CHAPTER 2

OPTICAL FIBERS

2.1 Introduction

The availability of low-loss fibers led to a revolution in the field of lightwave technology and started the era of fiber-optic communications. This chapter focuses on the role of optical fibers as a communication channel in lightwave systems.

2.2 Ray Transmission Theory

2.2.1 Total Internal Reflection

To consider the propagation of light within an optical fiber utilizing the ray theory model it is necessary to take account the ***refractive index*** of the dielectric medium. The refractive index of a medium is defined as ***the ratio of the velocity of light in a vacuum to the velocity of light in the medium***. A ray of light travels more slowly in an optically dense medium than in one that is less dense, and the refractive index gives a measure of this effect. When a ray is incident on the interface between two dielectrics of differing refractive indices (e.g. glass–air), refraction occurs, as illustrated in Figure 2.1 (a). It may be observed that the ray approaching the interface is propagating in a dielectric of refractive index n_1 and is at an angle φ_1 to the normal on the surface of the interface. If the dielectric on the other side of the interface has a refractive index n_2 which is less than n_1, then the refraction is such that the ray path in this lower index medium is at an angle φ_2 to the normal, where φ_2 is greater than φ_1. The angles of incidence φ_1 and refraction φ_2 are related to each other and to the refractive indices of the dielectrics by *Snell's* law of refraction, which states that:

$$n_1 \sin \emptyset_1 = n_2 \sin \emptyset_2 \qquad \text{(Equation 2.1)}$$

Or:

$$\frac{\sin \emptyset_1}{\sin \emptyset_2} = \frac{n_2}{n_1} \qquad \text{(Equation 2.2)}$$

