Spherical Aberration

- An "on-axis" aberration which arises from different radial zones on a optic producing a focus at different distances.

- By its geometrical definition, a parabola is free of spherical aberration (but guilty of others).

CENTER OR PARAXIAL RAYS DETERMINE F.L. and ARE STANDARD OF REFERENCE.

←S.A.→

EDGE RAYS FALL SHORT...THE LENS IS SPHERICALLY UNDER-CORRECTED

SPHERICAL ABERRATION

AS SHOWN, A POSITIVE LENS IS SPHERICALLY UNDER-CORRECTED. S.A. VARIES WITH f/VALUE OF LENS AND IS LESS FOR A SMALL APERTURE

Paraxial focal plane

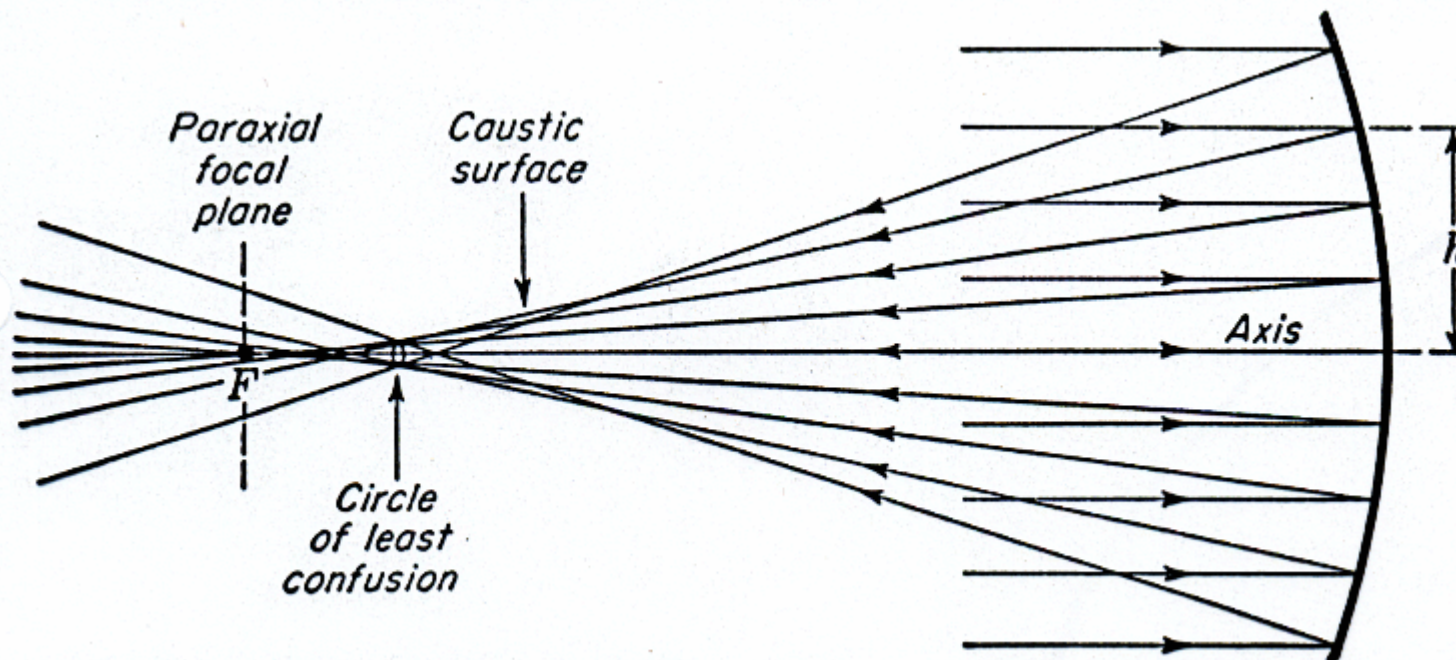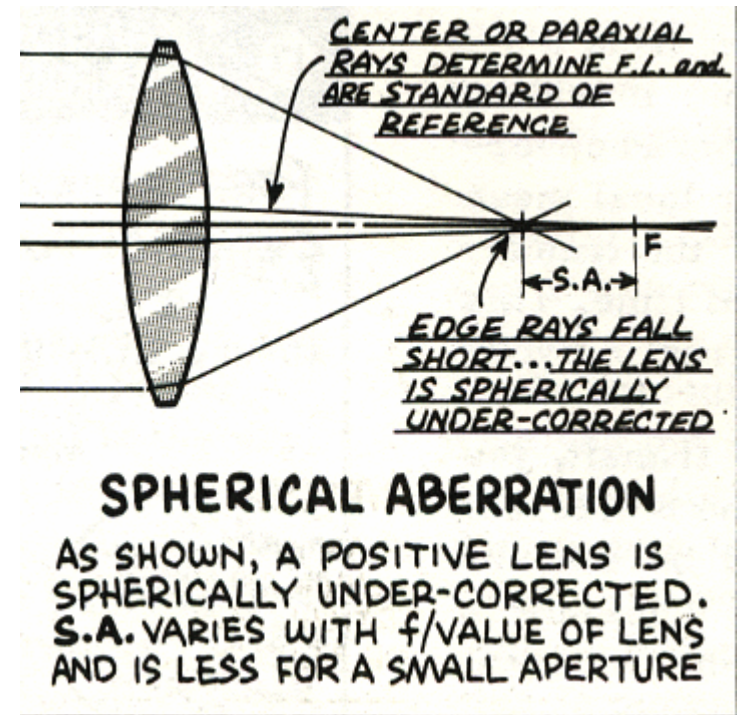Caustic surface

Axis

h

F

Circle of least confusion

FIG. 6K. Spherical aberration of a concave spherical mirror.

Quantifying Aberrations

- The paraxial approximation applies for cases where rays intersect optics at small angle, θ, so that $\sin \theta = \theta$.

- For larger angles, a more exact approximation yields

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \ldots$$

- Classical aberrations of optical systems can be traced to the behavior of the third order term in the expansion in a raytrace.

  - These 5 classical aberrations -- spherical, coma, astigmatism, distortion, and field curvature were enumerated by von Seidel and bear his name.

Quantifying Aberrations – Defining Surfaces

- Actual calculation of ray paths requires the mathematical definition of a surface.

- Spheres are easy...

  - In two dimensions
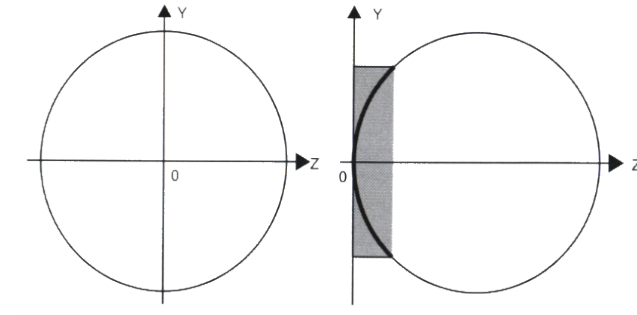
$$y^2 + z^2 = R^2 \qquad z^2 - 2zR + y^2 = 0$$

- More general conic sections are expressed by

$$(1-e^2)z^2 - 2zR + y^2 = 0 \qquad e^2 = \frac{(a^2-b^2)}{a^2} = -K$$
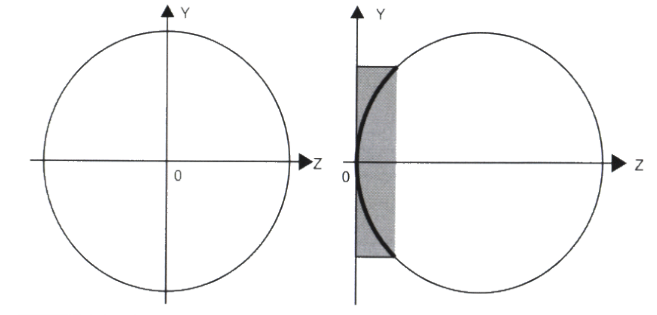
  - K is the conic constant (0=circle, 1=parabola, >1=hyperbola, 0-1 = ellipse)

Quantifying Aberrations – Defining Surfaces

- Solving the quadratic parameterizes z as a function of y – the shape of the surface as a function of ray height

$$(1-e^2)z^2 - 2zR + y^2 = 0$$

$$z = \frac{R}{P}\left[1 - \sqrt{1 - P\left(\frac{y}{R}\right)^2}\right] \qquad P=(1-e^2)$$

- which can be binomial expanded to

$$z \approx \frac{y^2}{2R} + \frac{Py^4}{8R^3} + \frac{P^2 y^6}{16R^5} + \frac{5P^3 y^8}{128R^7} + \ldots$$

- if P=1 this is the (messy) definition of a sphere

- deviations from spherical shape can be defined in terms of P (conics) or in terms of multipliers of the higher order even terms in y (aspheric coefficients).

Conic Sections

- The simplest surface to describe and manufacture is a sphere.

- Significant control of aberrations can be obtained by modifying spherical surfaces.

- Nearly perfect systems may be prescribed by using aspheres but they may be impossible to manufacture.

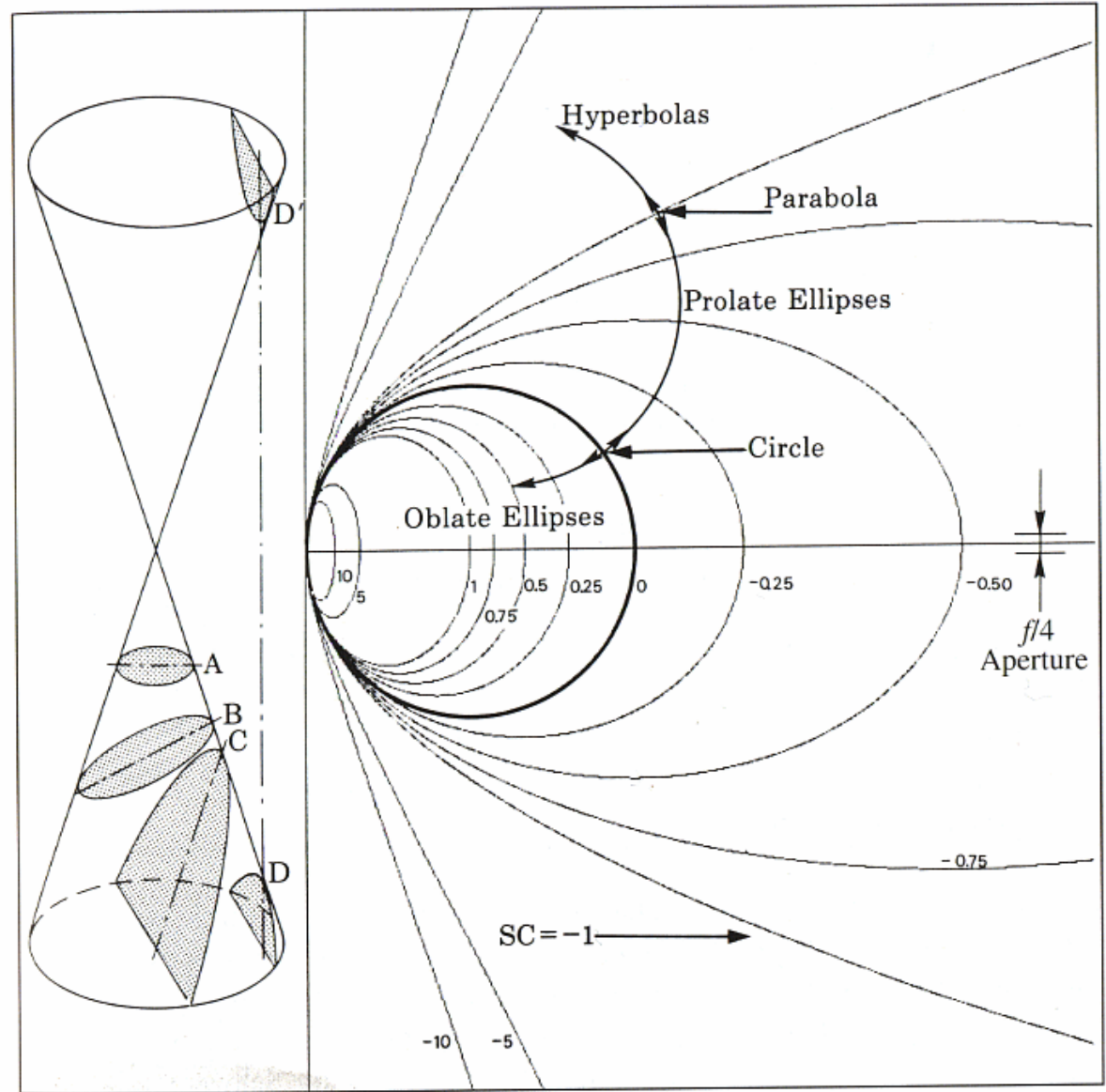- The most common aspheres are conics (parabolas, hyperbolas and ellipses)

Fig. 20.1 *Conic Sections.*

General Third Order Aberrations

Expanding beyond the linear approximation give the third-order Seidel aberration terms. The angular aberrations are:

$$AA = a_s \frac{y^3}{R^3} + a_c \frac{y^2\theta}{R^2} + a_a \frac{y\theta^2}{R} + a_{fc} \frac{y\theta^2}{R} + a_d\theta^3$$

- spherical aberration
- coma
- astigmatism
- field curvature
- distortion

AA = angular aberration (e.g. arcsec or radians)

a_i = constants

R = radius of curvature

y = height of ray

θ = angle of incidence of rays from object

Taking $R = f_n y$ then:

$$AA_{max} = \frac{a_s}{f_n^3} + a_c \frac{\theta}{f_n^2} + a_a \frac{\theta^2}{f_n} + a_{fc} \frac{\theta^2}{f_n} + a_d\theta^3$$

- spherical aberration
- coma
- astigmatism
- field curvature
- distortion

AA_{max} = AA for $y = y_{max} = D$
f_n = f-number
D = aperture diameter

Note: the "faster" the optical system, the greater the aberrations!

