Lens Aberrations Fundamentals

Optical Engineering

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Aberrations
(3rd order – Seidel)

Chromatic

$$\sin x = \underbrace{x - \frac{x^3}{3!}}_{3^{rd}\ \text{order}} + \frac{x^5}{5!} + O(x^7)$$

$$\cos x = \underbrace{1 - \frac{x^2}{2!}}_{3^{rd}\ \text{order}} + \frac{x^4}{4!} + O(x^6)$$

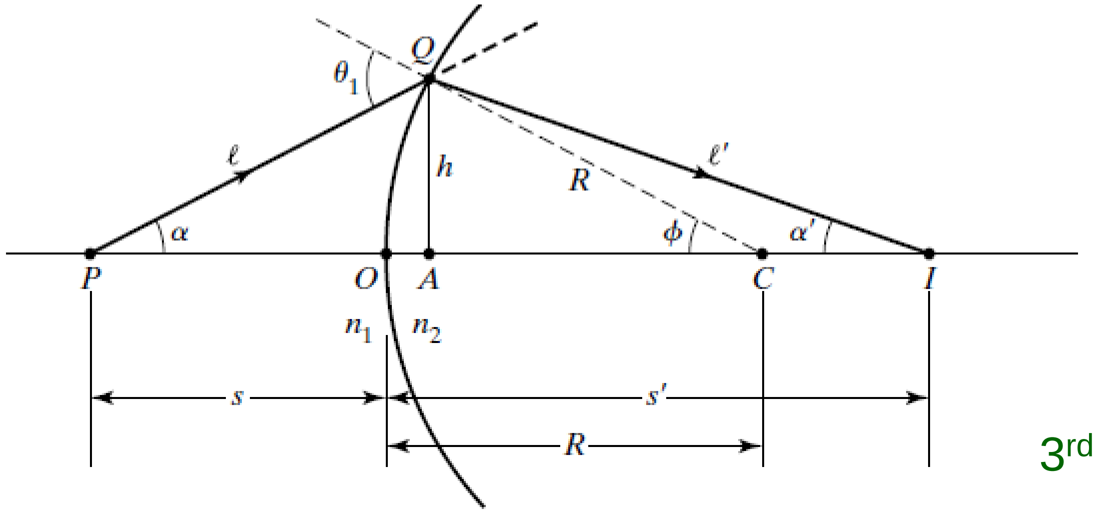
optics.hanyang.ac.kr/~shsong/20-Aberration%20theory.pdf

Monochromatic

Unclear Image →
- Spherical
- Coma
- Astigmatism

Deformation of Image →
- Field Curvature
- Distortion

Spherical Aberration (3rd order – Seidel)



$$a(Q) = (n_1\ell + n_2\ell') - (n_1 s + n_2 s')$$

3rd order – Seidel

$$\sin x \simeq x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

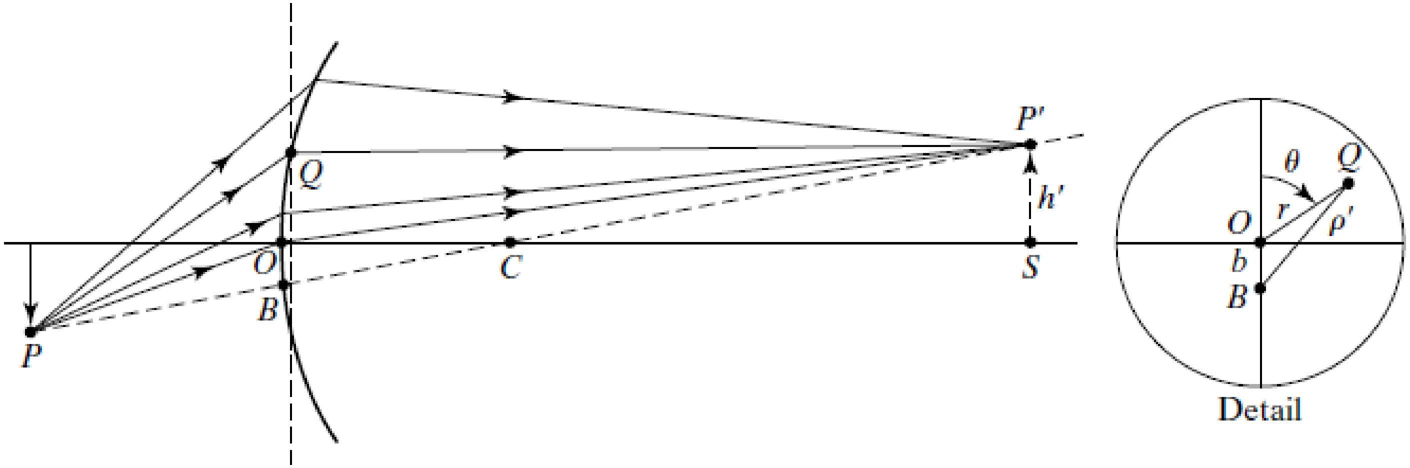
$$\cos x \simeq 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$(1+x)^{1/2} \simeq 1 + \frac{x}{2} - \frac{x^2}{8}$$

$$a(Q) = \frac{h^2}{2}\left[\frac{n_1}{s} + \frac{n_2}{s'} - \frac{n_2 - n_1}{R}\right] - \frac{h^4}{8}\left[\frac{n_1}{s}\left(\frac{1}{s} + \frac{1}{R}\right)^2 + \frac{n_2}{s'}\left(\frac{1}{s'} - \frac{1}{R}\right)^2\right]$$

F. L. Pedrotti et al., "Introduction to Optics", 3rd Ed., Prentice Hall, New Jersey,2006

Aberrations (3rd order – Seidel)



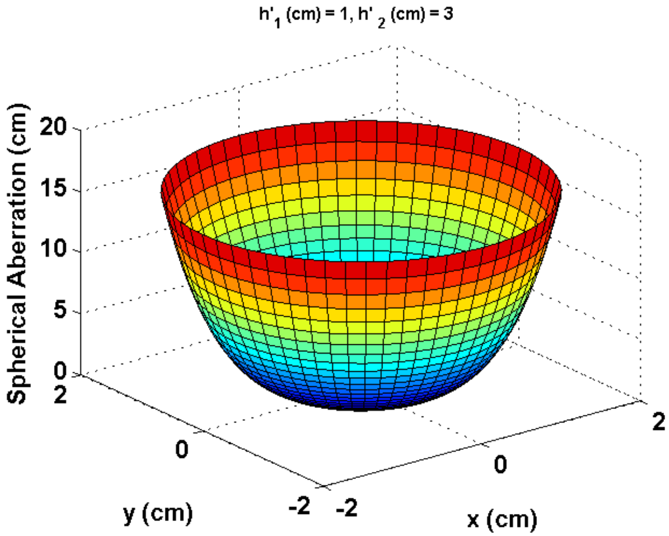
$$a(Q) = {}_0C_{40}r^4 + {}_1C_{31}h'r^3\cos\theta + {}_2C_{22}h'^2r^2\cos^2\theta + {}_2C_{22}h'^2r^2 + {}_3C_{11}h'^3r\cos\theta$$

r^4	:	Spherical Aberration
$h'r^3\cos\theta$	:	Coma
$h'^2r^2\cos^2\theta$	:	Astigmatism
h'^2r^2	:	Field Curvature
$h'^3r\cos\theta$	:	Distortion

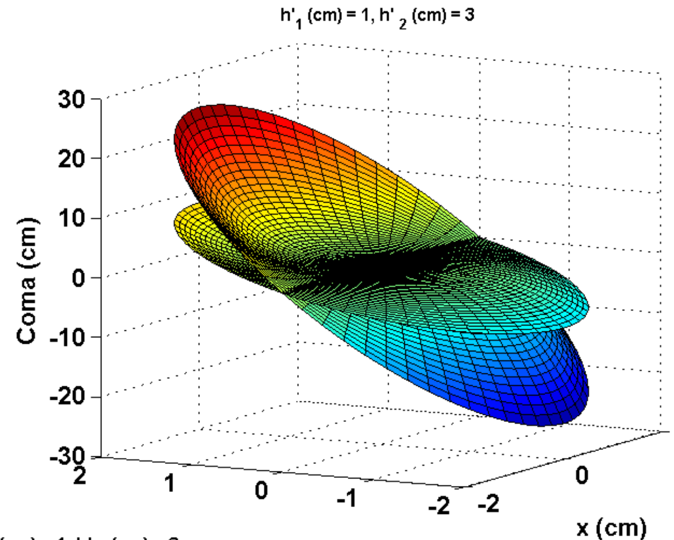
F. L. Pedrotti et al., "Introduction to Optics", 3rd Ed., Prentice Hall, New Jersey,2006

Aberrations (3rd order – Seidel)

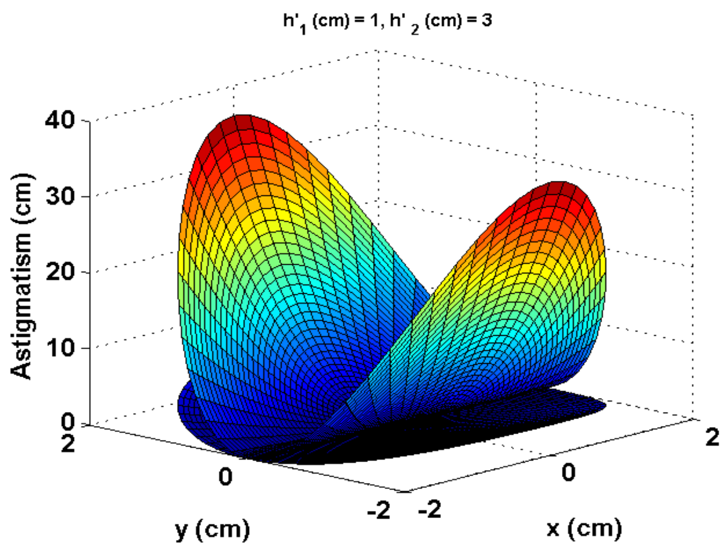
r^4 : Spherical Aberration



$h' r^3 \cos\theta$: Coma

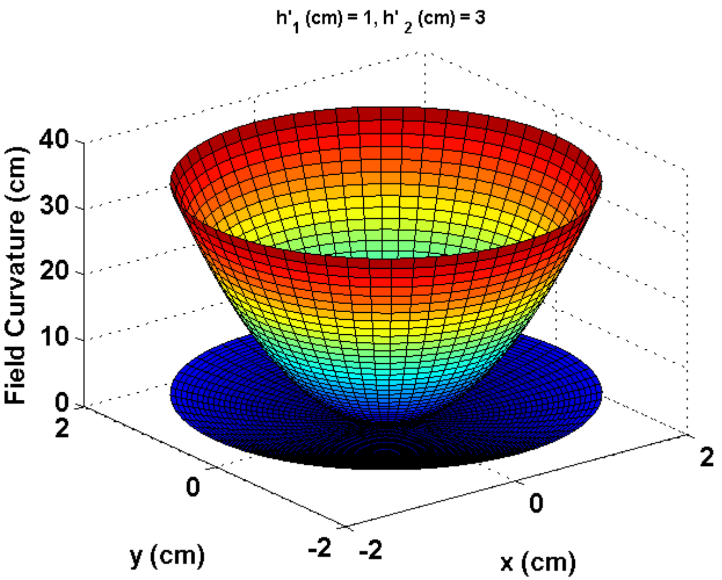


$h'^2 r^2 \cos^2\theta$: Astigmatism

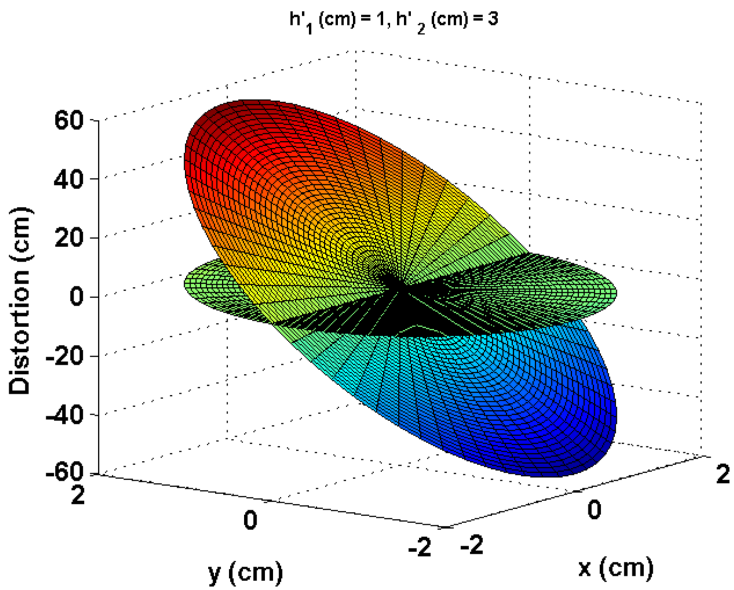


Aberrations (3rd order – Seidel)

