

GEOMETRIC OPTICS

Reflection and Refraction at a Plane Surface:

By an object we mean anything from which light rays radiate. This light could be emitted by the object itself if it is **self-luminous**, like the glowing filament of a light bulb. Alternatively, the light could be emitted by another source (such as a lamp or the sun) and then reflected from the object.

Point object that has no physical extent. **Real objects** with length, width, and height are called extended objects. Reflects part of the incident light, or a polished metal surface that reflects almost 100% of the light that strikes it. We will always draw the reflecting surface as a black line with a shaded area behind it, as in **Fig.1** Bathroom mirrors have a thin sheet of glass that lies in front of and protects the reflecting surface; we'll ignore the effects of this thin sheet. According to the law of reflection, all rays striking the surface are reflected at an angle from the normal equal to the angle of incidence. Since the surface is plane, the normal is in the same direction at all points on the surface, and we have specular reflection. After the rays are reflected, their directions are the same as though they had come from point **P'**. We call point **P** an object point and point **P'** the corresponding image point, and we say that the reflecting surface forms an image of point **P**. An observer who can see only the rays reflected

from the surface, and who doesn't know that he's seeing a reflection, thinks that the rays originate from the image point **P'**. The image point is therefore a convenient way to describe the directions of the various reflected rays, just as the object point **P** describes the directions of the rays arriving at the surface before reflection. we call **P'** an image point.

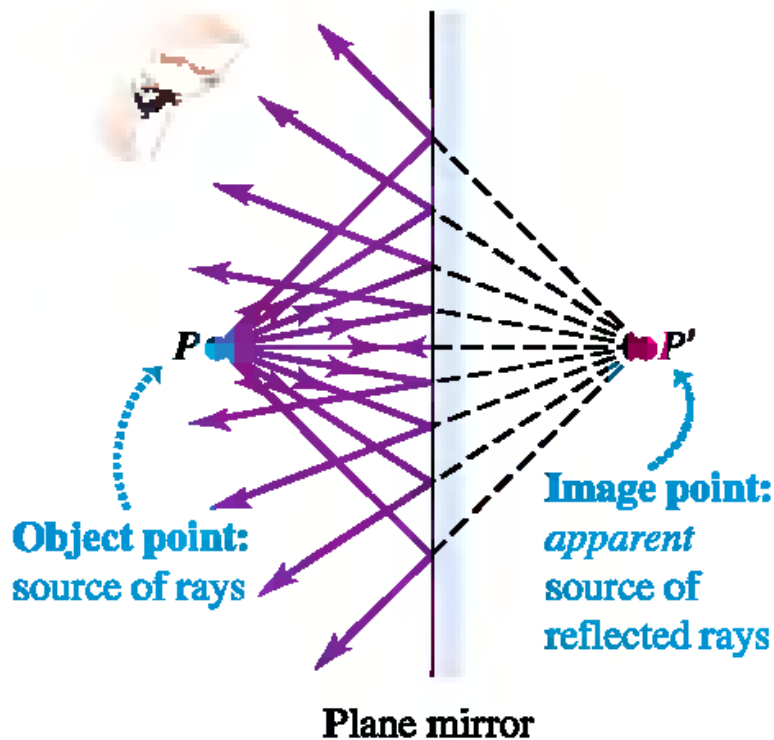
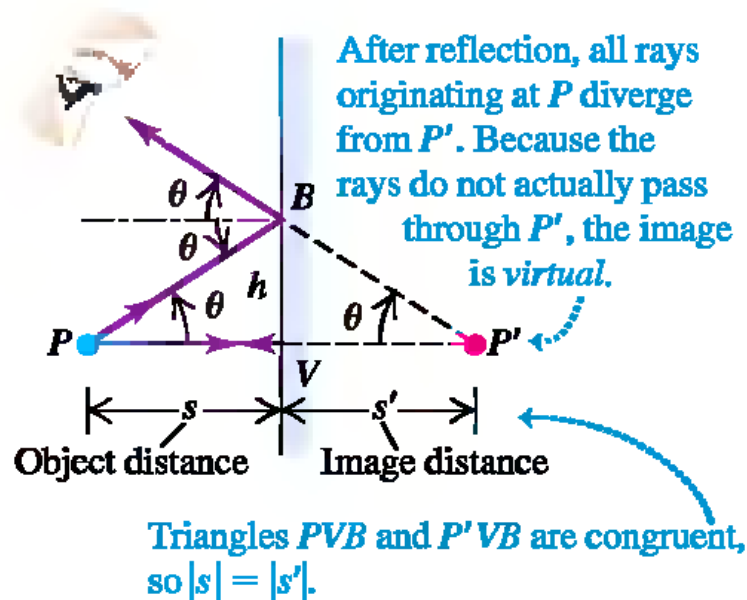


Image Formation by a Plane Mirror

Fig.2. The figure shows two rays diverging from an object point **P** at a distance s to the left of a plane mirror. We call s the object distance. The ray **PV** is incident normally on the mirror (that is, it is perpendicular to the mirror surface), and it returns along its original path.