Herter and Stacey, Cornell University

Spherical Aberration

- An "on-axis" aberration which arises from different radial zones on a optic producing a focus at different distances.

- By its geometrical definition, a parabola is free of spherical aberration (but guilty of others).
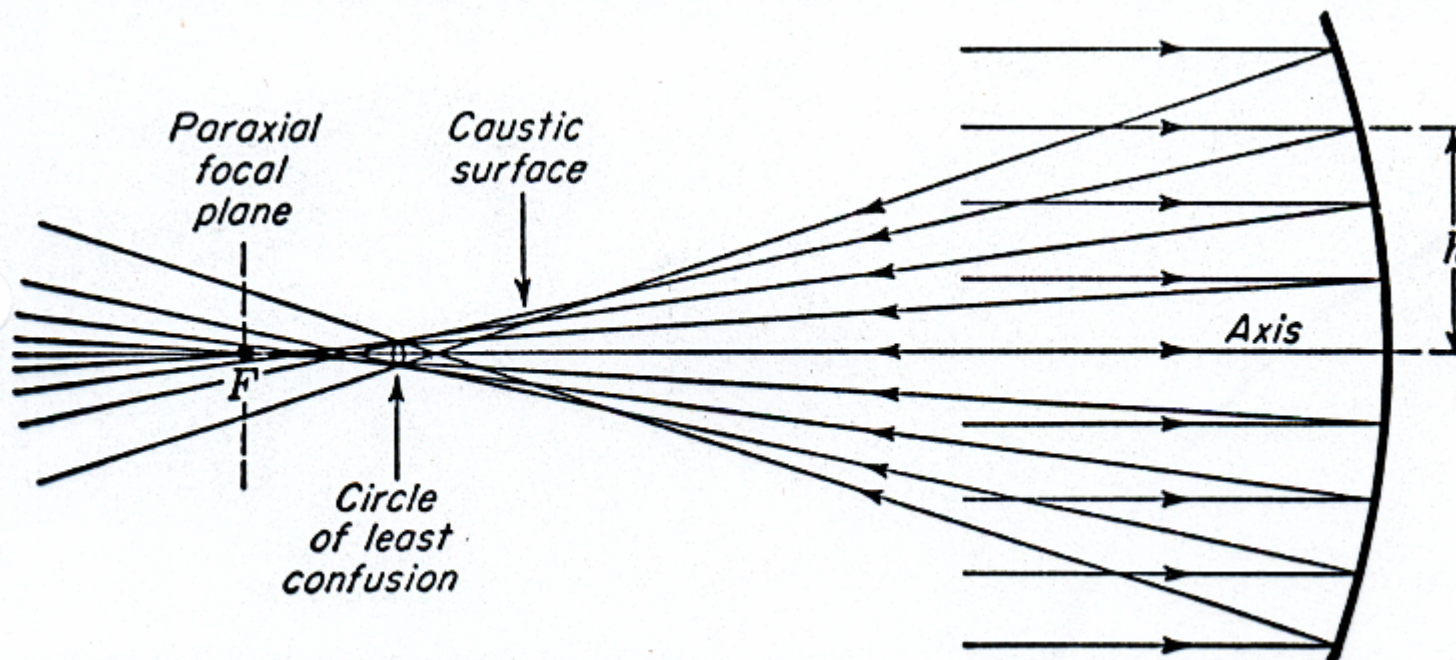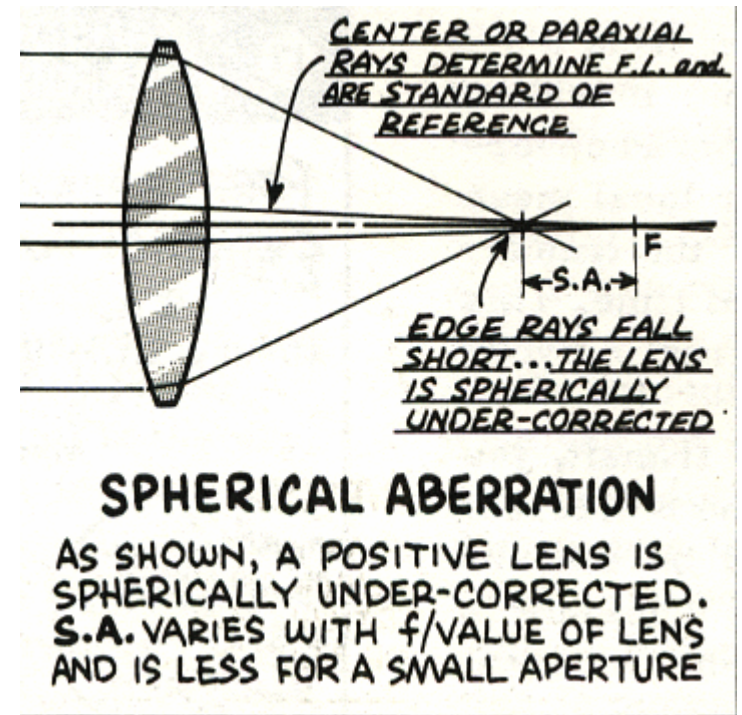

CENTER OR PARAXIAL RAYS DETERMINE F.L. and ARE STANDARD OF REFERENCE

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SPHERICAL ABERRATION

AS SHOWN, A POSITIVE LENS IS SPHERICALLY UNDER-CORRECTED. S.A. VARIES WITH f/VALUE OF LENS AND IS LESS FOR A SMALL APERTURE

Paraxial focal plane

Caustic surface

F

Circle of least confusion

h

Axis

FIG. 6K. Spherical aberration of a concave spherical mirror.

Spherical Aberration

- Spherical aberration can be quantified in three ways.

  - Transverse spherical aberration -- the ray height of the most offensive ray at the paraxial focal plane

  - Longitudinal spherical aberration -- the separation between "worst" and paraxial focus on the optical axis

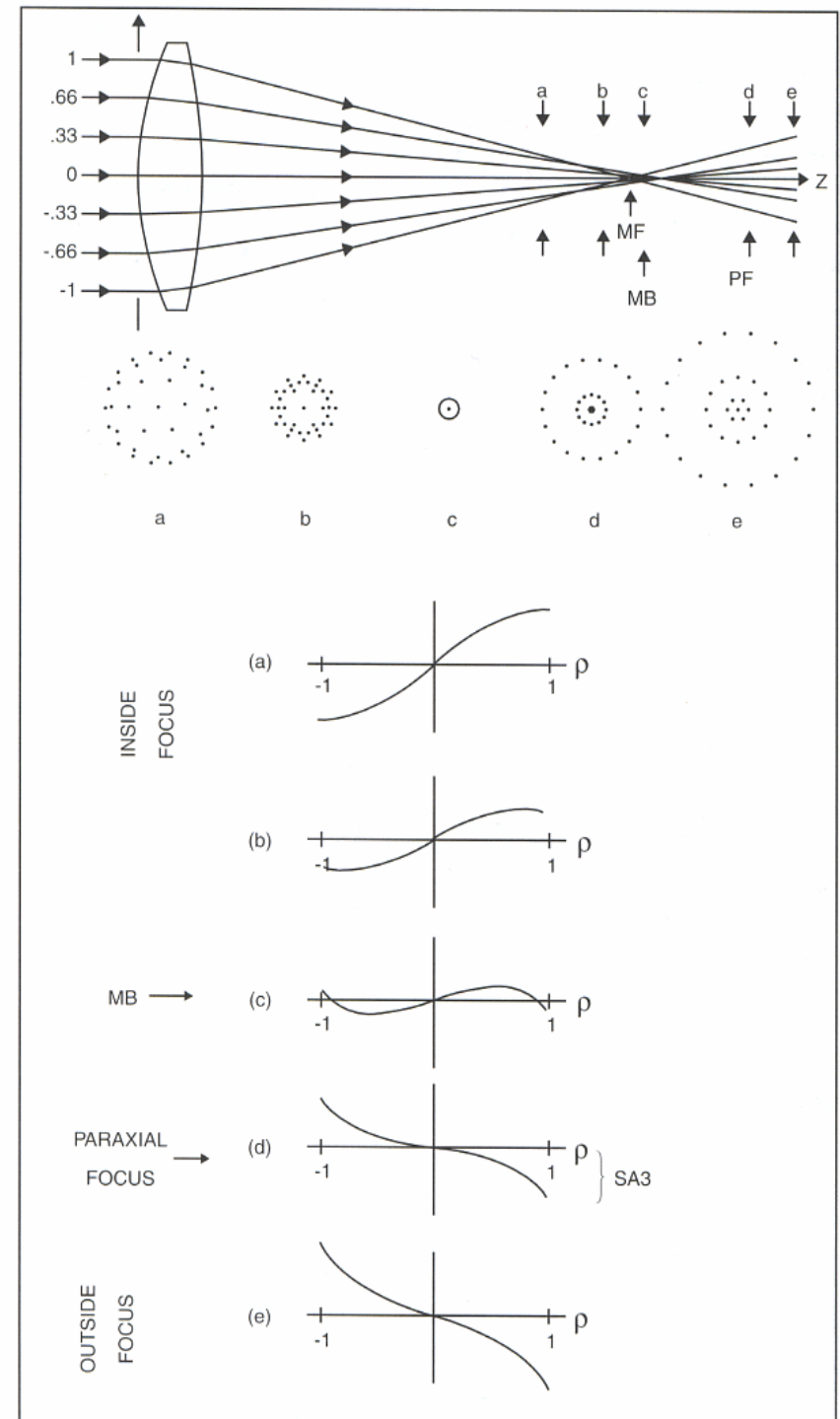 - The diameter of the circle of least confusion – possibly of greatest interest to an observer.

Fig. 7.6 *Spherical Aberration: meridional plane (top); spot diagrams (center); ray fan plots (bottom). Reprinted with permission from Rutten and van Venrooij,* Telescope Optics (Willmann-Bell, 1988).

Introduction to Lens Design (Geary)

Transverse Spherical Aberration

- Transverse SA is measured in terms of the most deviant ray height at the paraxial focus.