$h'^2 r^2 \quad : \quad \text{Field Curvature}$



$h'^3 r \cos\theta \quad : \quad \text{Distortion}$



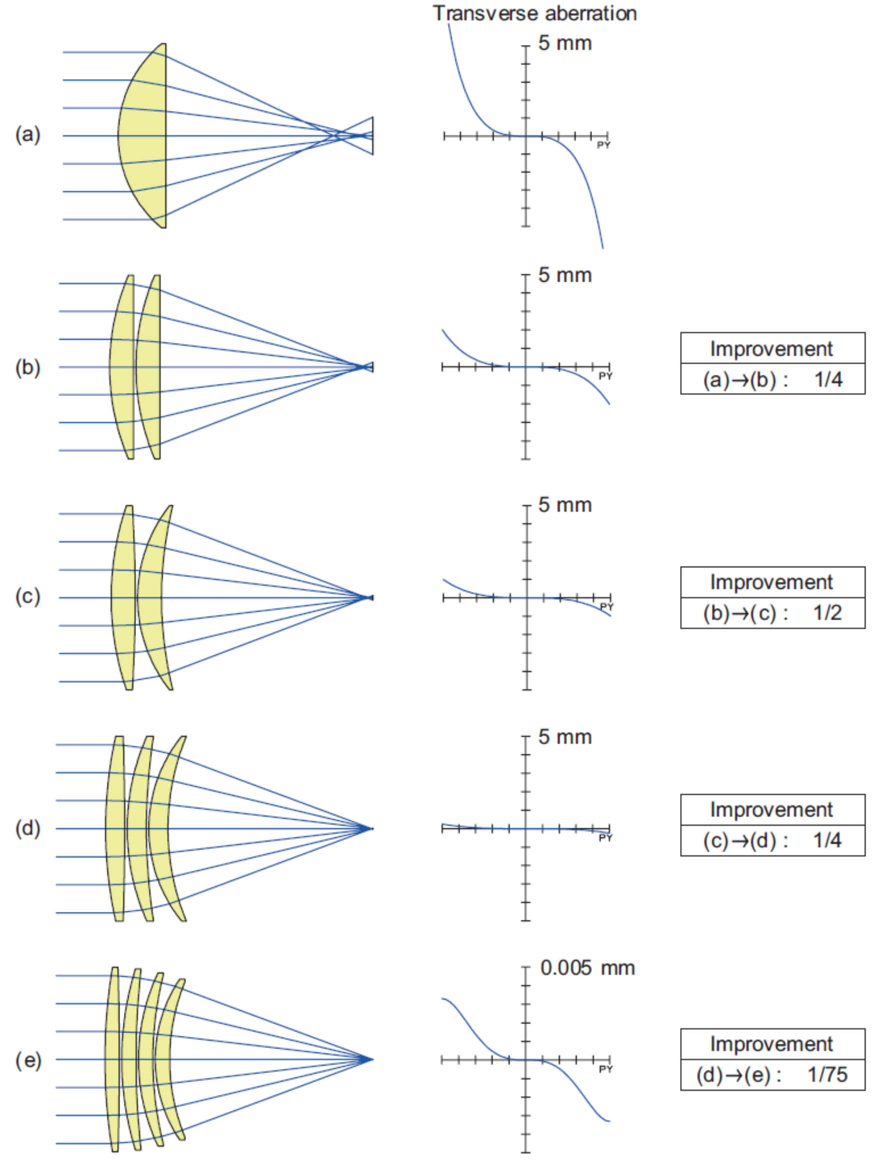
Lens Aberrations and Common Corrections

Aberration	Character	Correction
1. Spherical aberration	Monochromatic, on- and off-axis, image blur	Bending, high index, aspherics, gradient index, doublet
2. Coma	Monochromatic, off-axis only, blur	Bending, spaced doublet with central stop
3. Oblique astigmatism	Monochromatic, off-axis blur	Spaced doublet with stop
4. Curvature of field	Monochromatic, off-axis	Spaced doublet
5. Distortion	Monochromatic, off-axis	Spaced doublet with stop
6. Chromatic aberration	Heterochromatic, on- and off-axis, blur	Contact doublet, spaced doublet

http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/aberrcon.html

Lens Aberrations and Corrections Approaches

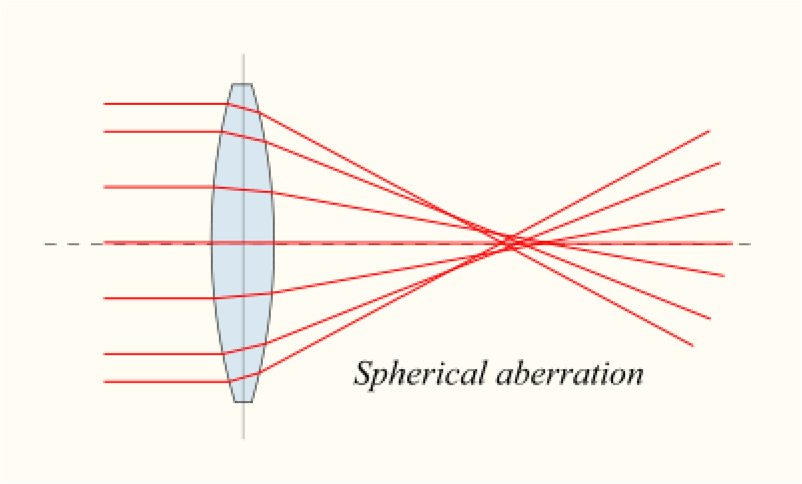
- Lens Bending
- Power Splitting
- Power Combination
- Distances
- Refractive Index
- Dispersion
- Gradient Index Materials
- Cemented Surfaces
- Aspherical Surfaces
- Diffractive Surfaces



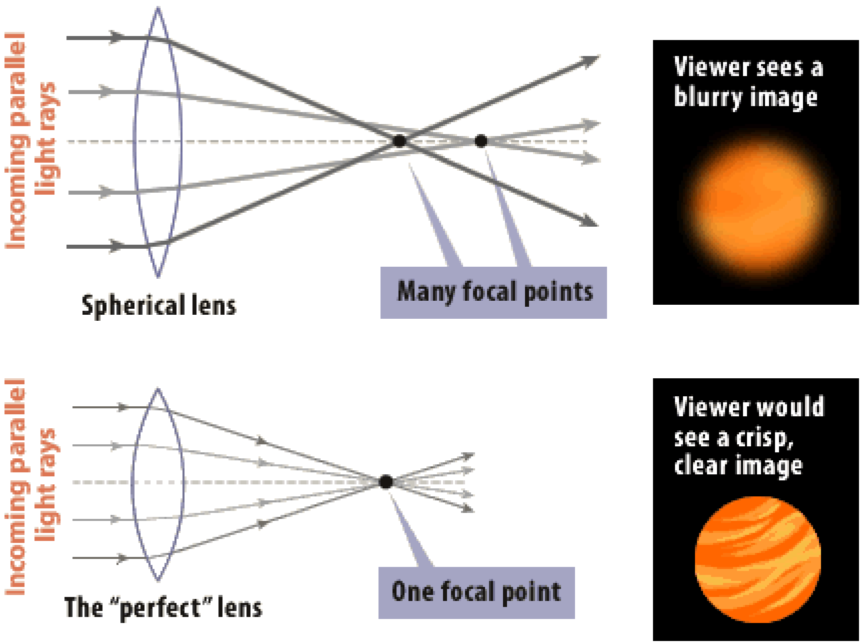
Example: Splitting and bending. In the examples shown *f ′ = 100 mm, the aperture is f/1.4, the refractive index n = 1.5.* Note the change in the scale for the transverse aberration diagram in case (e) with four elements. The transverse aberration is shown with respect to the Gaussian image plane.

Handbook of Optical Systems, Ed. H. Gross, vol. 3, 2007

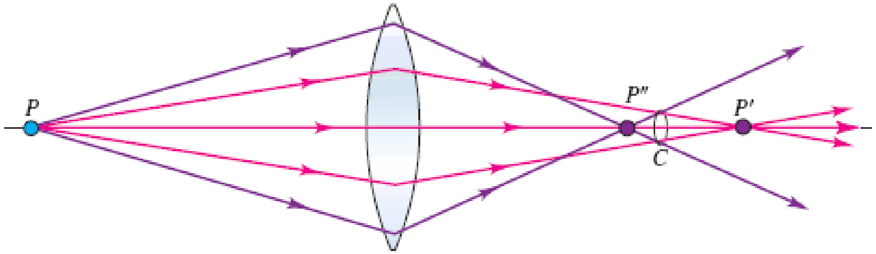
Spherical Aberration

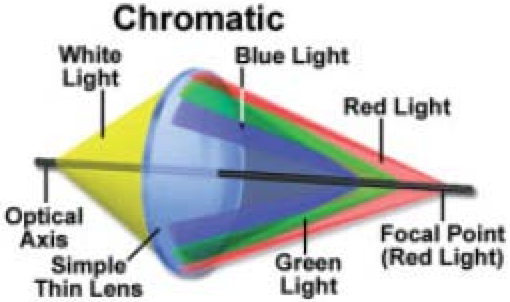


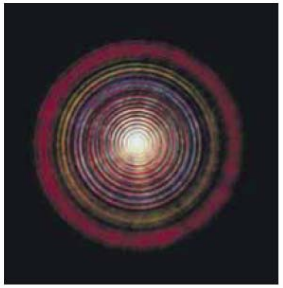
http://en.wikipedia.org/wiki/Lens_(optics)



http://amazing-space.stsci.edu/resources/explorations/groundup/lesson/basics/g11/







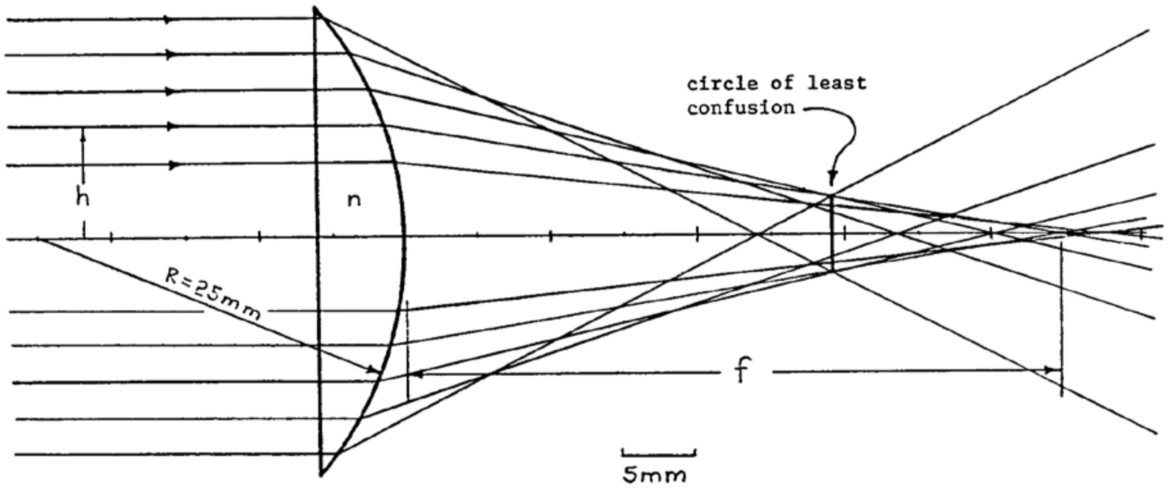
Spherical Aberration Example

https://www.wehelpchicagosee.com/blog/2015/01/28/high-definition-hd-vision-154460

Plano-Convex Lens

A plano-convex lens has a radius of curvature of R and an index of refraction, n. A collimated beam of light is incident upon the plano side of this lens in air as shown in the figure. Accounting for the plano-convex shape of the lens, the focal distance as defined in the figure, for an axial ray of displacement h from the principal axis is

$$f = \left[h/tan\left[sin^{-1}\left(\frac{nh}{R}\right) - sin^{-1}\left(\frac{h}{R}\right)\right]\right] - R + (R^2 - h^2)^{1/2}.$$



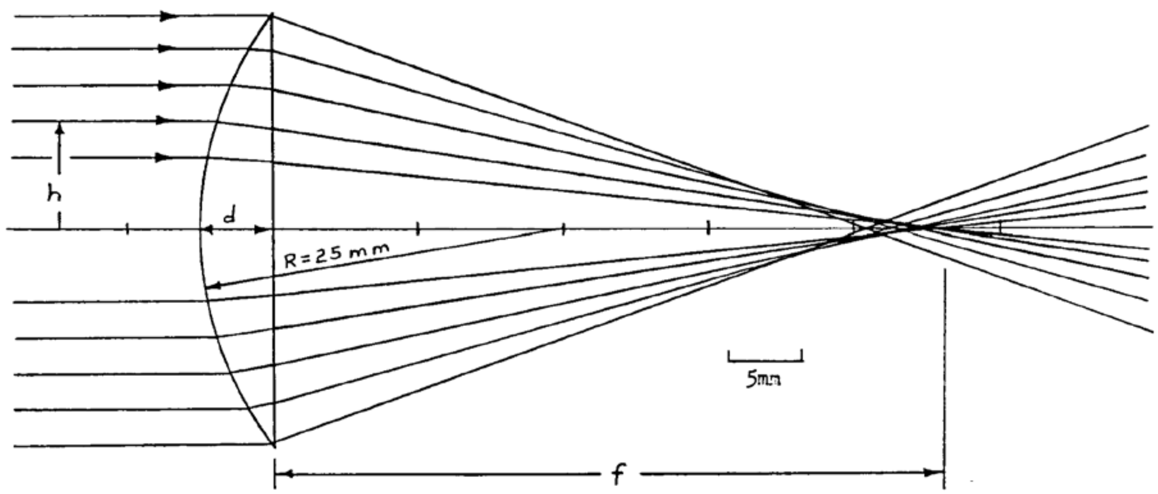
$h(mm)$	$f(mm)$
5.0	47.7
7.5	44.7
10.0	40.2
12.5	33.8
15.0	24.1

Fig. 1. Focusing of rays by plano-convex lens showing significant spherical aberration.

