha + B)) = Q \sin(\alpha + B)$

$126.8 = Q \sin(25 + 45)$

$$Q = \frac{126.8}{\sin 70} = 134.9 \text{ N}$$

$F_x = P + Q \sin(90 - (\alpha + B)) = P + Q \cos(\alpha + B)$

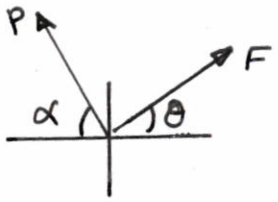
$271.9 = P + 134.9 \cos(25 + 45)$

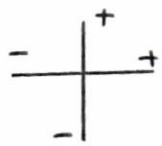
$P = 271.9 - 134.9 \cos 70 = 225.76 \text{ N}.$

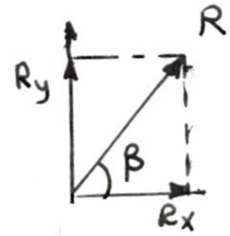
3.2 Resultant of force system

The resultant of a force system has been defined as the simplest force system which can replace the original system without changing its external effect on rigid body.

* Resultant of a Concurrent, Coplanar force system.







$$R_x = \Sigma F_x = F\cos\theta - P\cos\alpha$$

$$R_y = \Sigma F_y = F\sin\theta + P\sin\alpha$$

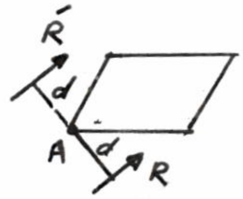
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\beta = \tan^{-1}\frac{R_y}{R_x}$$

* Resultant of a Non Concurrent, Coplanar force system.

a. Replace forces into its rectangular Components.
b. Summation of all x and y Components seperately.
c. Find the resultant from
   $R = \sqrt{R_x^2 + R_y^2}$ and its direction $\theta = \tan^{-1}\frac{R_y}{R_x}$

d. Take moment about any practical point.

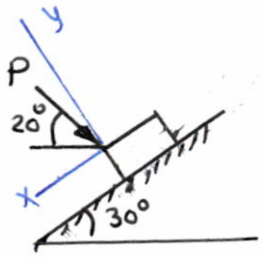


$$\Sigma M_A = R * d$$

then find (d)

if d is (-ve) then transfer the resultant to $\acute{R}$.

Example



The body on the 30° incline in figure is acted upon by a force P inclined at 20° with the horizontal. If P is resolved into components parallel and perpendicular to the incline and the value of the parallel components is 400 N, compute the value of the perpendicular component and that of P.

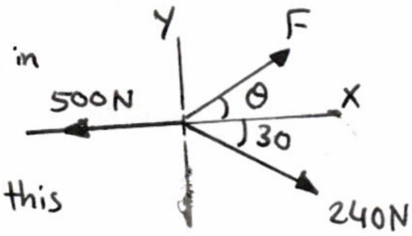
Solution

$$F_x = 400 = P\cos 50$$

$$P = \frac{400}{\cos 50} = 622.29 \text{ N}$$

$$F_y = P\sin 50 = 622.29 * \sin 50 = 476.7 \text{ N}$$

Example



The resultant of the concurrent force shown in figure is 300 N pointing up along the y axis. Compute the values of F and θ required to give this resultant.

Solution

Since the resultant is up along y axis $\Rightarrow \Sigma F_x = 0$

$$F\cos\theta + 240\cos 30 - 500 = 0$$

$$F\cos\theta - 292.15 = 0 \Rightarrow F\cos\theta = 292.15 \quad \text{----} ①$$

$$R = \Sigma F_y = F\sin\theta - 240\sin 30$$

$$300 = F\sin\theta - 120$$

$$F\sin\theta = 420 \text{ N} \quad \text{————} ②$$

divide ② by ① yields

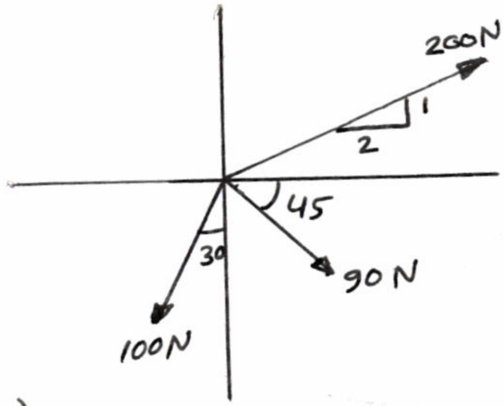
$$\frac{\cancel{F}\sin\theta}{\cancel{F}\cos\theta} = \frac{420}{292.15} \Rightarrow \tan\theta = 1.4376 \Rightarrow \theta = \tan^{-1} 1.4376 = 55.18^\circ$$

$$\therefore F = \frac{420}{\sin 55.18} = 511.6 \text{ N}.$$

HW Repeat this problem if the resultant is 400 N down to the right at 60° with the x-axis.

Example

Resolve the forces that shown in figure into their rectangular components and find the resultant and its direction.



Solution

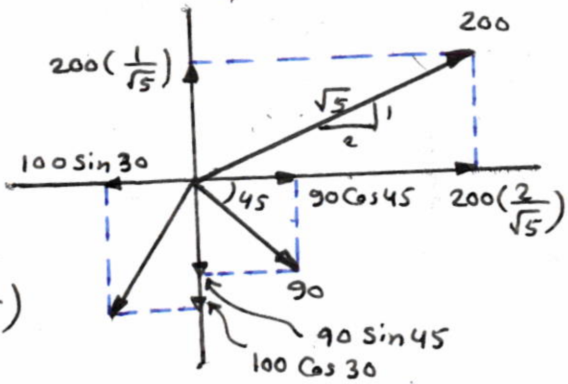
$\xrightarrow{+}$ $R_x = \Sigma F_x$

$= -100 \sin 30 + 90 \cos 45 + 200\left(\frac{2}{\sqrt{5}}\right)$

$= -50 + 63.64 + 178.88$

$= 192.5 \ N$

$\therefore R_x = 192.5 \ N \longrightarrow$



$+\uparrow R_y = \Sigma F_y$

$= -100 \cos 30 - 90 \sin 45 + 200\left(\frac{1}{\sqrt{5}}\right)$

$= -86.6 - 63.64 + 89.44$

$= -60.797 \ N$

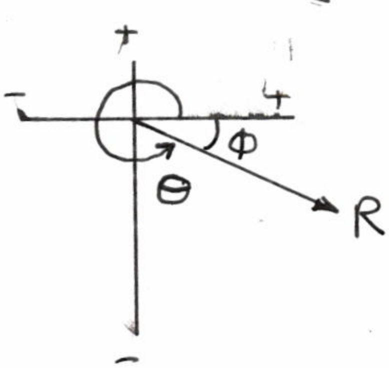
$\therefore R_y = 60.797 \ N \downarrow$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(192.5)^2 + (60.8)^2} = 201.87 \ N$

since R_x (+ve) & R_y (−ve) $\Rightarrow$ fourth quarter

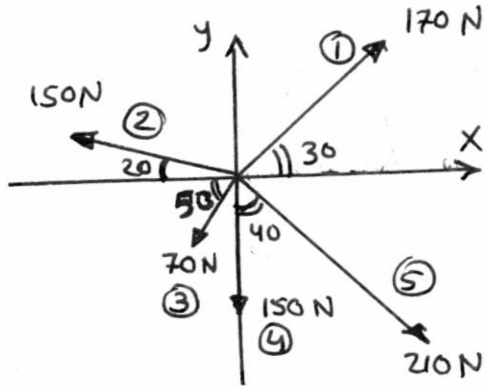
$\phi = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1}\left(-\frac{60.797}{192.5}\right) = -17.53^\circ$

$\theta = -17.53 + 360 = 342.47^\circ$.



Example

Find the magnitude and direction of the resultant force of the system of forces shown in figure using the method of resolution of forces « or summation of Components ».



Solution

① Numbering the forces.

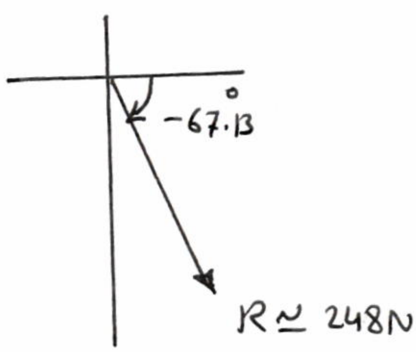
② Make a table as shown below.

Force	X-Component	Value	y-Component	Value
1	170 Cos 30	147.224	170 Sin 30	85
2	-150 Cos 20	-140.954	150 Sin 20	51.303
3	-70 Cos 50	-44.995	-70 Sin 50	-53.623
4	—	0	-150	-150
5	210 Sin 40	134.985	- 210 Cos 40	-160.869
SUM	Rx	96.26	Ry	-228.189

$$R = \sqrt{R_x^2 + R_y^2}$$

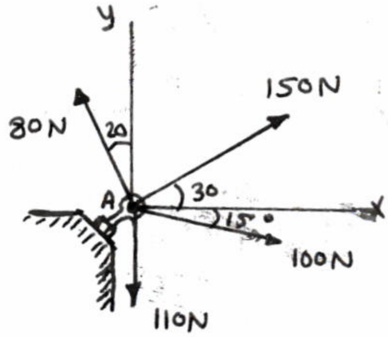
$$= \sqrt{(96.26)^2 + (228.19)^2} = 247.66 \text{ N}$$

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{(-228.19)}{96.26} = \tan^{-1}(-2.37) = -67.13^\circ$$

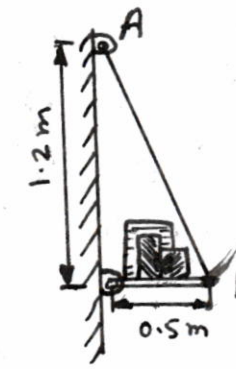


Problems

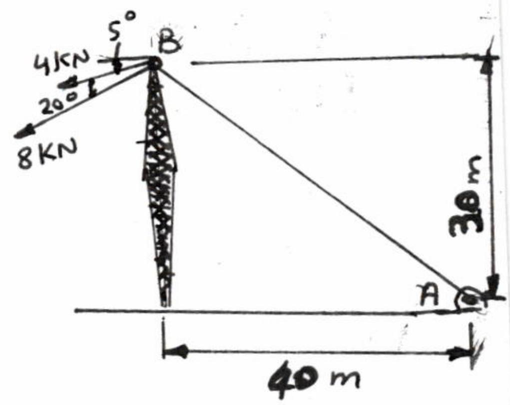
1. Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt. Ans [199.6N ∠ 4.1°]



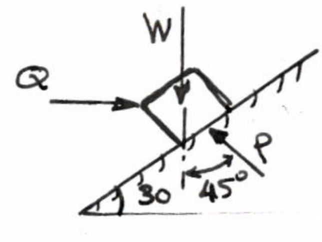
2. The tension in the support cable AB is 650N . Determine the horizontal and vertical components of the force acting on the pin at A .



3. Two cables which have known tensions are attached at point B . A third cable AB is used as a guy wire and is also attached at B . Determine the required tension in AB so that the resultant of the forces exerted by the three cables will be vertical.



4. The block shown in figure is acted upon by its weight W = 200 N, a horizontal force Q = 600 N, and the pressure P exerted by the inclined plane. The resultant R of these forces is up and parallel to the incline thereby sliding the block up it. Determine P and R

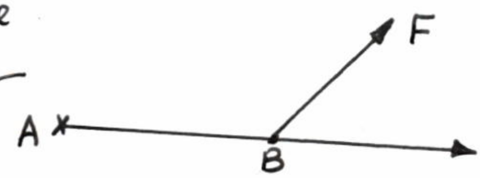


Hint: take one axis parallel to the incline. [Ans. R = 293 N]

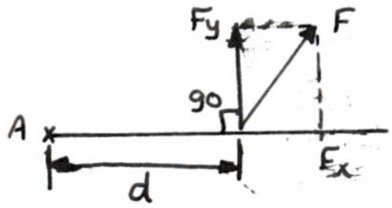
5. Two horses on opposite banks of a canal pull a barge moving parallel to the banks by means of two horizontal ropes. The tension in these ropes are 200 N and 240 N while the angle between them is 60°. Find the resultant pull on the barge and the angle between each of the ropes and the sides of the canal.

4. Moment of a force

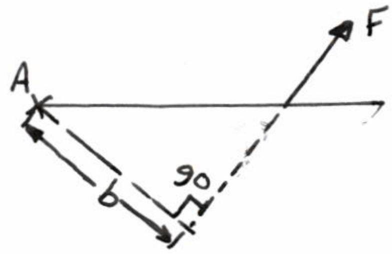
The moment of a force F with respect to a point A is defined as a vector with a magnitude equal to the product of perpendicular distance from A to F times the magnitude of F with direction perpendicular to the plane containing A and F and sense given by the direction of an advancing right-hand screw as it is turned about A in the direction indicated by F.



Moment of F about A $= F_y * d$
$= F * b$

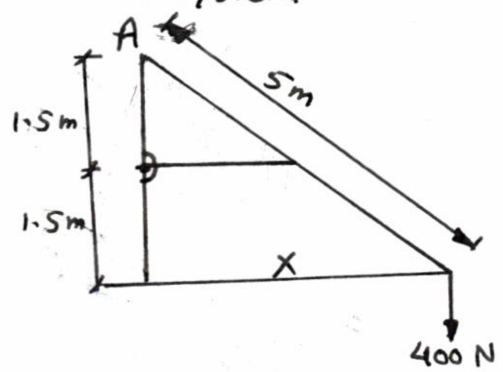


And it is shown as



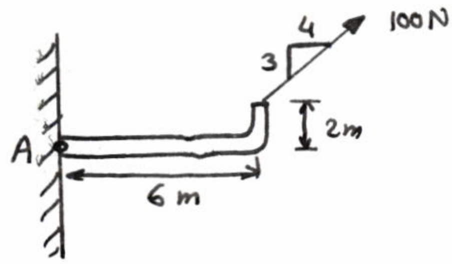
Examples

Determine the moment of a force with respect to point A for each system.



$X = \sqrt{5^2 - 3^2} = 4\ m$

$M_A = 400 * X = 400 * 4$
$= 1600\ N.m$



$/ = \sqrt{(3)^2 + (4)^2} = 5$

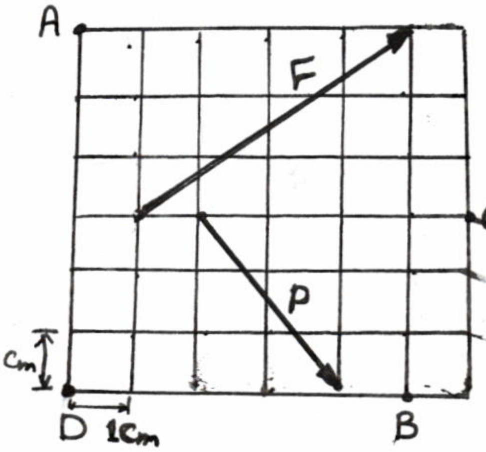
$\circlearrowleft + \ M_A = 100 (\frac{4}{5}) * 2 - 100 (\frac{3}{5}) * 6$
$= 160 - 360 = -200\ N.m$

$\therefore M_A = 200\ N.m$

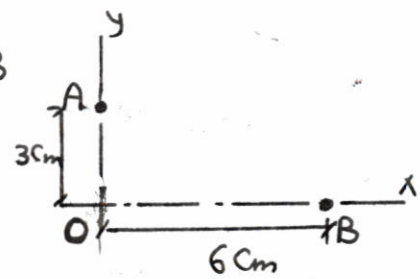
Problems

1. Two forces P and Q pass through a point A which is 4 m to the right of and 3 m above a moment Center O. Force P is 200 N directed up to the right at 30° with the horizontal and force P is 100 N directed up to the Left at 60° with the horizontal. Determine the moment of the resultant of these two forces with respect to O.

2. In figure assuming Clockwise moments as positive, Compute the moment of force F = 450N and of force P = 361 N about points A, B, C and D.



3. A force P passing through points A and B in figure has a Clockwise moment of 300 N.Cm about O. Compute the value of P.



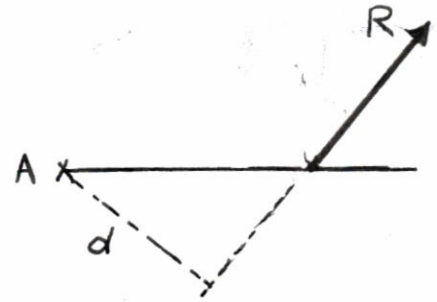
4. In the above figure, a force P intersects the X-axis at 4 Cm to the right of O. If the moment about A is 170 N.Cm Counter clock wise and its moment about B is 40 N.Cm Clock wise, determine its y intercept.

5. In figure above, the moment of a Certain force F is 180 N.Cm Clockwise about O and 90 N.Cm Counter clock wise about B. If its moment about A is zero, determine the force F.

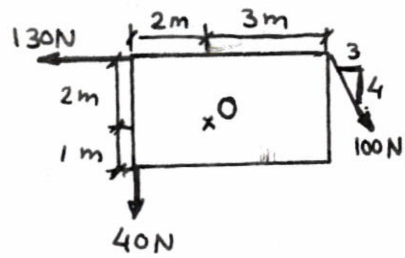
5. Location of Resultant

The location of the resultant force in a force system can be determined by using the base of moment of a force about a point.

$M_A = R * d$



Example: Determine the resultant of the Coplanar force system shown in figure and locate it with respect to point "O".