- The equation gives the TSA for a mirror of radius, R, with conic constant, K, as a function of ray height, y, from the optical axis.
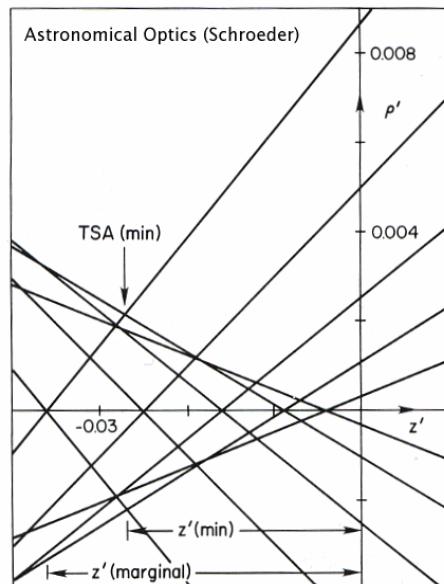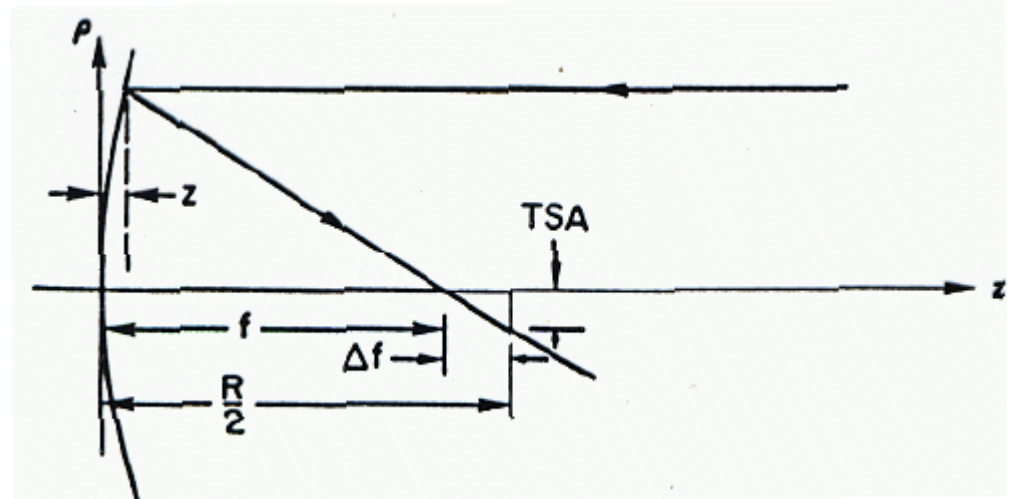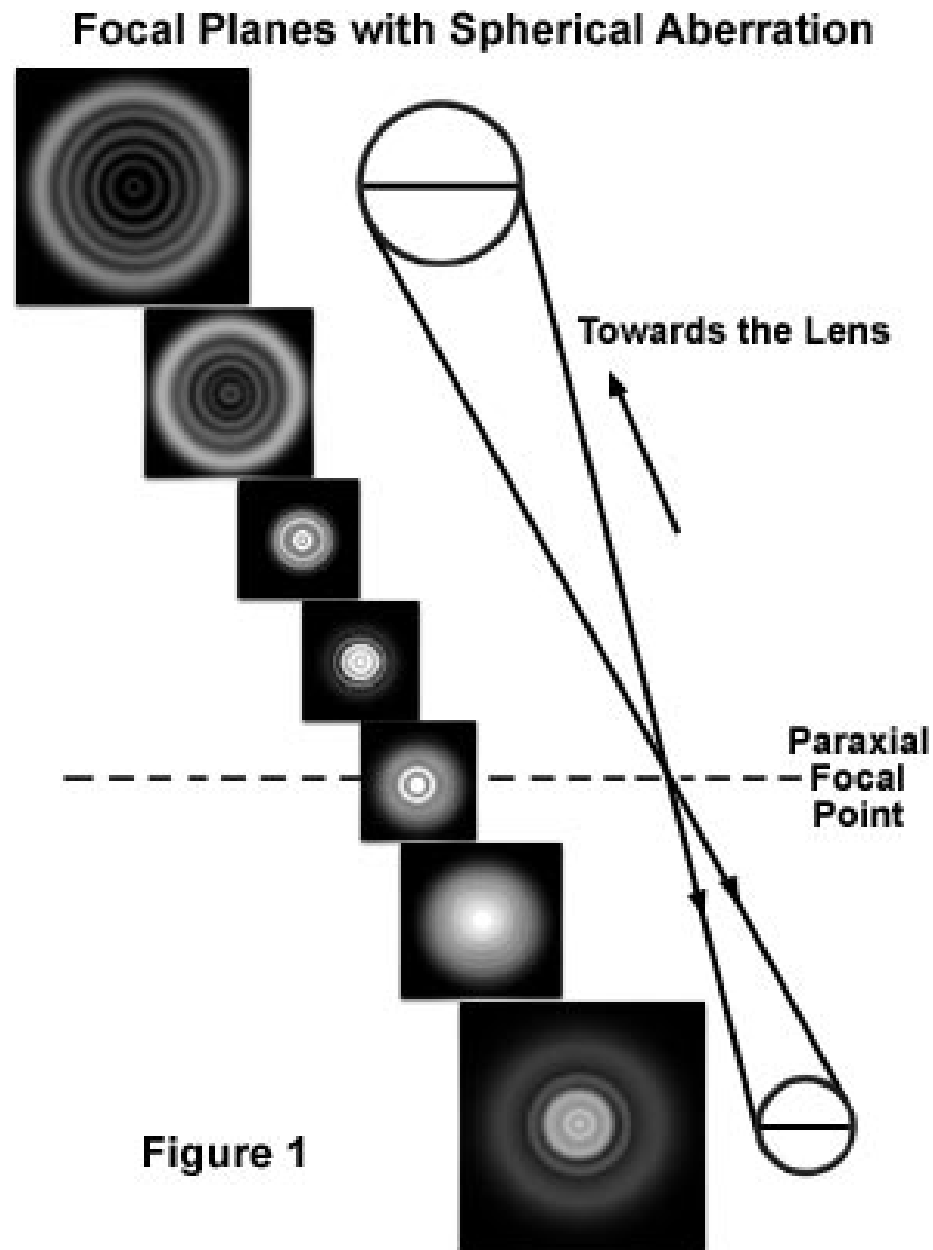

Fig. 4.5. Ray distribution near paraxial focus for image with spherical aberration. Paraxial focus is at (0, 0). See Eq. (4.2.8) for definition of parameters.

$$TSA = -(1+K)\frac{y^3}{2R^2} - 3(1+K)(3+K)\frac{y^5}{8R^4}$$
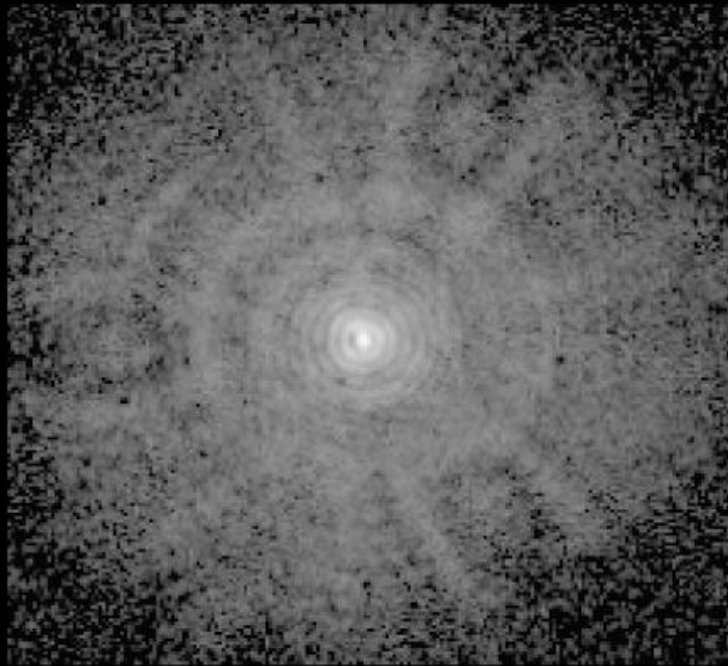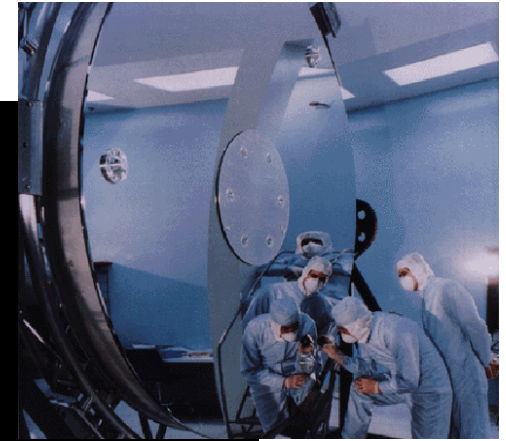
Recognizing Spherical Aberration

Focal Planes with Spherical Aberration

Towards the Lens

Paraxial Focal Point

Figure 1

Recognizing Spherical Aberration
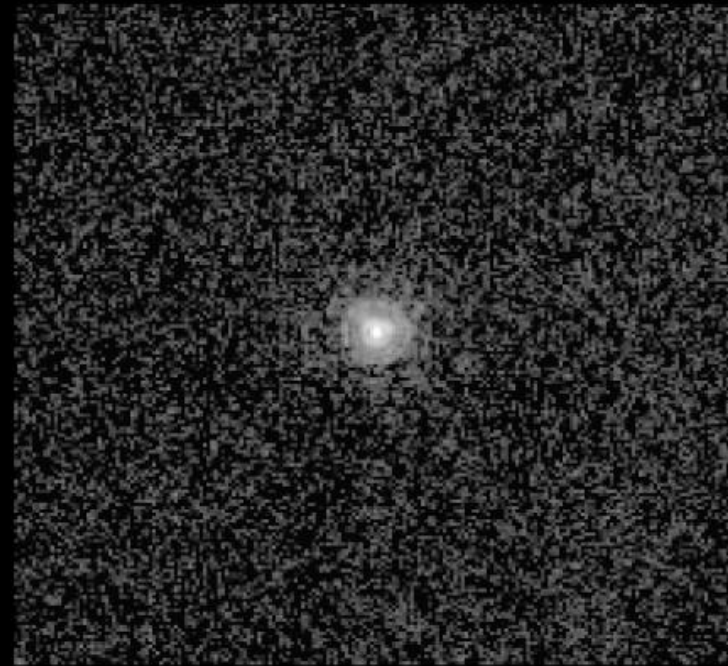

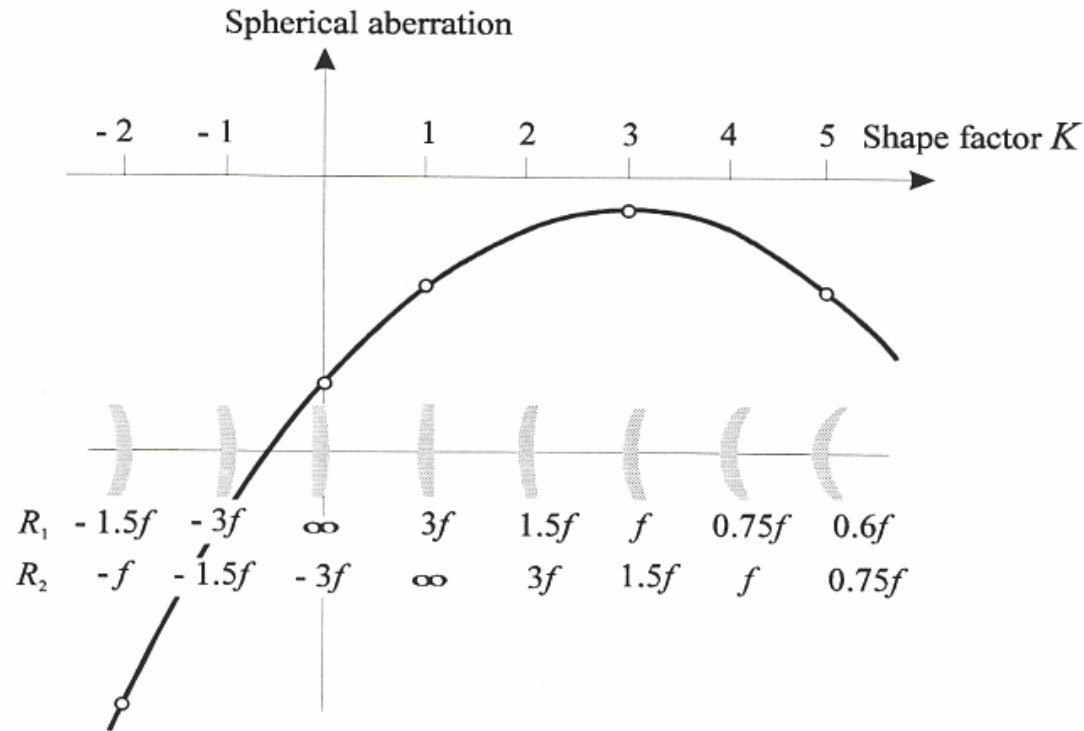HUBBLE SPACE TELESCOPE

FAINT OBJECT CAMERA

COMPARATIVE VIEWS OF A STAR

BEFORE COSTAR

AFTER COSTAR

Optical Design and Spherical Aberration Mitigation

FIG. 3.10 Change of spherical aberration with change of lens shape (lens bending) for a germanium lens with index $N = 4$. Minimum spherical aberration is obtained in this case when the first radius of the lens is equal to its focal length and the second radius is 1.5 times the focal length ($K = 3$).

Optical Design Fendamentals for Infrared Systems (Riedl)

- The power of optical design is illustrated by the control of spherical aberration provided by altering lens shape (a.k.a. "bending").

  - All of the illustrated lenses have the same focal length.

Coma

- Coma arises when incident rays are not parallel to the optical axis/normal.

    - Like spherical aberration, coma is manifested by different radial zones in the optic

    - Unlike spherical aberration, the images produced by the zones are in sharp focus at the paraxial focal plane.

- Each pair of symmetric points in each radial zone produces a sharp image, but since the lateral magnification is different for each pair each ring of incident rays forms an offset ring producing the classic "comma" image.
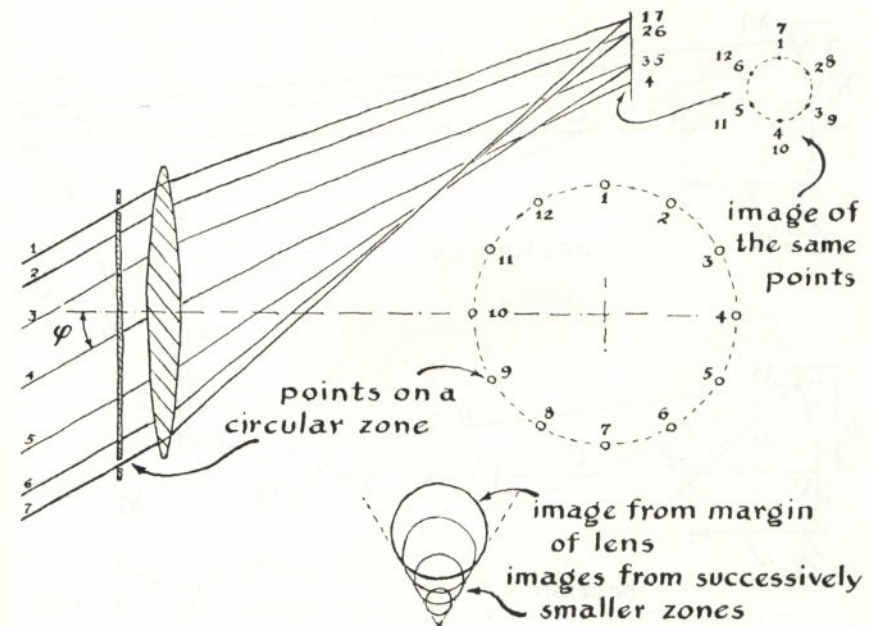

[§ 16-4]   Coma of a Parabolic Mirror   ·   357

image of the same points

points on a circular zone

image from margin of lens

images from successively smaller zones

FIG. 16-2   Nature of coma in a simple lens.