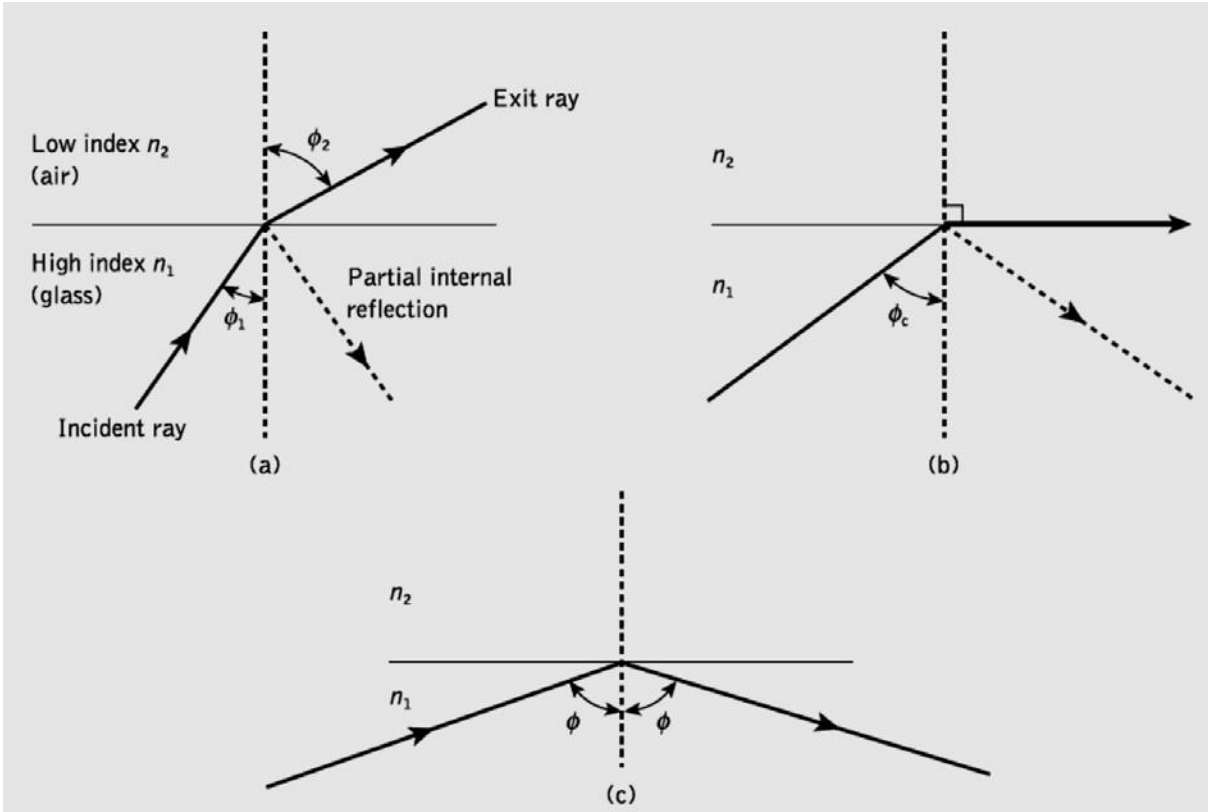

Figure 2.1: Light rays incident on a high to low refractive index interface (e.g. glass–air): (a) Refraction; (b) The limiting case of refraction showing the critical ray at an angle φ_c; (c) total internal reflection where $\varphi > \varphi_c$

It may also be observed in Figure 2.1 (a) that a small amount of light is reflected back into the originating dielectric medium (partial internal reflection). As $\boldsymbol{n_1}$ is greater than $\boldsymbol{n_2}$, the angle of refraction is always greater than the angle of incidence. Thus, when the angle of refraction is 90° and the refracted ray emerges parallel to the interface between the dielectrics, the angle of incidence must be less than 90°. This is the limiting case of refraction and the angle of incidence is now known as the **critical angle** φ_c, as shown in Figure 2.1 (b). The critical angle is defined as the angle of incidence *above* which total internal reflection occurs. From Equation (2.2) the value of the critical angle is given by:

$$\sin \phi_c = \frac{n_2}{n_1} \qquad \text{(Equation 2.3)}$$

At angles of incidence greater than the critical angle the light is reflected back into the originating dielectric medium (total internal reflection) with high efficiency (around 99.9%).

Hence, it may be observed in Figure 2.1 (c) that total internal reflection occurs at the interface between two dielectrics of differing refractive indices when light is incident on the dielectric of lower index from the dielectric of higher index, and the angle of incidence ray exceeds the critical value. This is the mechanism by which light at a sufficient shallow angle (less than 90° − φc) may be considered to propagate down an optical fiber with low loss.

Figure 2.2 illustrates the transmission of a light ray in an optical fiber via a series of total internal reflections at the interface of the silica core and the slightly lower refractive index silica cladding. The ray has an angle of incidence φ at the interface which is greater than the critical angle and is reflected at the same angle to the normal. The light ray is known as a **meridional ray** as it passes through the axis of the fiber core.

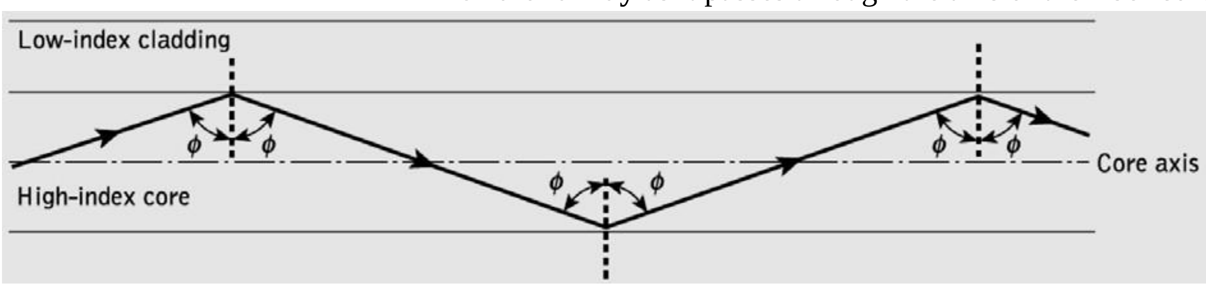

Figure 2.2: The transmission of a light ray in a perfect optical fiber

2.2.2 Acceptance Angle

The geometry concerned with launching a light ray into an optical fiber is shown in Figure 2.3, which illustrates a meridional ray *A* at the critical angle φ_c within the fiber at the core–cladding interface. It may be observed that this ray enters the fiber core at an angle θa to the fiber axis and is refracted at the air–core interface before transmission to the core–cladding interface at the critical angle.

