Astigmatism

When an object point lies an appreciable distance from the optical axis, the incident cone of rays will strike the lens asymmetrically, giving rise to a third primary aberration known as astigmatism. The word derives from the Greek a-, meaning not, and stigma, meaning spot or point. To facilitate its description, envision the meridional plane (also called the tangential plane) containing both the chief ray (i.e., the one passing through the center of the aperture) and the optical axis. The sagittal plane is then defined as the plane containing the chief ray, which, in addition, is perpendicular to the meridional plane (Fig. 6.27). Unlike the latter, which is unbroken from one end of a complicated lens system to the other, the sagittal plane generally changes slope as the chief ray is deviated at the various elements. Hence to be accurate

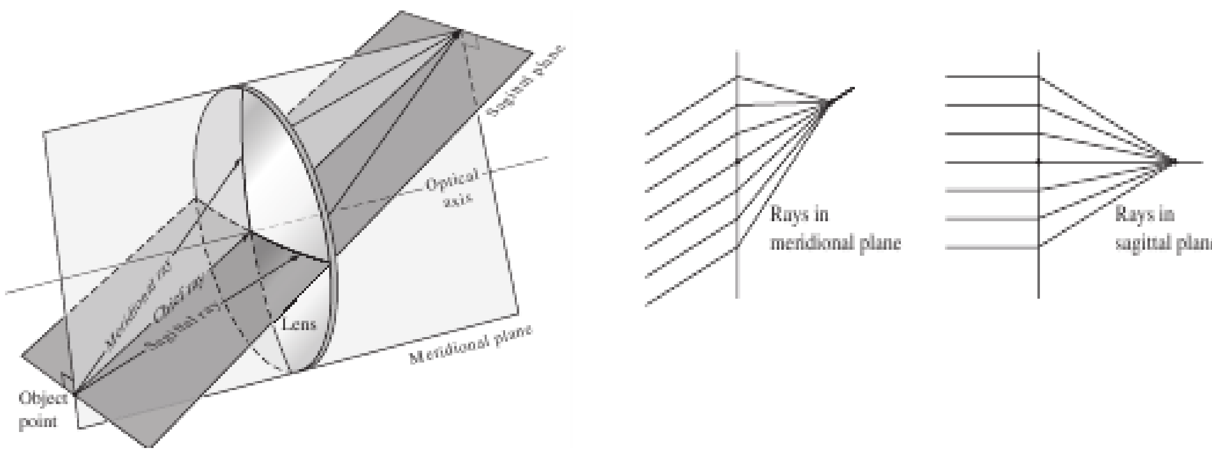


Figure 6.27 The sagittal and meridional planes.

we should say that there are actually several sagittal planes, one attendant with each region within the system. Nevertheless, all skew rays from the object point lying in a sagittal plane are termed sagittal rays.

In the case of an axial object point, the cone of rays is symmetrical with respect to the spherical surfaces of a lens. There is no need to make a distinction between meridional and sagittal planes. The ray configurations in all planes containing the optical axis are identical. In the absence of spherical aberration, all the focal lengths are the same, and consequently all rays arrive at a single focus. In contrast, the configuration of an oblique, parallel ray bundle will be different in the meridional and sagittal planes. As a result, the focal lengths in these planes will be different as well. In effect, here the meridional rays are tilted more with respect to the lens than are the sagittal rays, and they have a shorter focal length. It can be shown,* using Fermat's Principle, that the focal length difference depends effectively on the power of the lens (as opposed to the shape or index) and the angle at which the rays are inclined. This astigmatic difference, as it is often called, increases rapidly as the rays become more oblique, that is, as the object point moves farther off the axis, and is, of course, zero on axis.Having two distinct focal lengths, the incident conical bundle of rays takes on a considerably altered form after refraction (Fig. 6.28). The cross section of the beam as it leaves the lens is initially circular, but it gradually becomes elliptical with the major axis in the sagittal plane, until at the tangential or meridional focus FT, the ellipse degenerates into a “line” (at least in third-order theory). Actually, it’s a complicated elongated diffraction pattern that looks more linelike the more astigmatism is present. All rays from the object point traverse this “line,” which is known as the primary image. Beyond this point, the beam’s cross section rapidly opens out until it is again circular. At that location, the image is a circular blur known as the circle of least confusion. Moving farther from the lens, the beam’s cross section again deforms into a “line,” called the secondary image. This time it’s in the meridional plane at the sagittal focus, FS.

