(c) The maximum bit rate that may be obtained without substantial errors on the link assuming only intermodal dispersion;

(d) The bandwidth–length product corresponding to (c).

Solution:

$$\delta T_s \simeq \frac{L n_1 \Delta}{c} = \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8} = 300 \text{ ns}$$

$$\sigma_s = \frac{L n_1 \Delta}{2\sqrt{3}c} = \frac{1}{2\sqrt{3}} \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8} = 86.7 \text{ ns}$$

$$B_T(\max) = \frac{1}{2\tau} = \frac{1}{2\delta T_s} = \frac{1}{600 \times 10^{-9}} = 1.7 \text{ Mbit s}^{-1}$$

$$B_T(\max) = \frac{0.2}{\sigma_s} = \frac{0.2}{86.7 \times 10^{-9}} = 2.3 \text{ Mbit s}^{-1}$$

$$B_{opt} \times L = 2.3 \text{ MHz} \times 6 \text{ km} = 13.8 \text{ MHz km}$$

3.3.2 Chromatic dispersion (CD).

Chromatic or intramodal dispersion may occur in all types of optical fiber and results from the finite spectral linewidth of the optical source. Since optical sources do not emit just a single frequency, but a band of frequencies (in the case of the injection laser corresponding to only a fraction of a percent of the center frequency, whereas for the LED it is likely to be a significant percentage), then there may be propagation delay differences between the different spectral components of the transmitted signal.

This causes broadening of each transmitted mode and hence intramodal dispersion. The delay differences may be caused by the dispersive properties of the waveguide material (material dispersion) and also guidance effects within the fiber structure (waveguide dispersion).

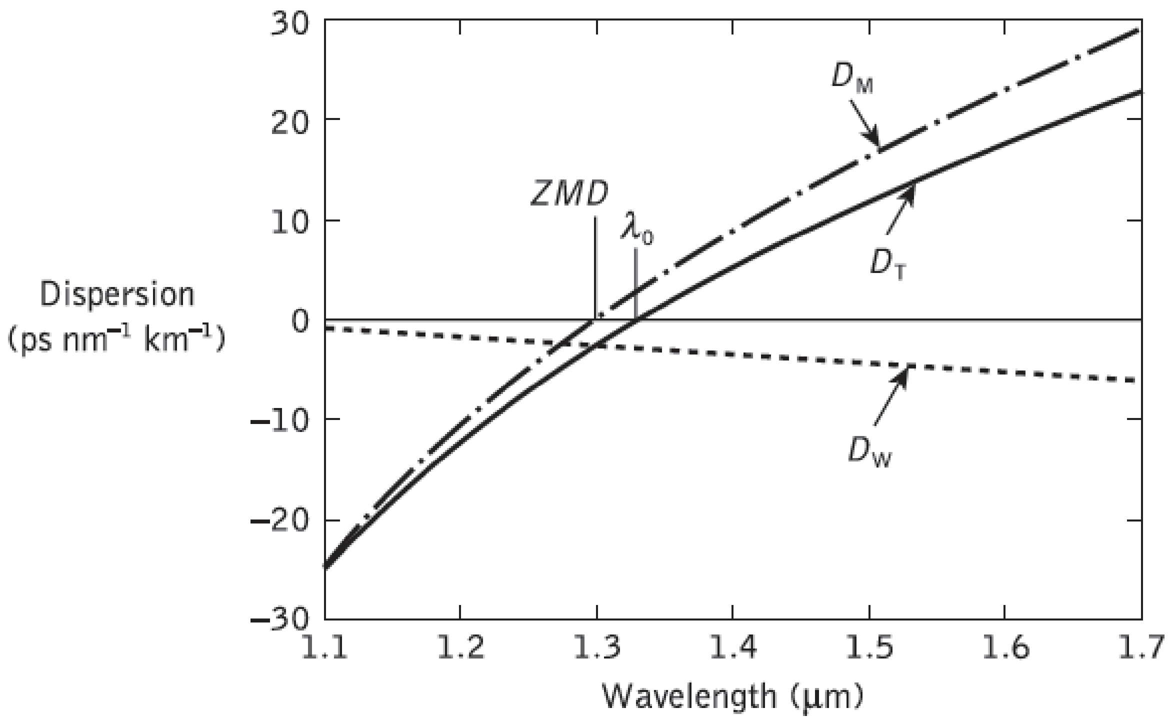
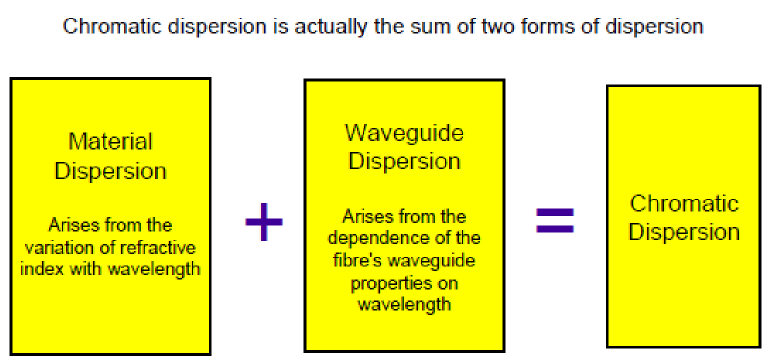