Convex-Plano Lens

A convex-plano lens has a radius of curvature of R, a thickness d, and an index of refraction, n. A collimated beam of light is incident upon the convex side of this lens in air as shown in the figure. Accounting for the convex-plano shape of the lens, the focal distance as defined in the figure, for an axial ray of displacement h from the principal axis is

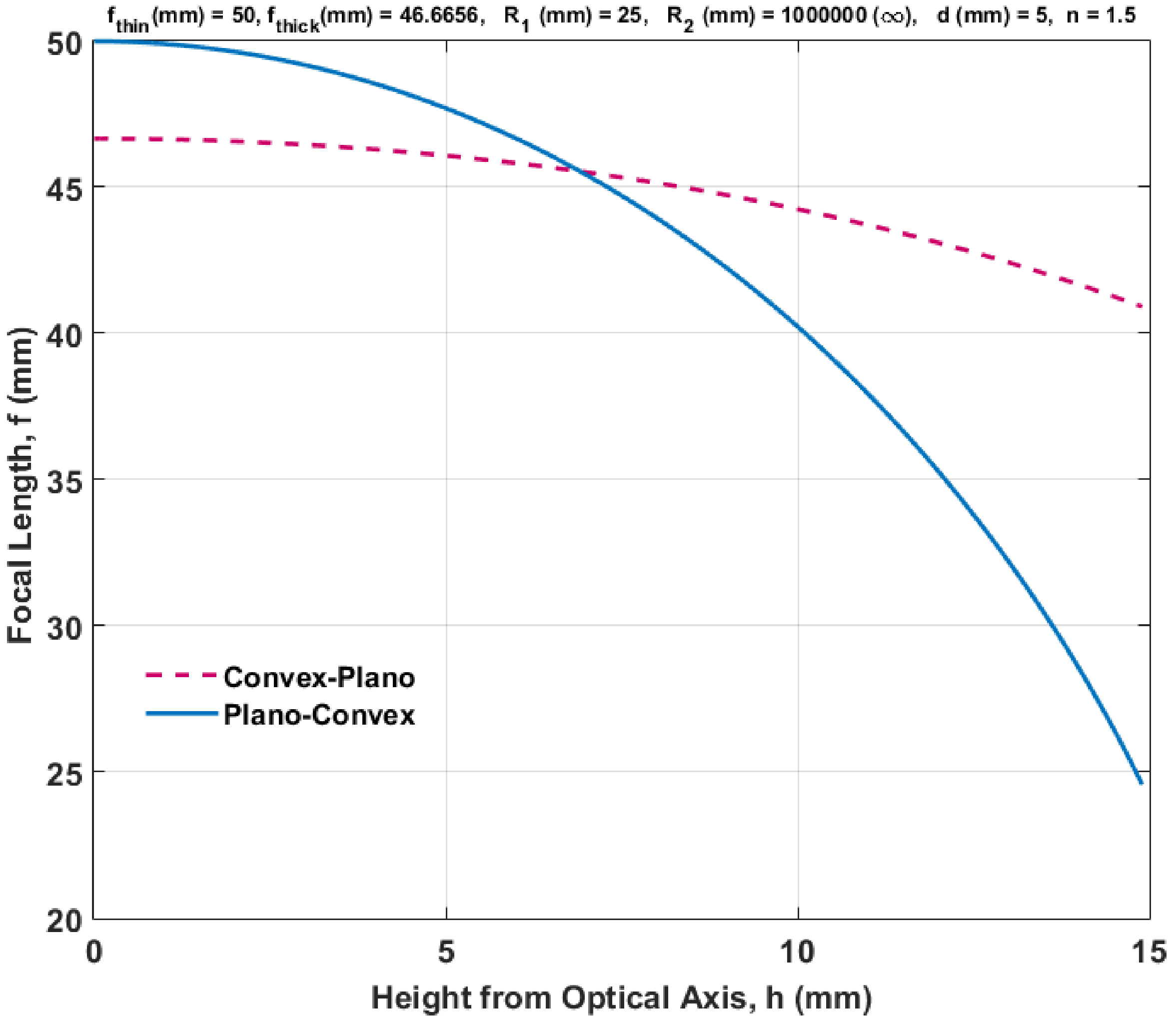
$$f = \frac{\left[\left(h/\{n\,sin[sin^{-1}(\frac{h}{R}) - sin^{-1}(\frac{h}{n\,R})]\}\right) + R - d\right] tan[sin^{-1}(\frac{h}{R}) - sin^{-1}(\frac{h}{n\,R})]}{tan\left(sin^{-1}\{n\,sin[sin^{-1}(\frac{h}{R}) - sin^{-1}(\frac{h}{n\,R})]\}\right)}.$$



$h(mm)$	$f(mm)$
5.0	46.1
7.5	45.3
10.0	44.3
12.5	42.8
15.0	40.8

Fig. 1. Focusing of rays by convex-plano lens showing relatively small spherical aberration.

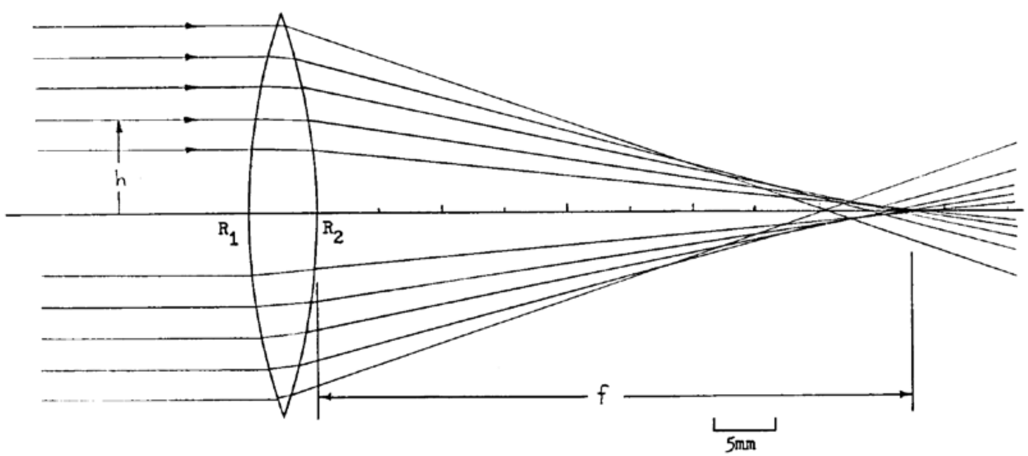
Comparison of the Spherical Aberration of a Plano-Convex Lens and a Convex-Plano Lens



Biconvex Lens

A biconvex lens has radii of curvature of R_1 and R_2, a center thickness of d, and an index of refraction n. If R_1 and R_2 are taken to be magnitudes, the focal distance of this biconvex lens in air for an axial ray of distance h from the principal axis is

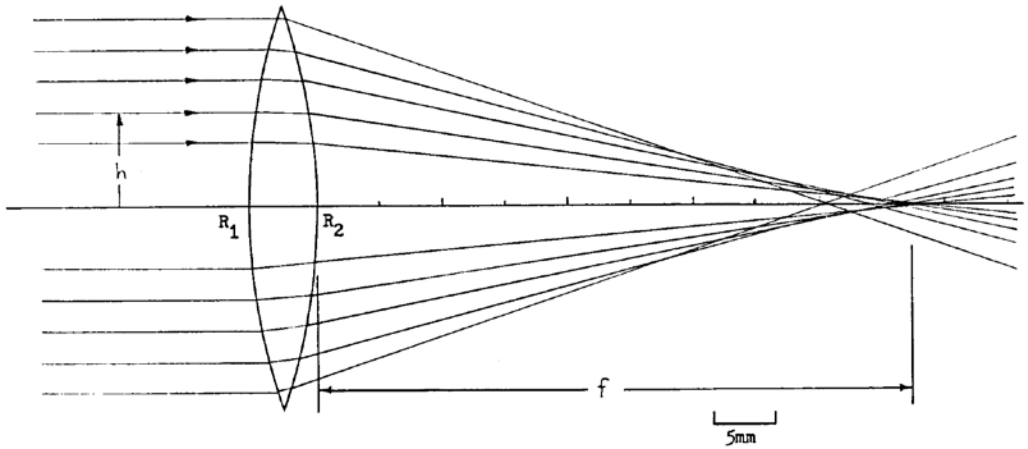
$$
\begin{aligned}
f = \Bigg\{ & n\left\{(R_1+R_2-d)\ sin\left[sin^{-1}\left(\frac{h}{R_1}\right) - sin^{-1}\left(\frac{h}{nR_1}\right)\right] + \frac{h}{n}\right\} \Bigg/ \\
& sin\left[\left(sin^{-1}\left\{\left(\frac{n}{R_2}\right)(R_1+R_2-d)\ sin\left[sin^{-1}\left(\frac{h}{R_1}\right) - sin^{-1}\left(\frac{h}{nR_1}\right)\right] + \frac{h}{R_2}\right\}\right)\right. \\
& -\left(sin^{-1}\left\{\left(\frac{1}{R_2}\right)(R_1+R_2-d)\ sin\left[sin^{-1}\left(\frac{h}{R_1}\right) - sin^{-1}\left(\frac{h}{nR_1}\right)\right] + \frac{h}{n\,R_2}\right\}\right) \\
& \left. + \left[sin^{-1}\left(\frac{h}{R_1}\right) - sin^{-1}\left(\frac{h}{nR_1}\right)\right]\right] \Bigg\} - R_2
\end{aligned}
$$

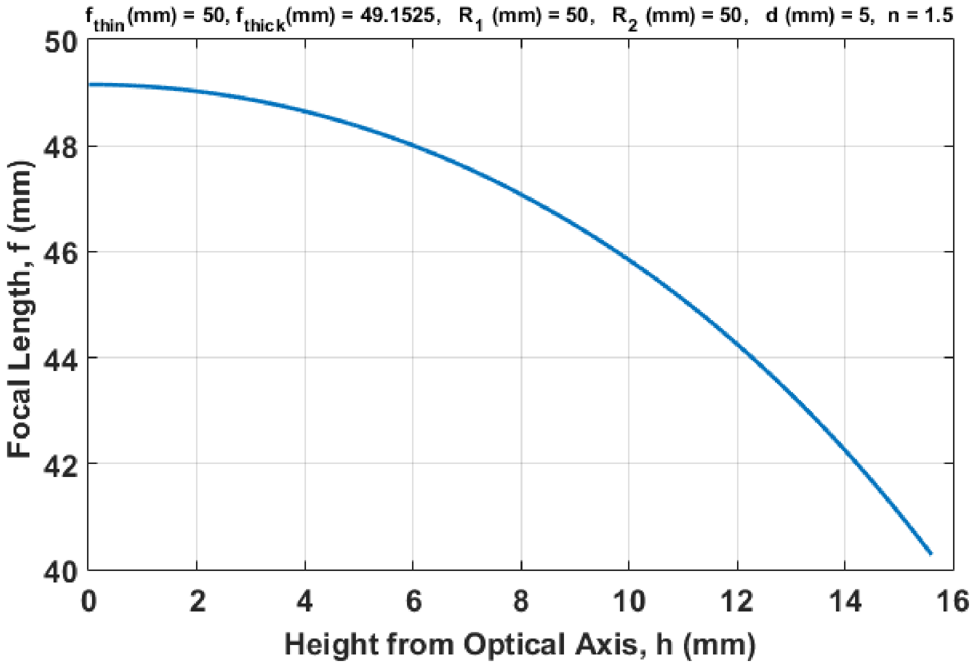


$h(mm)$	$f(mm)$
5.0	48.3
7.5	47.3
10.0	45.8
12.5	43.8
15.0	41.1

Fig. 1. Focusing of rays by biconvex lens showing spherical aberration.

