Coma

Coma, or comatic aberration, is an image-degrading, mono-chromatic, primary aberration associated with an object point even a short distance from the axis. Its origins lie in the fact that the principal "planes" can actually be treated as planes only in the paraxial region. They are, in fact, principal curved surfaces (Fig. 6.1). In the absence of SA, a parallel bundle of rays will focus at the axial point-Fi, a distance b.f.l. from the rear vertex. Yet the effective focal lengths, and therefore the transverse magnifications, will differ for rays traversing off-axis regions of the lens. When the image point is on the optical axis, this situation is of little consequence, but when the ray bundle is oblique and the image point is off-axis, coma will be evident. The dependence of MT on h, the ray height at the lens, is shown in Fig. 6.22a.

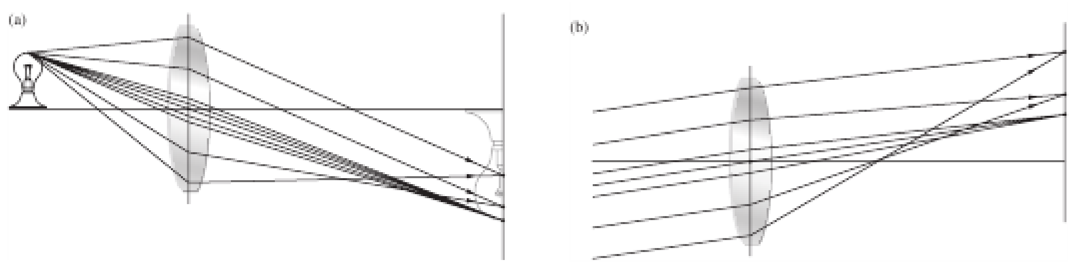

Figure 6.22 (a) Negative coma. (b) Positive coma.

Here meridional rays traversing the extremities of the lens arrive at the image plane closer to the axis than do the rays in the vicinity of the principal ray (i.e., the ray that passes through the principal points).

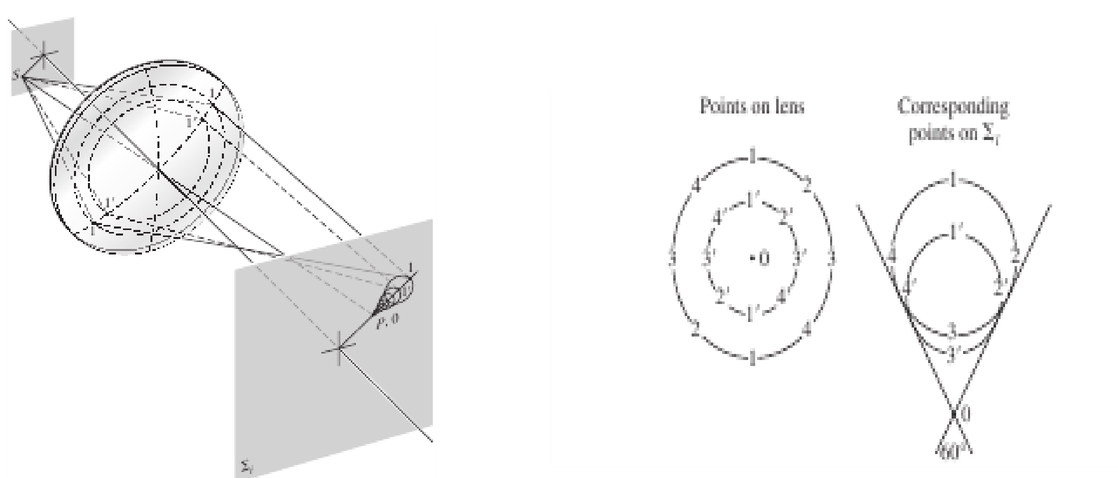

Figure 6.23 The geometrical coma image of a monochromatic point source. The central region of the lens forms a point image at the vertex of the cone

In this instance, the least magnification is associated with the marginal rays that would form the smallest image—the coma is negative. By comparison, the coma in Figs. 6.22b and c is positive because the marginal rays focus farther from the axis.Several non-meridional or skew rays are drawn from an extra- axial object point-S in Fig. 6.23 to illustrate the formation of the geometrical comatic image of a point. Observe that each circular cone of rays whose endpoints (1-2-3-4-1-2-3-4) form a ring on the lens is imaged in what H. Dennis Taylor called a comatic circle on Σ_i. This case corresponds to positive coma, so the larger the ring on the lens, the more distant its comatic circle from the axis. When the outer ring is the intersection of marginal rays, the distance from 0 to 1 in the image is the tangential coma, and the length from 0 to 3 on Σi is termed the sagittal coma. A little more than half of the energy in the image appears in the roughly triangular region between 0 and 3. The coma flare, which owes its name to its comet like tail, is often thought to be the worst of all aberrations, primarily because of its asymmetric configuration. It's not the purview of Geometrical Optics to be concerned with interference, but when light reaches the screen in Fig. 6.23, it's certainly to be expected. The coma cone, just like the Gaussian image point, is an oversimplification. The image point is really an image disk-ring system, and the coma cone is actually a complicated asymmetrical diffraction pattern. The more coma there is, the more the cone departs from the Airy pattern into an elongated structure of blotches and arcs that only vaguely suggests the disk-ring structure from which it evolved (Fig. 6.24).

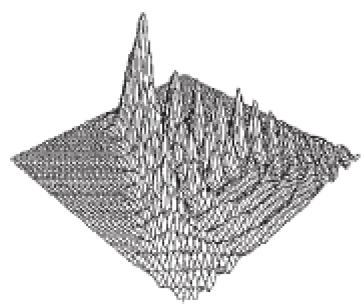

Figure 6.24 Third-order coma. (a) A computer-generated diagram of the image of a point source formed by a heavily astigmatic optical system. (OPAL Group, St. Petersburg, Russia.) (b) A plot of the corresponding irradiance distribution. (OPAL Group, St. Petersburg, Russia.)