Figure 3.10: The material dispersion parameter (D_M), the waveguide dispersion parameter (D_W) and the total dispersion parameter (D_T) as functions of wavelength for a conventional single-mode fiber



3.3.2.1 Material Dispersion

Pulse broadening due to material dispersion results from the different group velocities of the various spectral components launched into the fiber from the optical source. It occurs when the phase velocity of a plane wave propagating in the dielectric medium varies nonlinearly with wavelength, and a material is said to exhibit material dispersion when the second differential of the refractive index with respect to wavelength is not zero (i.e. $d^2n/d\lambda^2 \neq 0$).

The rms pulse broadening due to material dispersion is given by:

$$\sigma_m \simeq \frac{\sigma_\lambda L}{c}\left|\lambda\frac{d^2 n_1}{d\lambda^2}\right|$$

However, it may be given in terms of a material dispersion parameter M which is defined as:

$$M=\frac{1}{L}\frac{d\tau_m}{d\lambda}=\frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right|$$

Example 3.6

A glass fiber exhibit material dispersion given by |λ2(d2n1/dλ2) | of 0.025. Determine the material dispersion parameter at a wavelength of 0.85 μm, and estimate the pulse broadening per kilometer for a good LED source with an rms spectral width of 20 nm at this wavelength.

Solution:

$$M=\frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right|=\frac{1}{c\lambda}\left|\lambda^2\frac{d^2 n_1}{d\lambda^2}\right|$$

$$=\frac{0.025}{2.998\times10^5\times850}\ \text{s nm}^{-1}\ \text{km}^{-1}$$

$$=98.1\ \text{ps nm}^{-1}\ \text{km}^{-1}$$

$$\sigma_m \simeq \frac{\sigma_\lambda L}{c}\left|\lambda\frac{d^2 n_1}{d\lambda^2}\right|$$

$$\sigma_m \simeq \sigma_\lambda L M$$

Hence, the rms pulse broadening per kilometer due to material dispersion:

$$\sigma_m(1\ \text{km}) = 20\times1\times98.1\times10^{-12} = 1.96\ \text{ns km}^{-1}$$

3.3.2.2 Waveguide Dispersion

The waveguide of the fiber may also create chromatic dispersion. This results from the variation in group velocity with wavelength for a particular mode. Considering the ray theory approach, it is equivalent to the angle between the ray and the fiber axis varying with wavelength which subsequently leads to a variation in the transmission times for the rays, and hence dispersion. For a single mode whose propagation constant is β, the fiber exhibit waveguide dispersion when $d^2\beta/d\lambda^2 \neq 0$. Multimode fibers, where the majority of modes

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propagates far from the cutoff, are almost free of waveguide dispersion and it is generally negligible compared with material dispersion (≈0.1 to 0.2 ns km^{-1}). However, with single-mode fibers where the effects of the different dispersion mechanisms are not easy to separate, waveguide dispersion may be significant.

The final expression may be separated into three composite dispersion components in such a way that one of the effects dominates each term [Ref. 46]. The dominating effects are as follows:

1. The material dispersion parameter DM defined by $\lambda/c \mid d^2n/d\lambda^2 \mid$ where $n = n_1$ or n_2 for the core or cladding respectively.
2. The waveguide dispersion parameter DW, which may be obtained by:

$$D_W = -\left(\frac{n_1 - n_2}{\lambda c}\right) V \frac{d^2(Vb)}{dV^2}$$

where V is the normalized frequency for the fiber. Since the normalized propagation constant b for a specific fiber is only dependent on V, then the normalized waveguide dispersion coefficient $Vd^2(Vb)/dV^2$ also depends on V. This latter function is another universal parameter which plays a central role in the theory of single mode fibers.