Spherical Aberration of a Biconvex Lens

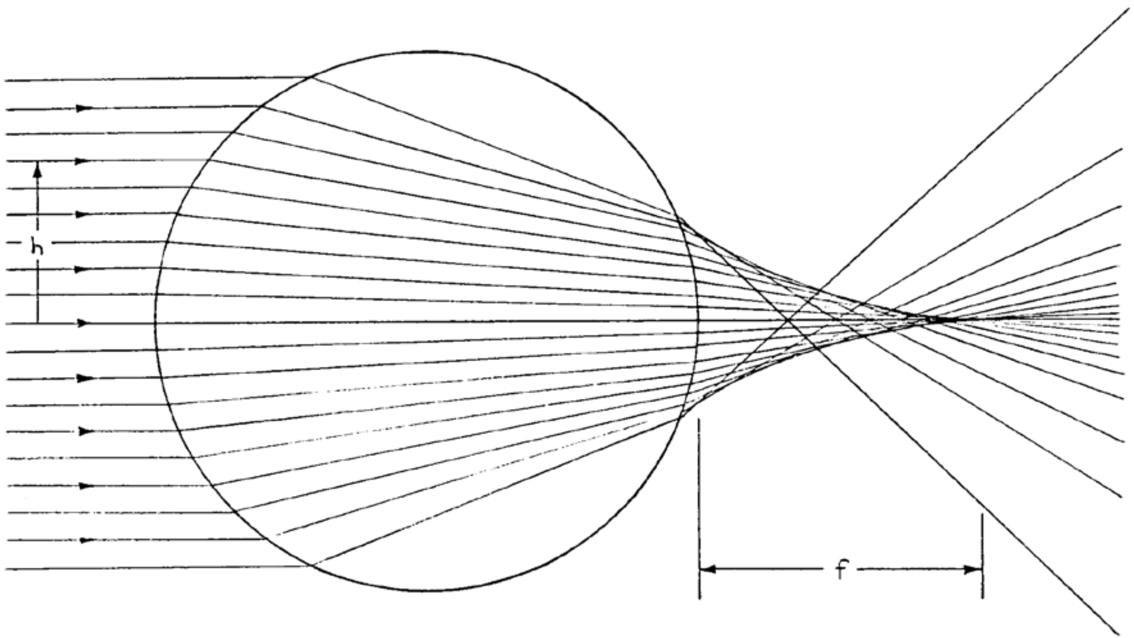




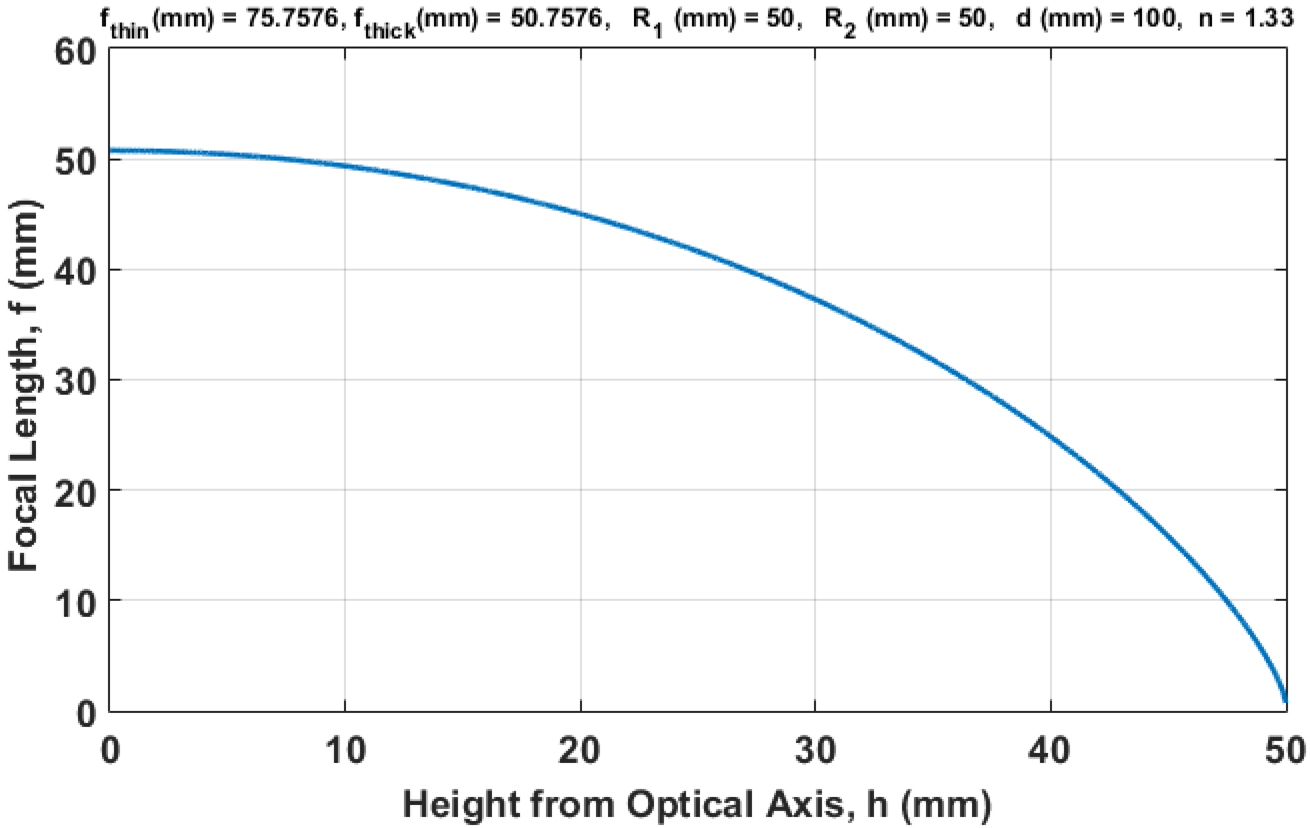
Sphere Lens

A sphere of radius R and an index of refraction n may be used as a lens. The focal distance of this lens in air for an axial ray of distance h from the principal axis is

$$f = R\left\{\left[\left(\frac{h}{r}\right) \Big/ \; sin\left\{2\left[sin^{-1}\left(\frac{h}{R}\right) - sin^{-1}\left(\frac{h}{nR}\right)\right]\right\}\right] - 1\right\}$$

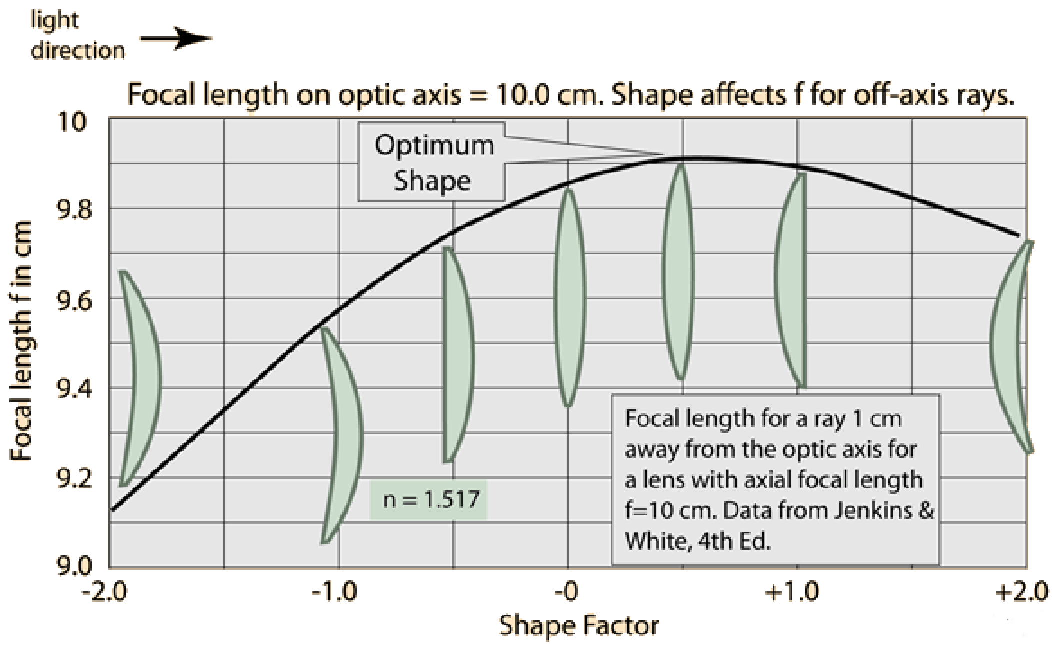


Spherical Aberration of a Dielectric Sphere



h (mm)	f (mm)
5	50.4
10	49.4
15	47.6
20	45.0
25	41.6
30	37.3
35	31.8
40	24.8
45	15.8

Spherical Aberration



http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/imggo/meniscus.gif

Shape Factor

$$\sigma = \frac{R_2 + R_1}{R_2 - R_1}$$

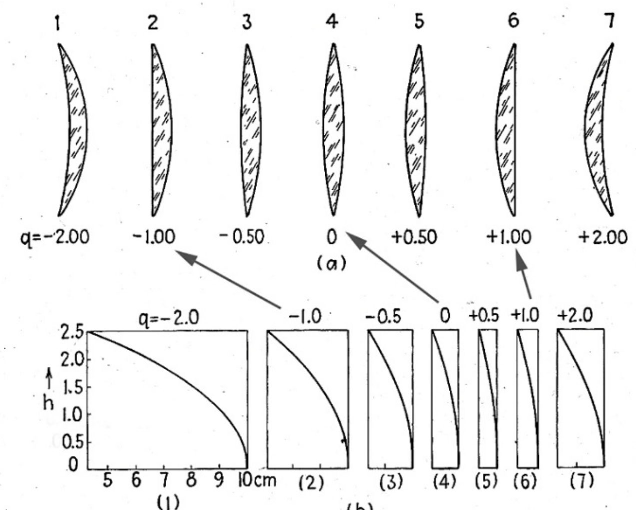
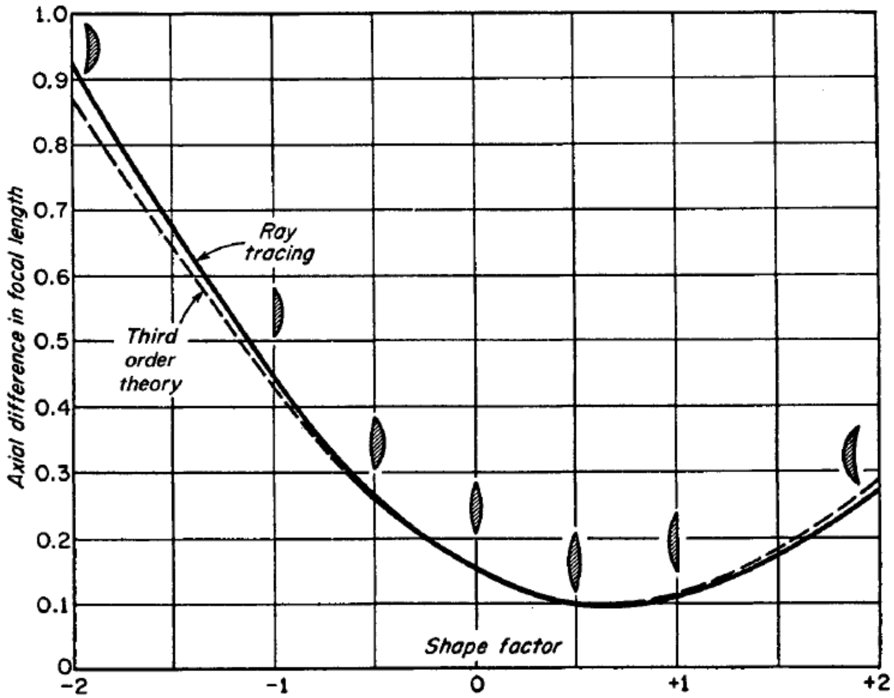


FIG. 9C (a) Lenses of different shapes but with the same power. The difference is one of "bending." (b) Focal length vs. radius for these lenses.

Spherical Aberration (3rd order)

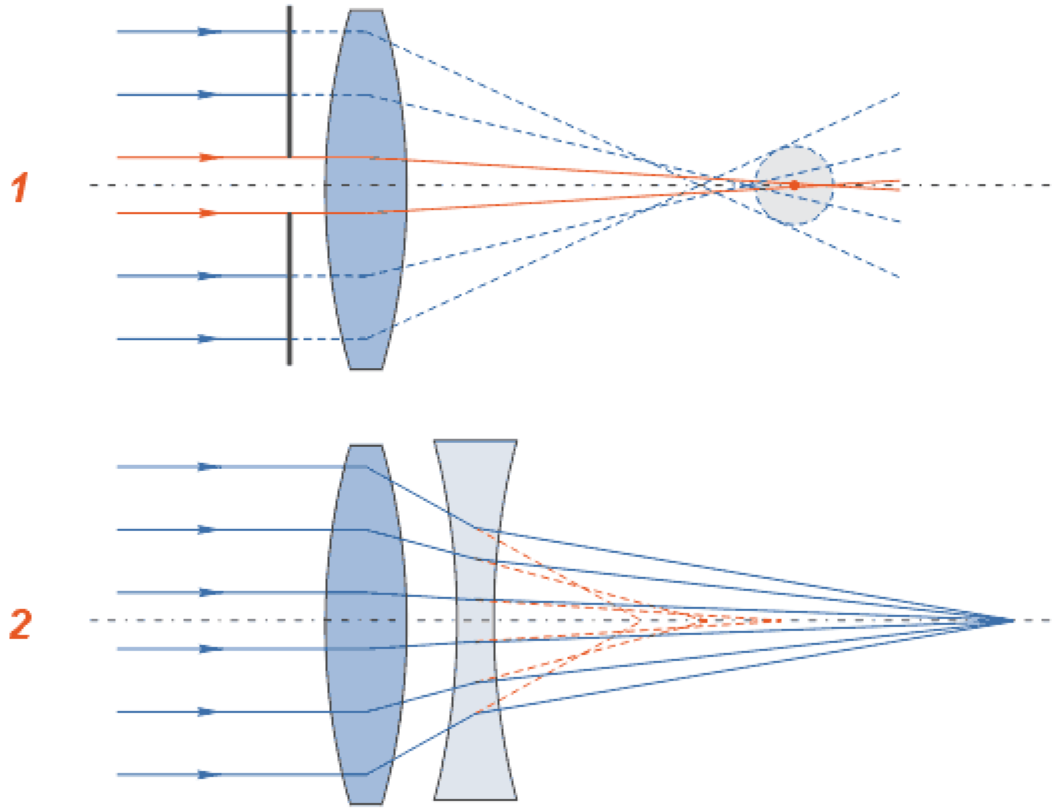
$$\frac{1}{s'(h)} - \frac{1}{s'(0)} = \frac{h^2}{8f^3}\frac{1}{n(n-1)}\left[\frac{n+2}{n-1}\sigma^2 + 4(n+1)p\sigma + (3n+2)(n-1)p^2 + \frac{n^3}{n-1}\right]$$