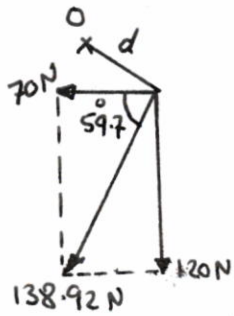
Solution

$\xrightarrow{+} R_x = -130 + 100\left(\frac{3}{5}\right) = -130+60 = -70 N$

$+\uparrow R_y = -40 - 100\left(\frac{4}{5}\right) = -40-80 = -120 N$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(70)^2+(120)^2} = 138.92 N$

$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\left(\frac{-120}{-70}\right) = 59.74^\circ$ (thrid quarter)



$\Sigma M_o = R * d$ (suppose it as in figure).

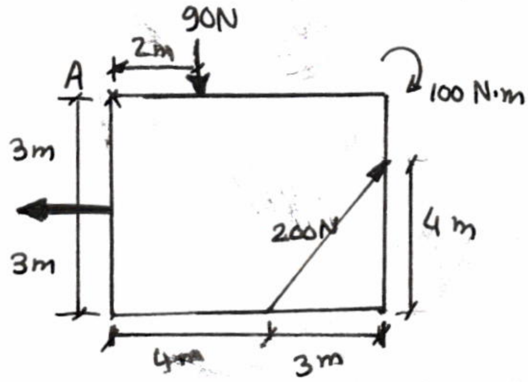
$\circlearrowright^{+}\ 138.9*d = 100\left(\frac{4}{5}\right)*3 + 100\left(\frac{3}{5}\right)*2 - 40*2 - 130*2$

$138.92 * d = 20 \Rightarrow d = 0.1439\ m.$

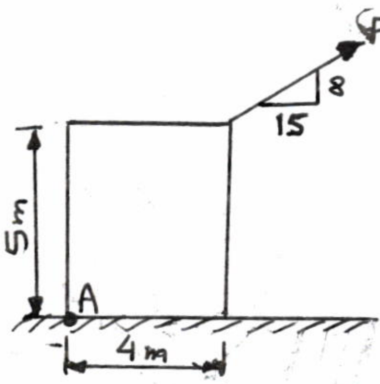
NB If we get d with -ve sign then

Problems

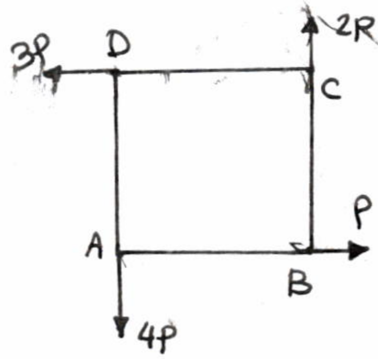
1. Determine the resultant of the force system shown in figure and locate it with respect to point A.



2. The magnitude of vertical component of the force in figure is 160 N.
   a. determine the force F (340 N)
   b. determine the moment of F w.r.t point A. (860 N.m)
   c. By means of principle of moments, determine the perpendicular distance from A to the line of action of F. (2.54 m)



3. The system of the given forces that shown in figure are acting along the four sides of square ABCD. Find the magnitude, direction and position (location) of the resultant force.



6. Couples

A Couple Consists of two forces which have equal magnitude and parallel lines of action but are opposite in sense.

The characteristics of a couple

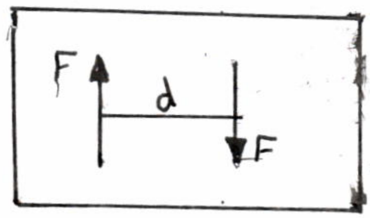
a) Its magnitude of the moment.

b) Slope of the plane of Couple.

c) the sense of rotation of the Couple.

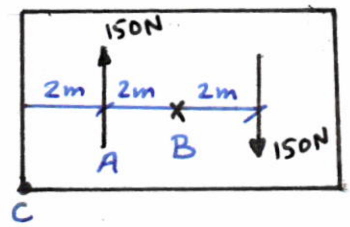
i.e

- Its magnitude is $F * d$
- at the same plane and Can be transmitted to any point in the plane.
- Its sense in this Case is ↻



Example

Determine the moment of the Couple in figure, with respect to point A, B & C.



Solution

$\circlearrowright + \; M_A = 150 * 4 = 600 \text{ N.m}$

$\circlearrowright + \; M_B = 150 * 2 + 150 * 2 = 600 \text{ N.m}$

$\circlearrowright + \; M_C = 150 * 6 - 150 * 2 = 600 \text{ N.m}$

Example By using the transformation of a Couple, replace the three Couples of fig. by one Couple with a force acting horizontally at A and B.

Solution

$\circlearrowright + \; M_{AB} = -300 + 400 - 200$

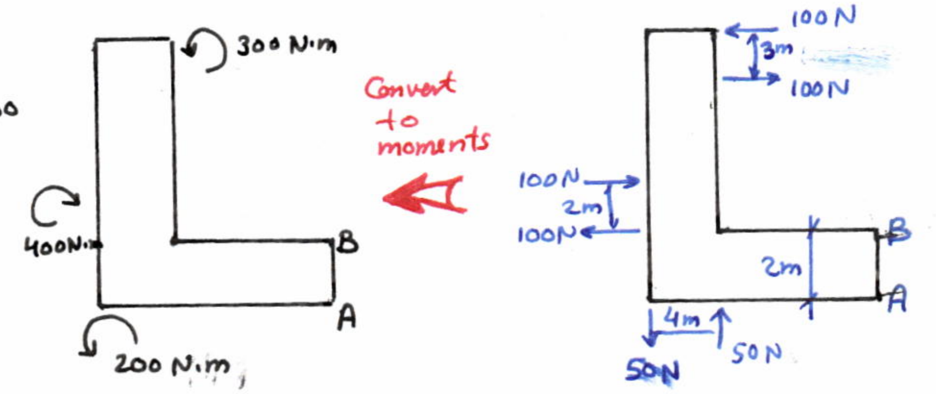
$= -100 \text{ N.m}$

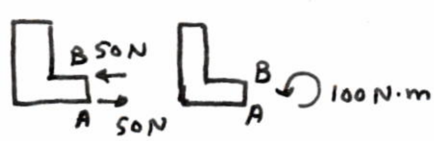
or $M_{AB} = 100 \text{N.m} \circlearrowleft$

$= F * \overline{AB}$

$= F * 2$

$\therefore F = 50 \text{ N}.$

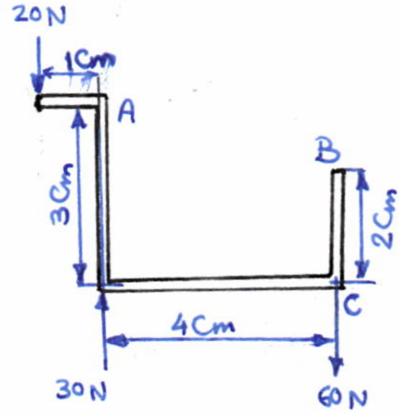




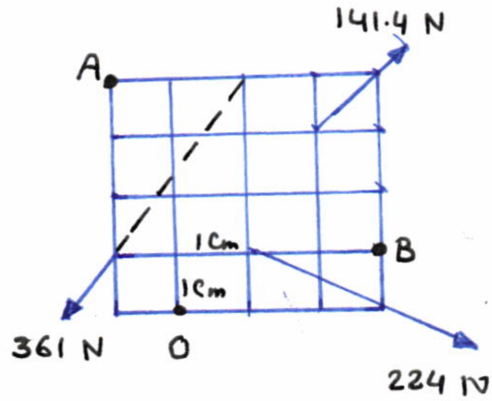
Problems

1. Replace the system of forces acting on the frame in figure by resultant R at A and a couple acting horizontally through B and C.

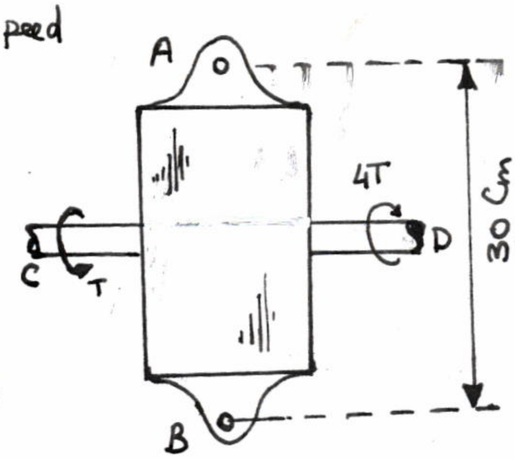
Ans: $R = 50N \downarrow$, $B = 110N \rightarrow$, $C = 110N \leftarrow$



2. Replace the system of forces shown in figure by an equivalent force through O and a couple acting through A and B. Solve if the forces of the couple are:
(a) horizontal. (b) vertical.



3. The figure represents the top view of a speed reducer which is geared for a four to one reduction in speed. The torque input at the horizontal shaft C is 100 N·m, the torque output at horizontal shaft D, because of the speed reduction, is 400 N·m. Compute the torque reaction at the mounting bolts A and B holding the reducer to the floor.



Ans: R=[redacted] directed vertically up at A and down at B.

7. Equilibrium.

7.1 Definition of equilibrium : Is the term used to designate the Condition where the resultant of a system of forces is Zero. A body is said to be in equilibrium when the force system acting upon it has a zero resultant.

The physical meaning of equilibrium of a body is that the body either is at rest or is moving in a straight line with Constant velocity.

7.2 Principle of equilibrium:

The following three principle are important from the subject point of view:-

a. Two force principle - It state « If a body in equilibrium is acted upon by two forces, then they must be equal, opposite and Collinear.»

b. Three force principle - It state « If a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and Collinear with the third force.».

c. Four force principle - It state « If a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and Collinear with the resultant of the other two.».

7.3 Conditions of equilibrium

Consider a body acted upon by system of forces. A little Consideration with show that as a result of these forces, the body may has one of the following states:-

a. The body may move in any one direction, it means that there is a resultant force acting on it, but if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero ; or; the horizontal component of all forces (ΣF_x) and vertical Component of all forces (ΣF_y) must be zero. Mathematically:-

$$\Sigma F_x = 0 \qquad \& \qquad \Sigma F_y = 0$$

b. The body may rotate about itself without moving, it means that there is a single resultant Couple acting on it with no resultant force, but if the body is to be at rest or in equilibrium, the moment of the Couple causing rotation must be zero; or; the resultant moment of all forces (ΣM) must be zero, Mathematically:-

$$\Sigma M = 0$$

c. The body may move in any one direction, and at the same time it may also rotate about itself, it means that there is a resultant force and also a resultant Couple acting on it, but if the body is to be at rest or in equilibrium, the resultant force causing movements and the resultant moment of the Couple causing rotation must be zero; or; (ΣF_x), (ΣF_y) and (ΣM) must be zero; mathematically :-

$$\Sigma F_x = 0 \quad , \quad \Sigma F_y = 0 \quad \& \quad \Sigma M = 0$$

* The above mentioned three equations are known as the Conditions of equilibrium. As a matter of fact, these Conditions helpus in finding out the reactions of forces at a particular point, when the body is in equilibrium (unknown reactions of forces).

7.4 Types of equilibrium

Stable equilibrium A body is said to be in stable equilibrium if it returns back to its original position, after it is slightly displaced from its position of rest

Unstable equilibrium A body is said to be in an unstable equilibrium, if it does not return back to its original position, and keeps farther away, after slightly displaced from its position of rest.

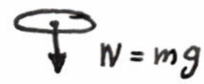
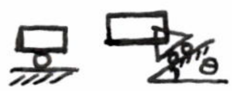
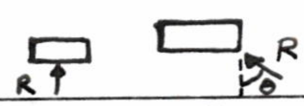
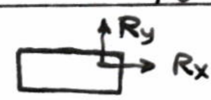
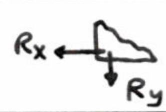
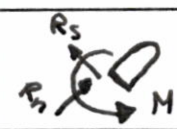
Neutral equilibrium A body is said to be in a neutral equilibrium if it occupies a new position (and remains at rest in this position) after slightly displaced from its position of rest.

※ Equilibrium When a system of forces acting on a body has no resultant, the body on which the force system acts is in equilibrium.

8. Free body diagram

It is a sketch of a body or a portion of a body or two or more bodies completely isolated or free from all other bodies, showing the force exerted by all other bodies on one being considered.

The table below shows some of the more common types of free body diagram.

Type of body removed	Sketch of reacting bodies	Action of body removed upon (FBD)
Earth		
Flexible cord, rope		
Smooth surface		
Roller or ball		
Smooth pin or hinge		
Knife edge		
Support for a beam or post fixed at the end		
Pin in smooth guide (slot)		

Procedue for solution of equilibrium Problems :-

1. Determine Carefully the given data and what results are required.
2. Draw (F.B.D) of the member or group of members on which some or all of the unknown forces are acting.
3. Note the number of independent equations of equilibrium available for the type of forces system involved.
4. Observe the type of forces system which acts on the FBD drawn.
5. Compare the number of unknowns on the FBD with a number of independent equations of equilibrium available for the force system.
6. a- If there are as many independent equations of equilibrium as unknown, proceed with the solution by writting and solving the equations of equilibrium.
   b- If there are more unknown to be evaluated than independent equations of equilibrium available, draw a free body diagram of another body and repeat steps 3, 4 and 5 for the second FBD drawn.
7. a- If there are as many independent equations of equilibrium as unknowns for the second FBD, proceed with the solution by writting and solving the necessary equations of equilibrium.
   b- If there are more unknowns to be evaluated than independent equations of equilibrium for the second FBD, Compare the total number of unknowns on both FBD's with the total number of independent equations of equilibrium available for both diagrams.
8. If there are still too many unknowns, then the problem is statically indeterminate, that is, not all the unknowns can be evaluated by statics alone.

The requirements for the static equilibrium of the four types of systems Considered are summarized as:-

① Colinear force system

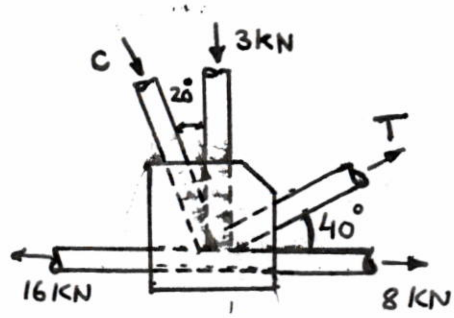
The algebric summation of forces along one known direction is zero.

$$\sum F = 0$$

② Concurrent - forces system

The algebric summation of forces along two perpendicular directions is zero.

$$\sum F_x = 0 \quad , \quad \sum F_y = 0$$



Example Determine the magnitudes of the forces "C" & "T" which, along with the other three forces shown, act on the bridge truss joint.

solution

$\sum F_x = 0$

$8 + T\cos 40 + C\sin 20 - 16 = 0$

$0.766T + 0.342C = 8$ ---- ①

$\sum F_y = 0$

$T\sin 40 - 3 - C\cos 20 = 0$

$0.643T - 0.94C = 3$ ------ ②

simultaneous solution of eqn ① & ② Yields

$T = 9.09$ KN & $C = 3.03$ KN

③ Parallel force system

ⓐ The algebric summation of forces along one known direction is zero.

& ⓑ The algebric summation of moments about any reference point in the system is zero.

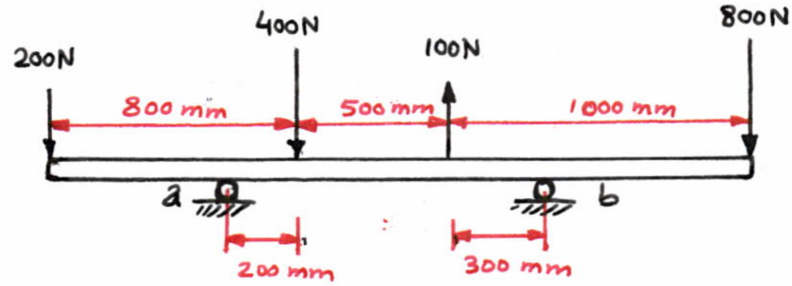
$$\sum F_x = 0 \quad \& \quad \sum M = 0$$

OR

$$\sum F_y = 0 \quad \& \quad \sum M = 0$$

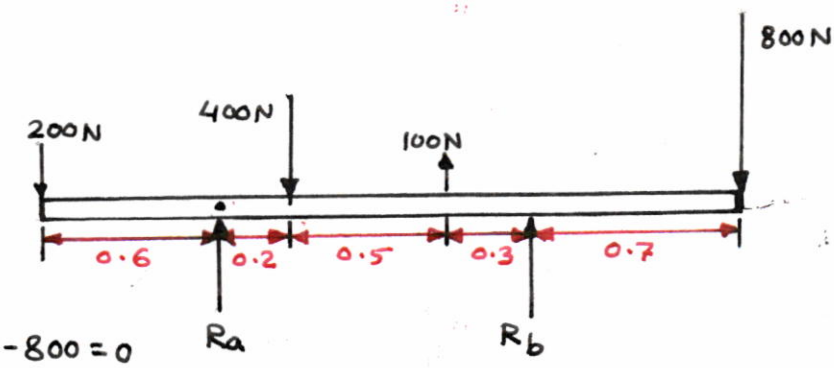
Example

The figure shows a laterally loaded beam which is supported by two rollers and in equilibrium. Find the values of the reaction forces R_a and R_b of the rollers on the beam. The beam is weightless.