Hence, any rays which are incident into the fiber core at an angle greater than θ_a will be transmitted to the core–cladding interface at an angle less than φ_c, and will not be totally internally reflected. This situation is also illustrated in Figure 2.3, where the incident ray *B* at an angle greater than θ_a is refracted into the cladding and eventually lost by radiation.

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Thus, for rays to be transmitted by total internal reflection within the fiber core they must be incident on the fiber core within an acceptance cone defined by the conical half angle θ_a. Hence θa **is the maximum angle to the axis at which light may enter the fiber in order to be propagated, and is often referred as the acceptance angle for the fiber.**

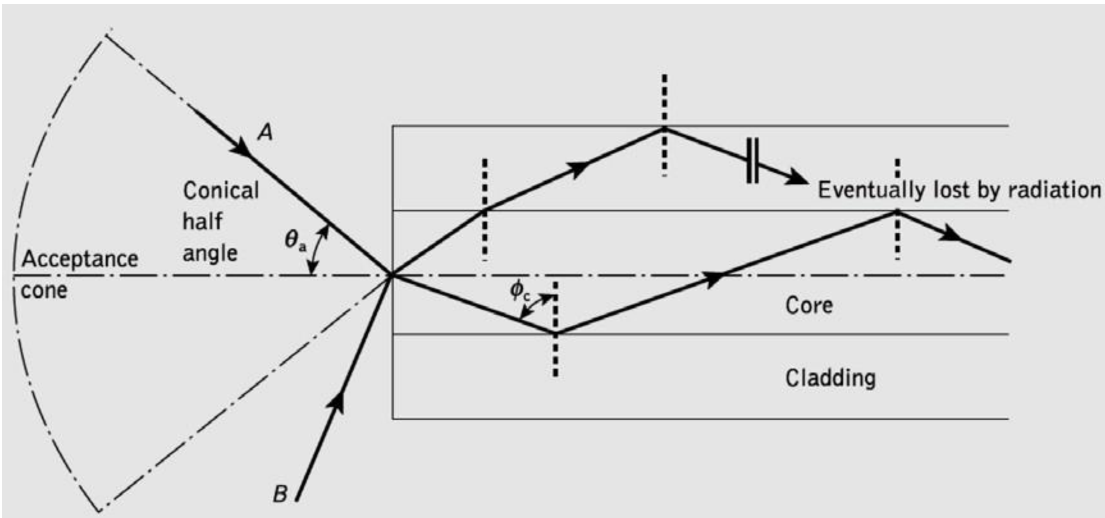

Figure 2.3: The acceptance angle θ_a when launching light into an optical fiber

2.2.3 Numerical Aperture

The numerical aperture (NA) of a fiber is defined **as the sine of the largest angle an incident ray can have for total internal reflectance in the core.** Rays launched outside the angle specified by a fiber's NA will excite radiation modes of the fiber. A higher core index, with respect to the cladding, means larger NA. However, increasing NA causes higher scattering loss from greater concentrations of dopant. A fiber's NA can be determined by measuring the divergence angle of the light cone it emits when all its modes are excited.

Figure 2.4 shows a light ray incident on the fiber core at an angle θ_1 to the fiberaxis, whichh is less than the **acceptance angle** for the fiber θ_a. The ray enters the fiber from a medium (air) of refractive index n_0, and the fiber core has a refractive index n_1, which is slightly greater than the cladding refractive index n_2.