$$p = \frac{s'-s}{s'+s}\,, \quad \sigma = \frac{R_2+R_1}{R_2-R_1}$$

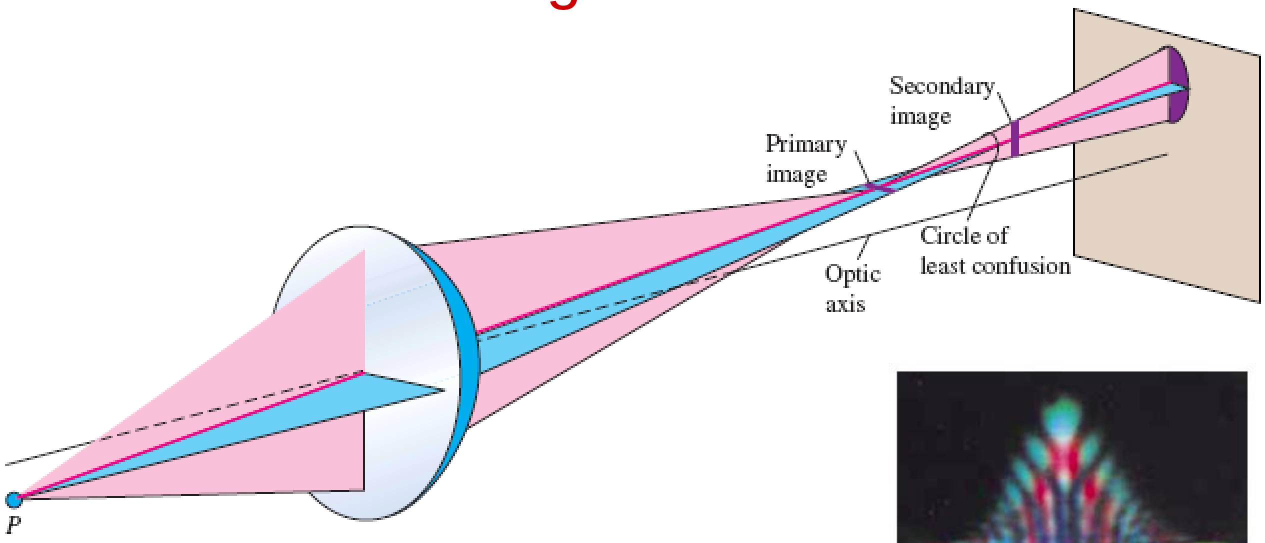


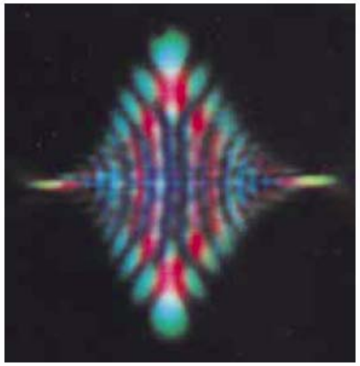
Jenkins and White, "Fundamentals of Optics", 4th Ed., McGraw-Hill, 2001

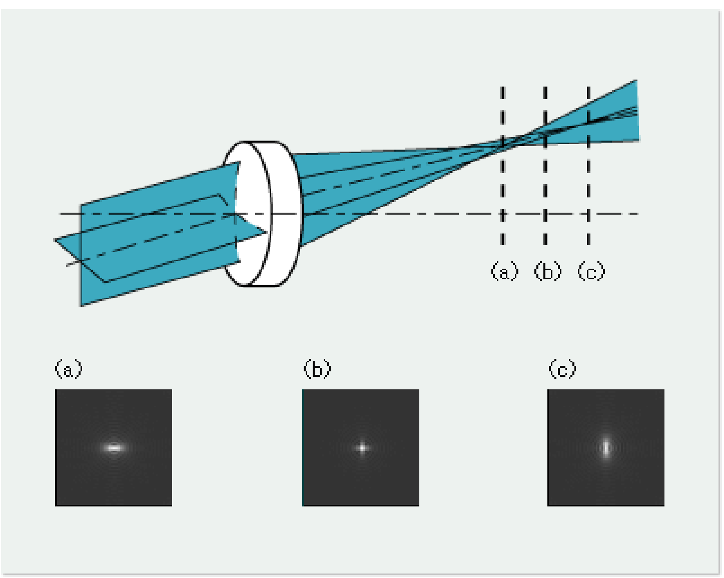
Spherical Aberration Correction

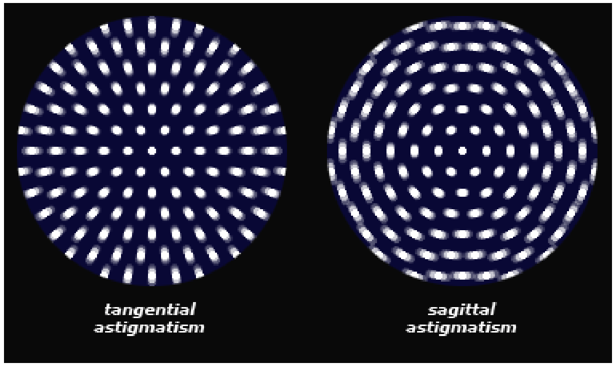


Astigmatism







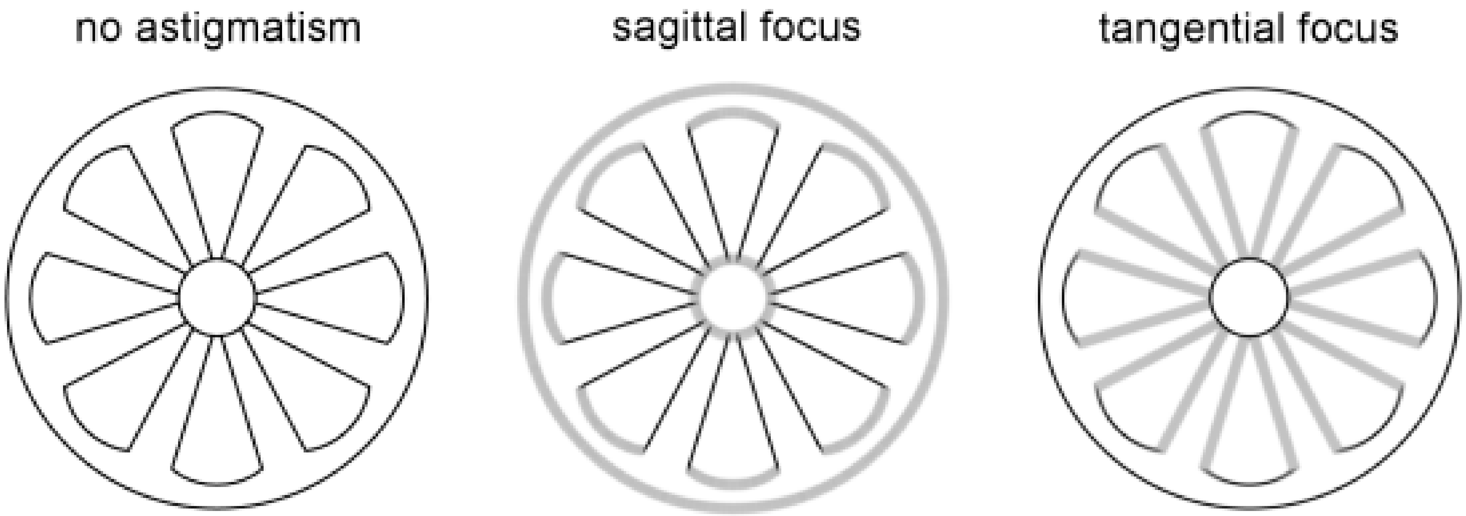


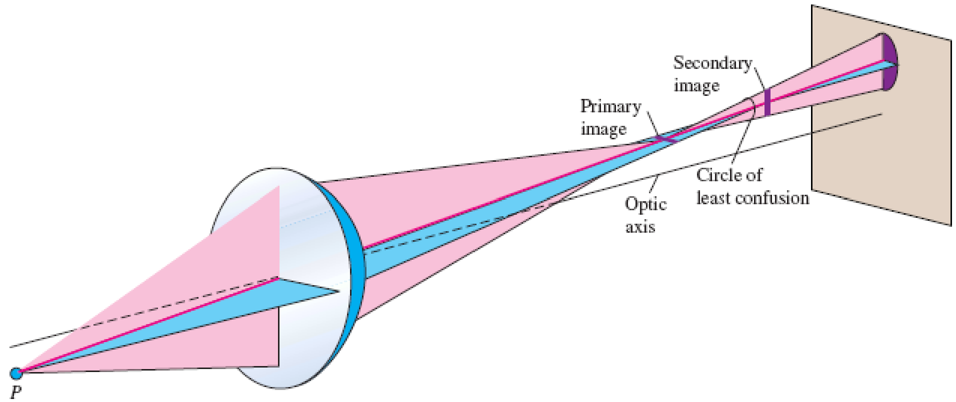
http://www.handprint.com/ASTRO/ae4.html

Astigmatism

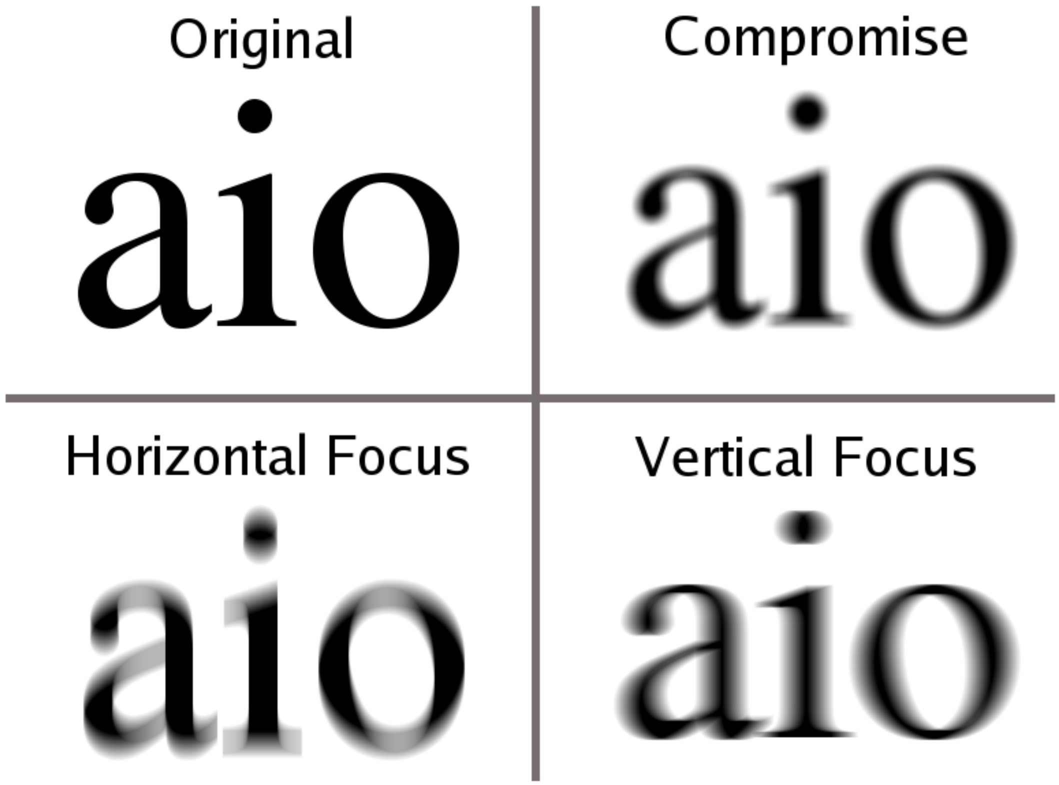
Horizontal Focus

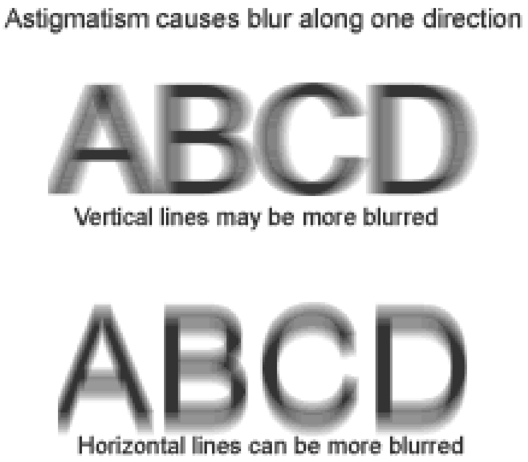
Vertical Focus





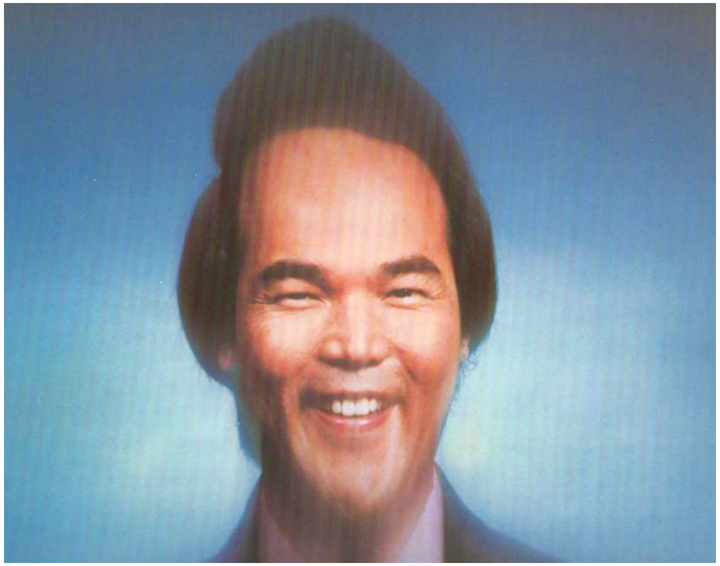
Astigmatism (Examples)