Solution

$\Sigma F_y = 0$



$R_a + R_b - 200 - 400 + 100 - 800 = 0$

$R_a + R_b = 1300$

$R_a = 1300 - R_b$ ------ (1)

$\Sigma M_a = 0$ (+)

$-(200 * 0.6) + (400 * 0.2) - (100 * 0.7) - (R_b * 1) + (800 * 1.7) = 0$

$-120 + 80 - 70 - R_b + 1360 = 0$

$R_b = 1360 + 80 - 120 - 70 = 1250\ N$

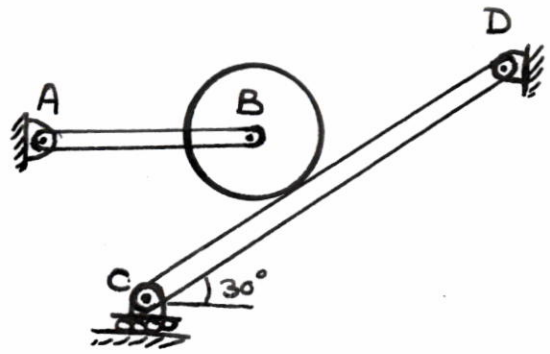
$\therefore R_a = 1300 - 1250 = 50\ N.$

HW Repeat the solution by taking the moment about (b).

(4) General two dimensional force system

$$\boxed{\Sigma F_x = 0, \quad \Sigma F_y = 0 \quad \& \quad \Sigma M = 0}$$

Example

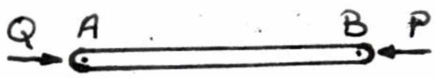


A 200 N Cylinder is supported by a horizontal rod AB and rests against the uniform bar CD which weight 100 N.

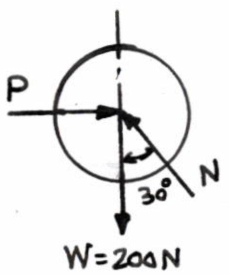
Draw the free body diagrams of (a) rod AB (b) the Cylinder (c) bar CD (d) the assembled cylinder and bar.

Solution

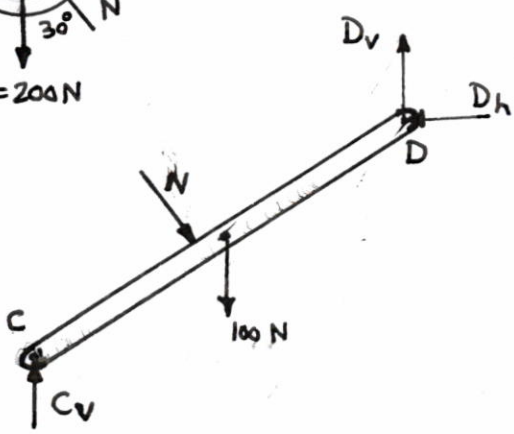
(a)



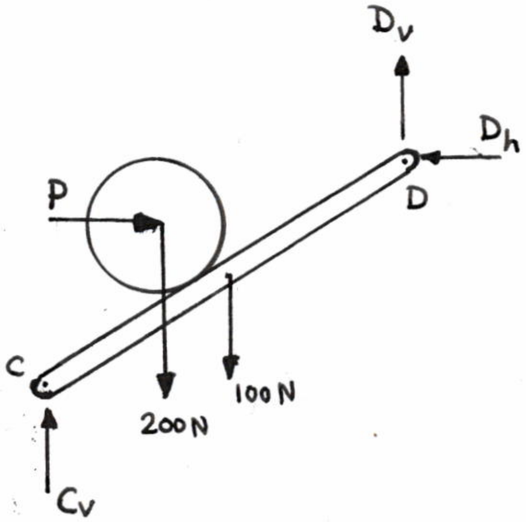
(b)



(c)

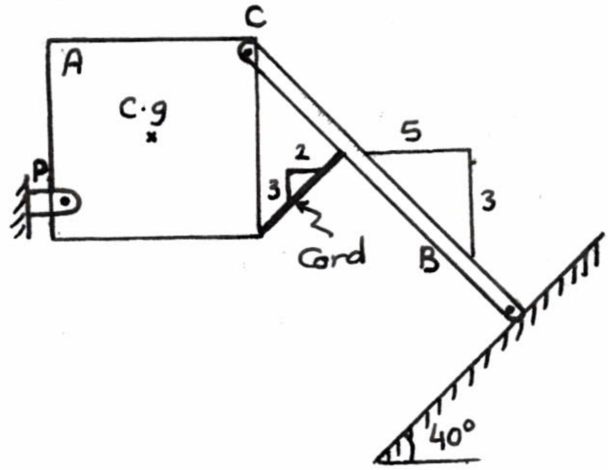


(d)

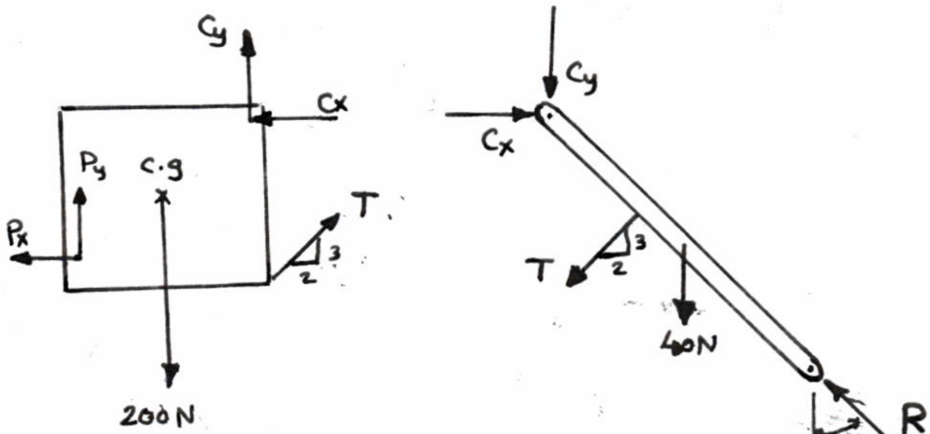


Example

A body "A" in figure below weights 200 N and the homogenous bar "B" weights 40 N, Draw F.B.D of each of the two bodies.

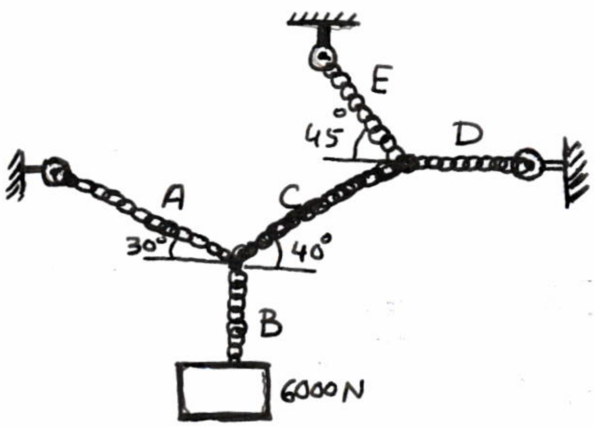


Solution

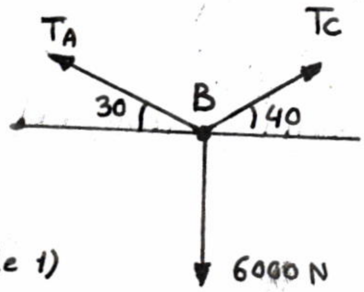


Example

Determine the tensile force in chain D.



Solution



F.B.D (node 1)

$\Sigma F_x = 0$

$-T_A \cos 30 + T_C \cos 40 = 0$ ---- (1)

$\Sigma F_y = 0$

$T_C \sin 40 + T_A \sin 30 - 6000 = 0$ --- (2)

from (1)

$T_C = T_A \dfrac{\cos 30}{\cos 40} = 1.13\, T_A$ --- (3)

sub in (2)

$1.13\, T_A \sin 40 + T_A \sin 30 = 6000$

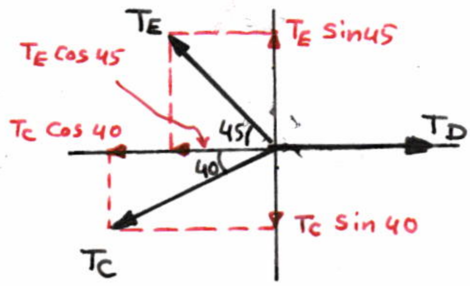
$1.227\, T_A = 6000$

$T_A = 4891.24$ N

sub in (3)

$T_C = 1.13 * 4891.24$

$= 5527.1$ N



F.B.D (node 2)

$\Sigma F_x = 0$

$T_D - T_E \cos 45 - T_C \cos 40 = 0$

$T_D = T_E \cos 45 + T_C \cos 40$

$= 0.707\, T_E + 4234$ ---- (4)

$\Sigma F_y = 0$

$T_E \sin 45 - T_C \sin 40 = 0$

$T_E = 5527.1 \dfrac{\sin 40}{\sin 45} = 5024.35$ N

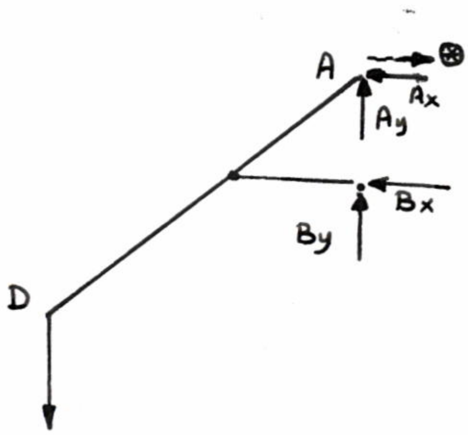
Sub. in (4)

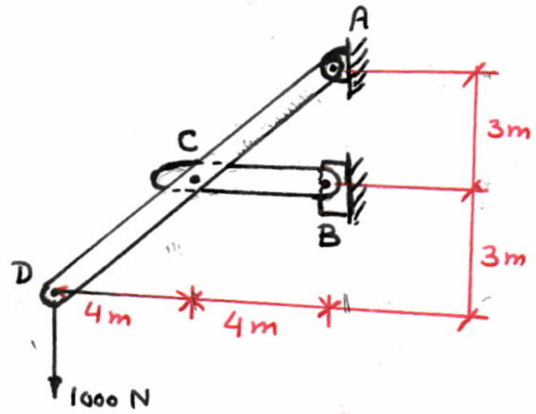
$T_D = 0.707 * 5024.35 + 4234$

$= 7786.2$ N.

Example

Determine the pin reaction at A on bar AD and at B&C.





$\Sigma M_B = 0$ ↻+

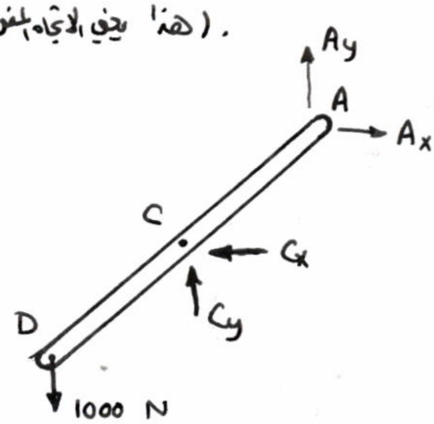
$-A_x * 3 - 1000(8) = 0$

$A_x = \dfrac{-1000(8)}{3} = -2666.7\ N \Rightarrow A_x = 2666.7\,N \rightarrow$ ⊛

$\Sigma F_x = 0 \ \xrightarrow{+}$

$-B_x + A_x = 0$

$B_x = A_x = 2666.7\,N \leftarrow$ (هنا يعني الاتجاه المفروض صحيح).



For member DCA

↻+ $\Sigma M_C = 0$

$A_x(3) - A_y(4) - 1000(4) = 0$

$A_y = \dfrac{-4000 + 8000}{4} = 1000\,N$

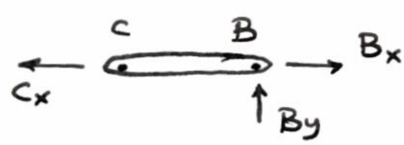
$+\uparrow \Sigma F_y = 0$

$1000 + C_y - 1000 = 0 \Rightarrow C_y = 0\,N$

$\xrightarrow{+} \Sigma F_x = 0$

$-C_x + A_x = 0 \Rightarrow A_x = C_x = 2666.7\,N \leftarrow$

For member CB

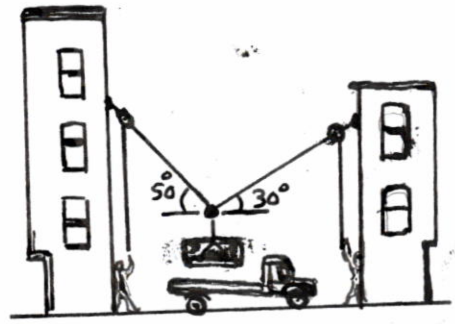


$B_y = C_y = 0$

$B_x = C_x = 2666.7\,N$

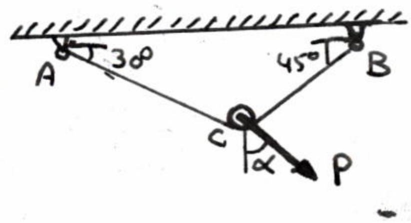
Problems

1. Consider a 75 kg Crate shown in the figure. This Crate was lying between two buildings, at it is being lifted onto a truck, which will remove it. The crate is supported by a vertical cable, which is joined at A to two ropes which pass over pulleys attached to the buildings at B and C. Determine the tension in each of the ropes AB and AC.



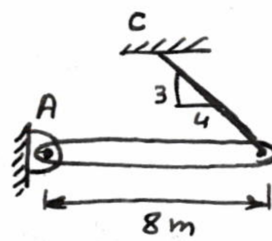
Ans. [$T_{AB} = 647N$, $T_{AC} = 480N$]

2. The force P is applied to a small wheel which rolls on the Cable ACB. Knowing that the tension in both parts of the Cable is 750 N, determine the magnitude and direction of P.



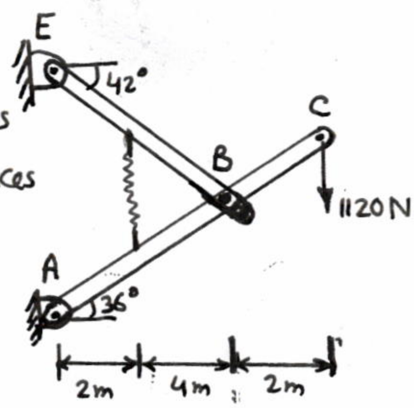
Ans. [$P = 913N$, $\alpha = 7.5°$]

3. The 100 N homogenous bar AB shown in figure is supported by a pin at A and the Cord BC. Determine the pin reaction at A on the Bar AB.



Ans. [$R_A = 83.3$ N, $\theta = 36.8$]

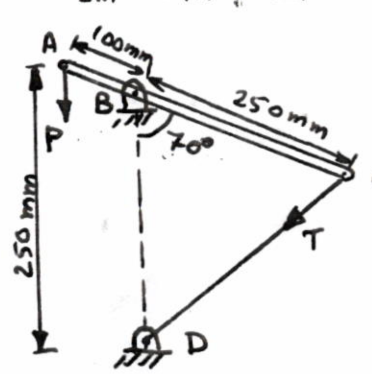
4. The tension in the spring in the pin-Connected structure shown in the figure is 540 N, the weights of the members and friction at all Contact surfaces can be neglected. Determine the horizontal and vertical Components of the pin reaction at B on member BE.



5. Knowing that the magnitude of vertical force P in figure is 400 N. Determine.

a- The reaction at B.

b- The tension in Cable CD. [183.6N]



9. Centroids of area & lines

$$A\bar{x} = \sum ax$$
$$A\bar{y} = \sum ay$$

The expression $A\bar{x}$ as well as $A\bar{y}$ is called the moment of area, can be defined as the product of the area multiplied by the perpendicular distance from the center of area to the axis of moments.

$$\bar{x} = \frac{\sum ax}{A} \quad \& \quad \bar{y} = \frac{\sum ay}{A}$$

This gives a method of locating a point called the centroid of area, "the point corresponding to the center of gravity of plate of infintesimal thickness.

The term " Centroid" rather than "Center of gravity" is used where refering to area (as well as to lines) because these figures don't have weight.

when referring to lines, the centroid may be determined by similar means

$$\bar{x} = \frac{\sum Lx}{L} \quad , \bar{y} = \frac{\sum Ly}{L}$$

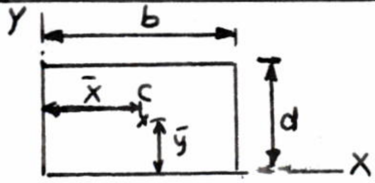
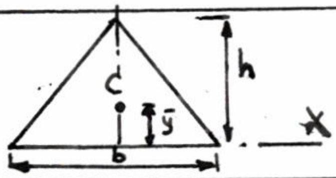
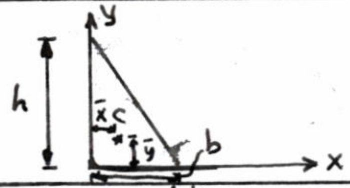
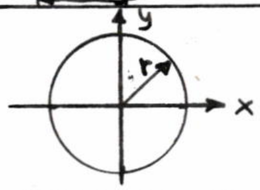
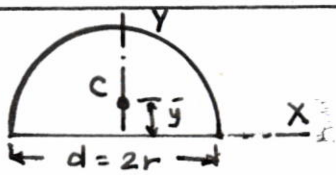
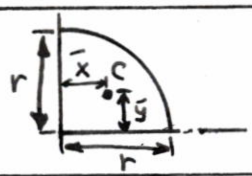
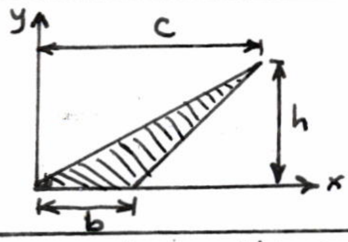
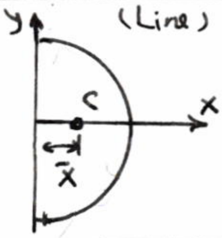
Also the Centroids can be determined by integration, and the equations for determining the Centroid of an area would become

$$A\bar{x} = \int x\, dA$$
$$A\bar{y} = \int y\, dA$$

and for determining the Centroid of a line,

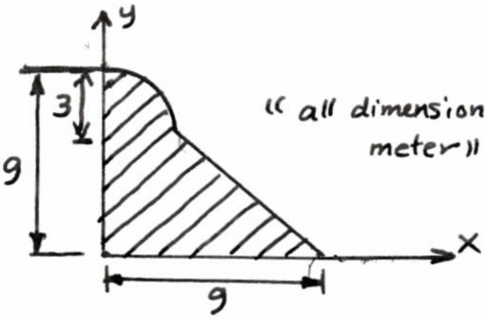
$$L\bar{x} = \int x\, dL$$
$$L\bar{y} = \int y\, dL$$

The general forms that give area, $\bar{x}$ and $\bar{y}$ are shown in the following table.