The image of a point source formed by a slightly astigmatic optical system (□ 0.2l), in the vicinity of the circle of least confusion, looks very much like the Airy disk-ring pattern, but it’s somewhat asymmetrical. As the amount of astigmatism increases (upwards of roughly 0.5l), the biaxial asymmetry becomes more apparent. The image transforms into a complex distribution of bright and dark regions (resembling the Fresnel diffraction patterns for rectangular openings, p. 523) and only very subtly retains a curved structure arising from the circular aperture. Remember that in all of this we are assuming the absence of SA and coma.

Since the circle of least confusion increases in diameter as the astigmatic difference increases (i.e., as the object moves farther off-axis), the image will deteriorate, losing definition around its edges. Observe that the secondary “line” image will change in orientation with changes in the object position, but it will always point toward the optical axis; that is, it will be radial. Similarly, the primary “line” image will vary in orientation, but it will remain normal to the secondary image. This arrangement causes the interesting effect shown in Fig. 6.29 when the object is made up of radial and tangential elements. The primary and secondary images are, in effect, formed of transverse and radial dashes, which increase in size with distance from the axis. In the latter case, the dashes point like arrows toward the center of the image ergo, the name sagittal. The existence of the sagittal and tangential foci can be verified directly with a fairly simple arrangement. Place a positive lens with a short focal length (about 10 or 20 mm) in the beam of a He-Ne laser. Position another positive test lens with a somewhat longer focal length far enough away so that the now diverging beam fills that lens. A convenient object, to be located between the two lenses, is a piece of ordinary wire screening (or a transparency). Align it so the wires are hrizontal (x) and vertical (y). If the test lens is rotated roughly 45□ about the vertical (with the x-, y-, and z-axes fixed in the lens), astigmatism should be observable. The meridional is the xz-plane (z being the lens axis, now at about 45□ to the laser axis), and the sagittal plane corresponds to the plane of y and the laser axis. As the wire mesh is moved toward the test lens, a point will be reached where the horizontal wires are in focus on a screen beyond the lens, whereas the vertical wires are not. This is the location of the sagittal focus. Each point on the object is imaged as a short line in the meridional (horizontal) plane, which accounts for the fact that only the horizontal wires are in focus. Moving the mesh slightly closer to the lens will bring the vertical lines into clarity while the horizontal ones are blurred. This is the tangential focus. Try rotating the mesh about the central laser axis while at either focus.

Note that unlike visual astigmatism (p. 222), which arose from an actual asymmetry in the surfaces of the optical system, the third-order aberration by that same name applies to spherically symmetrical lenses.

Mirrors, with the singular exception of the plane mirror, suffer many of the same monochromatic aberrations as do lenses. Thus, although a paraboloidal mirror is free of SA for an infinitely distant axial object point, its off-axis imagery is quite poor due to astigmatism and coma. This strongly restricts its use to narrow field devices, such as searchlights and astronomical telescopes. A concave spherical mirror shows SA, coma, and astigmatism. Indeed, one could draw a diagram just like Fig. 6.28 with the lens replaced by an obliquely illuminated spherical mirror. Incidentally, such a mirror displays ppreciably less SA than would a simple convex lens of the same focal length.

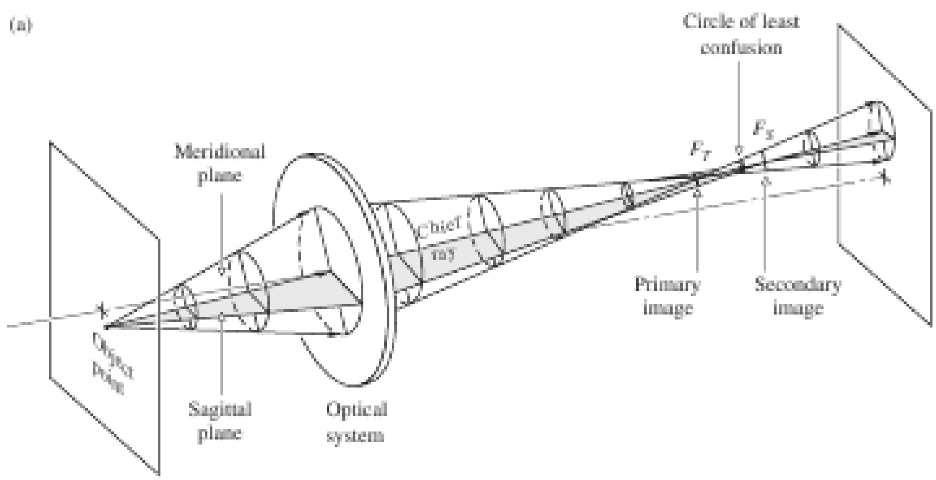


Figure 6.28 Astigmatism. The light from a monochromatic point source is elongated by an astigmatic lens