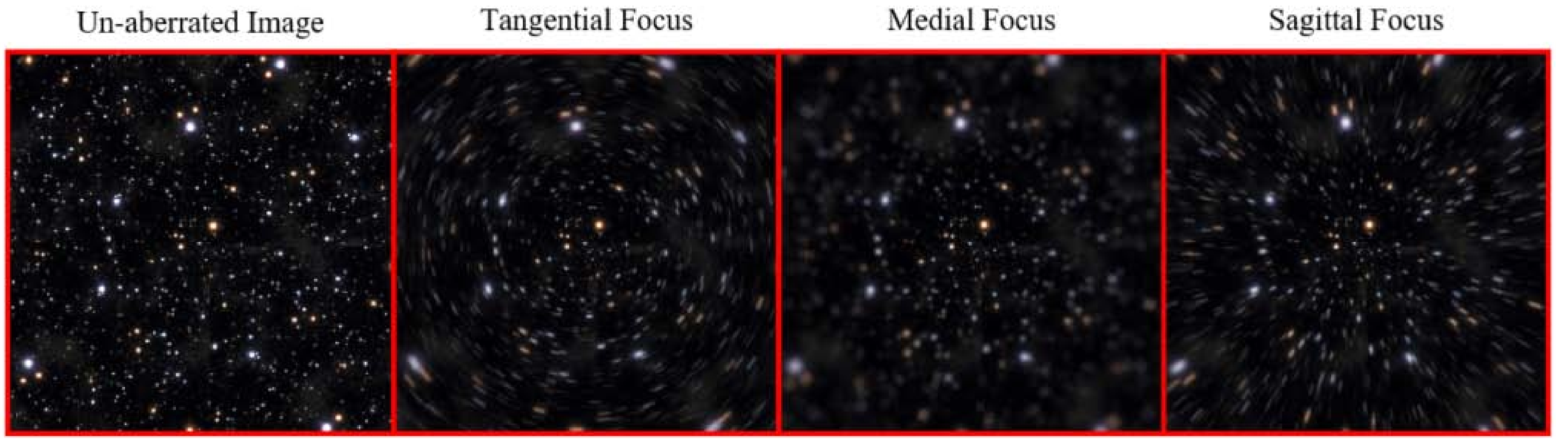




http://acuitylaservision.com/category/all-about-eyes/medical-blog-acuity/page/3/

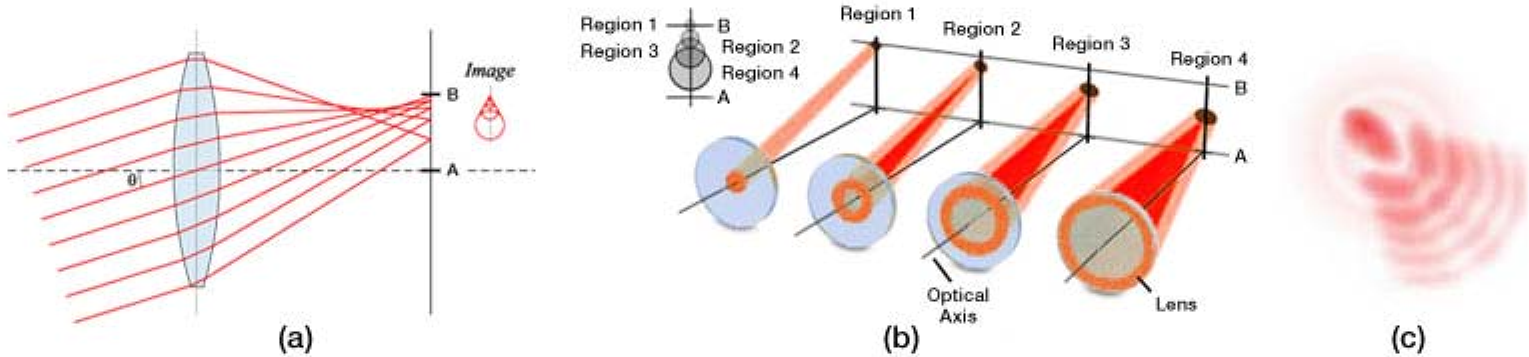
Astigmatism Causes Blurring of the Image





http://eckop.com/wp-content/uploads/2017/05/Fig-1.14-Tangential-and-Sagittal-Foci.png

Coma



http://www.cheyennezjphotographer.com/photography/lenses-2/

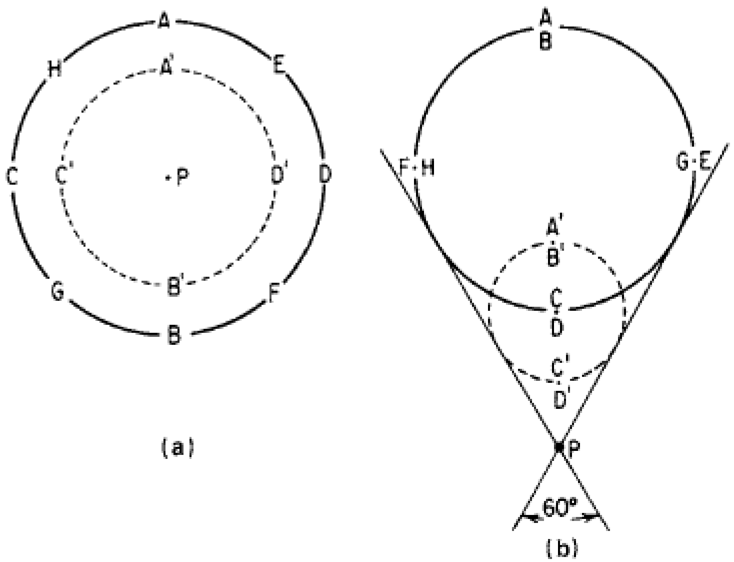
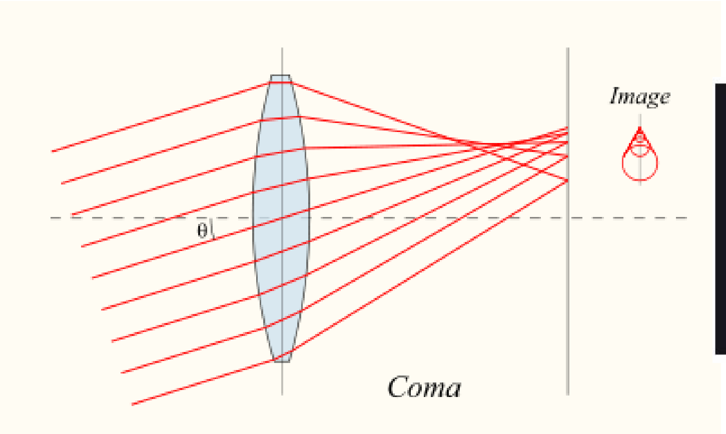
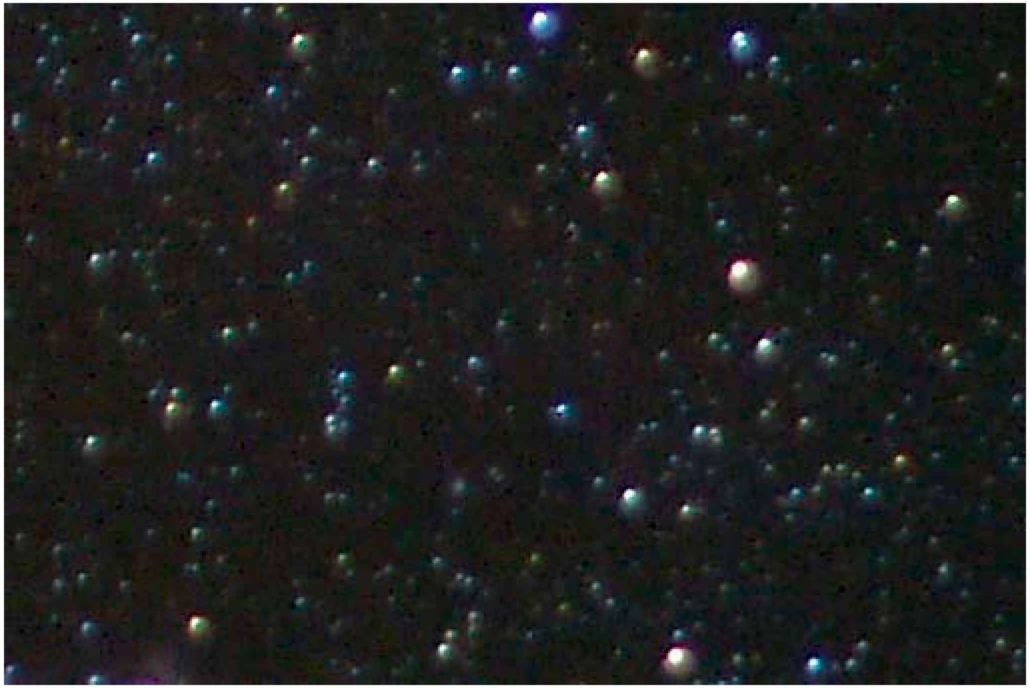


Figure 5.6 The relationship between the position of a ray in the lens aperture and its position in the coma patch. (a) View of the lens aperture with rays indicated by letters. (b) The letters indicate the positions of the corresponding rays in the image figure. Note that the diameters of the circles in the image are proportional to the *square* of the diameters in the aperture.

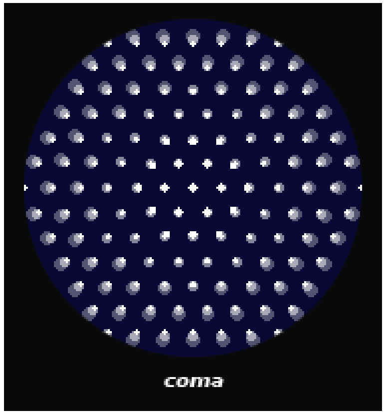


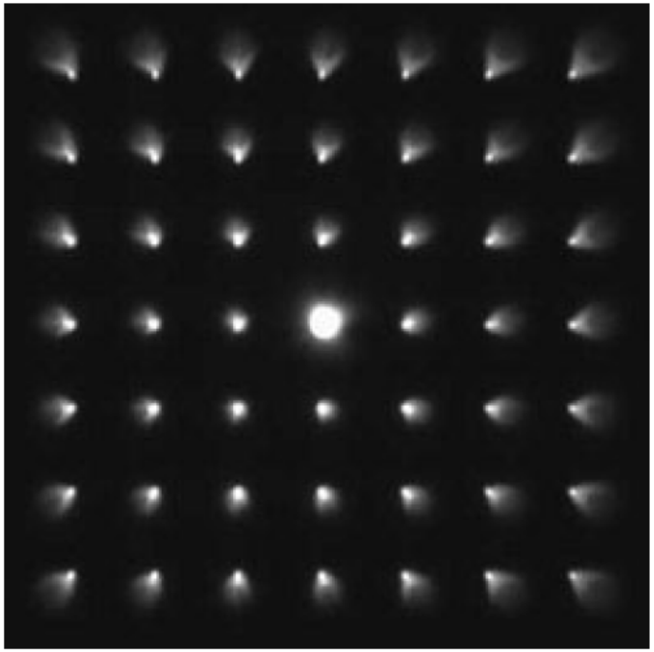
http://en.wikipedia.org/wiki/Lens_(optics)

Coma (Examples)