TABLE (*) Centroids for Common geometric shapes			
Shape	Area / Length	$\bar{x}$	$\bar{y}$
	$b*d$	$\frac{b}{2}$	$\frac{d}{2}$
	$\frac{b*h}{2}$	0	$\frac{h}{3}$
	$\frac{b*h}{2}$	$\frac{b}{3}$	$\frac{h}{3}$
	πr^2	0	0
	$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$ (0.424r)
	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$ (0.424r)	$\frac{4r}{3\pi}$ (0.424r)
	$\frac{bh}{2}$	$\frac{b+c}{3}$	$\frac{h}{3}$
	πr	$\frac{2r}{\pi}$	0

Example

Determine the Centroid of the shaded area that shown in figure.



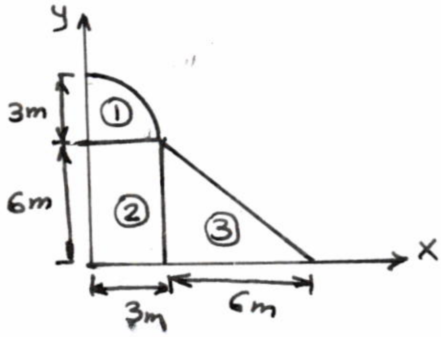
« all dimensions in meter »

Solution

① Divide the shaded area into a common portions.

② put the dimensions as shown in the figure.

③ make a table as shown below.



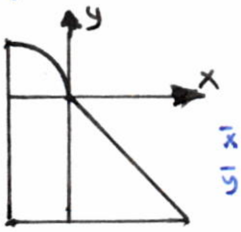
Component	area (m^2)	$\bar{x}$ (m)	$\bar{y}$ (m)
① 	$\frac{\pi r^2}{4} = \frac{\pi * 9}{4} = 7.07$	$0.424r = 1.272$	$0.424 + 6 =$ $1.272 + 6 = 7.272$
② 	$b*h = 3*6 = 18$	$\frac{b}{2} = \frac{3}{2} = 1.5$	$\frac{h}{2} = \frac{6}{2} = 3$
③ 	$\frac{bh}{2} = \frac{6*6}{2} = 18$	$\frac{b}{3} + 3 = \frac{6}{3} + 3 = 5$	$\frac{h}{3} = \frac{6}{3} = 2$
SUM	43.07		

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{7.07 * 1.272 + 18 * 1.5 + 18 * 5}{43.07} = 2.925 \text{ m}$$

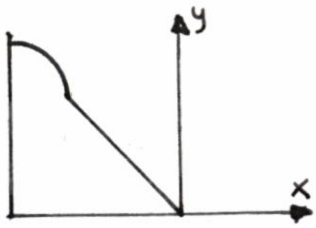
$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{7.07 * 7.272 + 18 * 3 + 18 * 2}{43.07} = 3.283 \text{ m}$$

H.W

1. Repeat the above example for the following drawings.



$\bar{x} = -0.075$m
$\bar{y} = -2.717$m

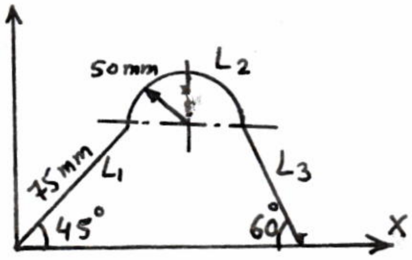


$\bar{x} = -6.075$ m
$\bar{y} = 3.283$ m

2. Repeat the above example for line (not area).

Example

A slender homogenous wire of uniform cross section is being in the form shown in figure. Determine the centroid of the wire with respect to the given axes.



Solution height of L_1 = height of L_3 (why?).

$h_{L_1} = 75 \sin 45 = 53.033$ mm.

$L_3 = \frac{h_{L_1}}{\sin 60} = \frac{53.033}{\sin 60} = 61.237$ mm

Comp.	Length (mm)	$\bar{X}$ (mm)	$\bar{y}$ (mm)
L_1	75	$(1/2)\cos 45 = 26.516$	$(1/2)\sin 45 = 26.516$
L_2	$\pi r = 157.143$	$75\cos 45 + 50 = 103.033$	$\frac{2r}{\pi} + L_1 \sin 45 = 84.851$
L_3	61.237	168.342	$\frac{L_3}{2}\sin 60 = 26.516$
$\sum$	293.38		

$$\bar{x}_{L_3} = L_1 \cos 45 + 2r + \frac{L_3 \cos 60}{2} = 75 \cos 45 + 100 + \frac{61.237}{2} \cos 60$$

$$= 168.342 \text{ mm.}$$

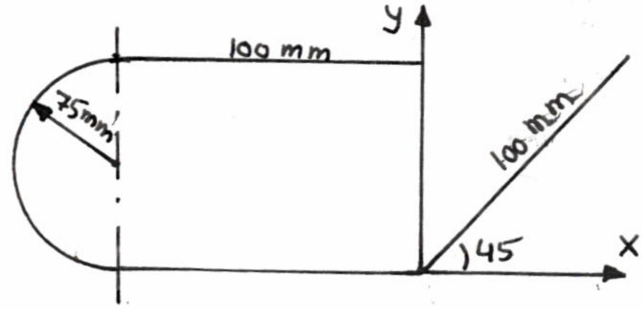
$$\bar{X} = \frac{L_1 X_1 + L_2 X_2 + L_3 X_3}{L_1 + L_2 + L_3} = \frac{75 * 26.516 + 157.143 * 103.033 + 61.237 * 168.342}{293.38}$$

$$= 97.104 \text{ mm.}$$

$$\bar{y} = \frac{\Sigma L y}{\Sigma L} = \frac{75 * 26.516 + 157.143 * 84.851 + 61.237 * 26.516}{293.38} = 57.762 \text{ mm}$$

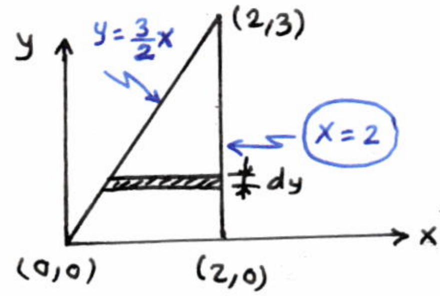
HW 1. Repeat the preceed example for area.

2. Locate the centroid of the bar built up shown in figure.



Example

Using integration, determine the Centroid of the area shown.



Solution

$$y = mx = \frac{y_2-y_1}{x_2-x_1}x = \frac{3-0}{2-0}x = \frac{3}{2}x$$

$$\int dA = \int x\,dy = \int_0^3 (x_2-x_1)\,dy$$

$$= \int_0^3 \left(2-\frac{2}{3}y\right)dy = \left[2y - \frac{2y^2}{6}\right]_0^3 = \left[2*3 - \frac{9}{3}\right] = 6-3 = 3\ m^2$$

$$\int x\,dA = \int \overbrace{\left(\frac{x_2+x_1}{2}\right)}^{x} \cdot \overbrace{(x_2-x_1)\,dy}^{dA}$$

$$= \int_0^3 \left(\frac{2+\frac{2y}{3}}{2}\right)\cdot\left(2-\frac{2y}{3}\right)dy = \int_0^3 \left(\frac{6+2y}{6}\right)\left(\frac{6-2y}{3}\right)dy$$

$$= \frac{1}{18}\int_0^3 (36-4y^2)\,dy = \frac{1}{18}\left[36y - \frac{4y^3}{3}\right]_0^3$$

$$= \frac{1}{18}\left[36*3 - \frac{4*3^3}{3}\right] = \frac{1}{18}\left[108-36\right] = \frac{72}{18} = 4\ m^3$$

$$\therefore \bar{x} = \frac{\int x\,dA}{\int dA} = \frac{4}{3}\ m$$

※ note that it is $= \frac{2b}{3}$

$$\int y\,dA = \int_0^3 y\left(2-\frac{2y}{3}\right)dy = \int_0^3 \left(2y - \frac{2y^2}{3}\right)dy$$

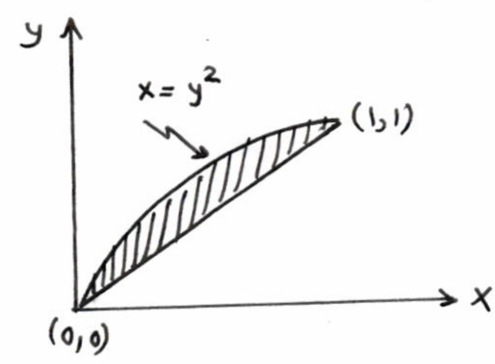
$$= \left[y^2 - \frac{2y^3}{9}\right]_0^3 = 9 - \frac{2*27}{9} = 9-6 = 3\ m^3$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{3}{3} = 1\ m.$$

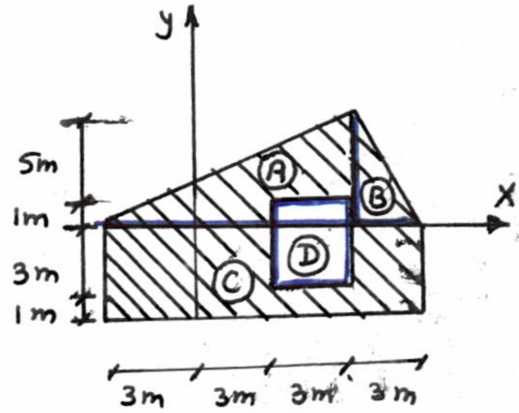
※ note that it is $= \frac{h}{3}$

HW Locate the Centroid of the shaded area (all dimensions are in meter).

Ans: (0.4, 0.5)



Example



Determine the Coordinate of the Centroid of the shaded area shown in figure.

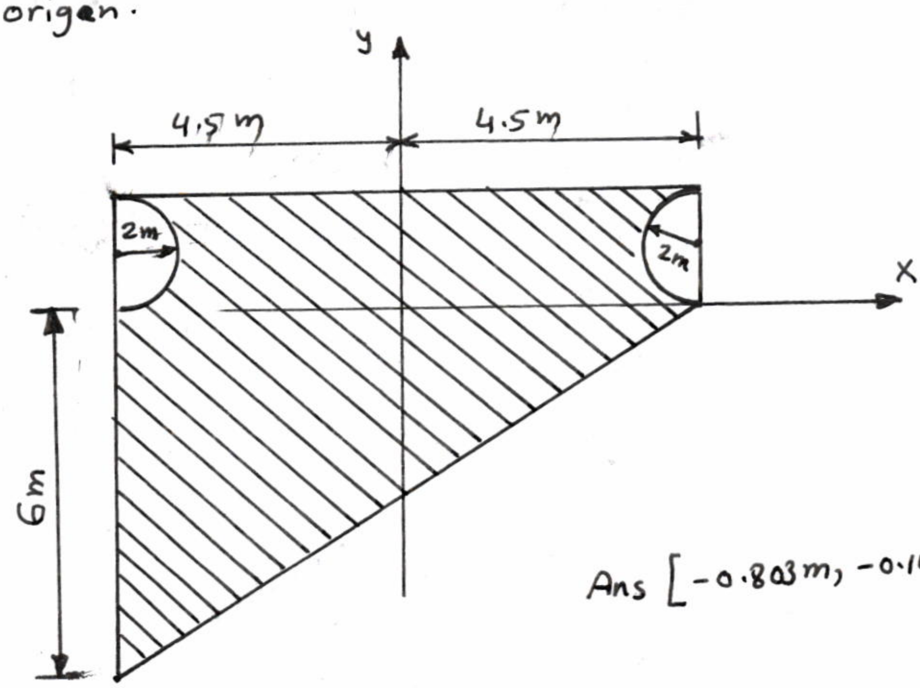
Solution

symbol	Area (m^2)	$\bar{X}$ (m)	M_y (m^3)	$\bar{y}$ (m)	M_x (m^3)
A	$\frac{6\times9}{2}=27$	$\frac{2\times9}{3}-3=3$	81	$\frac{6}{3}=2$	54
B	$\frac{3\times6}{2}=9$	$1+6=7$	63	$\frac{6}{3}=2$	18
C	$4\times12=48$	$\frac{12}{2}-3=3$	144	$-(\frac{4}{2})=-2$	-96
D	$-(4\times3)=-12$	$\frac{3}{2}+3=4.5$	-54	$-(\frac{4}{2}-1)=-1$	12
Σ	72		234		-12

$$\bar{X}=\frac{\Sigma M_y}{\Sigma A}=\frac{234}{72}=3.25\ m.$$

$$\bar{y}=\frac{\Sigma M_x}{\Sigma A}=\frac{-12}{72}=-0.1667\ m.$$

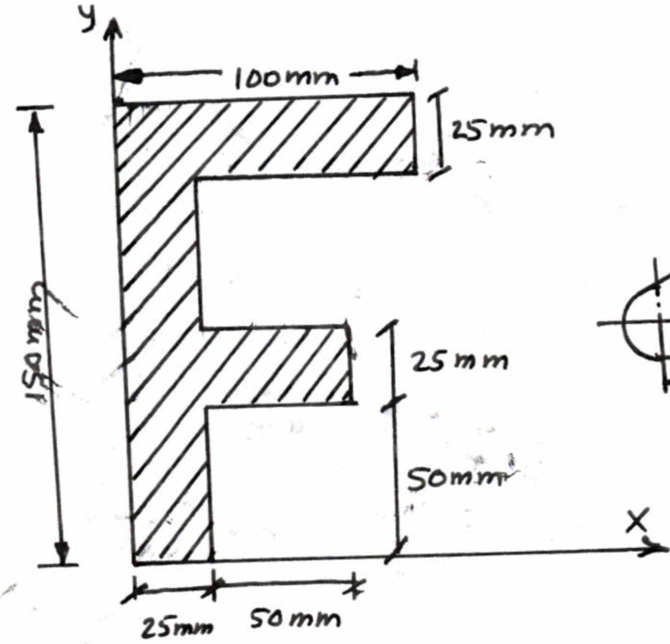
HW Locate the Centroid of the shaded area shown in figure with respect to the origin.