Coma

Fig. 7.7 *Coma: meridional plane (top); spot diagrams (center); ray fan plots (bottom). Reprinted with permission from Rutten and van Venrooij,* Telescope Optics *(Willmann-Bell, 1988).*

Introduction to Lens Design (Geary)

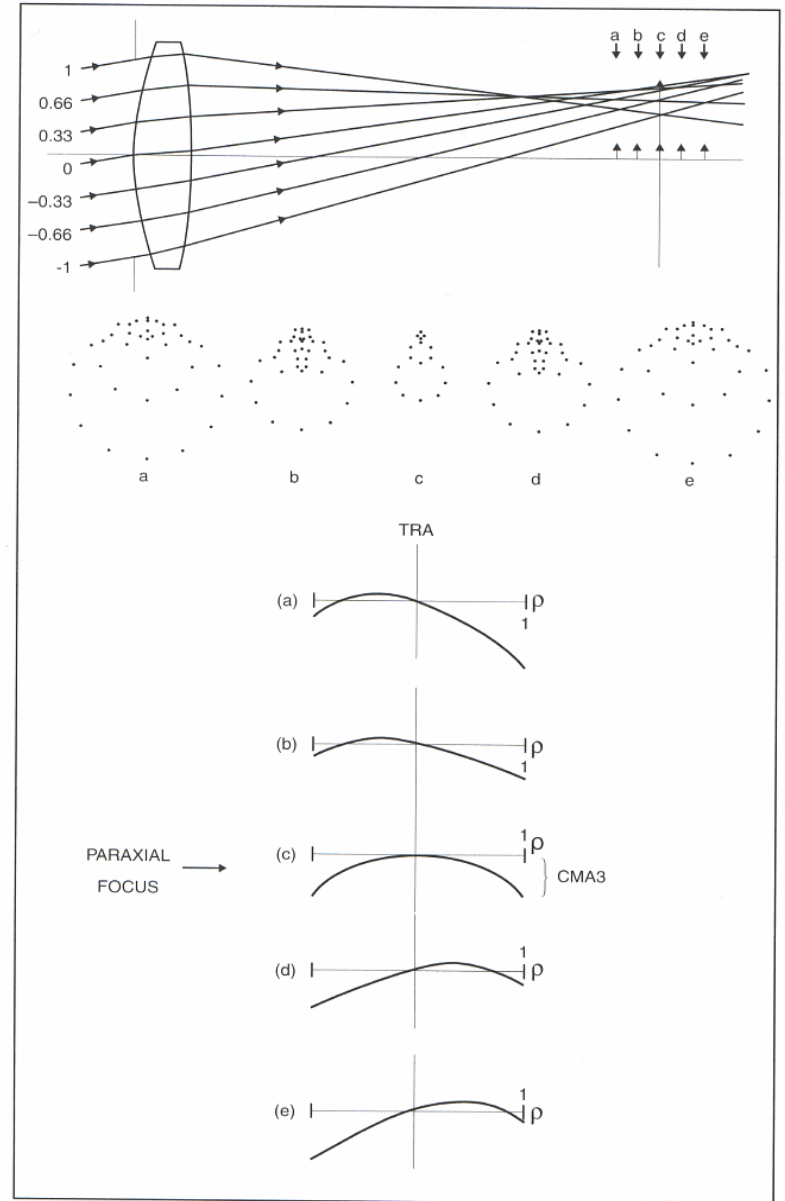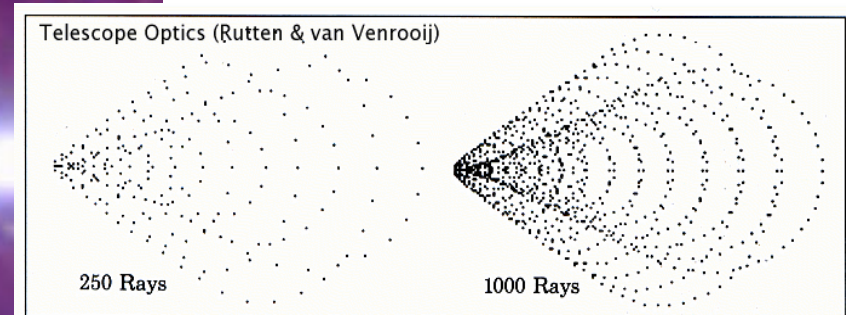**Fig. 7.8** *Paraxial image of specific zones when coma is present.*
Introduction to Lens Design (Geary)

Coma in Practice

Telescope Optics (Rutten & van Venrooij)

250 Rays

1000 Rays

Fig. 4.3 *Spot Diagrams for a 200 mm f/8 Newtonian, 12.5 mm Off-axis.*

Optimizing for Coma via Bending

- In a simple lens spherical aberration and coma cannot be minimized simultaneously (but close)
  - The optimal shape is close to plano-convex
  - but not that this is different from convex-plano ... direction matters!
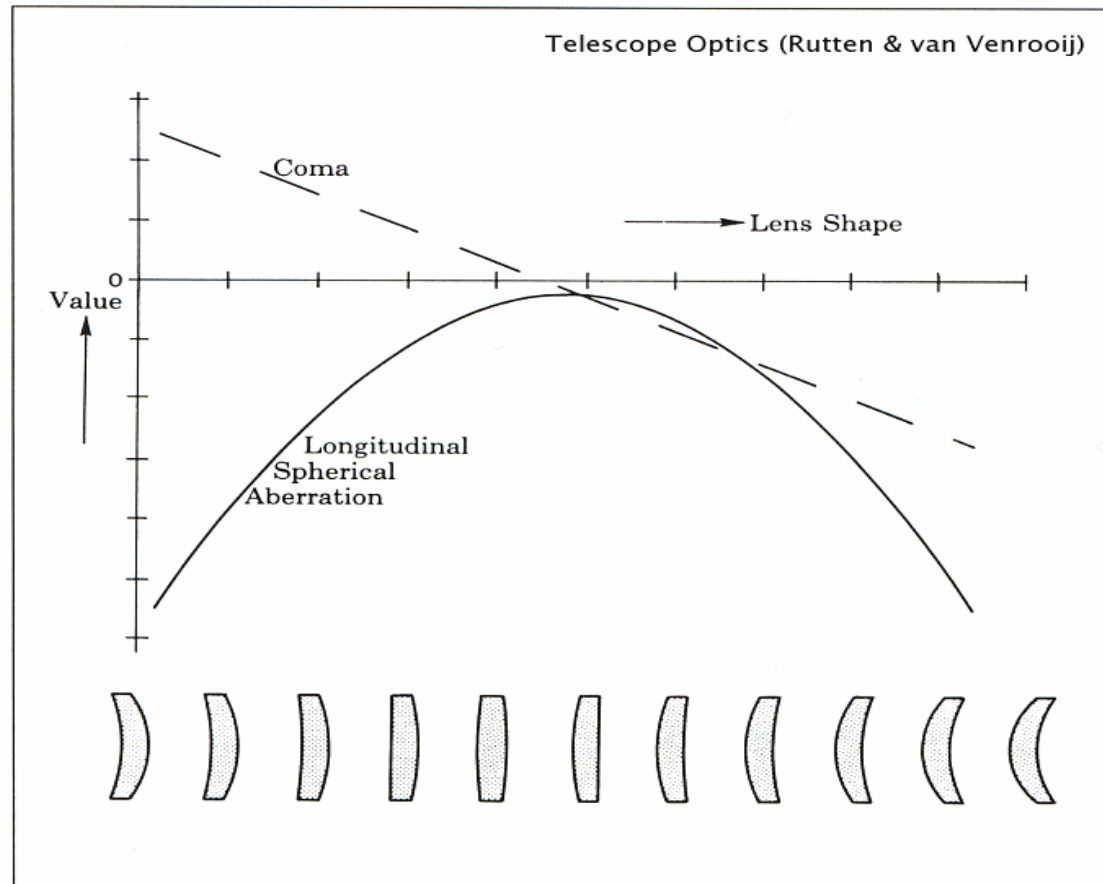

Fig. 6.2 *Spherical Aberration and Coma of a Simple Lens.*

Astigmatism

- Coma was described in terms of zonal/axial symmetry

- Astigmatism addresses the broken symmetry introduced from the off-axis perspective

  - The optical axis and "chief ray" define a plane – the tangential plane.

    - Ray fans in and parallel to this plane behave differently than ray fans lying in and parallel to the perpendicular "sagittal" plane

    - In particular, the two planes focus at different distances producing sharp perpendicular "line" images at two depths with a circle of least confusion in between.
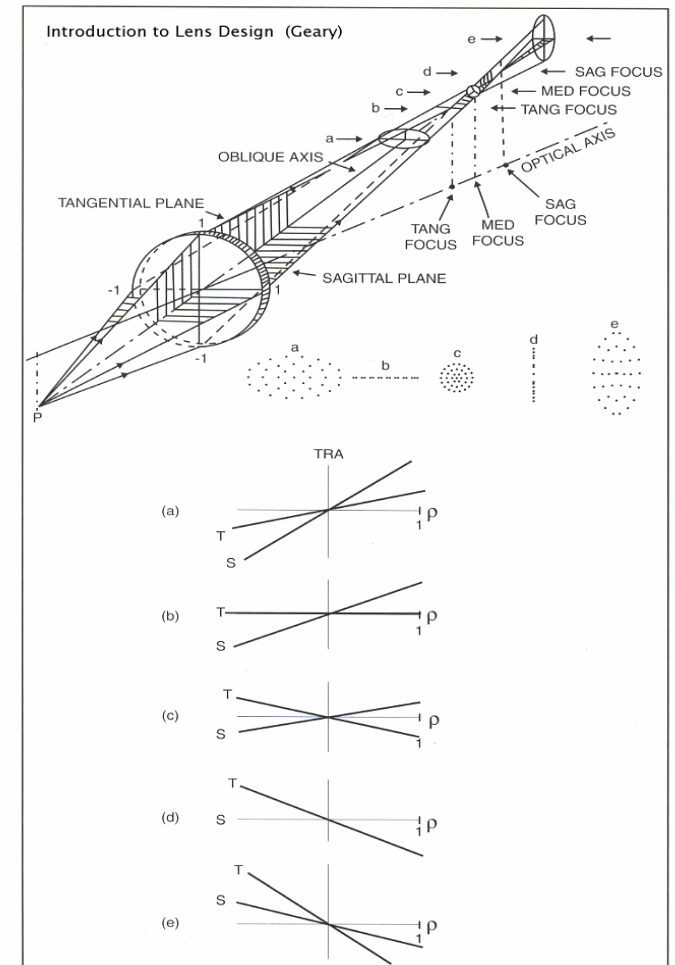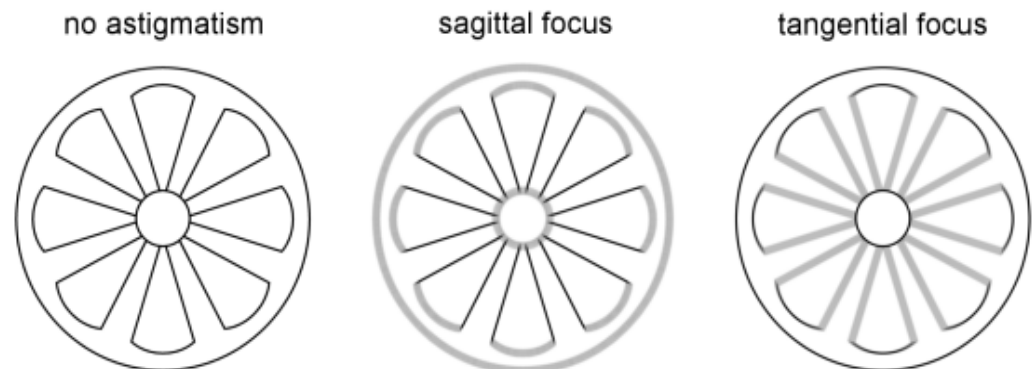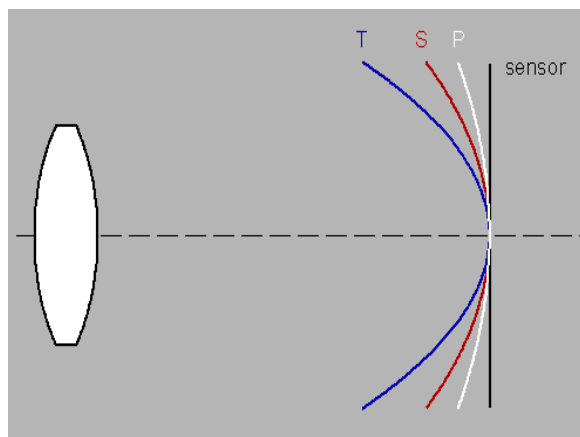

Fig. 7.9 *Astigmatism: 3-D view (top); spot diagrams (center); ray fan plots (bottom). Reprinted with permission from Rutten and van Venrooij, Telescope Optics (Willmann-Bell, 1988).*

http://www.microscopyu.com/tutorials/java/aberrations/astigmatism/index.html
http://www.vanwalree.com/optics/astigmatism.html

Astigmatism

- Coma was described in terms of zonal/axial symmetry

- Astigmatism addresses the broken symmetry introduced from the off-axis perspective
  - The optical axis and "chief ray" define a plane – the tangential plane.
    - Ray fans in and parallel to this plane behave differently than ray fans lying in and parallel to the perpendicular "sagittal" plane
    - In particular, the two planes focus at different distances producing sharp perpendicular "line" images at two depths with a circle of least confusion in between.