The tangential and sagittal image shells are separated by $2AC = u_p^2 f$ The circle of beast confusion is located halfway between and has a diameter of

$$B = 2\,TAC = u_p^2\, y. \qquad (3.9)$$

Dividing this by the focal length of the lens, the angular blur spot due to astigmatism is

$$\beta_{ast} = \frac{B}{f} = \frac{u_p^2}{2(f/\#)}. \qquad (3.10)$$

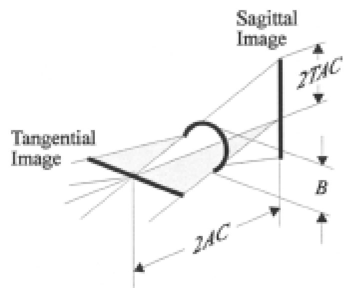


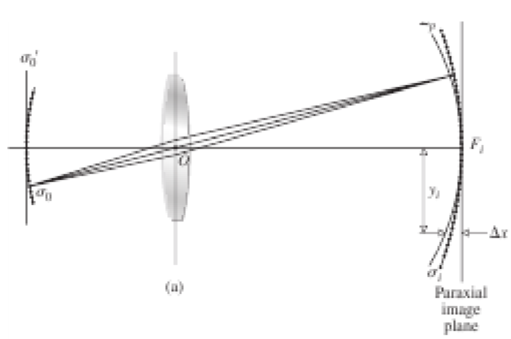
FIG. 3.13 Astigmatism, minimum blur spot, size, and location.

Field Curvature

Suppose we had an optical system that was free of all the aberrations thus far considered. There would then be a one-to-one correspondence between points on the object and image surfaces (i.e., stigmatic imagery). We mentioned earlier (Section 5.2.3) that a planar object normal to the axis will be imaged approximately as a plane only in the paraxial region. At finite apertures the resulting curved stigmatic image surface is a manifestation of the primary aberration known as Petzval field curvature, after the Hungarian mathematician Josef Max Petzval (1807–1891). The effect can readily be appreciated by examining Figs. 5.21 (p. 171) and 6.30. A spherical object segment σo is imaged by the lens as a spherical segment σi, both centered at O. Flattening out σo into the plane $\sigma o'$ will cause each image point to move toward the lens along the concomitant chief ray, thus forming a paraboloidal Petzval surface ΣP. Whereas the Petzval surface for a positive lens curves inward toward the object plane, for a negative lens it curves outward away from that plane. Evidently, a suitable combination of positive and negative lenses will negate field curvature. Indeed, the displacement Δx of an image point at height yi on the Petzval surface from the paraxial image plane is given by

$$\Delta x = \frac{y_i^2}{2} \sum_{j=1}^{m} \frac{1}{n_j f_j} \qquad (6.49)$$

where nj and fj are the indices and focal lengths of the m thin lenses forming the system. This implies that the Petzval surface



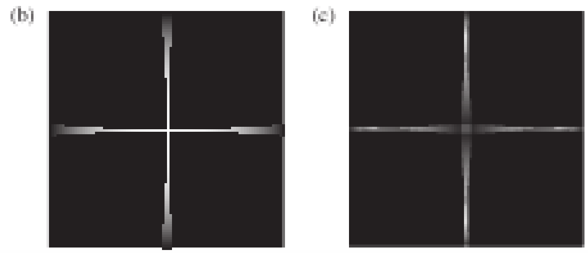


Figure 6.30 Field curvature. (a) When the object corresponds to σo', the image will correspond to surface ΣP. (b) The image formed on a flat screen near the paraxial image plane will be in focus only at its center. (E.H.) (c) Moving the screen closer to the lens will bring the edges into focus. (E.H.)

will be unaltered by changes in the positions or shapes of the lenses or in the location of the stop, as long as the values of nj and ƒj are fixed. Notice that for the simple case of two thin lenses (m = 2) having any spacing, Δx can be made zero provided that

$$\frac{1}{n_1 f_1} + \frac{1}{n_2 f_2} = 0$$

or, equivalently,

$$n_1 f_1 + n_2 f_2 = 0 \qquad (6.50)$$

This is the so-called Petzval condition. As an example of its use, suppose we combine two thin lenses, one positive, the other negative, such that ƒ1 = -ƒ2 and n1 = n2. Since

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad [6.8]$$

$$f = \frac{f_1^2}{d}$$

the system can satisfy the Petzval condition, have a flat field, and still have a finite positive focal length. In visual instruments a certain amount of curvature can be tolerated, because the eye can accommodate for it. Clearly, in photographic lenses field curvature is most undesirable, since it has the effect of rapidly blurring the off-axis image when the film plane is at Fi. An effective means of nullifying the inward curvature of a positive lens is to place a negative field flattener lens near the focal plane. This is often done in projection and photographic objectives when it is not otherwise practicable to meet the Petzval condition (Fig. 6.31). In this position the flattener will have little effect on other aberrations.