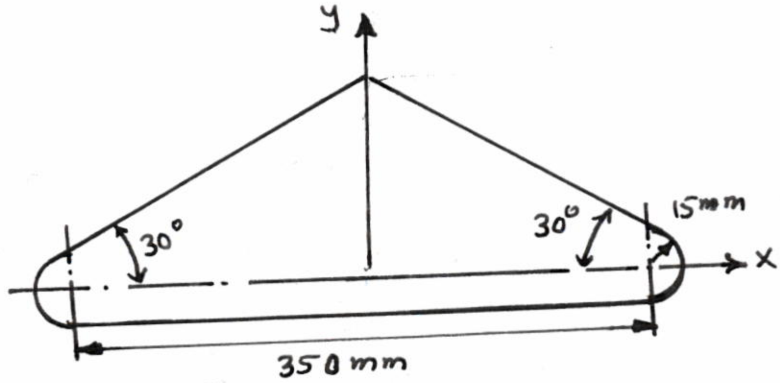


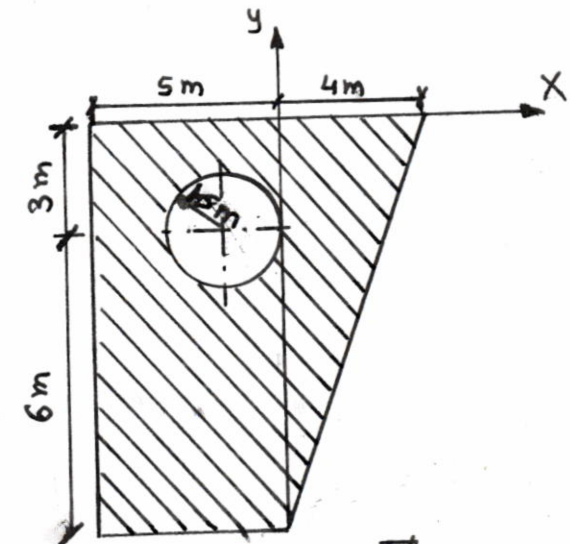
Ans [$-0.803m$, $-0.1416m$]

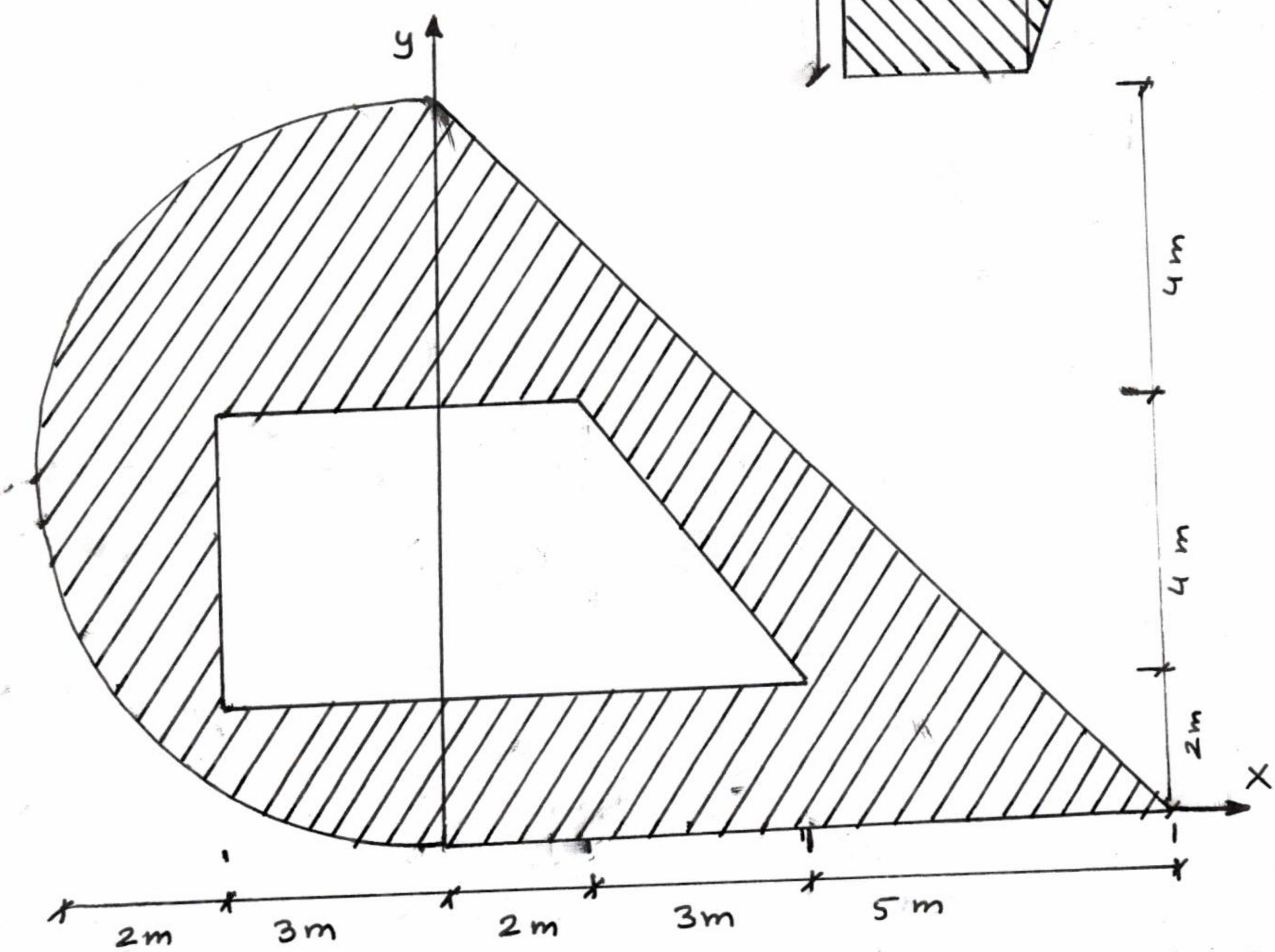
Problems

Locate the centroids of each of the following figures with respect to the axes shown.



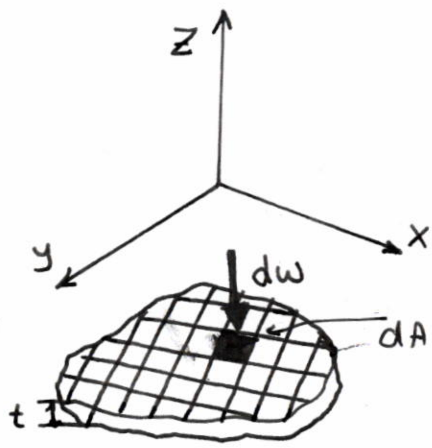






10. Center of gravity of a body

The body Consist of an infinite number of particles. A general procedure of Calculating C.G (center of gravity) Can be done Considering the plane :-



where

$dw = \gamma t \, dA$

dw - is the weight of an element.

t - is the thickness of an element.

dA - is the area of the element.

$\therefore W = \int \gamma t \, dA$

$M_x = \int y \gamma t \, dA$

But $M_x = \bar{y} w$ and $w = \int \gamma t \, dA$

$$\therefore \bar{y} = \frac{\int y \gamma t \, dA}{\int \gamma t \, dA} \quad ----(1)$$

In the same mannar

$$\bar{x} = \frac{\int x \gamma t \, dA}{\int \gamma t \, dA} \quad ---(2)$$

$$\bar{z} = \frac{\int z \gamma t \, dA}{\int \gamma t \, dA} \quad ---(3)$$

If γt = Constant, then γt Can be taken outside the integral

homogenous plate (γ), Constant thickness (t)

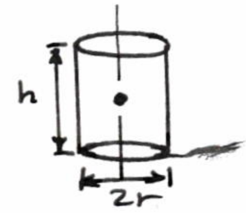
$$\bar{y} = \frac{\int y \, dA}{\int dA}, \quad \bar{x} = \frac{\int x \, dA}{\int dA} \quad \& \quad \bar{z} = \frac{\int z \, dA}{\int dA}$$

For 3D the center of gravity of solid bodies (such as hemispheres), cylinders, right circle solid cones —— etc) is found out in the same way as that of plane figures. The only difference between the plane figures and solid bodies, is that in the case of solid bodies, we calculate volumes instead of areas.

The volume of few solid bodies are given below:-

Vol. of Cylinder, $V = \pi r^2 h$

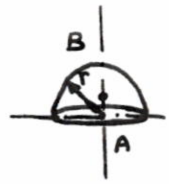
$\bar{y} = \frac{h}{2}$



Vol of hemisphere $= \frac{2}{3}\pi r^3$

$\bar{y} = \frac{5}{8} r$ from B

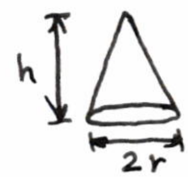
or $\bar{y} = \frac{3}{8} r$ from A



Vol. of right circular solid cone;

$V = \frac{\pi}{3} r^2 h$

$\bar{y} = \frac{h}{4}$



where

"r" = radius of the body

"h" = height of the body

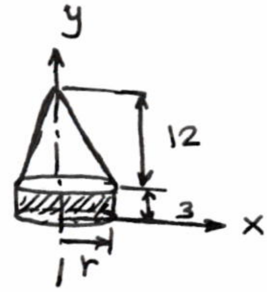
N.B Sometimes the densities of the two solids are different. In such a case, we calculate the weights instead of volumes, and the center of gravity of the body is found out as usual.

Example

A solid body is formed by joining the base of a right circular cone of height "H" to the equal base of a right circular cylinder of height "h".

Calculate the distance of the center of gravity (C.g) of the solid from its plane face, when H = 12 Cm and h = 3 Cm.

Solution



$\bar{X} = 0$ (why?)

$$V_{\triangle} = \frac{\pi}{3} r^2 h = \frac{\pi r^2}{3} * 12 = 4\pi r^2 \ Cm^3$$

$$\bar{y}_{\triangle} = \frac{h}{4} + 3 = 3+3 = 6 \ Cm$$

$$V_{\square} = \pi r^2 h = 3\pi r^2 \ Cm^3$$

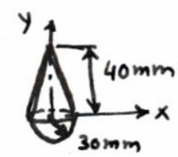
$$\bar{y}_{\square} = \frac{h}{2} = \frac{3}{2} \ Cm$$

location of C.g ($\bar{X} = 0$) &

$$\bar{y} = \frac{\sum y_i V_i}{\sum V_i} = \frac{4\pi r^2 * 6 + 3\pi r^2 * \frac{3}{2}}{4\pi r^2 + 3\pi r^2} = \frac{28.5 \pi r^2}{7 \pi r^2} = 4.07 \ Cm.$$

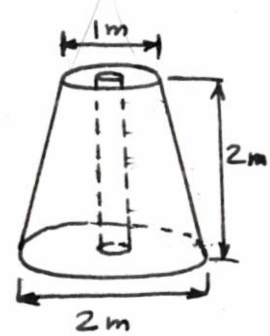
Problems

1. A body consists of a right circular solid cone of height 40 mm and radius of 30 mm placed on a solid hemisphere of radius 30 mm of the same material. find the position of C.g of the body.



2. A frustrum of a solid right circular cone has an axial hole of 50 cm diameter as shown in figure. Determine the C.G of the body.

Ans. [$\bar{y} = 0.76$ m]



11. Moment of Inertia

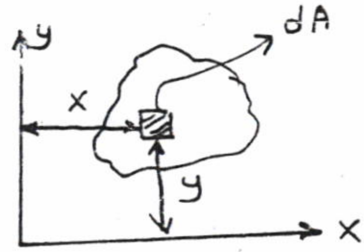
11.1 Definition

Many engineering formula, such as those relating of strength beams, Columns, deflection of beams involve the use of mathematical expression of the form ($\int \rho^2 dA$), where ρ is the ⊥ distance from" to the axis of inertia.

The integral appears so frequently that it has been named "moment of inertia", that has mathematical expression usually denoted by the symbol "I".

The mathematical definition of moment of inertia;

$I = \int \rho^2 dA$ indicates that an area is divided into small pa such as "dA", and each area is multiplied by the square of its mome arm about the reference axis. Thus, as shown in figure below, if the Coordinates of the center of the differential area dA are (x, y), the mo of inertia about x-axis is the summation of product of each area dA by the square of its moment arm y. This gives:-



$$I_x = \int y^2 dA$$

and for y-axis

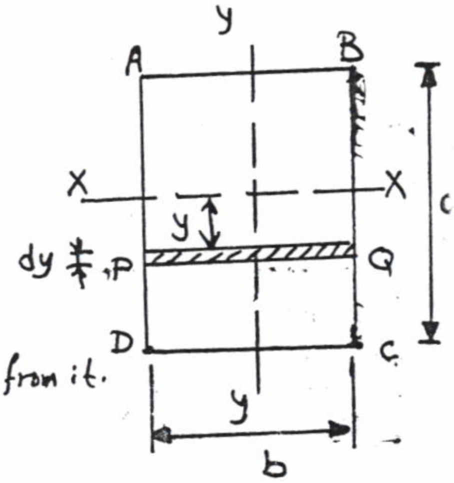
$$I_y = \int x^2 dA$$

The moment of inertia (of area) is sometimes called the second moment of area, because each differential area multiplied by it moment arm gives the moment of area; when multiplied a second tim by its moment arm it gives the moment of inertia. The term second moment of area is preferable to the expression moment of inertia when applied to an area.

The units of "I" of a plane area depends upon the units of the area and the length, as:-

If the area is in m^2 and length in m, then I is expressed in m^4
If the area is in Cm^2 and length in Cm, then I is expressed in Cm^4
If the area is in mm^2 and length in m, then I is expressed in mm^4

11.2 <u>Moment of inertia (by method of integration) of a rectangular section.</u>



Consider a rectangular section ABCD as shown in figure.

Let b = width of the section.

d = depth of the section.

Now consider a strip PQ of thickness dy parallel to X-axis and at a distance "y" from it.

Area of the strip $= b \cdot dy$

I at the strip about x-axis is $= b \cdot dy \cdot y^2$

I of the whole section can be found out by integration for wh length of section i.e from $(-d/2)$ to $(d/2)$ as shown.

$$I_{xx} = \int_{-d/2}^{d/2} b y^2 dy = b \int_{-d/2}^{d/2} y^2 dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2} = \frac{bd^3}{12}$$

Similarly

$$I_{yy} = \frac{db^3}{12}$$

11.3 <u>M.I of hollow rectangular section</u>

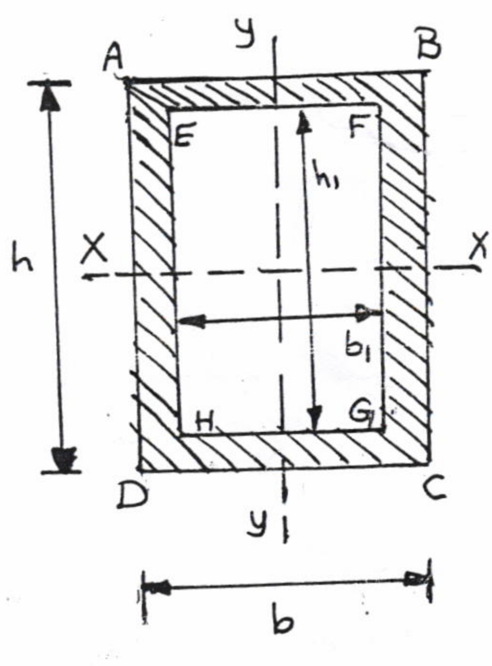
Let

b = width AB of outer rectangle.

h = depth BC of the outer rectangle.

b_1 = width of inner rectangle.

h_1 = depth of inner rectangle.



$$I_{xx\ \text{outer section}} = \frac{bh^3}{12}$$

&

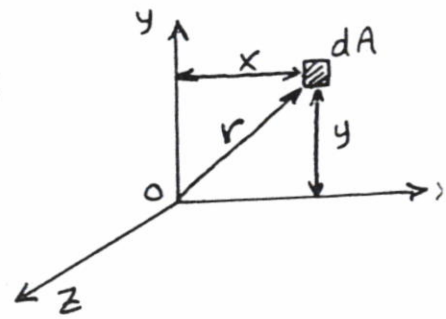
$$I_{xx\ \text{inner section}} = \frac{b_1 h_1^3}{12}$$

$$\therefore I_{xx\ \text{whole section}} = \frac{bh^3}{12} - \frac{b_1 h_1^3}{12}$$

Similarly

$$I_{yy} = \frac{hb^3}{12} - \frac{h_1 b_1^3}{12}$$

11.4 Theorem of perpendicular axis



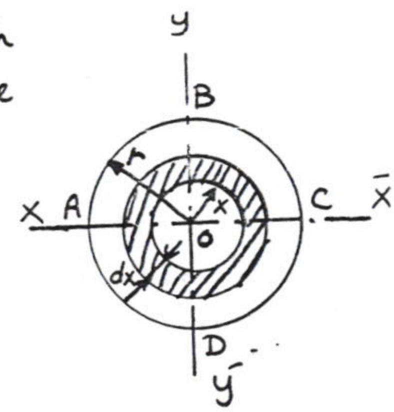
It states " If I_{xx} and I_{yy} be the moments on inertia of a plane section about two perpendicular axes meeting at O, the moment of inertia I_{zz} abut the axis z-z, perpendicular to the plane and passing through the intersection of x-x and y-y is given by the relation below. This moment of inertia I_{zz} is Called "polar moment of inertia", and denoted by the symbol "J".

In the figure, the moment of an area is the x-y plane with respe to z-axis is

$$J_z = \int r^2 dA = \int (x^2+y^2)\,dA = \int x^2\,dA + \int y^2\,dA$$

i.e $\boxed{J_z = I_{xx} + I_{yy}}$

11.5 M.I of a circular section.



Consider a circle ABCD of radius (r) with center O, and $x-\bar{x}$, $y-\bar{y}$ be two axis of reference through O as shown in figure.

Consider an element ring of radius x and thickness dx.

Therefore, area of the ring is

$$dA = 2\pi x\,dx$$

∴ M.I of ring about $x-\bar{x}$ and $y-\bar{y}$ axis

$$I_{xx} \text{ or } I_{yy} = 2\pi x\,dx\,(x^2) = 2\pi x^3 dx$$

M.I of whole section about the center of gravity (central axis) found by integrating along r:

$$J_{zz} = \int_0^r 2\pi x^3 dx = 2\pi \int_0^r x^3 dx = \frac{\pi}{2} r^4 = \frac{\pi d^4}{32}$$

from perpendicular axis theorem, that $J_{zz} = I_{xx} + I_{yy}$

but I_{xx}, I_{yy} are identical, then

$$I_{xx} = I_{yy} \Rightarrow J_{zz} = 2I_{xx} = 2I_{yy}$$

$$\therefore I_{xx} = I_{yy} = \frac{J_{zz}}{2} = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$$

HW Find an expression for J_{zz} of hollow circular section.

11.6 Theorem of Parallel axis

It states "If the moment of inertia of a plane area abo[ut] an axis through its Center of gravity, be denoted by I_G or I', th[e] moment of inertia of the area about axis AB, Parallel to the first, an[d] a distance "h" from the Center of gravity is given by

$$I_{AB} = I_G + Ah^2$$ where

I_{AB} = MI of the area about an axis AB.

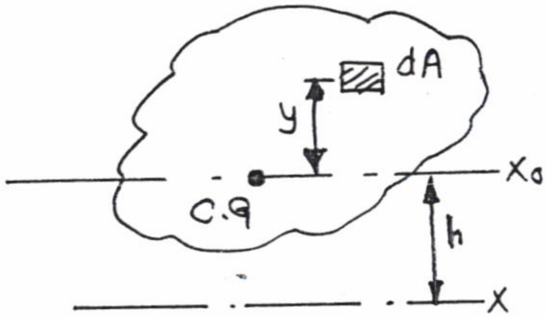
I_G = MI of the area about its C.G.

A = area of the section.

h = distance between C.G of the section and the axis AB.

Proof.