Like SA, coma is dependent on the shape of the lens. Thus a strongly concave positive-meniscus lens with the object at infinity will have a large negative coma. Bending the lens so that it becomes planar-convex , then equiconvex , convex-planar , and finally convex-meniscus will change the coma from negative, to zero, to positive. The fact that it can be made exactly zero for a single lens with a given object distance is quite significant. The particular shape it then has (so = oo) is almost convex-planar and nearly the configuration for minimum SA. It is important to realize that a lens that is well corrected for the case in which one conjugate point is at infinity (so = 00) may not perform satisfactorily when the object is nearby. One would operating at finite conjugates, to combine two infinite conjugate corrected lenses, as in Fig. 6.25.

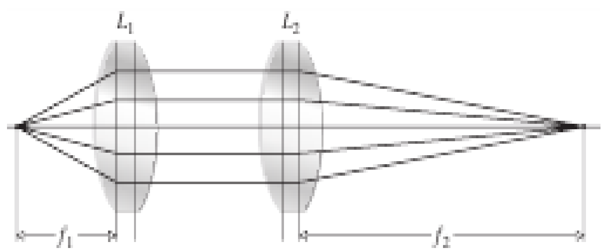

Figure 6.25 A combination of two infinite conjugate lenses yielding a system operating at finite conjugates.

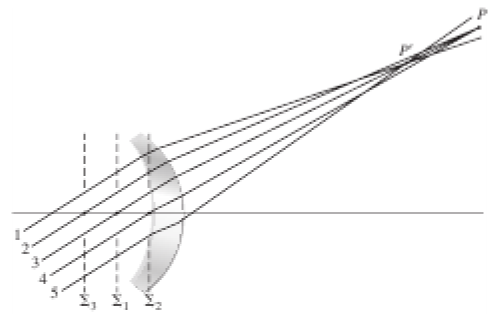

Figure 6.26 The effect of stop location on coma.

In other words, since it is unlikely that a lens with the desired focal length, which is also corrected for the particular set of finite conjugates, can be obtained ready-made, this back-to-back lens approach is an appealing alternative. Coma can also be negated by using a stop at the proper location, as William Hyde Wollaston (1766–1828) discovered in 1812. The order of the list of primary aberrations (SA, coma, astigmatism, Petzval field curvature, and distortion) is significant, because any one of them, except SA and Petzval curvature, will be affected by the position of a stop, but only if one of the preceding aberrations is also present in the system. Thus, while SA is independent of the location along the axis of a stop, coma will not be, as long as SA is present. This can be appreciated by examining the representation in Fig. 6.26. With the stop at Σ1, ray-3 is the chief ray and there is SA but no coma; that ray, and the rays on either side of it, such as 3 and 5, meet above, not on it—there is positive coma. With the stop at Σ 3, rays-1 and -3 intersect below the chief ray, 2, and there is negative coma. In this way, controlled amounts of the aberration can be introduced into a compound lens in order to cancel coma in the system as a whole.

The optical sine theorem is an important relationship that must be introduced here even if space precludes its formal proof. therefore do well, when using off-the-shelf lenses in a system

It was discovered independently in 1873 by Abbe and Helmholtz, although a different form of it was given 10 years earlier by R. Clausius (of thermodynamics fame). In any event, it states that

$$n_o y_o \sin\alpha_o = n_i y_i \sin\alpha_i \qquad (6.47)$$

where no, yo, ao and ni, yi, ai are the index, height, and slope angle of a ray in object and image space, respectively, at any aperture size* (Fig. 6.9). If coma is to be zero,

$$M_T = \frac{y_i}{y_o} \qquad [5.24]$$

must be constant for all rays. Suppose, then, that we send a marginal and a paraxial ray through the system. The former will comply with Eq. (6.47), the latter with its paraxial version (in which sin ao = aop, sin ai = aip). Since MT is to be constant over the entire lens, we equate the magnification for both marginal and paraxial rays to get

$$\frac{\sin \alpha_o}{\sin \alpha_i} = \frac{\alpha_{op}}{\alpha_{ip}} = \text{constant} \qquad (6.48)$$

which is known as the Sine Condition. A necessary criterion for the absence of coma is that the system meet the Sine Condition. If there is no SA, compliancy with the Sine Condition will be both necessary and sufficient for zero coma. It's an easy matter to observe coma. In fact, anyone who has focused sunlight with a simple positive lens has no doubt seen the effects of this aberration. A slight tilt of the lens, so that the nearly collimated rays from the Sun make an angle with the optical axis, will cause the focused spot to flare out into the characteristic comet shape.

As already mentioned and repeated in here Fig. 3.12, the third-order coma patch resembles the shape of a comet. Even though sagittal coma CC is only one third in size of the tangential coma, the triangle covered by the sagittal dimension contains about 55% of all the energy from the imaged object point.

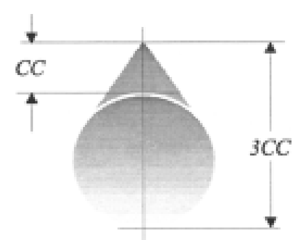

FIG. 3.12 Third-order coma patch.

If we use the amount of sagittal coma as a measure of the blur spot, its angular extent can be expressed by

$$\beta_{coma_{minsph}} = \frac{CC}{f} = \frac{u_p}{16(N+2)(f/\#)^2}, \qquad (3.8)$$

where u_P is the angle of the principal ray for the specified off—axis point. Notice Eq. (3.8) is for a thin lens, shaped for minimum spherical aberration.