Assuming the entrance face at the fiber core to be normal to the axis, then considering the refraction at the air–core interface and using *Snell's* law given by Equation (2.1):

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$$n_0 \sin\theta_1 = n_1 \sin\theta_2 \qquad \text{(Equation 2.4)}$$

Considering the right–angled triangle ABC indicated in Figure 2.4, then:

$$\emptyset = \frac{\pi}{2} - \theta_2 \qquad \text{(Equation 2.5)}$$

where $\emptyset$ is greater than the critical angle at the core–cladding interface. Hence Equation (2.4) becomes:

$$n_0 \sin\theta_1 = n_1 \cos\emptyset \qquad \text{(Equation 2.6)}$$

Using the trigonometrical relationship $\sin^2\emptyset + \cos^2\emptyset = 1$, Equation (2.6) may be written in the form:

$$n_0 \sin\theta_1 = n_1 (1 - \sin^2\emptyset)^{\frac{1}{2}} \qquad \text{(Equation 2.7)}$$

When the limiting case for total internal reflection is considered, $\emptyset$ becomes equal to the critical angle for the core–cladding interface and is given by Equation (2.3). Also in this limiting case $\boldsymbol{\theta_1}$ becomes the acceptance angle for the fiber $\boldsymbol{\theta_a}$. Combining these limiting cases into Equation (2.7) gives:

$$n_0 \sin\theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}} \qquad \text{(Equation 2.8)}$$

Equation (2.8), apart from relating the acceptance angle to the refractive indices, serves as the basis for the definition of the important optical fiber parameter, the **numerical aperture (NA)**.

Hence the ***NA*** is defined as:

$$NA = n_0 \sin\theta_a = (n_1^2 - n_2^2)^{\frac{1}{2}} \qquad \text{(Equation 2.9)}$$

Since the *NA* is often used with the fiber in the air where $\boldsymbol{n_0}$ is unity, it is simply equal to ($\sin\theta_a$). It may also be noted that incident meridional rays over the range $0 \le \theta_1 \le \theta_a$ will be propagated within the fiber. The *NA* may also be given in terms of the relative refractive index difference (Δ) between the core and the cladding which is defined as:

$$NA = n_1 (2\Delta)^{\frac{1}{2}} \qquad \text{(Equation 2.10)}$$

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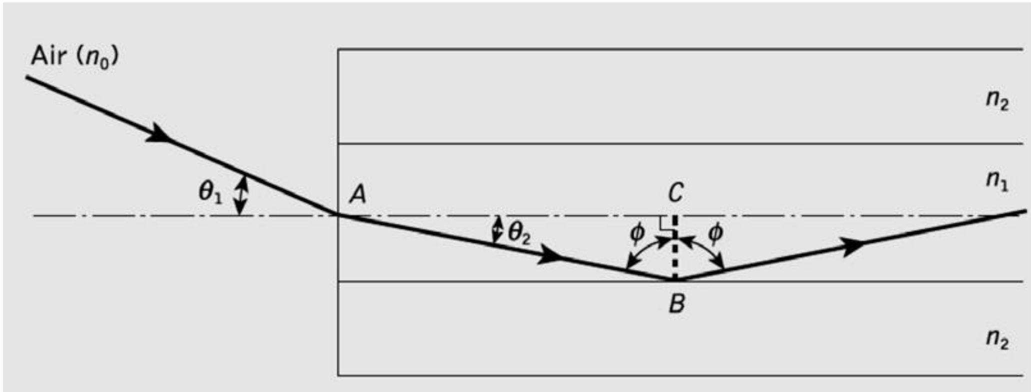

Figure 2.4: The ray path for a meridional ray launched into an optical fiber in air at an input angle less than the acceptance angle for the fiber

Example 2.1

A silica optical fiber with a core diameter large enough to be considered by the ray theory has a core refractive index of 1.5 and a cladding refractive index of 1.47, determine:

(a) The critical angle at the core–cladding interface.

(b) The NA of the fiber.

(c) The acceptance angle in air to the fiber.

Solution:

(a) The critical angle φ_c at the core–cladding interface is given by Equation (2.3) where:

$$\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.47}{1.5}\right)$$

$$= 78.5^o$$

(b) From Equation (2.9) the NA is:

$$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} = (1.5^2 - 1.47^2)^{\frac{1}{2}}$$

$$= (2.25 - 2.16)^{\frac{1}{2}} = 0.3$$

(c) Considering Equation (2.9) the acceptance angle in air θa is given by:

$$\theta_a = \sin^{-1} NA = \sin^{-1}(0.3)$$

$$= 17.4^o$$