As shown in figure



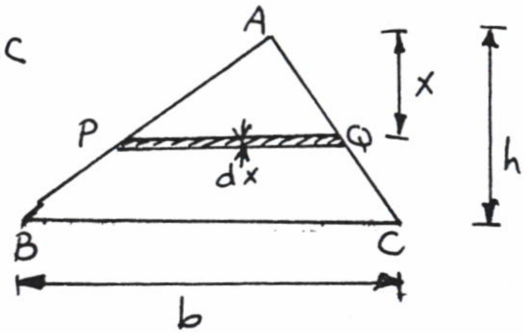
$$I_x = \int (y+h)^2 dA$$
$$= \int (y^2 + 2yh + h^2)\, dA$$
$$= \int y^2 dA + \int 2yh\, dA + \int h^2 dA$$
$$= \int y^2 dA + 2h \int y\, dA + h^2 \int dA$$
$$= I_G + 0 + h^2 A$$
$$\therefore I_x = I_G + h^2 A$$

* $\int y\, dA = 0$ is the algebric sum of moments of all the areas abou[t] an axis through C.G of the section is equal to $A\bar{y}$ where $\bar{y}$ is th[e] distance between the section and the axis passing through C.G which equal zero.

11.7 M.I of triangular section

Consider a triangular section ABC whose M.I is required to be found

let b = base, h = height



Now consider small strip PQ of thickness dx at the distance of x from A as shown in figure.

From a similarity of two triangles ABC & APQ,

$$\frac{\overline{PQ}}{\overline{BC}} = \frac{x}{h}$$

$$\overline{PQ} = \frac{\overline{BC} \cdot x}{h} = \frac{b \cdot x}{h}$$

area of strip PQ $= \frac{bx}{h} dx$

M.I of strip $= \frac{bx}{h} dx (h-x)^2 dx$ (about the base BC)

then

M.I of whole section about base BC is

$$I_{BC} = \int_0^h \frac{bx}{h} (h-x)^2 dx = \frac{b}{h} \int_0^h x (h^2 - 2hx + x^2) dx$$

$$= \frac{b}{h} \int_0^h (xh^2 - 2hx^2 + x^3) dx = \frac{b}{h} \left[\frac{x^2 h^2}{2} - \frac{2hx^3}{3} + \frac{x^4}{4} \right]_0^h$$

$$= \frac{b}{h} \left[\frac{h^4}{2} - \frac{2h^4}{3} + \frac{h^4}{4} \right] = \frac{bh^3}{2} - \frac{2bh^3}{3} + \frac{bh^3}{4} = \frac{6bh^3 - 8bh^3 + 3bh^3}{12}$$

$$\therefore \boxed{I_{BC} = \frac{bh^3}{12}}$$

we know that the distance between C.G of triangle and bas is $d = h/3$

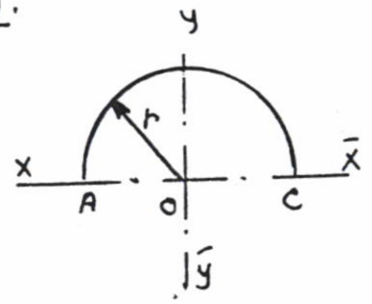
$\therefore$ M.I of section about axis pass through C.G and parallel to x-axis

$$I_G = I_{BC} - A d^2 = \frac{bh^3}{12} - \frac{bh}{2} \left(\frac{h}{3}\right)^2$$

$$\boxed{I_G = \frac{bh^3}{36}}$$

11.8 M.I of Semicircle (Semicircular section).

Let r = radius of semicircle.



M.I of semicircle about the base AC is half of M.I of circular section about AC, i.e

$$I_{AC} = \frac{1}{2}\frac{\pi}{4}r^4 = \frac{\pi}{8}r^4$$

Then

M.I of semicircle about axis passes through centroid of section I_G i:

$I_G = I_{AC} - Ah_g^2$ $\quad A = \frac{\pi r^2}{2}$ & $h = 0.424\,r$

$$I_G = \frac{\pi}{8}r^4 - \frac{\pi r^2}{2}(0.424\,r)^2$$

$I_G = 0.11\,r^4$ about x-axis

& $I_G = \frac{\pi}{8}r^4$ about y-axis (because h=0 i.e the centroid lie

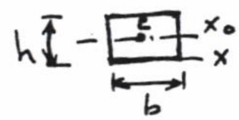
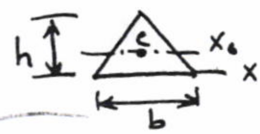
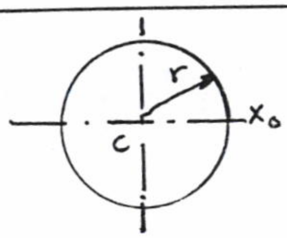
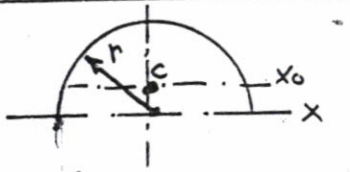
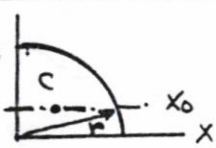
11.9 M.I of a Composite section

1. Split up the given section into plane area (i.e rectangle, circle --- etc
2. Find out the moment of inertia of these areas about their respective centroids.
3. Transfer these moments of inertia about the required axis (AB) by the theorem of parallel axis, i.e

$$I_{AB} = I_G + Ah^2$$

4. The moment of inertia of the given section may now be obtained by algebric sum of the moment of inertia about the required axis.

The table below shows some of Common figures moment of inertia. ([illegible].)

Figure	$I_{x_o} = I$	I_x
	$\frac{bh^3}{12}$	$\frac{bh^3}{3}$
	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
	$\frac{\pi r^4}{4}$	$\frac{\pi r^4}{4}$ $J = \frac{\pi r^4}{2} = 2I$
	$0.11\, r^4$	$\frac{\pi r^4}{8}$
	$\bar{I}_x = \bar{I}_y = 0.055\, r^4$	$I_x = I_y = \frac{\pi r^4}{16}$

12- Friction

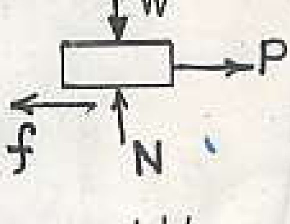
When a body slides or tends to slide on other body, the force tangent to the contact surface which resists the motion, or the tendency toward motion, of one body relative to the other is defined as friction.

It has advantages and disadvantages.

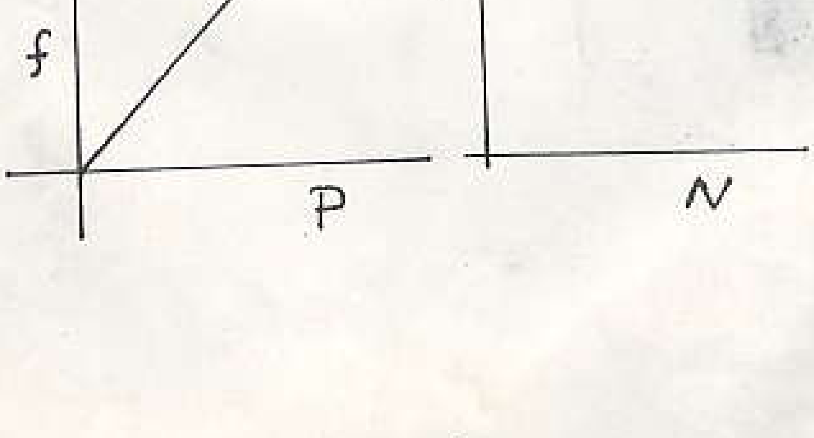
If the contact surface (area) between surfaces is assumed smooth then there is no friction. There are 2 types of friction.

1) static friction. When there is no relative motion between surfaces.
2) Kinetic friction. when there is relative motion between the bodies.

The static friction force is always the minimum force required to maintain equilibrium or prevent relative motion between the bodies, and the kinetic friction varies some what with velocity. see figure.



W: Weight
P: force applied
N: Normal force
f: friction force



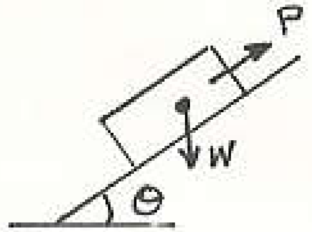
Example

What is the range of P to impend motion.

$W = 100N$

$\theta = 30^\circ$

$\mu = 0.2$

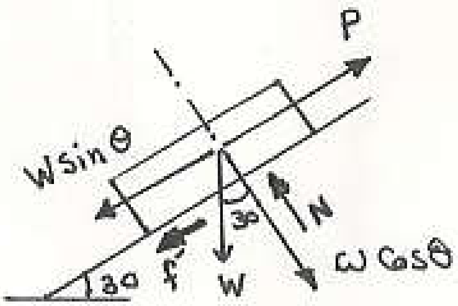


N.B

* the friction force is opposite to the direction of motion

Solution

to impends motion upward.



$P = W\sin\theta + f$ —— ①

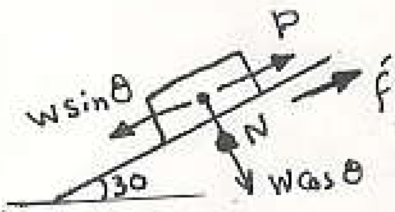
$N = W\cos\theta$ —— ②

$\mu = \dfrac{f}{N} \Rightarrow f = \mu N$

$\therefore P = 100 \sin 30 + 0.2\ (100) \cos 30$

$= 50 + 17.32 = 67.32\ N$

to impends motion downward



$P = W\sin\theta - f$

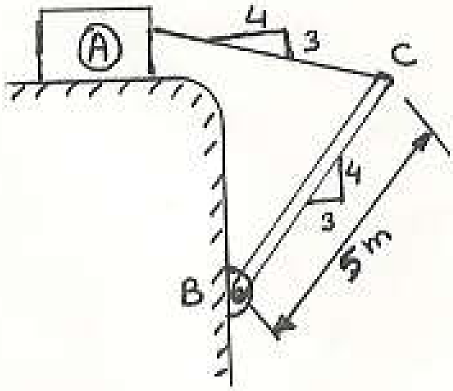
$= W\sin\theta - \mu N$

$= W\sin\theta - \mu W\cos\theta$

$= 50 - 0.2(100)\cos 30 = 32.68\ N$

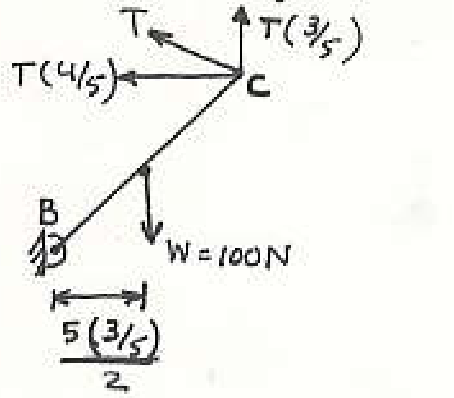
Example

The coefficient of friction μ between the 50 N body A and the plane is 0.5. The bar BC weight is 100 N. Determine the forces acting on body A.



Solution

First, draw the free body diagram (BC)



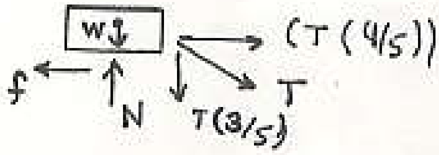
$\circlearrowleft + \ \Sigma M_B = 0$

$$100(1.5) - T(4/5)(4) - T(3/5)(3) = 0$$

$$T = 30\,N.$$

F.B.D for (A)

\# (one case of motion is to the right) why?



$\Sigma F_y = 0$

$$W + 30\,(3/5) = N$$

$$N = 50 + 18 = 68\,N$$

$$f = T(4/5) = 24\,N$$

and for test if body impend motion.

$$\acute{f} = \mu N = 0.5(50+18) = 34\,N$$

This is the value of friction force to impend motion

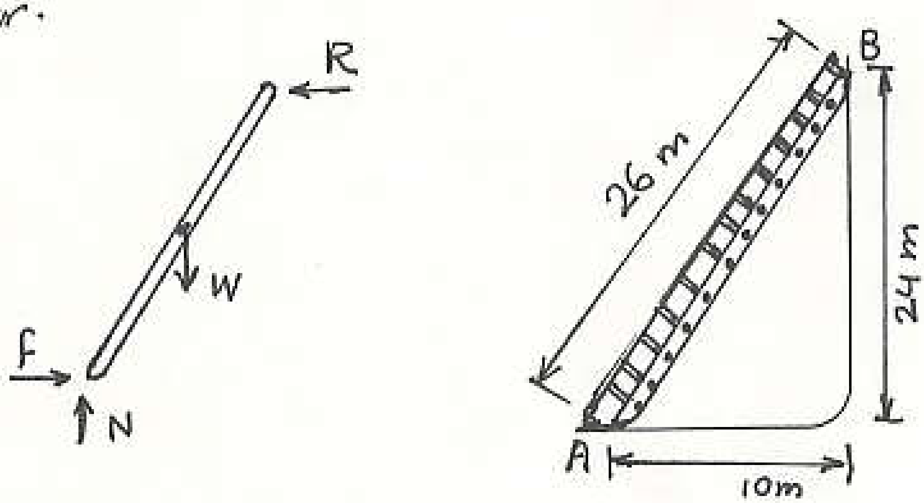
Then $T(4/5) = 34\,N \Rightarrow T = 42.5\,N$ (required T to impend motion).

__HW__ find the mass of BC to impend motion

(ans: 14.45 kg)

Example

A 26m ladder weighs 50 N is placed against a smooth vertical wall, its lower end is at 10 m from the wall. (μ) Coefficient of friction between the ladder and the floor is 0.3. Determine the frictional force at A acting on the ladder.



$\Sigma F_y = 0$

$N - W = 0 \Rightarrow N - 50 = 0 \Rightarrow N = 50\,N \uparrow$

$\Sigma M_B = 0$

$f(24) + 50(5) - N(10) = 0$

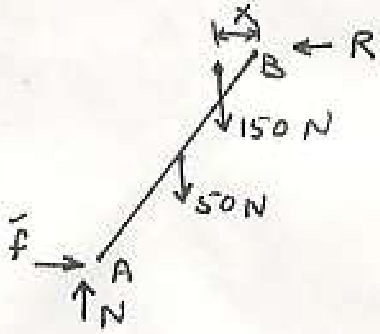
$f = 10.42\,N \rightarrow$

to impend motion

$f \equiv \bar{f} = \mu N = 0.3(50) = 15\,N$

then the ladder will not slip.

If in the above example there is a boy of weight 150 N is climb the ladder. Determine the distance from the boy to the wall when the ladder start to slip.



$\Sigma F_y = 0 \Rightarrow N - 50 - 150 = 0$

$N = 200\,N \uparrow$

$\bar{F} = \mu N = 0.3(200) = 60\,N$

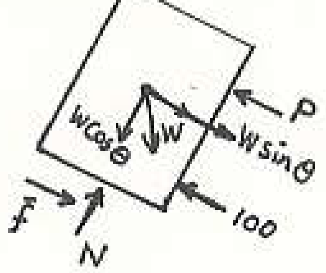
$\Sigma M_B = 0$

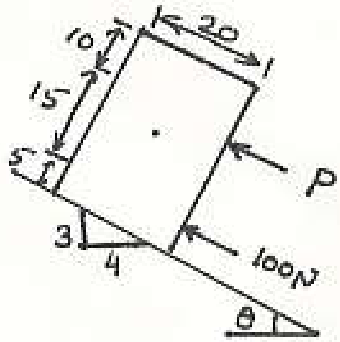
$150(X) + 50(5) - 10(200) + 24(60) = 0$

$X = 2.02\,m$

Example

The homogeneous block weighs 2500 N. The Coefficient of friction between the plane and the block is 0.3. Determine the range of values of P for which the block is in equilibrium.





All Dimensions in «Cm»

$\cos\theta = 4/5$

$\sin\theta = 3/5$

to impend motion upward.

$\Sigma F_y = 0$

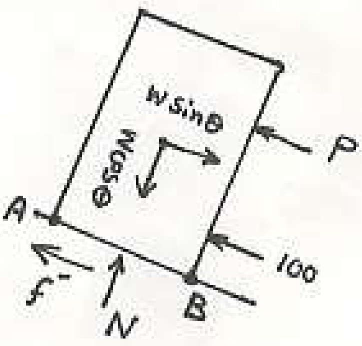
$N = W\cos\theta = 2500(4/5) = 2000\ N$

$f = \mu N = 0.3(2000) = 600\ N$

$\Sigma F_x = 0$

$P = 600 + 2500(3/5) - 100 = 2000\ N.$

Now, If P is small so that the body will impends motion downward then:



$\Sigma F_x = 0$

$f + P + 100 = W\sin\theta$

$P = 2500(3/5) - 100 - \mu N$

$= 1500 - 100 - 0.3(2500)(4/5)$

$= 1500 - 100 - 600 = 800\ N$

$\Sigma M_A = 0$ this is to test if the block will turn over.

$$-P(0.2) - 100(0.05) + 2500(4/5)(0.1) + 2500(3/5)(0.15) = 0$$

$$P = \frac{-5 + 200 + 225}{0.2} = 2100\ N.$$

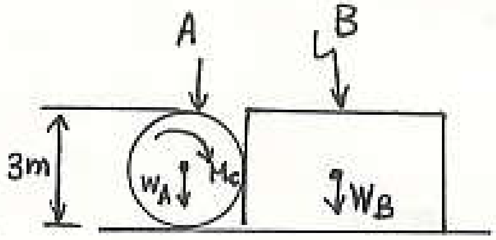
$\because 2100 > 2000$ the the body will move upward before it turns over.

Example

Body A is a homogeneous cylinder weighing 500 N, and body B weighs 900 N. The coefficient of friction for all contact surfaces of body A is 0.4, and between body B and the plane it is 0.1

Determine couple M_C that will cause body B to have impending motion.