Astigmatism

- Astigmatism (like coma and SA) is signed. Combining under and over corrected elements can lead to (imperfect) cancellation of the aberration.

- The goal in this cancellation is making the sagittal and tangential planes coincident.
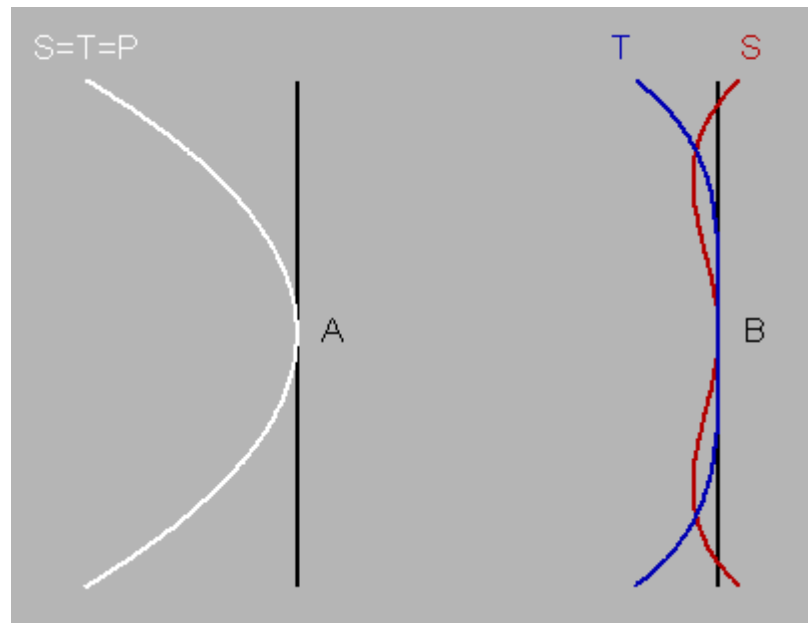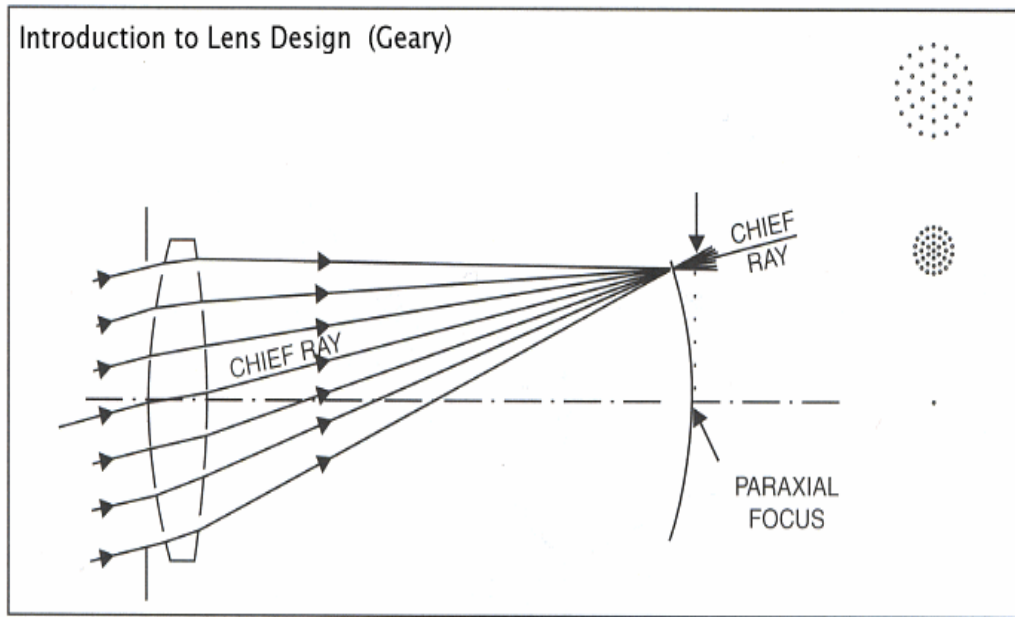
Curvature of Field and Distortion

Introduction to Lens Design (Geary)

CHIEF RAY

CHIEF RAY

PARAXIAL FOCUS

Fig. 7.11 *Meridional plot illustrating field curvature. Reprinted with permission from Rutten and van Venrooij,* Telescope Optics *(Willmann-Bell, 1988).*
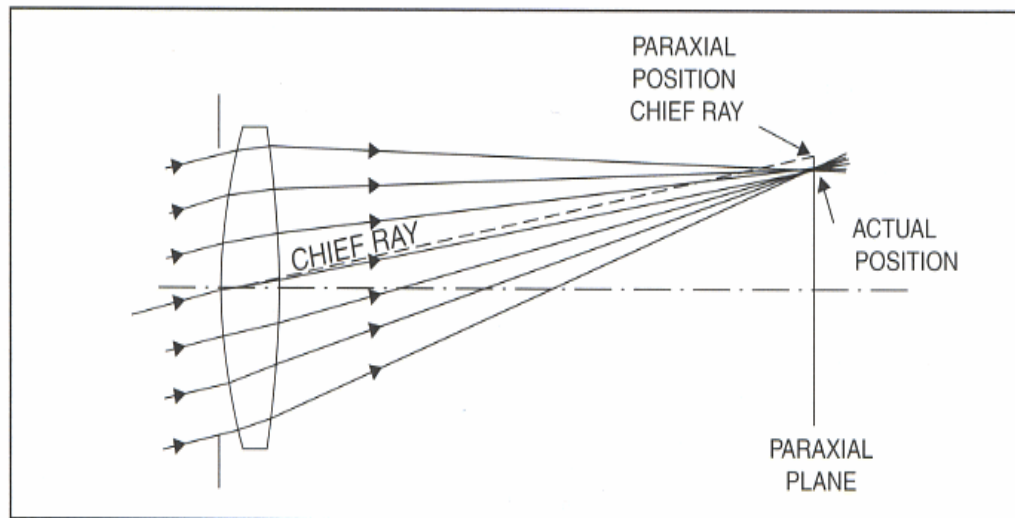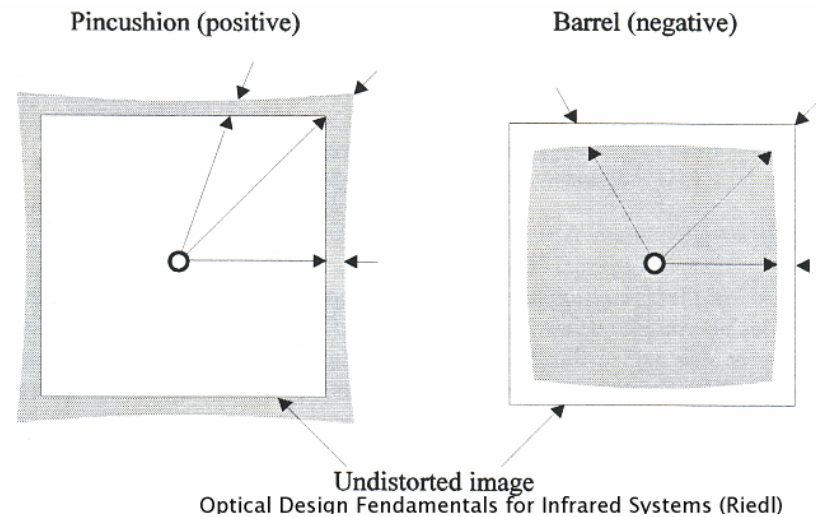

PARAXIAL POSITION CHIEF RAY

CHIEF RAY

ACTUAL POSITION

PARAXIAL PLANE

Fig. 7.12 *Meridional plot illustrating distortion. Reprinted with permission from Rutten and van Venrooij,* Telescope Optics *(Willmann-Bell, 1988).*

Pincushion (positive)   Barrel (negative)

Undistorted image
Optical Design Fendamentals for Infrared Systems (Riedl)

Field Flattening

- Curvature of field can be corrected by adding an element directly ahead of the focal plane which serves to introduce a "delay" in focus as a function of distance from the optical axis.

- Little additional aberration can be introduced since each point source "beam" sees a locally flat optic.

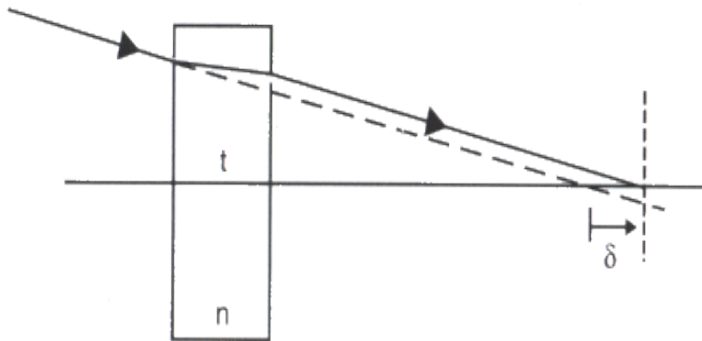Geary Introduction to Lens Design

Fig 14.8 *Focus shift introduced by a parallel plate in a converging beam.*
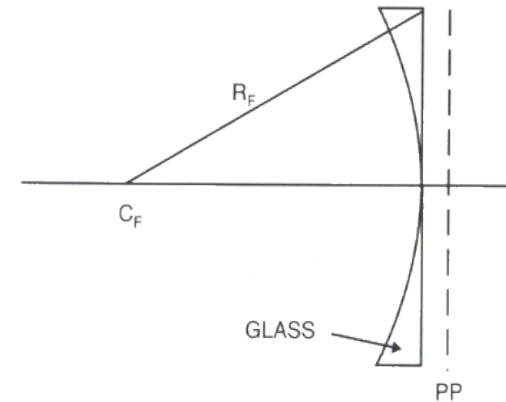
Fig 14.10 *Idealistic field flattener.*

Chromatic Aberration

- A lens' focal length depends on the refractive index of the lens material.

  - Refractive index (both fortunately and unfortunately) is a function of wavelength.

  - Only one wavelength can be exactly in focus at a time.

- Imaging systems often function over broad bandpasses (e.g. K-band spans 2.0 – 2.4 um)

- Optical design mixes materials (e.g. crown and flint glass in a traditional achromat) to mitigate chromatic aberration.

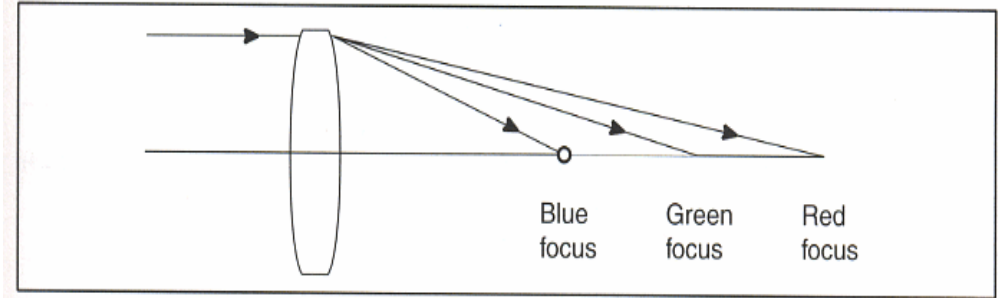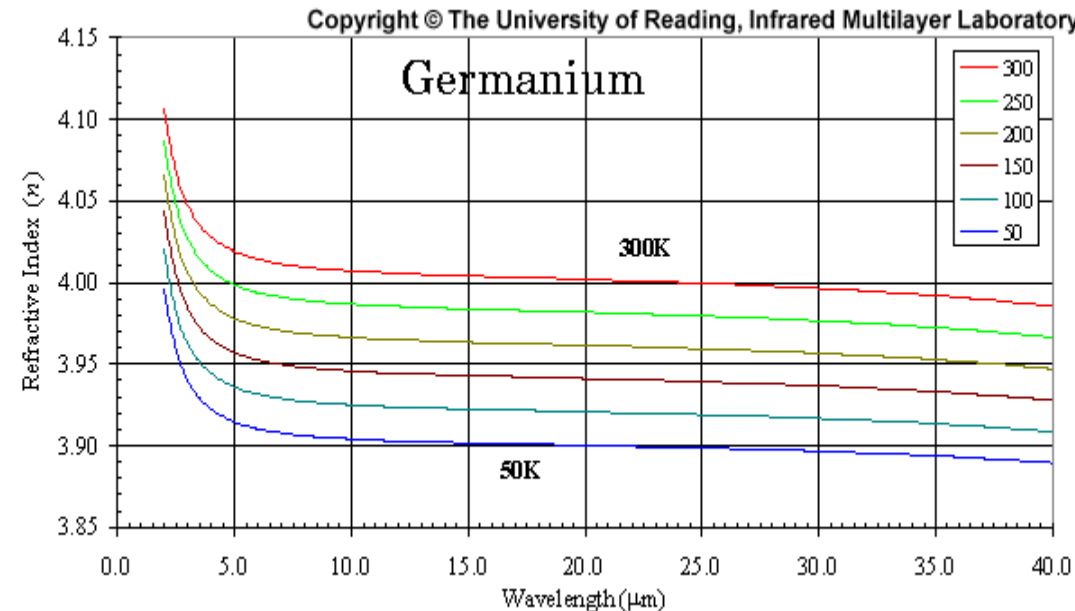- All-reflective optical systems cannot suffer chromatic aberration.