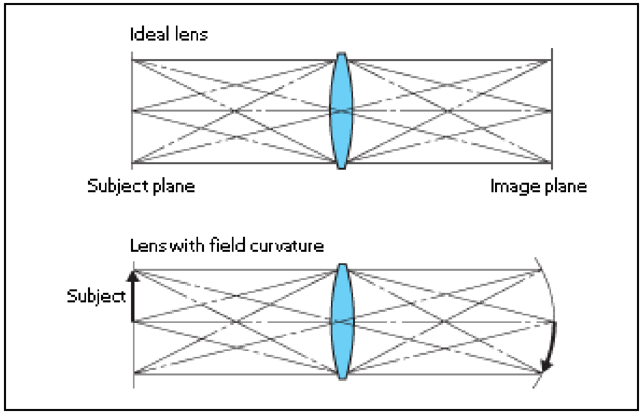


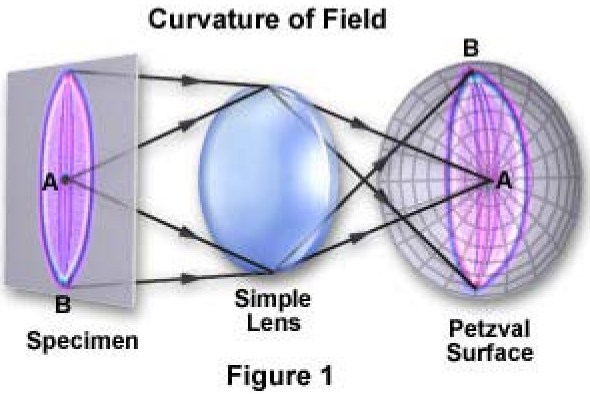
http://www.handprint.com/ASTRO/ae4.html



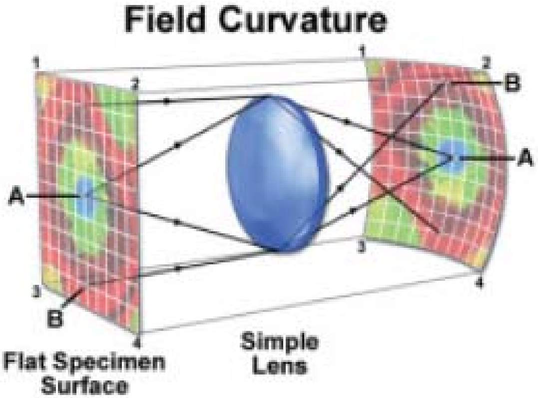


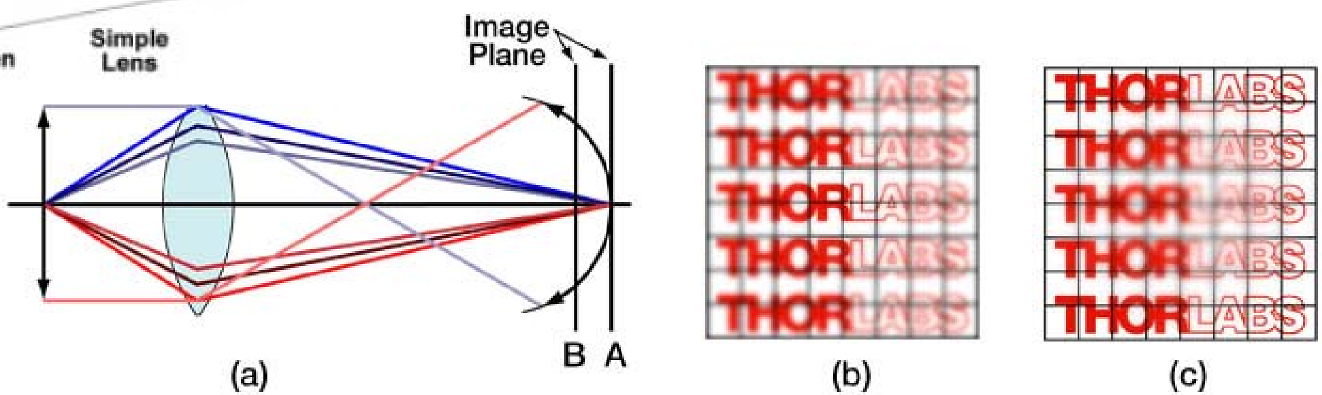
Field Curvature





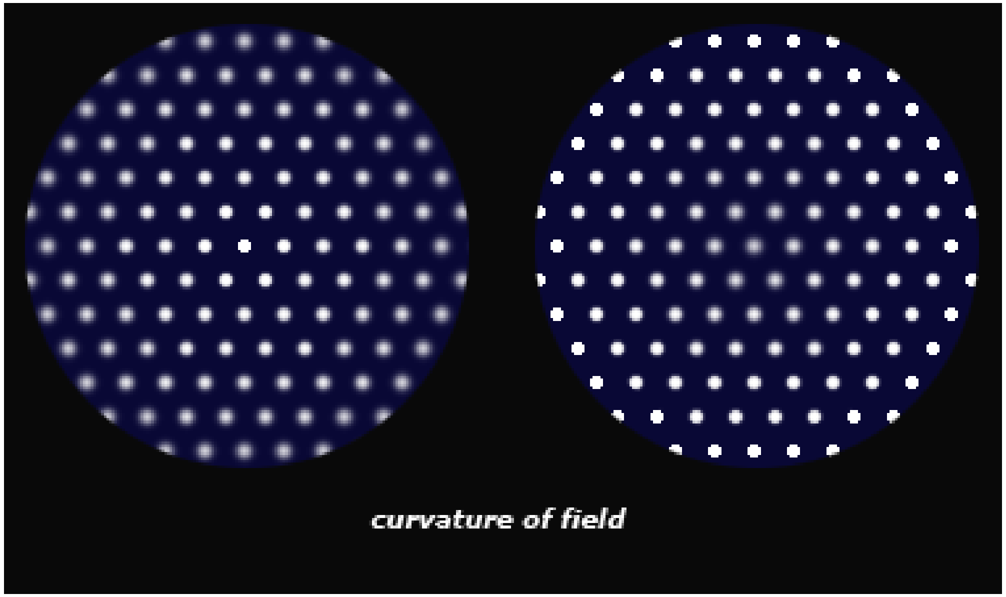
http://www.microscopyu.com/tutorials/java/aberrations/curvatureoffield/





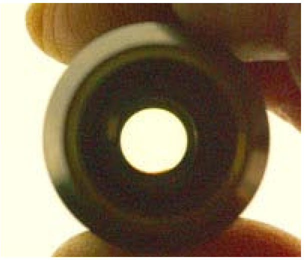
http://www.cheyennezjphotographer.com/photography/lenses-2/

Field Curvature

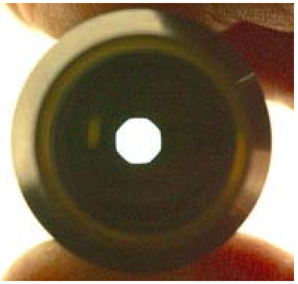


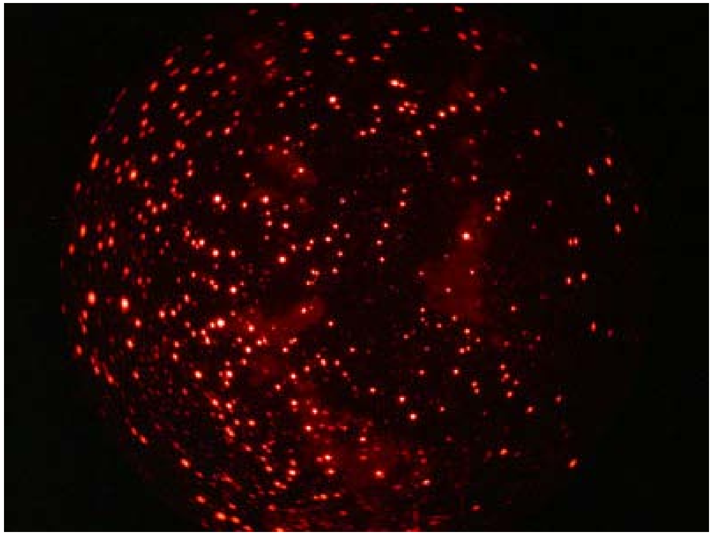
http://www.handprint.com/ASTRO/ae4.html

Field Curvature



Notice that with a closed iris the distortion is greatly reduced, however the light gathering power of the lens is also reduced. Under dim fluorescence conditions the decrease in pixel intensity may be unacceptable.





Distortion

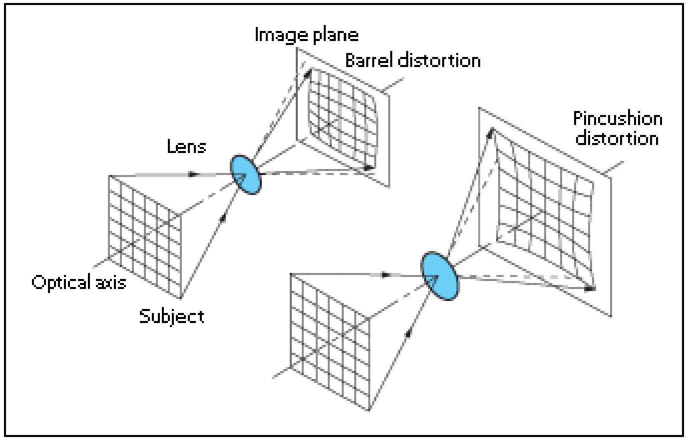
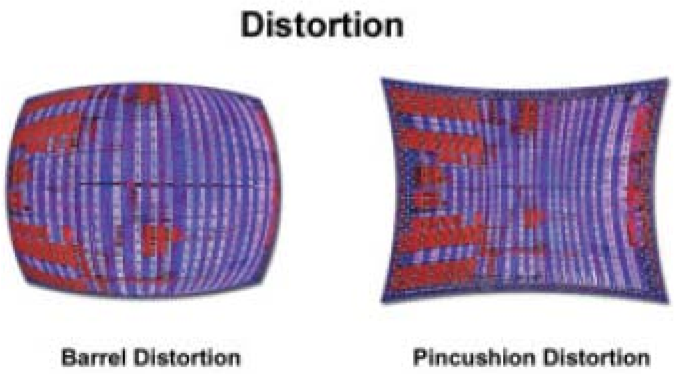
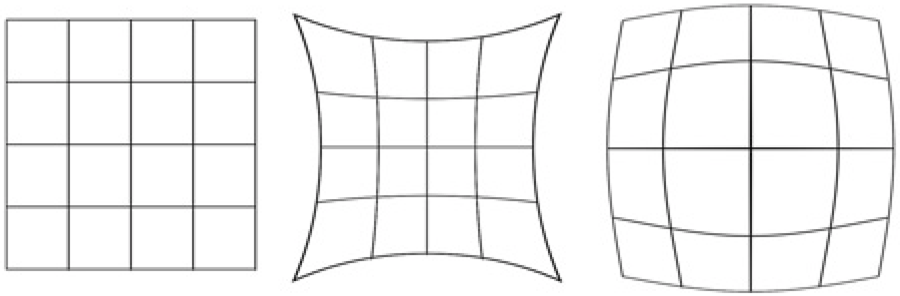


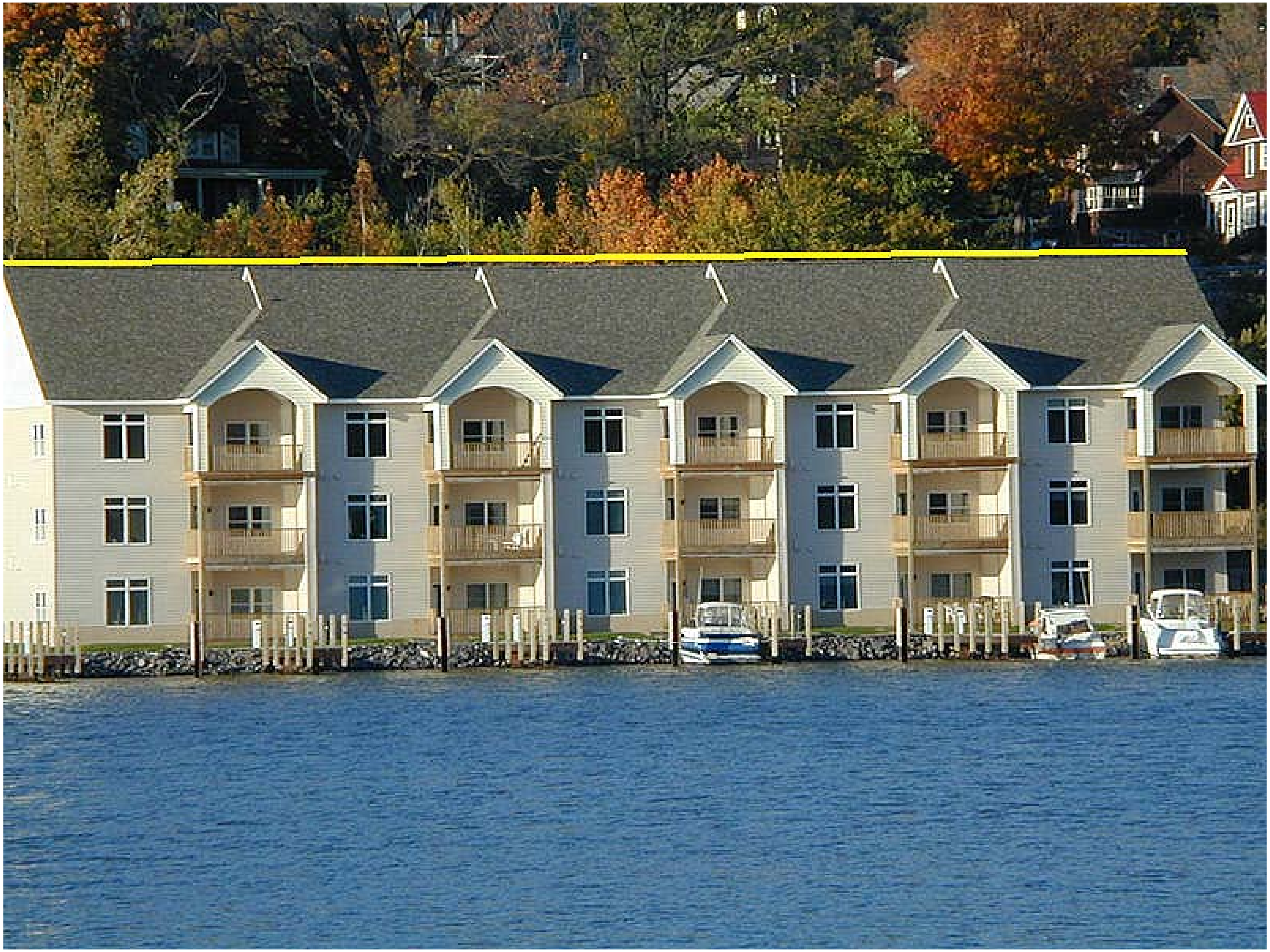
Image plane
Barrel distortion
Pincushion distortion
Lens
Optical axis
Subject



Distortion
Barrel Distortion
Pincushion Distortion

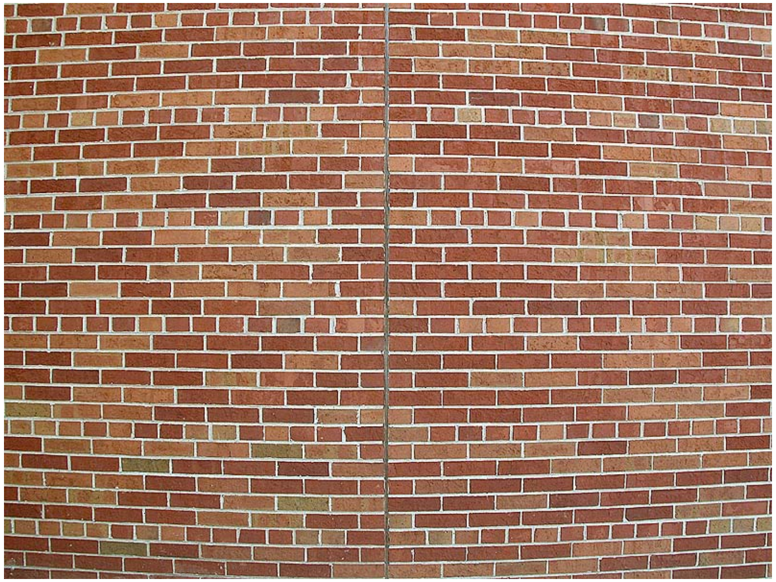


Pincushion distortion aberration



Pincushion distortion aberration

Barrel distortion aberration



Distortion

Pincushion distortion aberration

Barrel distortion aberration

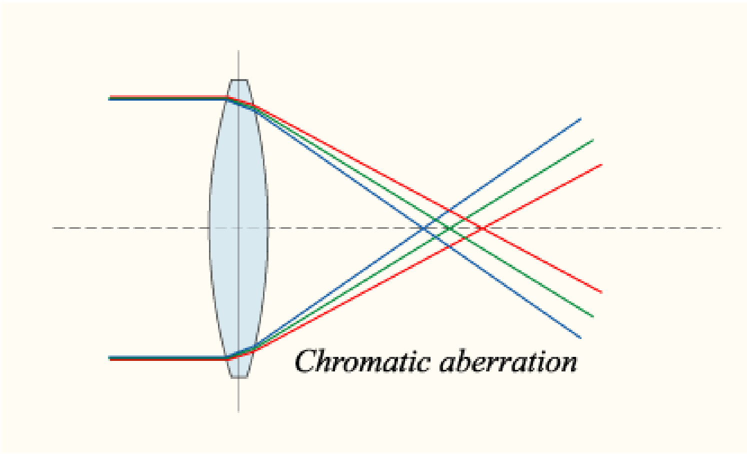
http://www.azurephotonics.com/22.files/image010.jpg