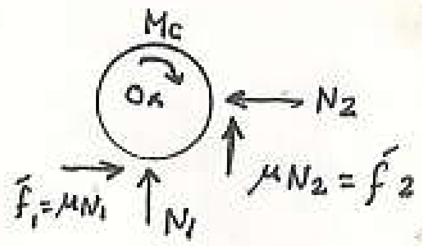
Solution



* If the cylinder will sliding before it moves rightward

$\circlearrowright \Sigma M_O = 0$

$$-\mu N_1 (1.5) - \mu N_2 (1.5) + M_C = 0 \quad \text{— ①}$$



$\Sigma F_y = 0$

$$-W_A + N_1 + \mu N_2 = 0 \quad \text{— ②}$$

$\Sigma F_x = 0$

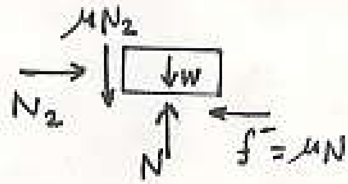
$$\mu N_1 = N_2 \quad \text{— ③}$$

$\therefore N_1 = 431.03 N$; $N_2 = 172.413 N$: $M_C = 362$ N.m .

* If Block B move first.

$$N_2 = \bar{f} = \mu N \quad \text{— ④}$$

$$\mu N_2 + W - N = 0 \quad \text{— ⑤}$$



$N_2 = 187.5 N$; $N = 937.5 N$

Back to cylinder (eqn. ①)

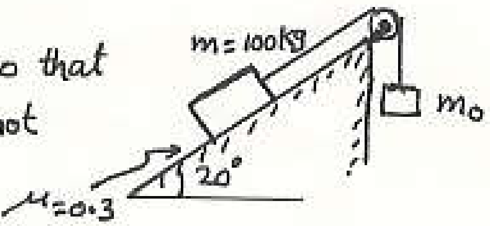
$$M_C - N_2 (1.5) - 0.6 (N_2) = 0$$

$$M_C = 2.1 N_2 = 393.75 \text{ N.m}$$

$\because M_{C\,slip} < M_{C\,move} \Rightarrow$ the body will slip before impends motion of block B.

Problems

1. What is the range of mass m_o so that the 100 kg mass shown in figure cannot move upward or downward.



Ans: (m_o 6 kg → 62.4 kg).

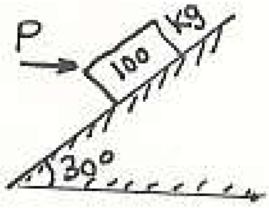
2- Find the friction on the body if

a) P = 500 N

b) P = 100 N

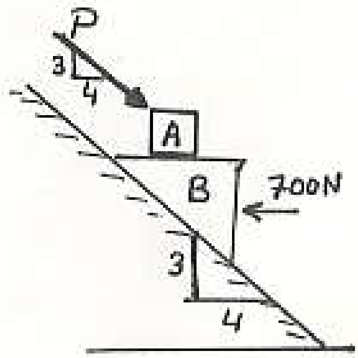
take μ = 0.17

ans: (a) 134 N ↙ (b) 163 N ↗

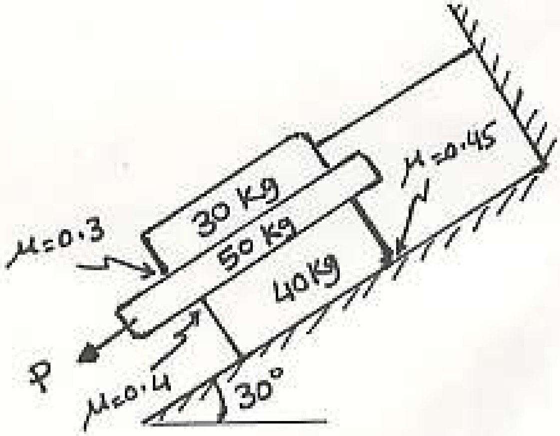


3. Body A weighs 700 N, Body B weighs 900 N. The coefficient of friction for all surfaces of contact is 0.3. Determine the force P that will cause motion of A to impend.

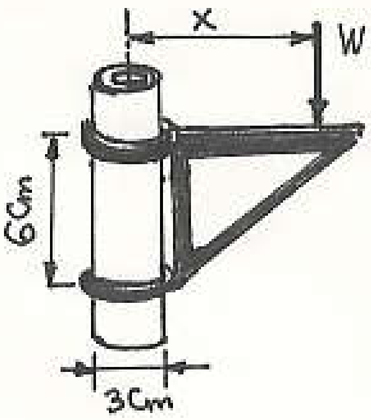
Ans: P = 110 N ↘



4. Three blocks are arranged as shown in figure below. The upper one is forbeden from movement by a rope. Determine the maximum value of P to impend motion. (Ans: P = 93.8 N)

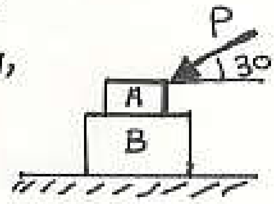


5. The movable bracket shown may be placed at any height on the 3 Cm diameter pipe. If the Coefficient of static friction between the pipe and bracket is 0.25, determine the minimum distance X at which the load W can be supported. Neglect the weight of bracket.

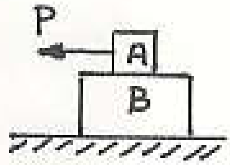


((Ans: 12 Cm))

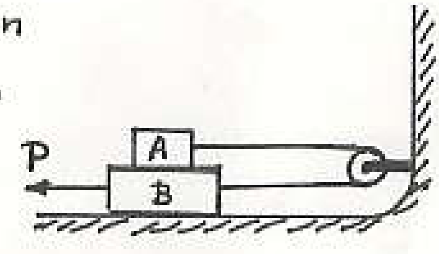
6. Find P so that block B will move first assume all surface friction $\mu = 0.3$, $W_A = 200$ N, $W_B = 100$ N. (Ans: P= 83.8N)



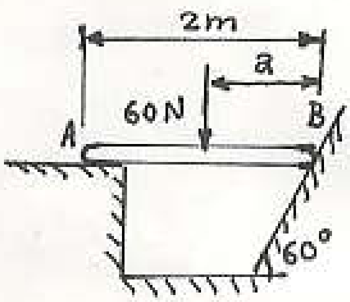
7. Find P so that block A will move first, $\mu_s = 0.3$, $W_A = 200$ N & $W_B = 100$ N. What is the magnitude of P required to move block B firstly? Ans: (P = 60N, 90N).



8. Find force P required to impend motion of block B. $W_A = 200$N, $W_B = 100$N, $\mu = 0.2$, neglect friction in pully. (Ans: 140N)



9. The rod AB rests on a horizontal surface at A and against slipping surface at B if $\mu_s = 0.25$ at A & B. Find the minimum distance a for equilibrium, neglect the weight of the rod. (Ans a = 1.61 m)



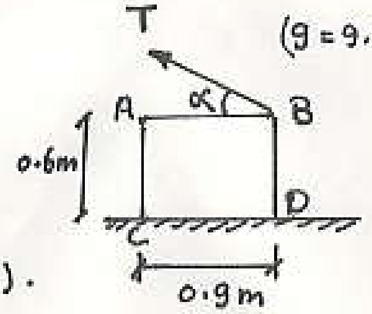
10. A packing crate of mass 30kg. The coefficient of friction between the crate and the floor is 0.35 if $\alpha = 30°$ determine.

a) The tension T required to move the crate.

b) Whether the crate will slide or tip

ans: (98.9 N, slides since T= 140N to tip).

(g = 9.8 m/sec)



2. Dynamics

It is a part of mechanics dealing with the analysis of bodies in motion.

It is divided into two parts.

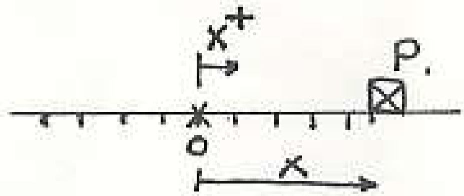
1. Kinematics, which is the study of motion without reference to the Cause of the motion.
2. Kinetics, which is the study of the relation existing between the force acting on a body and the motion of the body.

Kinematics

2.1 Rectilinear motion of particles:

If a particle moving a long a straight line is said to be in rectilinear motion.

then,



Average velocity $= \bar{v}$

$$= \frac{\Delta x}{\Delta t} \quad \left(\frac{m}{sec}\right)$$

Instantaneous velocity $= v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

Also,

Average acceleration $= \bar{a} = \frac{\Delta v}{\Delta t} \quad (m/sec^2)$

Instantaneous acceleration $= a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Example

Consider a particle moving in a straight line and assume that its position is defined by:

$$x = 6t^2 - t^3$$

where, t in sec, x in meters.

Determine ① the time for max. velocity.

② max. velocity.

③ displacement at max. velocity.

④ Distance traveled by the particle from $t = 2$ sec to $t = 6$ sec.

Solution

① $x = 6t^2 - t^3$ ------ ①

$v = \frac{dx}{dt} = 12t - 3t^2$ ------ ②

If $v = f(t)$ and we went to find time for max. velocity then

$\frac{dv}{dt} = 0 = 12 - 6t \Rightarrow t = \frac{12}{6} = \underline{\underline{2\ \text{sec}}}$ (this is the time for max velocity).

② put $t = 2$ sec in eqn ② above to get max. velocity.

$v_{max} = 12(2) - 3(2^2) = 24 - 12 = \underline{\underline{12\ \text{m/sec}}}$

③ $x = 6(2^2) - (2^3) = 6 \times 4 - 8 = 24 - 8 = 16$ m.

④ test motion (ذهاب وعودة)

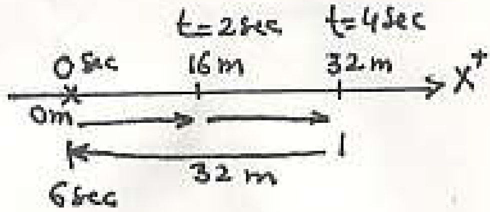
$\frac{dx}{dt} = 0 = 12t - 3t^2 \Rightarrow t = 4$ sec for max displacement.

$X_{at\ 2sec} = 6(2^2) - (2^3) = 24 - 8 = 16$ m.

$X_{at\ 4sec} = 6(4^2) - (4^3) = 96 - 64 = 32$ m.

$X_{at\ 6sec} = 6(6^2) - (6^3) = 0$ m.

Distance $= |16| + |32| = \underline{\underline{48\ \text{m}}}$



Example

The position of a particle which moves along a straight line is defined by the relation

$$x = t^3 - 6t^2 - 15t + 40$$

where : t in sec, x in meters.

Determine ① The time at which the velocity will be zero.
② the position and distance traveled by the particle at that time.
③ The acceleration at that time.
④ distance traveled by the particle from $t=4$ sec to $t=6$ sec.

Solution

① $\frac{dx}{dt} = 0 = 3t^2 - 12t - 15 \Rightarrow t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$\underline{\underline{t = 5 \text{sec}}}$ $\quad t = -1$ sec (ignore).

② $x = t^3 - 6t^2 - 15t + 40 = (5)^3 - 6(5)^2 - 15(5) + 40$

$= 125 - 150 - 75 + 40$

$= -60$ m.

motion test. $\frac{dx}{dt} = 0$ & from ① $t = 5$ sec, so

distance traveled $= X_{t=5} - X_{t=0} = -60 - 40 = -100$ m.

where, $X_{t=5} = -60$ m & $X_{t=0} = 40$ m.

③ $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \Rightarrow \frac{dx}{dt} = 3t^2 - 12t - 15$

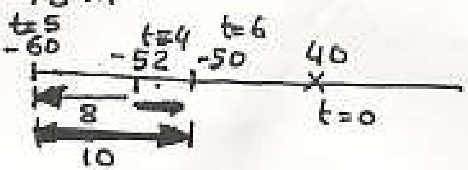
$a = \frac{d^2x}{dt^2} = 6t - 12$ $\quad$ then $a_{t=5\text{sec}} = 6(5) - 12 = 18$ m/sec^2

④ Distance traveled $= (X_{t=5} - X_{t=4}) + (X_{t=6} - X_{t=5})$

$X_{t=4} = -52$ m, $X_{t=5} = -60$ m, $X_{t=6} = -50$ m.

$\therefore$ Distance traveled $= |-60 + 52| + |-50 + 60|$

$= 8 + 10 = 18$ m.



H.W

① The motion of a particle is defined by the relation.

$$x = \frac{1}{3}t^3 - 3t^2 + 8t + 2$$

where, x in meters, t in sec.

Determine

① when the velocity is zero. (2 sec, 4 sec)

② the position and the total distance traveled when the acceleration is zero (8m, 7.33m)

② The motion of a particle is defined by the relation.

$$x = t^2 - 10t + 30$$

where, x in meters, t in sec.

Determine

① when the velocity is zero. (5 sec)

② the position and total distance traveled when $t = 8$ sec.

($x_{t=8} = 14$ m, Distance = 39 m).

2.2 Determination of the motion of a particle

From ① we see that the motion as a function between position and time. In practice It is defined as a relation between acceleration and time, position or velocity. These relations are usually obtained from experiment.

$\boxed{A}$ if $a = f(t)$

$$a = \frac{dv}{dt} \Rightarrow dv = a\,dt = f(t)\,dt \quad \text{and by } \int$$

$$\int_{v_0}^{v} dv = \int_0^t f(t)\,dt \quad \text{———} \boxed{A} \quad \text{see } Ex_A.$$

$$\therefore v - v_0 = \int_0^t f(t)\,dt$$

note that this relation applied for $a = 0$, $a = \text{Constant}$.

$\boxed{B}$ $a = f(x)$

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx}$$

$$v\,dv = a\,dx = f(x)\,dx$$

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx \quad \text{———} \boxed{B} \quad \text{see } Ex_B.$$

$$\frac{1}{2}(v^2 - v_0^2) = \int_{x_0}^{x} f(x)\,dx$$

$\boxed{C}$ $a = f(v)$

$$a = f(v) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{f(v)}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)} \quad \text{———} \boxed{C}$$

or $f(v) = v\dfrac{dv}{dx} \Rightarrow dx = \dfrac{v\,dv}{f(v)}$

$$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{f(v)} \quad \text{———} \boxed{C} \quad \text{see } Ex_C.$$

EX A The acceleration of a particle is defined by the relation

$$a = 18 - 6t^2$$

where, a - acceleration in m/s^2, t - time in sec.

assume at $t=0$, $v=0$, $X = 100$ m Determine

ⓐ the time when the velocity is again zero.

ⓑ the position and velocity when $t = 4$ sec.

ⓒ total distance traveled by particle from $t=0$ to $t=4$ sec.

Solution

$$a = \frac{dv}{dt} = 18 - 6t^2 \Rightarrow dv = 18dt - 6t^2 dt$$

$$\int_{v_0}^{v} dv = \int_0^t 18\,dt - \int_0^t 6t^2 dt \Rightarrow v - v_0 = 18t - 2t^3$$

but at $t=0$, $v=0 \Rightarrow v_0 = 0$

$$\therefore v = 18t - 2t^3 \quad \text{——} ①$$

ⓐ for $v=0$ sub in ①

$$0 = 18t - 2t^3 \Rightarrow t^2 - 9 = 0 \Rightarrow \underline{t = 3\,sec} \text{ (Ignore -ve value).}$$

ⓑ $v = 18t - 2t^3$ to find v at $t = 4$ sec.

$$v = 18(4) - 2(4)^3 = \underline{-56\ m/sec}$$

$$v = \frac{dx}{dt} = 18t - 2t^3 \Rightarrow dx = 18t\,dt - 2t^3 dt$$

$$\int_{x_0}^{x} dx = \int_0^t 18t\,dt - \int_0^t 2t^3 dt$$

$$X - X_0 = 9t^2 - \frac{t^4}{2} \Rightarrow X - 100 = 9t^2 - \frac{t^4}{2} \text{ —}$$

$$X = 100 + 9t^2 - \frac{t^4}{2} \quad \text{——} ②$$

$$X_{4sec} = 100 + 9(4)^2 - \frac{(4)^4}{2} = 100 + 144 - 128 = 116\ m$$

ⓒ Since $\frac{dx}{dt} = 0$ in ② lead to $t = 3$ sec see ⓐ

then total distance traveled =

$$|X_{t=4} - X_{t=3}| + |X_{t=3} - X_{t=0}| = |116 - 140.5| + |140.5 - 100|$$
$$= 65\ m.$$

Ex B The acceleration of a particle is defined by the relation

$$a = 21 - 12X^2$$

where a - acceleration m/s^2, X - distance in meters.

The particle starts with no initial velocity at $X=0$

Determine (a) the velocity when $X = 1.5$ m.

(b) the position where the velocity is again zero.

(c) the position where the velocity is maximum.

Solution

$$a = \frac{dv}{dt} = \frac{dv}{dX} \cdot \frac{dX}{dt} = v\frac{dv}{dX}$$

$$\int_{v_0}^{v} v\,dv = \int_{X_0}^{X} f(x)\,dX = \int_{X_0}^{X} (21 - 12X^2)\,dX$$

$$\frac{1}{2}(v^2 - v_0^2) = \int_{X_0}^{X} (21 - 12X^2)\,dX$$

but $v_0 = 0$ at $X = 0$ then.

$$v^2 = 42X - 8X^3 \quad \text{—— (1)}$$

(a) $v^2 = 42(1.5) - 8(1.5)^3 = 63 - 27 = 36$

$\therefore v = \pm 6$ m/sec.