3. A profile dispersion parameter DP which is proportional to $d\Delta/d\lambda$.

Example 3.7

A typical single-mode fiber has a zero-dispersion wavelength of 1.31 μm with a dispersion slope of 0.09 ps nm^{-2} km^{-1}. Compare the total first-order dispersion for the fiber at the wavelengths of 1.28 μm and 1.55 μm. When the material dispersion and profile dispersion at the latter wavelength are 13.5 ps nm^{-1} km^{-1} and 0.4 ps nm^{-1} km^{-1}, respectively, determine the waveguide dispersion at this wavelength.

Solution:

The total first-order dispersion for the fiber at the two wavelengths may be obtained from:

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$$D_T(1280\ \text{nm}) = \frac{\lambda S_0}{4}\left[1-\left(\frac{\lambda_0}{\lambda}\right)^4\right]$$
$$= \frac{1280 \times 0.09 \times 10^{-12}}{4}\left[1-\left(\frac{1310}{1280}\right)^4\right]$$
$$= -2.8\ \text{ps nm}^{-1}\ \text{km}^{-1}$$

and:

$$D_T(1550\ \text{nm}) = \frac{1550 \times 0.09 \times 10^{-12}}{4}\left[1-\left(\frac{1310}{1550}\right)^4\right]$$
$$= 17.1\ \text{ps nm}^{-1}\ \text{km}^{-1}$$

The total dispersion at the 1.28 μm wavelength exhibits a negative sign due to the influence of the waveguide dispersion. Furthermore, as anticipated the total dispersion at the longer wavelength (1.55 μm) is considerably greater than that obtained near the zero-dispersion wavelength. The waveguide dispersion for the fiber at a wavelength of 1.55 μm is given by:

$$D_W = D_T - (D_M + D_P)$$
$$= 17.1 - (13.5 + 0.4)$$
$$= 3.2\ \text{ps nm}^{-1}\ \text{km}^{-1}$$

3.3.3 Polarization mode dispersion (PMD)

Polarization mode dispersion (PMD) is a form of modal dispersion where two different polarizations of light in a waveguide, which normally travel at the same speed, travel at different speeds due to random imperfections and asymmetries, causing random spreading of optical pulses. Unless it is compensated, which is difficult, this ultimately limits the rate at which data can be transmitted over a fiber.

In practice, fibers are not perfectly circular symmetric, and the two orthogonally polarized modes have slightly different propagation constants; that is, practical fibers are slightly birefringent. Since the light energy of a pulse propagating in a fiber will usually be split between these two modes, this birefringence gives rise to pulse spreading. This phenomenon is called PMD. This is similar, in principle, to pulse spreading in the case of multimode fibers, but the effect is much weaker. PMD is illustrated in Figure 3.11 and Figure 3.12. The assumption here is that the propagation constants of the two polarizations are constant throughout the length of the fiber. If the difference in propagation constants is denoted by Δ

then the time spread, or differential group delay (DGD) due to PMD after the pulse has propagated through a unit length of fiber is given by

$$\Delta\tau =$$

Δ��/ω

A typical value of the DGD is $\Delta\tau$ = 0.5 ps/km, which suggests that after propagating through 100 km of fiber, the accumulated time spread will be 50 ps comparable to the bit period of 100 ps for a 10 Gb/s system. This would effectively mean that 10 Gb/s transmission would not be feasible over any reasonable distances due to the effects of PMD.

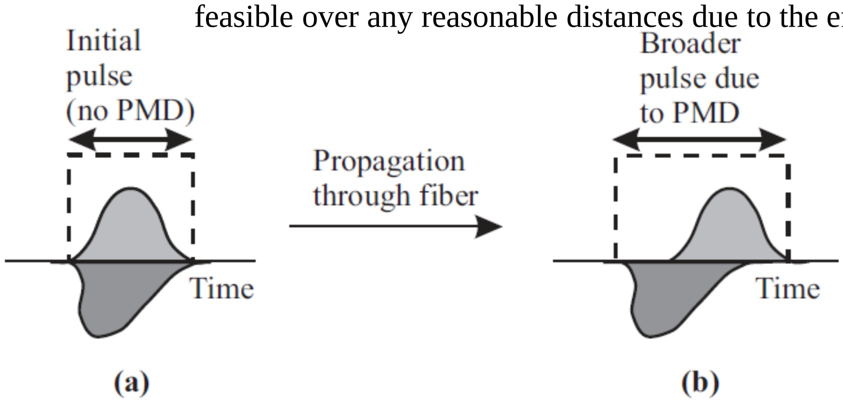

Figure 3.11: Illustration of pulse spreading due to PMD [13].