Fig. 16.1 *Dispersion in a lens.*
Introduction to Lens Design (Geary)

Copyright © The University of Reading, Infrared Multilayer Laboratory

http://www.cyber.rdg.ac.uk/ISP/infrared/library/technical_data/infrared_materials/ge3.htm

Chromatic Aberration

- The lensmaker's equation quantifies the difference in focus as a function of wavelength.

- Longitudinal chromatic aberration depends strongly on the dispersion of the material $dn/d\lambda$

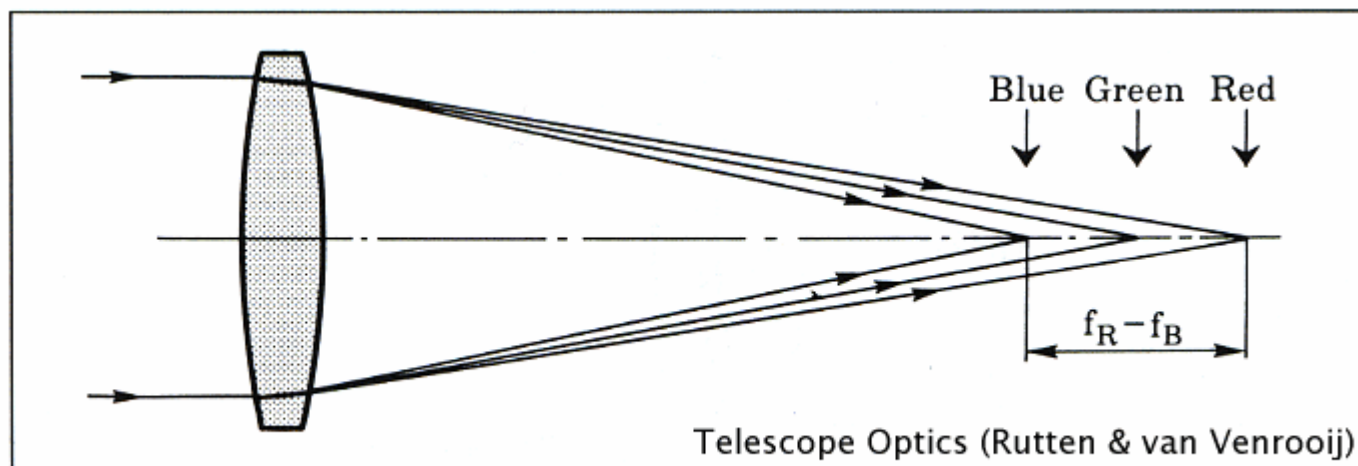$$\frac{1}{f_\lambda} = (n_\lambda - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Blue Green Red

$f_R - f_B$

Telescope Optics (Rutten & van Venrooij)

Fig. 21.12 *Longitudinal Chromatic Aberration for a Simple Lens.*

Controlling Chromatic Aberration

- Split the lens into two components (use additional surfaces to control classical aberrations).

- Make lenses out of materials with different dispersive properties

  - The Negative lens has a higher refractive index to control spherical aberration (compensating for its weaker power).

    - Zero spherical aberration can be achieved at only one color. The same is true of chromatic aberration.
    - The doublet is far from perfect and some are more perfect than others.

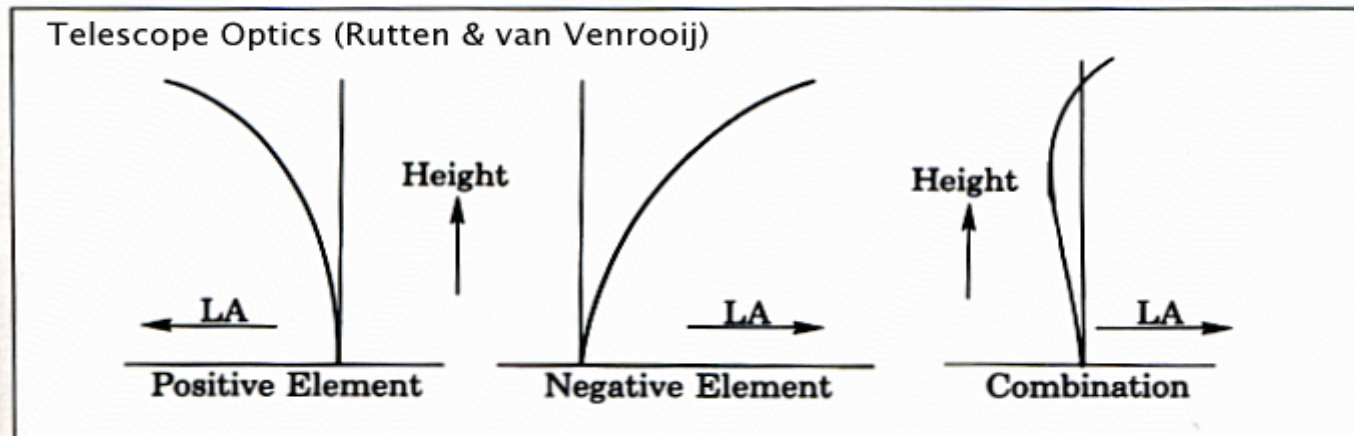Cemented   Fraunhofer   Steinheil
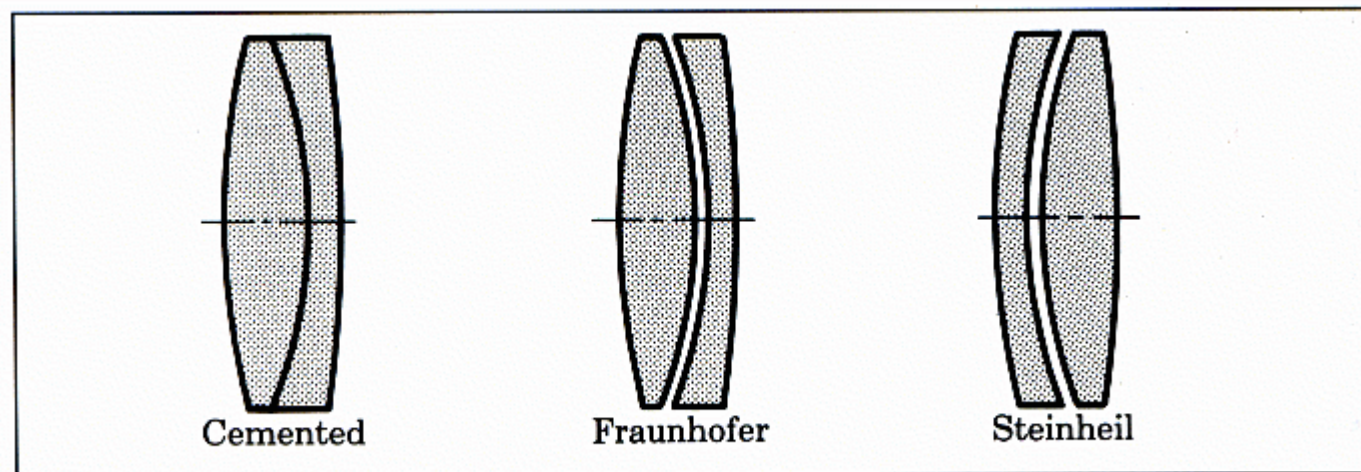
Fig. 6.5 *Achromatic Doublet Designs.*

Controlling Chromatic Aberration

- Split the lens into two components (use additional surfaces to control classical aberrations).

- Make lenses out of materials with different dispersive properties

    - The Negative lens has a higher refractive index to control spherical aberration (compensating for its weaker power).

        - Zero spherical aberration can be achieved at only one color. The same is true of chromatic aberration.
        - The doublet is far from perfect and some are more perfect than others.

Telescope Optics (Rutten & van Venrooij)

Height

LA

Positive Element

Height

LA

Negative Element

LA

Combination

Fig. 6.3 *Correction of Longitudinal Spherical Aberration (LA) in Doublet Objective.*

Controlling Chromatic Aberration

- The uncorrected chromatic aberration in a double achromat is called "secondary spectrum".

  - It can be minimized by appropriate material choices which

    - maximize the difference in Abee number (inverse of broadband dispersion) between the two materials

    - minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)
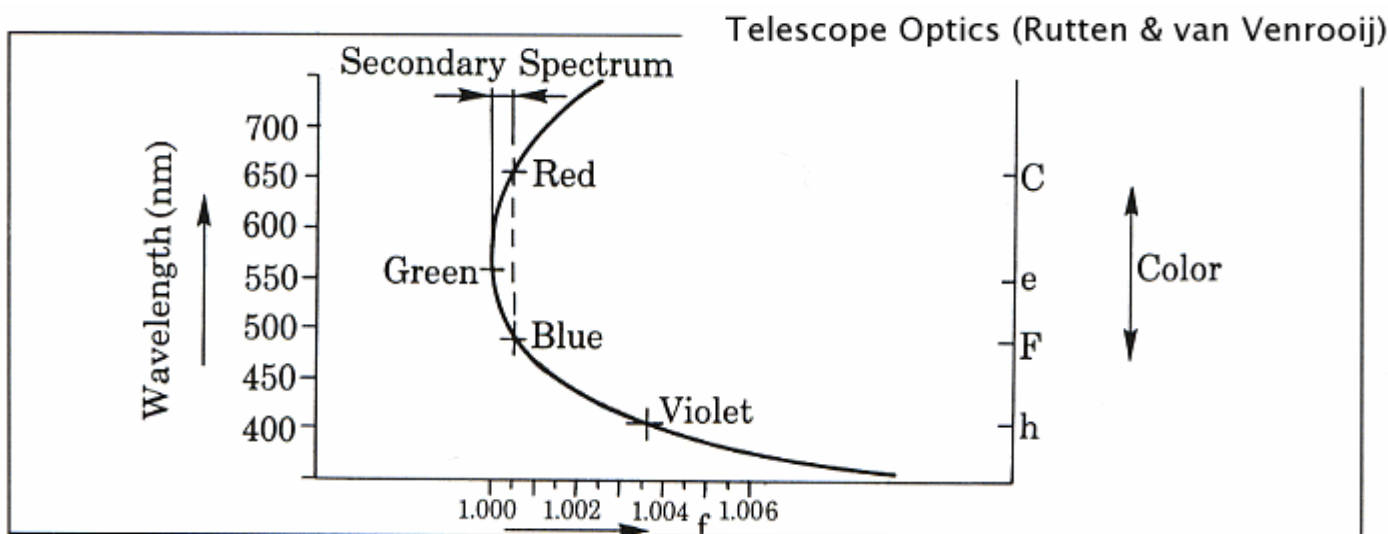