Distortion Aberrations

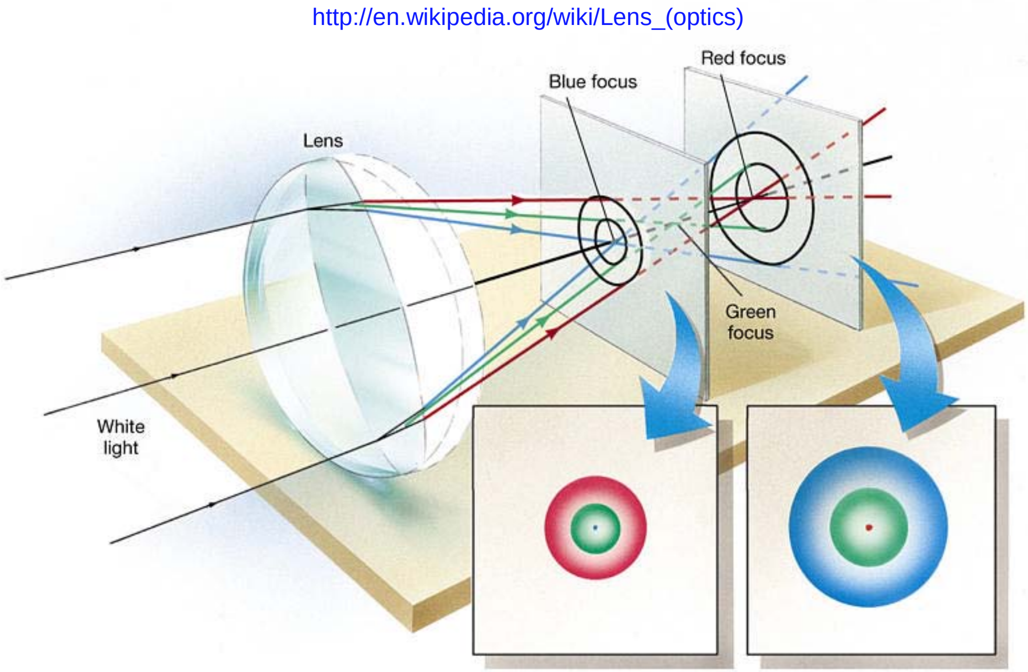
Barrel distortion aberration

Pincushion distortion aberration

Chromatic Aberration

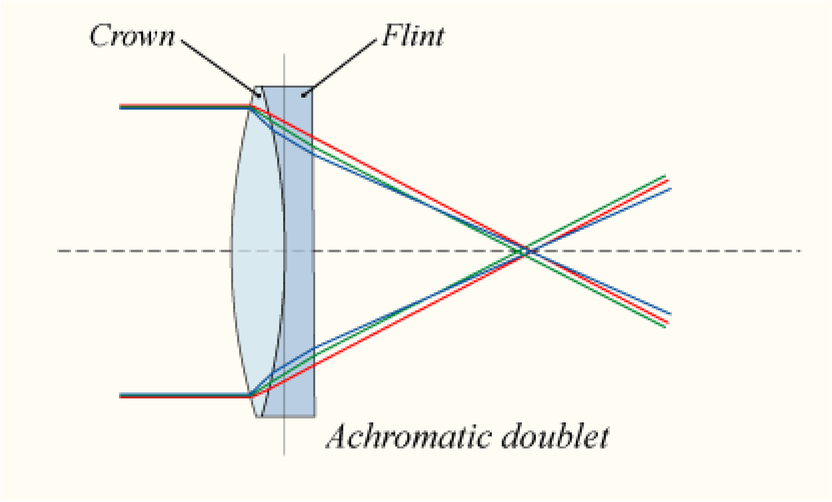


http://en.wikipedia.org/wiki/Lens_(optics)

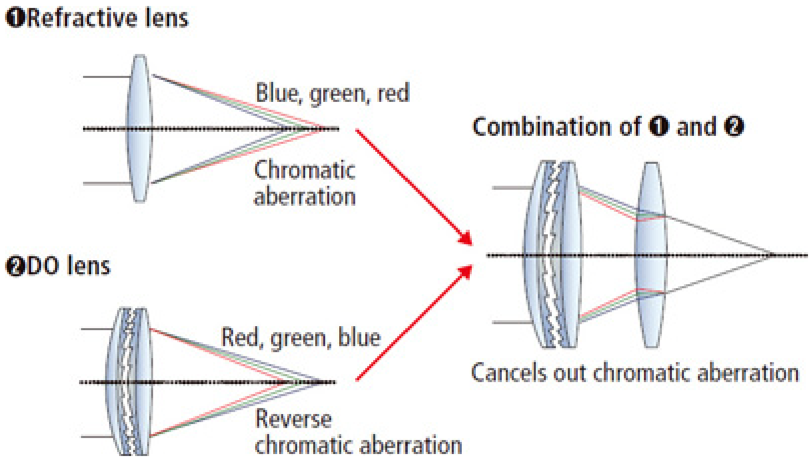


http://pages.uoregon.edu/jimbrau/BraulmNew/Chap05/FG05_05.jpg

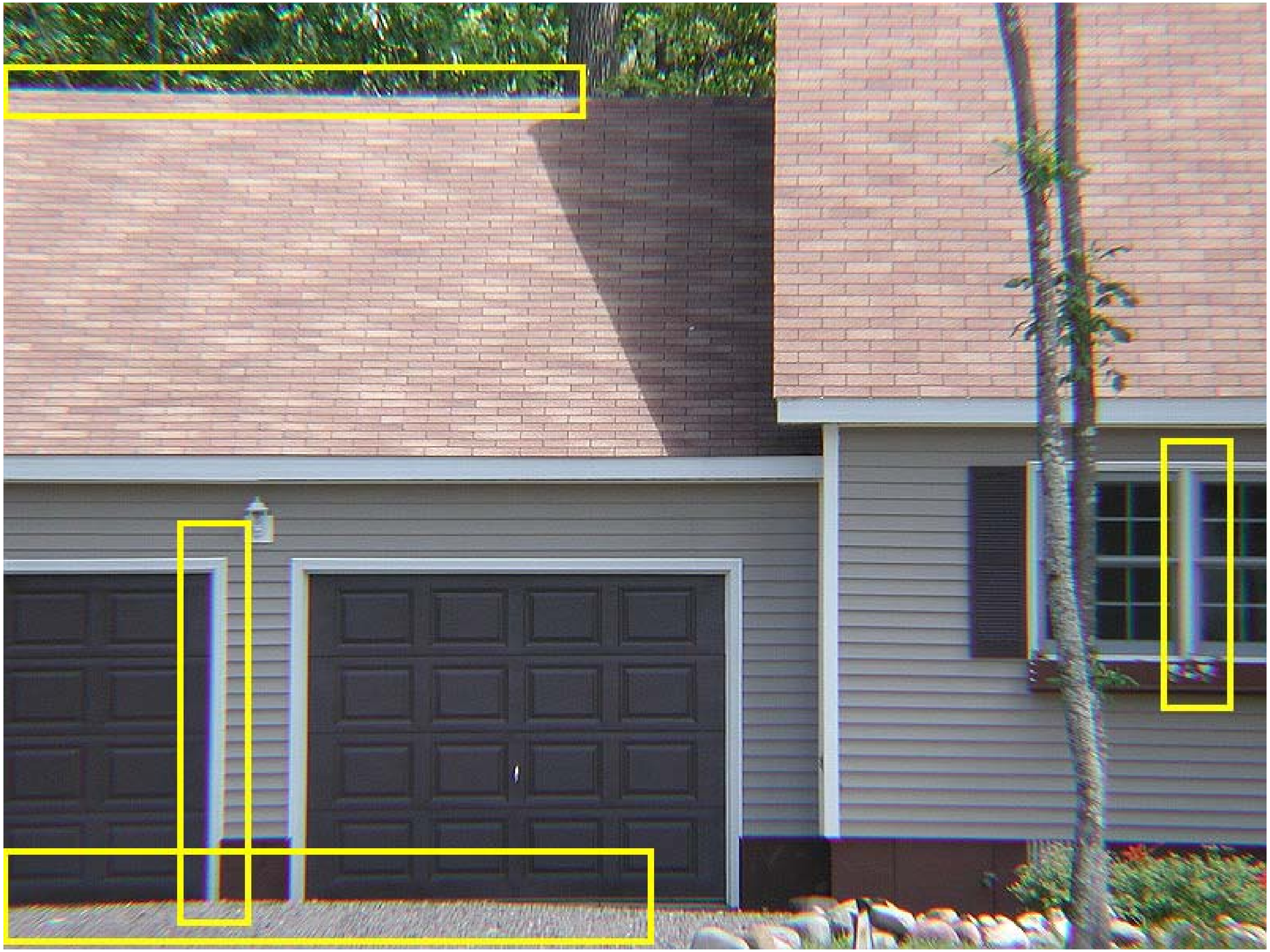
Chromatic Aberration Correction



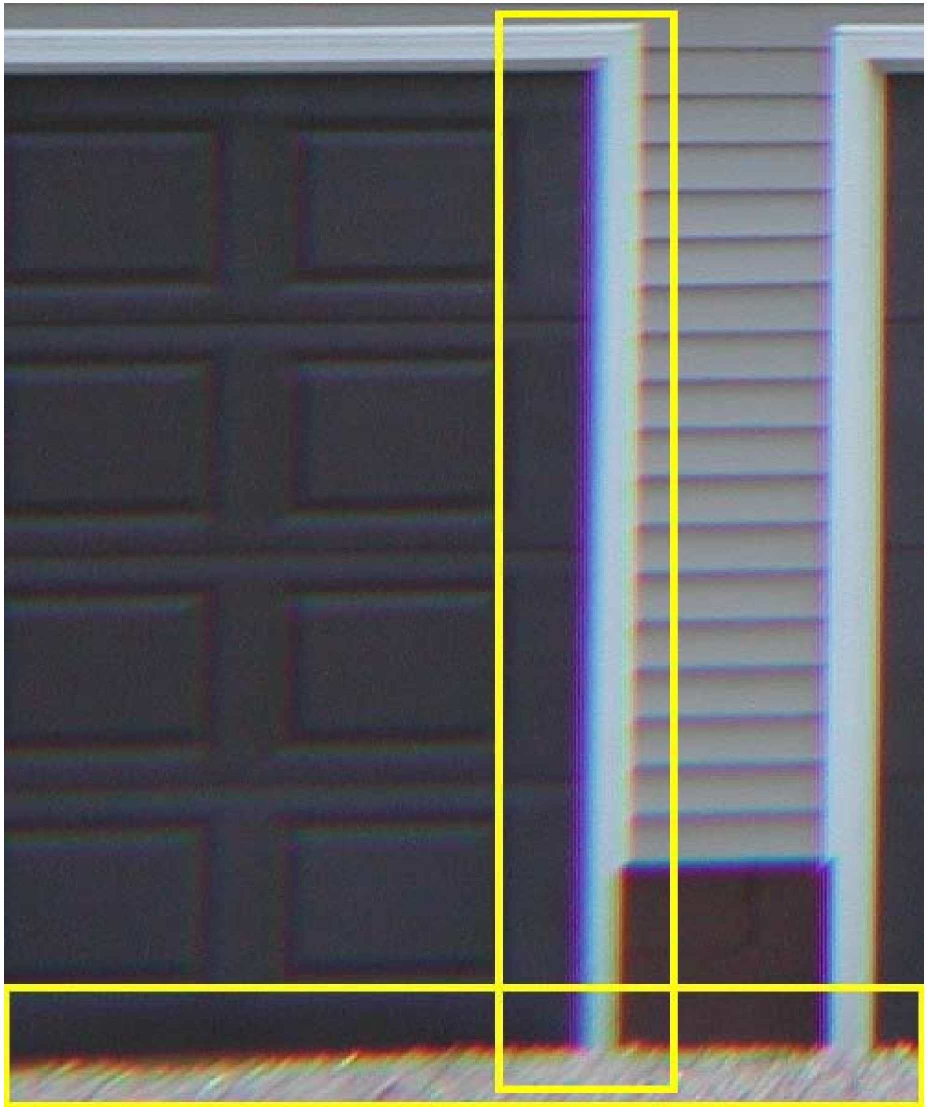
http://en.wikipedia.org/wiki/Lens_(optics)



Chromatic Aberration



Chromatic Aberration



Chromatic Aberration