Astigmatism is intimately related to field curvature. In the presence of the former aberration, there will be two paraboloidal image surfaces, the tangential, ΣT, and the sagittal, ΣS (as in Fig. 6.32). These are the loci of all the primary and secondary images, respectively, as the object point roams over the object plane. At a given height (yi), a point on ΣT always lies three times as far from ΣP as does the corresponding point on ΣS, and both are on the same side of the Petzval surface (Fig. 6.32). When there is no astigmatism, ΣS and ΣT coalesce onΣP. It is possible to alter the shapes of ΣS and ΣT by bending or relocateing the lenses or by moving the stop. The configuration of Fig. 6.32b is known as an artificially flattened field. A stop in front of an inexpensive meniscus box camera lens is usually arranged to roduce just this effect. The surface of least confusion, ΣLC, is planar, and the image there is tolerable, losing definition at the margins because of the astigmatism. That is to say, although their loci form ⍰LC, the circles of least confusion increase in diameter with distance off the axis. Modern good-quality photographic objectives are generally anastigmats; that is, they are designed so that ΣS and ΣT cross each other, yielding an additional off-axis angle of zero astigmatism. The Cooke Triplet, Tessar, Orthometer, and Biotar (Fig. 5.115) are all anastigmats, as is the relatively fast Zeiss Sonnar, whose residual astigmatism is illustrated graphically in Fig. 6.33. Note the relatively flat field and small amount of astigmatism over most of the film plane.

Let’s return briefly to the Schmidt camera shown in Fig. 5.125 (p. 239), since we are now in a better position to appreciate how it functions. With a stop at the center of curvature of the spherical mirror, all chief rays, which by definition pass through C, are incident normally on the mirror. Moreover, each pencil of rays from a distant object point is symmetrical about its chief ray. In effect, each chief ray serves as an optical axis, so there are no off-axis points and, in principle, no coma or astigmatism. Instead of attempting to flatten the image surface, the designer has coped with curvature by simply shaping the film plate to conform with it.

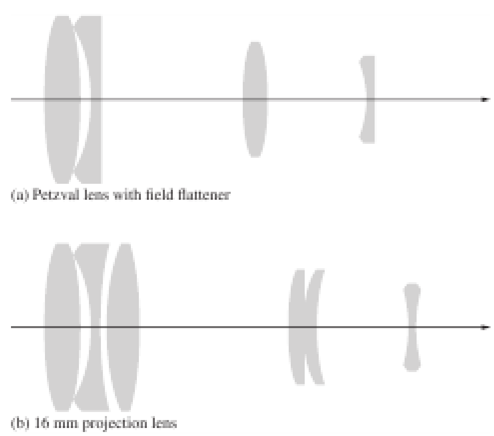


Figure 6.31 The field flattener.

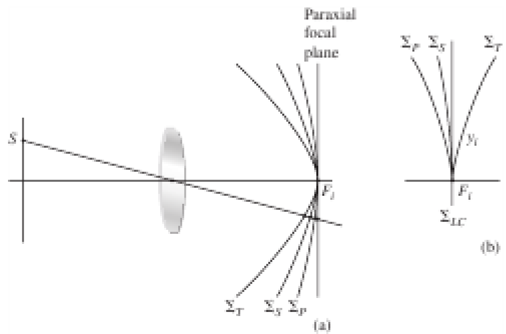


Figure 6.32 The tangential, sagittal, and Petzval image surfaces

The Petzval surface (Fig. 3.14) is actually parabolic in shape and is expressed as Petzval contribution PC by

$$PC = -\frac{u_p'^2 f}{2N} . \qquad (3.11)$$

The vertex radius of this parabola is called the Petzval Radius p. Its measure for a thin lens is

$$\rho = -Nf . \qquad (3.12)$$

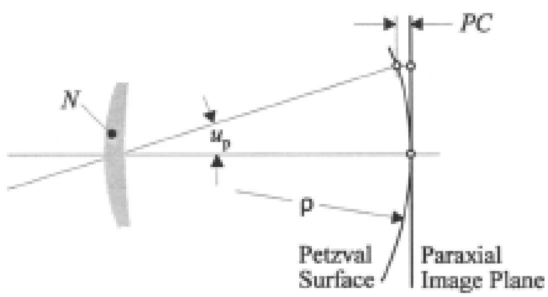


FIG. 3.14 Inward-curving Petzval surface for positive singlet.