Fig. 6.6 *Color Curve for a Normal Visually Corrected Doublet.*

$$\nu_d = \frac{n_d - 1}{n_F - n_C}$$
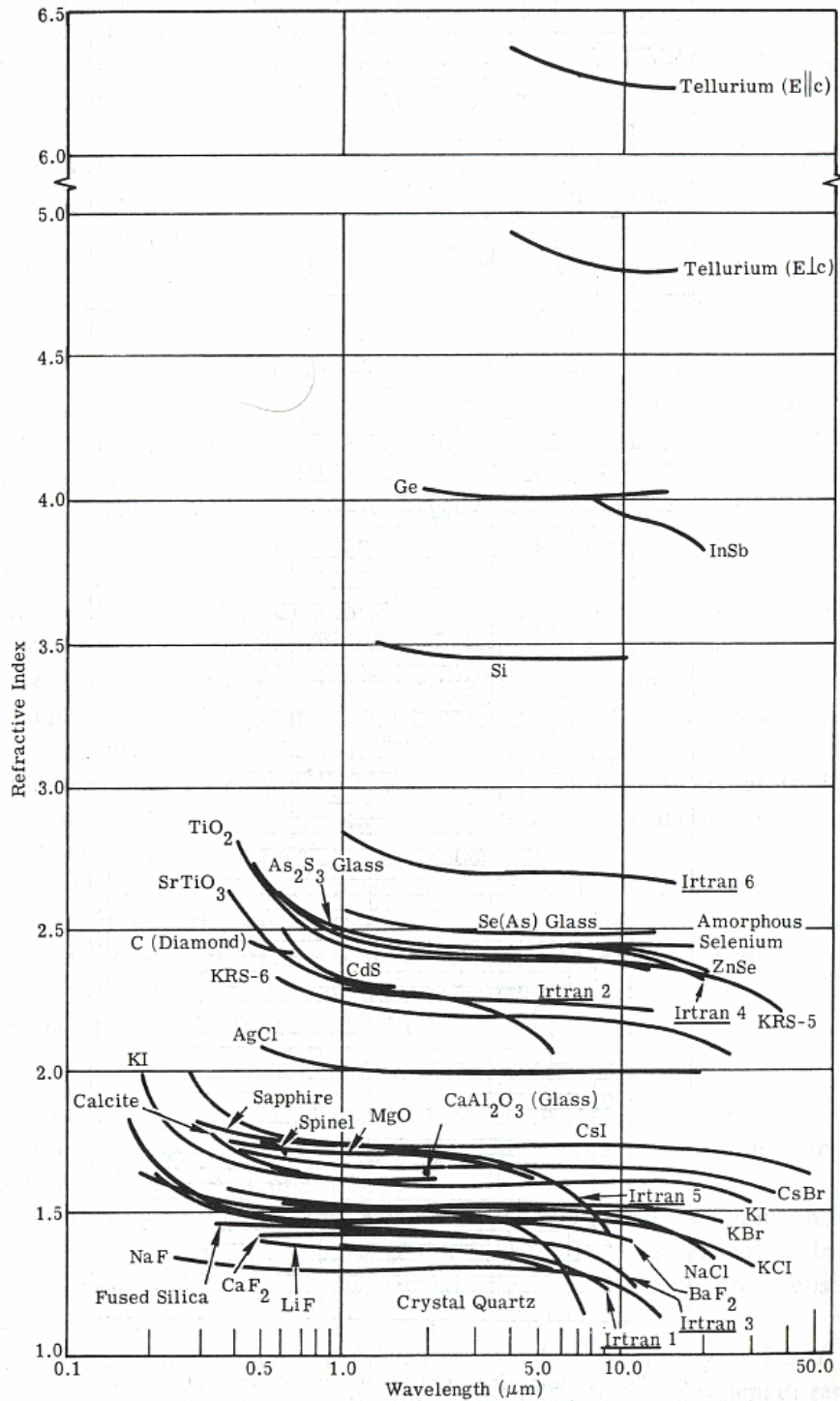
Abee Number

Refractive Indices and Dispersions

Fig. 7-6. Refractive index values. Irtran 6 is no longer manufactured by the Eastman Kodak Co. Irtran® is a registered trademark of the Eastman Kodak Co.
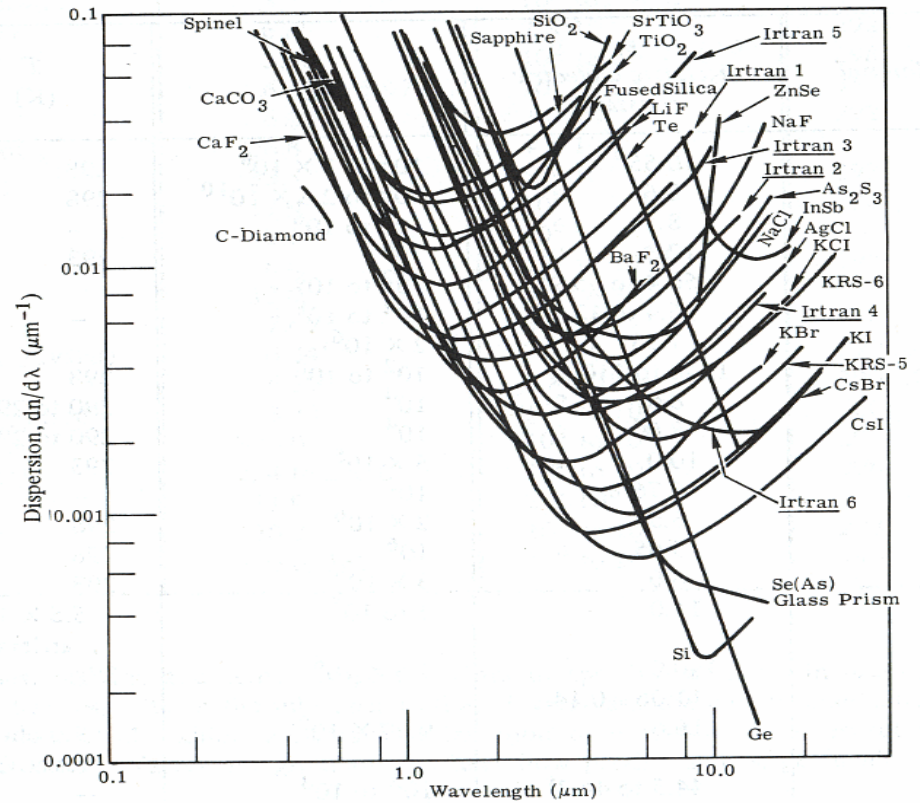

Fig. 7-7. $dn/d\lambda$ versus λ for selected materials. Irtran 6 is no longer manu-factured by the Eastman Kodak Co. Irtran® is a registered trademark of the Eastman Kodak Co.

Controlling Chromatic Aberration

- The uncorrected chromatic aberration in a double achromat is called "secondary spectrum".

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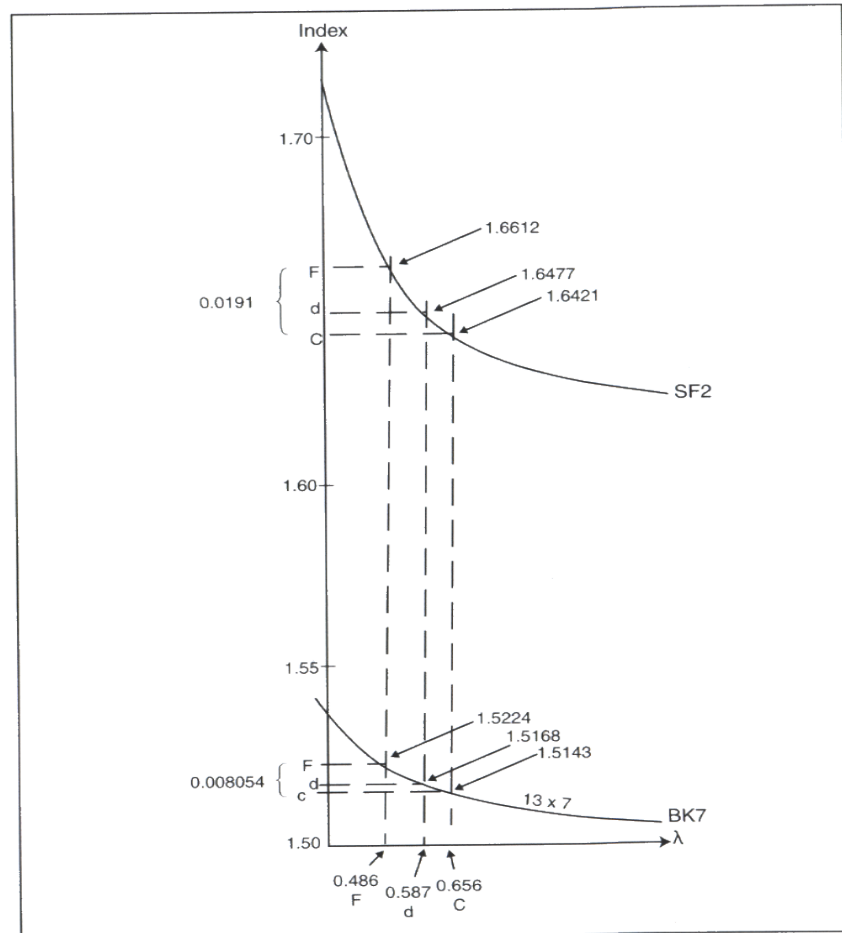

Fig. 18.1 *Index vs. wavelength for BK7 and SF2.*

$$P_{Fd} = \frac{n_F - n_d}{n_F - n_C} \left.\vphantom{\frac{n_F - n_d}{n_F - n_C}}\right\}$$
$$P_{dC} = \frac{n_d - n_C}{n_F - n_C}$$

(18.2)

Controlling Chromatic Aberration

- The uncorrected chromatic aberration in a double achromat is called "secondary spectrum".

  - It can be minimized by appropriate material choices which

    - maximize the difference in Abee number (inverse of broadband dispersion) between the two materials

    - minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)
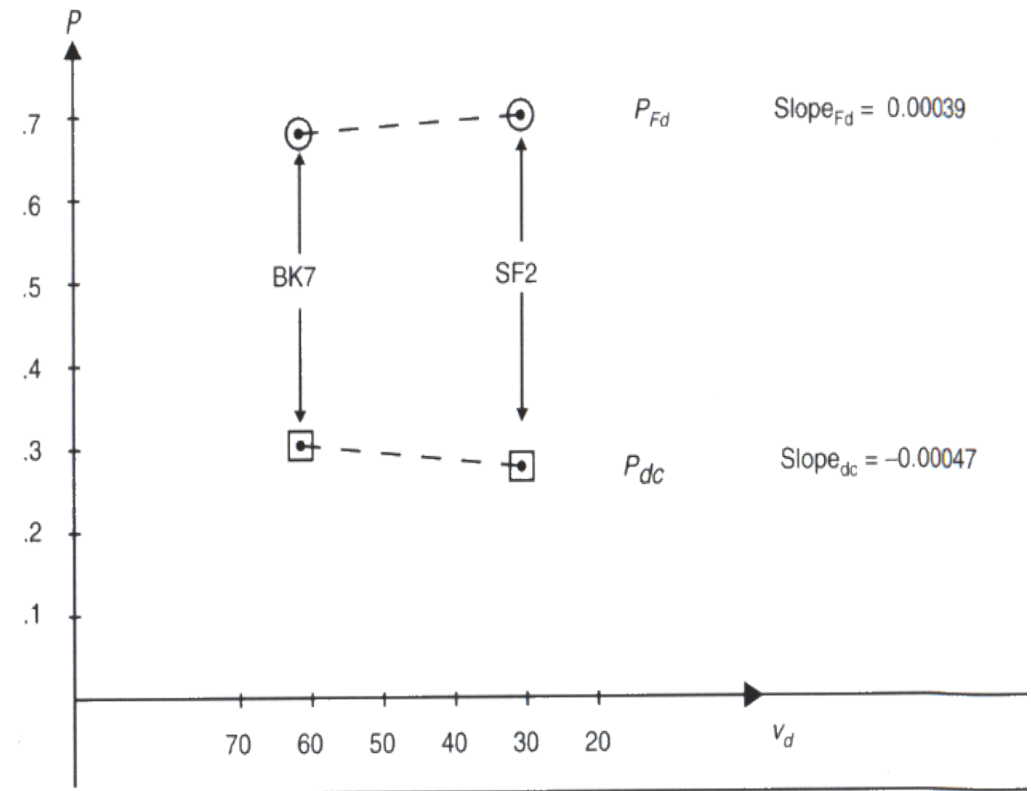

Fig. 18.2 *Partial dispersion plot for BK7 and SF2.*

Controlling Chromatic Aberration

- The uncorrected chromatic aberration in a double achromat is called "secondary spectrum".

  - It can be minimized by appropriate material choices which

    - maximize the difference in Abee number (inverse of broadband dispersion) between the two materials

    - minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)
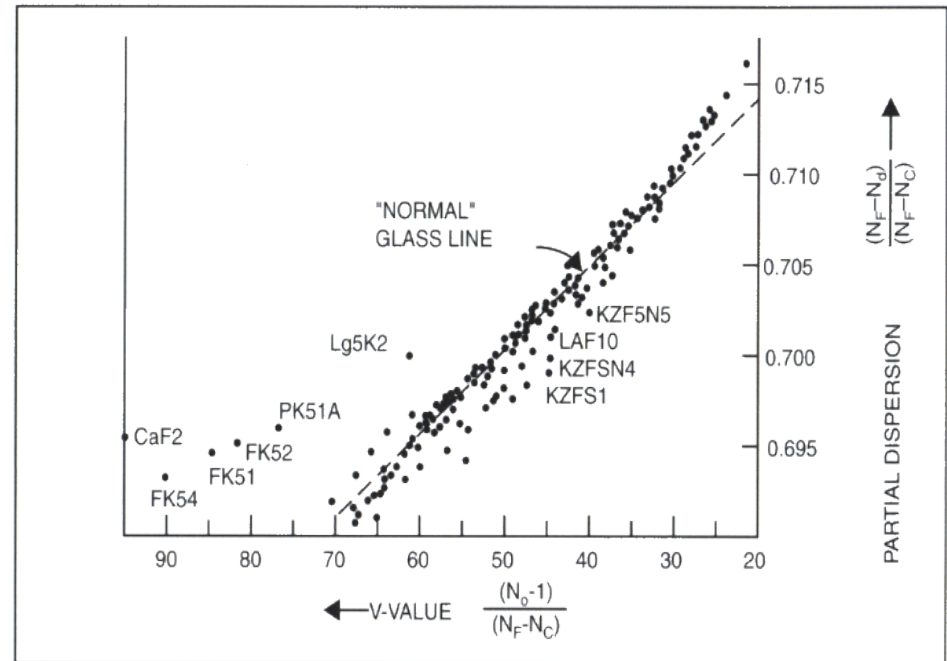

198 *Chapter 18 Secondary Color*

Fig. 18.3 *Partial dispersion vs. Abbe number for F-d light. From W. Smith,* Modern Lens Design *(McGraw-Hill, 1992). Reprinted with permission of the McGraw-Hill Companies.*

Controlling Chromatic Aberration

- The uncorrected chromatic aberration in a double achromat is called "secondary spectrum".