(b) $v = 0$ sub in (1)

$0 = 42X - 8X^3 \Rightarrow 21 - 4X^2 = 0$, $X^2 = \frac{21}{4}$, $X = \pm\sqrt{\frac{21}{4}}$

$\therefore X = \pm 1.73$ m, the value is $X = 1.73$ m since -ve value is impossible. (see eqn (1)) (!)

(c) derive eq (1) with respect to x

$2v\,dv = 42\,dX - 24X^2\,dX$

$2v\frac{dv}{dX} = 42 - 24X^2 = 0$

$\therefore 42 - 24X^2 = 0 \Rightarrow X^2 = \frac{42}{24} \Rightarrow X = \pm\sqrt{\frac{42}{24}}$

$X = \pm 1.205$ m But the -ve value is impossible see eq (1). then

$X = 1.205$ m

2-3 Uniform Rectilinear motion

It means that acceleration is zero (i.e $a=0$) which means $v =$ Constant.

$$v = \frac{dx}{dt} = \text{const.}$$

$$dx = v\,dt \Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt$$

$$x - x_0 = vt \Rightarrow x = x_0 + vt \quad \text{——— (1)}$$

2-4 Uniform accelerated rectilinear motion

$$a = \text{Constant} = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = \int_0^t a\,dt$$

$$v - v_0 = at$$

$$v = v_0 + at \quad \text{——— (2)}$$

But $v = \frac{dx}{dt}$

$$\frac{dx}{dt} = v_0 + at \Rightarrow \int_{x_0}^{x} dx = \int_0^t v_0\,dt + \int_0^t at\,dt$$

$$\therefore x - x_0 = v_0 t + \frac{at^2}{2}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \text{——— (3)}$$

Also $a = v\frac{dv}{dx}$

$$v\,dv = a\,dx$$

$$\int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx$$

$$\frac{1}{2}(v^2 - v_0^2) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad \text{——— (4)}$$

It is to be note that eqs 2,3 & 4 are used when the acceleration of a particle is known to be constant and when the velocity is constant we use eq (1)

Ex: C

The acceleration of a particle falling through atmosphere is defined by the relation $a = g(1 - K^2 v^2)$.

Knowing that the particle starts at $t = 0$ & $x = 0$ with no initial velocity.

ⓐ write an equation for the velocity $\rightarrow f(t)$

ⓑ write an equation for the velocity $\rightarrow f(x)$

Solution

ⓐ $a = f(v)$

$$f(v) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{f(v)}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)} = \int_0^V \frac{dv}{g(1-K^2V^2)}$$

$$\therefore gt = \int_0^V \frac{dv}{1-K^2V^2}$$

(HW: Complete the integration)
Hint: Let $K^2V^2 = \cos^2\theta$ or $\sin^2\theta$)

ⓑ $a = f(v) = v\frac{dv}{dx}$

$$dx = \frac{v\,dv}{f(v)} = \frac{v\,dv}{g(1-K^2V^2)}$$

$$\int_{x_0}^{x} g\,dx = \int_{v_0}^{v} \frac{v\,dv}{1-K^2V^2}$$

$$gx = \int_0^v \frac{v\,dv}{1-K^2V^2}$$

(HW: Complete the integration)

HW① The acceleration of a particle is defined by the relation

$a = -Kx^{-2}$ where a in m/s^2, x in meters.

the particle starts with no initial velocity at $x = 12$ m, and its velocity is 8 m/sec when $x = 6$ m.

Determine ⓐ the value of K. [384 m^3/s^2]

ⓑ the velocity of the particle when $x = 3$m [13.86 m]

Example

The acceleration of a particle is defined by the relation:

$a = -0.0125\, v^2$

where a - acceleration m/s^2, v - velocity m/sec.

If the particle is given an initial velocity v_o at $x=0$ find the distance traveled

a) before the velocity drops to half the initial one.

b) before it comes to rest (i.e $v_f = 0$)

Solution

$$a = f(v) = v\frac{dv}{dx}$$

$$\int_{x_o}^{x} dx = \int_{v_o}^{v} \frac{v\,dv}{f(v)} \Rightarrow x - x_o = -\int_{v_o}^{v} \frac{v\,dv}{0.0125\, v^2}$$

$$\therefore x = -\frac{1}{0.0125}\left[\ln v - \ln v_o\right]$$

(a)

If $v = \frac{1}{2} v_o$ then

$$x = -\frac{1}{0.0125}\left[\ln\left(\frac{\frac{1}{2}v_o}{v_o}\right)\right] = 55.45\,m$$

(b)

If $v = 0$ then $\ln 0 = \infty$

$$x = +\infty$$

HW(2): The acceleration of a particle is defined by the relation.

$a = -10v$

Knowing that at $t=0$ the velocity is $30\ m/sec$ find

a) the distance that the particle will travel before coming to rest.

b) time required for the particle to come to rest.

c) the time required for the velocity to be reduced 1% of its initial value.

These two situation (3, 4) is define the movement of a projectile, it may be an electron or a solid rockets. Two component of the velocity one with x-axis where $a_x = 0$ (eqn 1) and with respect to y-axis where a_y = Const = $-g$ (eq 2, 3 & 4).

Projectile

In case of the motion of a projectile it may be shown.

$a_x = 0$, $a_y = -g$ where $g = 9.81 \simeq 10\ m/sec^2$

neglect the resistance of the air to the body of projectile then in x- direction

$$x = x_0 + v_{0x} t \quad — ⑤$$ i.e constant acceleration

in y - direction.

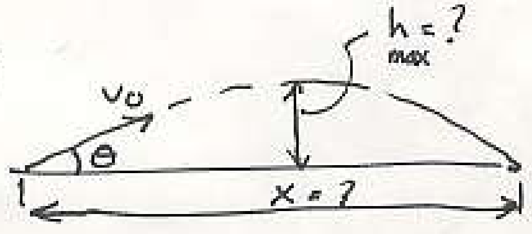
$$v = v_{oy} - gt \quad — ⑥$$ since $a_y = -g$.

$$y = y_0 + v_{oy}\, t - \frac{1}{2} g t^2 \quad — ⑦$$

$$v^2 = v_{oy}^2 - 2g(y - y_0) \quad — ⑧$$

HW A projectile is fired with an initial velocity of v_0 (m/sec) upward at an angle (θ) with the horizontal. Find the horizontal distance covered before the projectile returns to its original Level. Also determine the maximum height attained by the projectile.

[Hint]



Ans: $\left[h = \dfrac{v_0^2 \sin^2\theta}{2g} \;;\; x = \dfrac{v_0^2 \sin 2\theta}{g} \right]$

Example

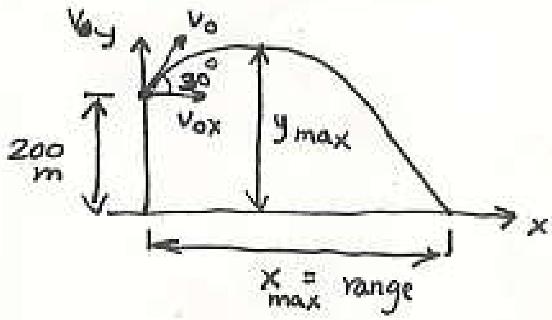
A projectile is fired from the edge of 200 m Cliff with an initial velocity of 180 m/sec at an angle of 30° with horizontal neglect air resistance find.

(a) the horizontal distance from the gun to the point where the projectile hits the ground.

(b) the greatest elevation above the ground reached by the projectile.

(c) velocity at which it hits the ground (take $g = 10\ m/sec^2$)

Solution



$V_{ox} = V_0 \cos 30 = 155.88\ m/s.$

$V_{oy} = V_0 \sin 30 = 90\ m/s.$

$a_x = 0, \quad a_y = -10\ m/s^2$

$y = y_0 + v_{oy} t - \frac{1}{2} g t^2$

to hit the ground

$y = 0 = 200 + 90t - 5t^2 \quad —①$

$t^2 - 18t - 40 = 0$

$(t-20)(t+2) = 0$

$\therefore t = 20 sec$ (time to hit the ground)

Range = X_{max}

$X_{max} = \cancel{X_0}^{0} + v_{ox} t = 155.8(20) = 3117.6\ m.$

max elevation at $\frac{dy}{dt} = 0$ (from eq ①)

$\frac{dy}{dt} = 0 = 90 - 10t \Rightarrow t = \frac{90}{10} = 9\ sec.$

$y_{max} = 200 + 90(9) - 5(9)^2 = 605\ m.$

$V_y = V_{oy} - gt = 90 - 10(20) = -110\ m/s$

$V_x = 155.88\ m/s.$

$V = \sqrt{V_x^2 + V_y^2} = 190\ m/sec$

$\theta = \tan^{-1} \frac{V_y}{V_x} = 35^\circ.$

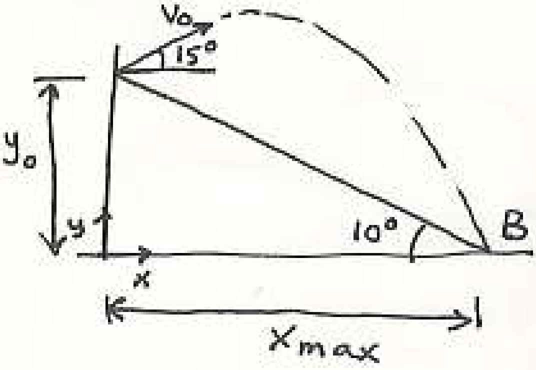
Example

Standing on the side of the hill, a person shoots an arrow with an initial velocity of 76 m/sec at an angle of 15° with the horizontal. Find.

ⓐ The horizontal distance x_{max} traveled by the arrow before it strikes the ground at B.

ⓑ max elevation with respect to B reached by the arrow.

ⓒ velocity at which the arrow hits the ground.



Solution

$\tan 10 = \dfrac{y_o}{x_{max}}$

$y_o = x_{max} \tan 10$

$x_{max} = v_{ox} t + \cancel{x_o}^{\,0} = v_{ox} t$

$y = y_o + v_{oy} t - \frac{1}{2} g t^2$ —— ①

$v_{ox} = 76 \cos 15 = 73.41$ m/s

$v_{oy} = 76 \sin 15 = 19.67$ m/s

when the arrow hits ground. $(y = 0)$

$\therefore\ 0 = y_o + v_{oy} t - 5t^2$

$x_{max} \tan 10 + V_{oy} t - 5t^2 = 0$

$v_{ox} t \tan 10 + v_{oy} t - 5t^2 = 0$

$73.41\, t \tan 10 + 19.67 t - 5t^2 = 0$

$32.614\, t - 5t^2 = 0$

$t = 6.52$ sec.

$\therefore\ x_{max} = 73.41\ (6.5) = 478.8$ m.

Differentiate eq 1 with t & ≡ to zero.

$\dfrac{dy}{dt} = 0 = 0 + v_{oy} - gt$

$t = \dfrac{v_{oy}}{g} = \dfrac{19.67}{10} = 1.967$ sec.

$y_{max} = (x_{max} \tan 10) + v_{oy} t - 5t^2$

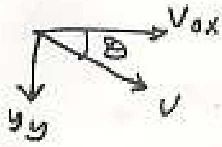
$= 84.4326 + 19.67 * 1.967 - 5\,(1.967)^2 = 103.77$ m.

$V = \sqrt{V_{ox}^2 + V_y^2}$

$V_y = V_{oy} - gt$

$= 19.67 - 10\,(6.523)$

$= -45.56$ m/sec



$V = \sqrt{(45.56)^2 + (73.41)^2}$

$= 86.398$ m/sec

$\theta = \tan^{-1} \dfrac{V_y}{V_x}$

$= 31.8°$

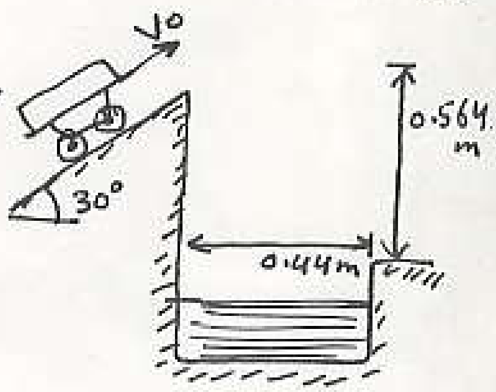
Problems

1. A stone is thrown from a hill at an angle of 60° to the horizontal with an initial velocity of 30 m/sec. After hitting level ground at the base of the hill the stone has covered a horizontal distance of 150 m. How high is the hill? (-230.69 m)

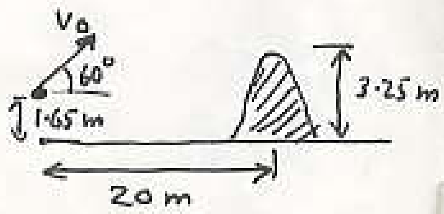
2. A shell leaves a mortar with a muzzle velocity of 150 m/s directed upward at 60° with the horizontal. Determine the position of the shell and its resultant velocity 20 sec after firing. How high will it rise? [1500 m, 100 m/s, 860 m].

3. A projectile is fired with an initial velocity of 63 m/sec upward at an angle of 30° to the horizontal from a point 85 m above a level plain. What horizontal distance will it cover before it strikes the level plain? [462 m]
   Repeat the problem if the projectile is fired downward at 30° to the horizontal level [111.65 m].

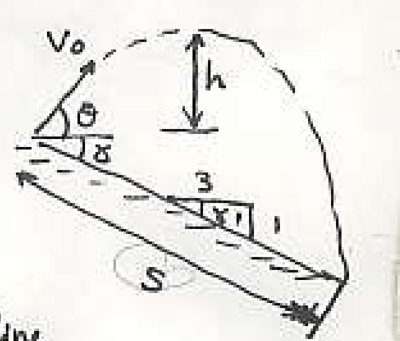
4. The car shown in fig is just to clear the wate-filled gap.
   Find the take-off velocity v_0.
   [1.244 m/sec].



5. A ball is thrown so that it just clears a 3.25 m fence 20 m away. If it left the hand 1.65 m above the ground & at angle of 60° to the horizontal, what was the initial velocity of the ball? [15.41 m/sec]

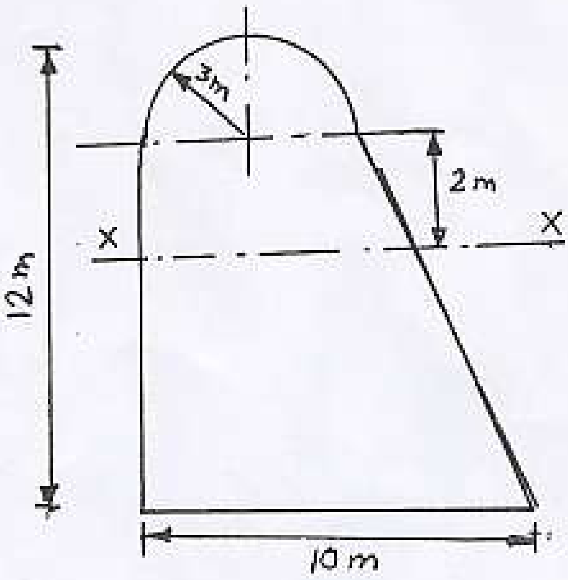


6. Determine the distance S at which a ball thrown with a velocity v_0 of 32.8 m/sec. at an angle $\theta = \tan^{-1}\frac{3}{4}$ will strike the incline shown in figure. [160 m]
   For the same figure, a ball thrown down the incline strikes it at a distance $S = 83.5$ m. If the ball rises to a maximum height $h = 21.31$ m above the point of release, Compute its initial velocity v_0 and inclination θ. [25.45 m/sec, 53.1°]



Example

Determine the moment of inertia for the Composite area shown in figure w.r.t the x-axis.



Solution

$I_{xx} = I_{\Delta} + I_{\square} + I_{\bigcirc}$

$I_{\Delta} = I'_{\Delta} + (Ah^2)_{\Delta}$

$I_{\square} = I'_{\square} + (Ah^2)_{\square}$

$I_{\bigcirc} = I'_{\bigcirc} + (Ah^2)_{\bigcirc}$

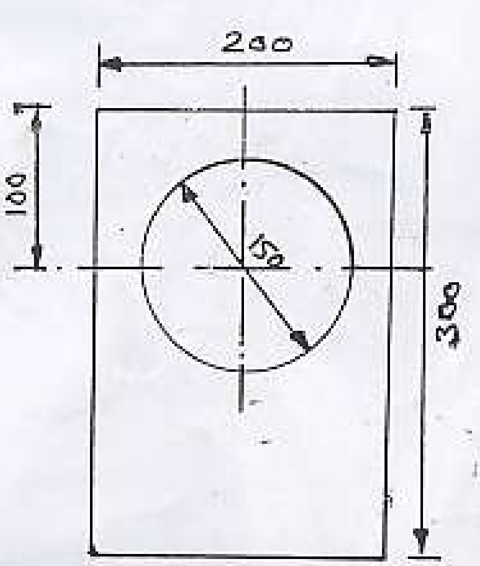
$I'_{\square} = \frac{bd^3}{12}$, $I'_{\Delta} = \frac{bh^3}{36}$, $I'_{\bigcirc} = 0.11 r^4$

Item	I'	Area	h	h^2	Ah^2
rectangle	364.5	54	-2.5	6.25	337.5
Semicircle	8.91	14.14	3.272	10.706	151.38
triangle	81	18	-4	16	288
SUM	454.41				776.88

$I_{xx} = 454.41 + 776.88 = 1231.29 \ m^4$

HW-1 Find the moment of inertia of a hollow section shown in figure about an axis passing through its Centroid and parallel to x-axis.

(all dimensions in mm).



HW-2 Find the Centroid for the figure in the above example w.r.t x-axis shown

[Ans: $\bar{x} = 0.9054 m$, $\bar{y} = -1.866 m$]