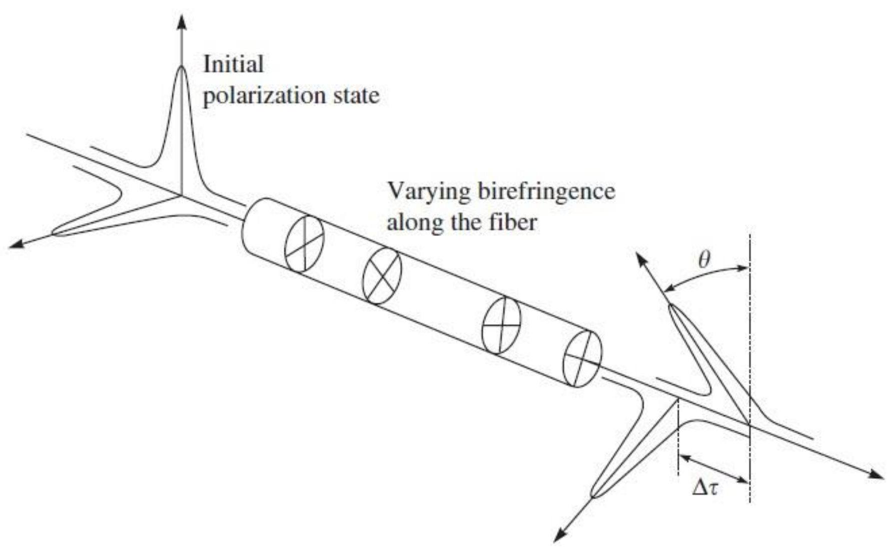

Figure 3.12: Variation in the polarization states of an optical pulse as it passes through a fiber that has varying birefringence along its length [14].

PMD is not a fixed quantity, but fluctuates with time due to factors such as temperature variations and stress changes on the fiber. It varies as the square root of distance and thus is

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specified as a maximum value in units of ps/$\sqrt{km}$. A typical value D_{PMD} is $= 0.05$ ps/$\sqrt{km}$. Since PMD is a statistically variable parameter, it is more difficult to control than chromatic dispersion, which has a fixed value.

Total Dispersion Calculation

If t_{mod}, t_{CD}, and t_{PMD} are the modal, chromatic, and polarization mode dispersion times, respectively, then the total dispersion t_T can be calculated from the relationship [14]:

$$t_T = \sqrt{(t_{\text{mod}})^2 + (t_{\text{CD}})^2 + (t_{\text{PMD}})^2}$$

Example 3.8

Consider a single-mode fiber for which D_{CD} = 2 ps/(km.nm) and D_{PMD} = 0.1 ps/(km.nm). If a transmission link has a length L = 500 km and uses a laser source with a spectral emission width of $\Delta\lambda$ = 0.01 nm, then we have $t_{mod} = 0$, $t_{CD} = D_{CD} \times L \times \Delta\lambda = 10$ ps, and $t_{PMD} = D_{PMD} \times \sqrt{L} = 2.24$ ps. Thus:

$$t_T = \sqrt{(10\,\text{ps})^2 + (2.24\,\text{ps})^2} = 10.2\,\text{ps}$$

If t_T can be no more than 10 percent of a pulse width, then the maximum data rate $R_{\max}$ that can be sent over this 500-km link is $R_{\max} = 0.1/t_T = 9.8$ Gbps (gigabits per second).

Example 3.9

A multimode graded index fiber exhibits total pulse broadening of 0.1 μs over a distance of 15 km. Estimate:

(a) The maximum possible bandwidth on the link assuming no intersymbol interference;
(b) The pulse dispersion per unit length;
(c) The bandwidth–length product for the fiber.