    - It can be minimized by appropriate material choices which

        - maximize the difference in Abee number (inverse of broadband dispersion) between the two materials

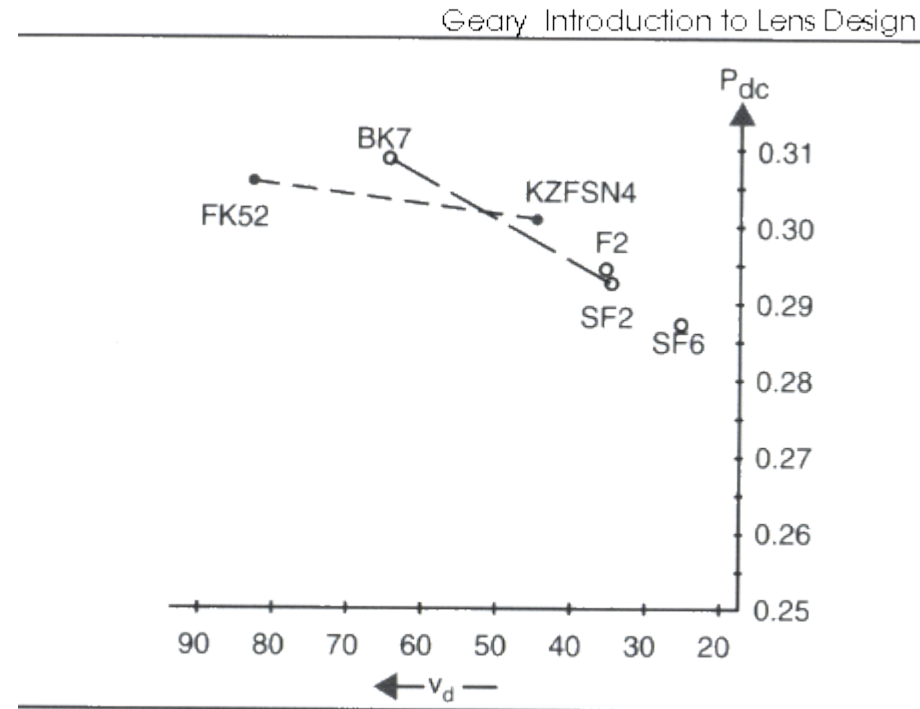   - minimize the difference in partial dispersion (curvature of the dispersion vs. wavelength)

Geary Introduction to Lens Design

Fig. 18.6 *Partial dispersion plot for selected glasses.*

Controlling Chromatic Aberration

The spot diagrams at right represent three different achromatic doublet objectives of varying quality. Today's Cadillac of refractors contain a fluorite element ($$$).

Note that optics designed for the human eye have only weak chromatic constraints.
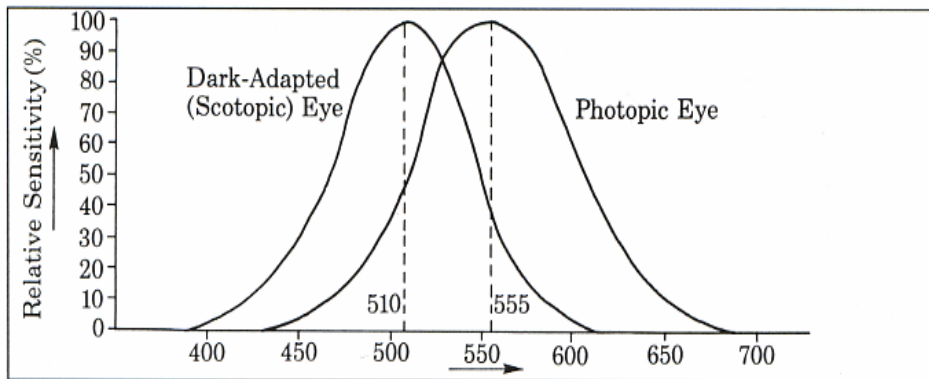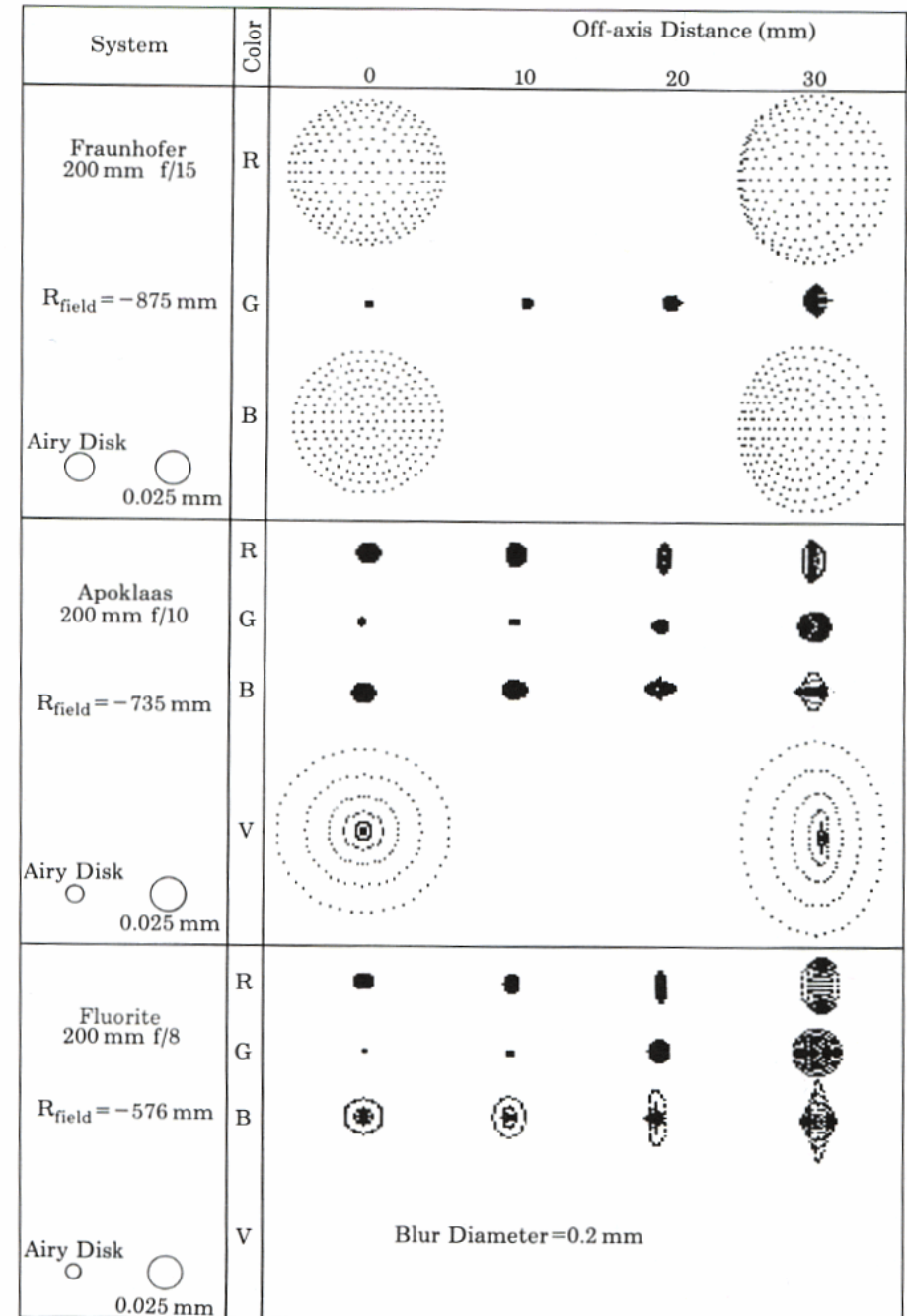

Telescope Optics (Rutten & van Venrooij)

Fig. 6.10 *Spot Diagrams for Various 200 mm Refractor Objectives.*
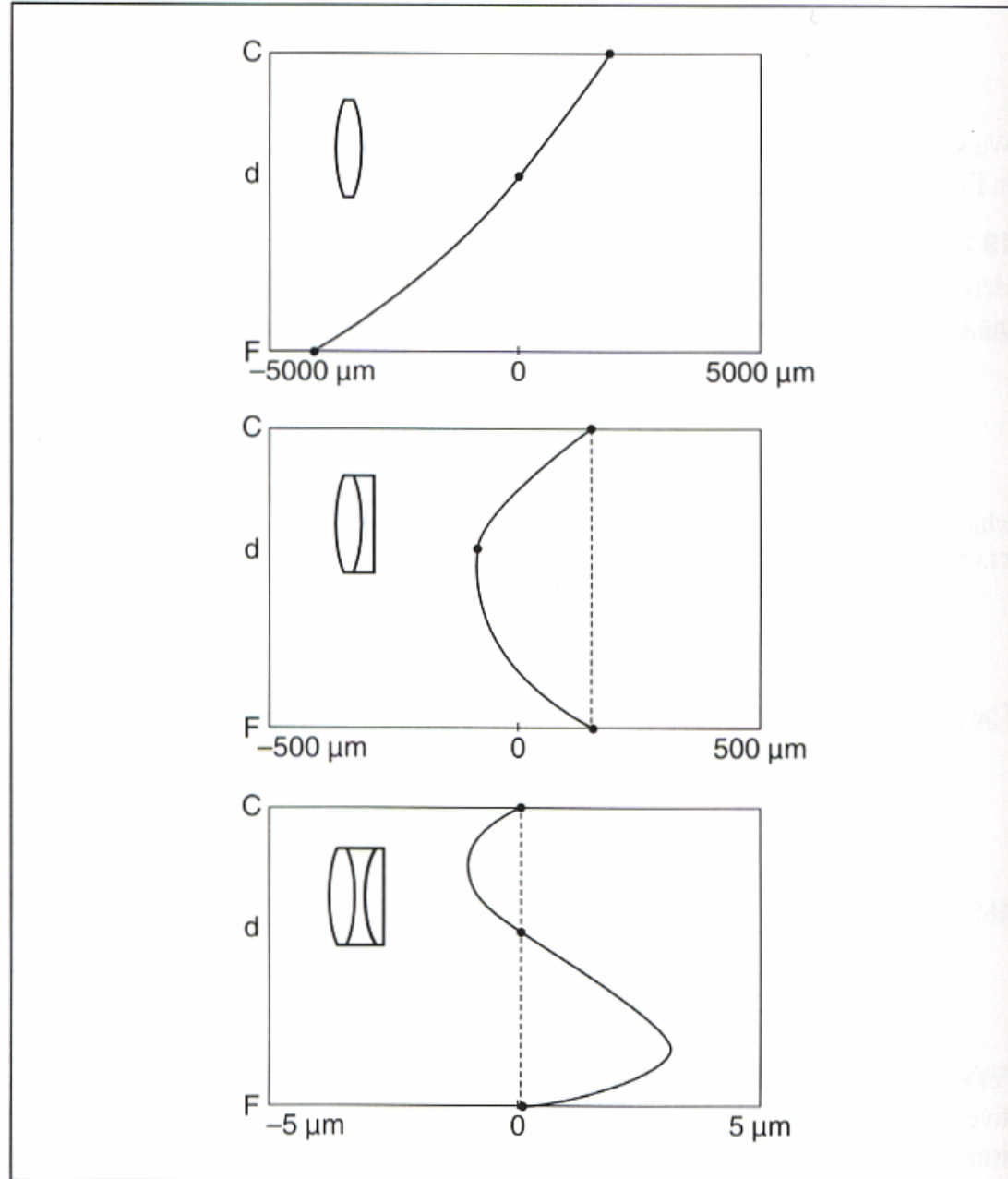
Chromatic Aberration Correction

Fig. 19.2 *The color correction path.*

Classical Optical Design

- With raytracing/computers as a tool, a designer can leverage the many degrees of freedom in a system to minimize image aberration.

- Degrees of freedom include:

Optical Materials	Reflective vs. Refractive
Number of surfaces/lenses	Curvature of surfaces
Lens thickness	Lens Separation
Custom Surfaces (aspheres)	
Stops (although not usually in Astronomy)	
Tilts	

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