Solution:

(a) The maximum possible optical bandwidth which is equivalent to the maximum possible bit rate, assuming no ISI may be obtained, where:

$$B_{opt} = B_T = \frac{1}{2\tau} = \frac{1}{2 \times 0.1 \times 10^{-6}} = 5\ MHz$$

(b) The dispersion per unit length may be acquired simply by dividing the total dispersion by the total length of the fiber:

$$\text{Dispersion} = \frac{0.1 \times 10^{-6}}{15} = 6.67\ ns.km^{-1}$$

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(c) The bandwidth–length product can be obtained by simply multiplying the maximum bandwidth for the fiber link by its length. Hence:

$$����_{oo} \times L = 5 \times 15 = 75\ M$$

$$MMk\mu$$

3.4 PROBLEMS

3.4.1 The mean optical power launched into an optical fiber link is 1.5 mW and the fiber has an attenuation of 0.5 dB km^{-1}. Determine the maximum possible link length without repeaters (assuming lossless connectors) when the minimum means optical power level required at the detector is 2 μW. Ans. (57.5 km)

3.4.2

The numerical input/output mean optical power ratio in a 1 km, length of optical fiber is found to be 2.5. Calculate the received mean optical power when a mean optical power of 1 mW is launched into a 5 km length of the fiber (assuming no joints or connectors).

3.4.3

Ans. (10 μW)

A 15 km optical fiber link uses fiber with a loss of 1.5 dB km^{-1}. The fiber is jointed every kilometer with connectors which give an attenuation of 0.8 dB each. Determine

3.4.4 the minimum mean optical power which must be launched into the fiber in order to maintain a mean optical power level of 0.3 μW at the detector. Ans. (703 μW)

3.4.5 Discuss absorption losses in optical fibers, comparing and contrasting the intrinsic and extrinsic absorption mechanisms.

3.4.6 Briefly describe linear scattering losses in optical fibers with regard to: (a) Rayleigh scattering; (b) Mie scattering.

respectively. Silica has an isothermal compressibility of 7×10^{-11} m^2 N^{-1} and an estimated afictive temperature of 1400 K. Determine the theoretical attenuation in

The photoelastic coefficient and the refractive index of silica are 0.286 and 1.46

decibels per kilometer due to the fundamental Rayleigh scattering in silica at optical wavelengths of 0.85 and 1.55 μm. Boltzmann's constant is 1.381×10^{-23} J K^{-1}. Ans. (1.57 dB km^{-1}, 0.14 dB km^{-1})

3.4.7 A $K_2O–SiO_2$ glass core optical fiber has an attenuation resulting from Rayleigh scattering of 0.46 dB km^{-1} at a wavelength of 1 μm. The glass has an estimated effective temperature of 758 K, isothermal compressibility of 8.4×10^{-11} m^2 N^{-1}, and a photoelastic coefficient of 0.245. Determine from theoretical considerations the refractive index of the glass. Ans. (1.49)

3.4.8 Compare stimulated Brillouin and stimulated Raman scattering in optical fibers, and indicate the way in which they may be avoided in optical fiber communications.

3.4.9 Explain what is meant by the critical bending radius for an optical fiber.

3.4.10 The threshold optical powers for stimulating Brillouin and Raman scattering in a single-mode fiber with a long 8 μm core diameter are found to be 190 mW and 1.7 W, respectively, when using an injection laser source with a bandwidth of 1 GHz. Calculate the operating wavelength of the laser and the attenuation in decibels per kilometer of the fiber at this wavelength. Ans. (1.5 μm, 0.30 dB km^{-1})

3.4.11 The threshold optical power for stimulated Brillouin scattering at a wavelength of 0.85 μm in a long single-mode fiber using an injection laser source with a bandwidth of 800 MHz is 127 mW. The fiber has an attenuation of 2 dB km^{-1} at this wavelength. Determine the threshold optical power of stimulated Raman scattering within the fiber at a wavelength of 0.9 μm assuming the fiber attenuation is reduced to 1.8 dB km^{-1} at this wavelength. Ans. (2.4 W)

3.4.12 A multimode graded index fiber has a refractive index at the core axis of 1.46 with a cladding refractive index of 1.45. The critical radius of curvature which allows large bending losses to occur is 2.56 μm when the fiber is transmitting light of a particular wavelength. Determine the wavelength of the transmitted light. Ans. (0.86 μm)

3.4.13 A single-mode step index fiber with a core refractive index of 1.49 has a critical bending radius of 10.4 mm when illuminated with light at a wavelength of 1.30 μm. If the cutoff wavelength of the fiber is 1.15 μm calculate its relative refractive index difference. Ans. (0.47%)

3.4.14 An 8 km optical fiber link without repeater uses multimode graded index fiber, which has a bandwidth–length product of 400 MHz km. Estimate the total pulse broadening on the link. Ans. (10 ns)

3.4.15 A step-index multimode glass fiber has a core diameter of 50 μm and cladding refractive index of 1.45. If it is to have a limiting intermodal dispersion and total broadening of a light pulse δT of 10 ns, find its acceptance angle for a 1 km fiber length.

Ans: 5.35º