The ray **PB** makes an angle θ with **PV**. It strikes the mirror at an angle of incidence θ and is reflected at an equal angle with the normal. When we extend the two reflected rays backward, they intersect at point **p'**, at a distance s' behind the mirror. We call s' the image distance. The line between **P** and **P'** is perpendicular to the mirror. **P** and **P'** are at equal distances from the mirror, and s and s' have equal magnitudes. The image point **P'** is located exactly opposite the object point **P** as far behind the mirror as the object point is from the front of the mirror.



Sign Rules

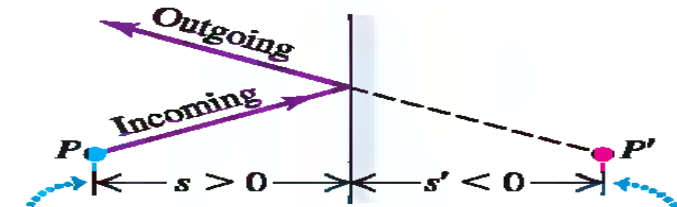
1. Sign rule for the object distance: When the object is on the same side of the reflecting or refracting surface as the incoming light, the object distance s is positive; otherwise, it is negative.
2. Sign rule for the image distance: When the image is on the same side of the reflecting or refracting surface as the outgoing light, the image distance s' is positive; otherwise, it is negative.
3. Sign rule for the radius of curvature of a spherical surface: When the center of curvature C is on the same side as the outgoing light, the radius of curvature is positive; otherwise, it is negative.

The object distance s is positive because the object point P is on the incoming side (the left side) of the reflecting surface. The image distance s' is negative because the image point P' is not on the outgoing side (the

left side) of the surface. The object and image distances s and s' are related simply by :

$$\square \quad s = -s' \quad \square \text{(plane mirror)}$$

(a) Plane mirror



In both of these specific cases:

Object distance s is positive because the object is on the same side as the incoming light.

Image distance s' is negative because the image is NOT on the same side as the outgoing light.

(b) Plane refracting interface

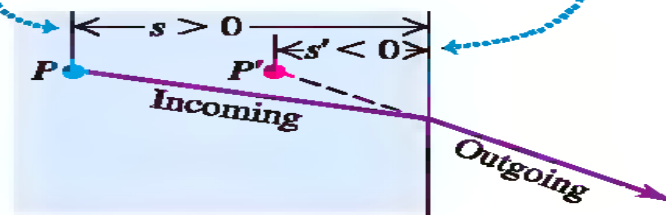


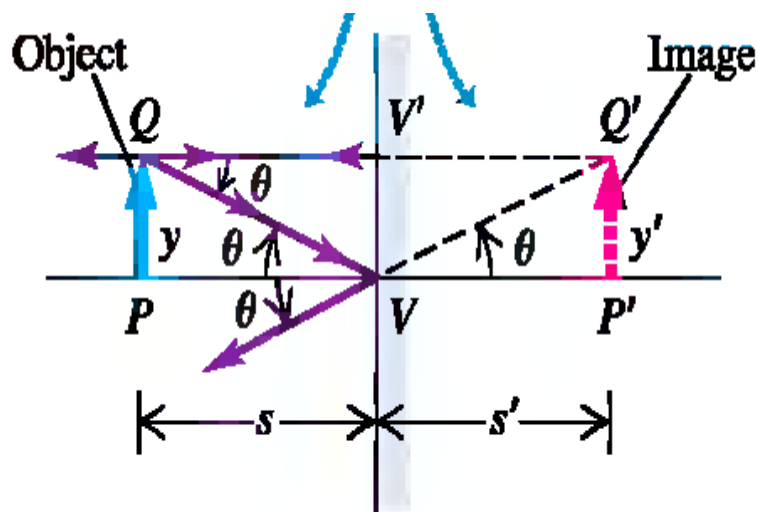
Image of an Extended Object: plane Mirror

Next we consider an extended object with finite size. For simplicity we often consider an object that has only one dimension, like a slender arrow, oriented parallel to the reflecting surface; an example is the arrow PQ in Fig.4. The distance from the head to the tail of an arrow oriented in this way is called its height; the height is y . The image formed by such an extended object is an extended image to each point on the object, there corresponds a point on the image. Two of the rays from Q are shown; all the rays from Q appear to diverge from its image point Q' after reflection. The image of the arrow is the line P' Q', with height y' . orientation, and $y = y'$.

The ratio of image height to object height, y'/y , in any image-forming situation is called the **lateral magnification m** that is:

$$\square \quad \mathbf{m = y' / y} \quad \square \text{(lateral magnification)}$$

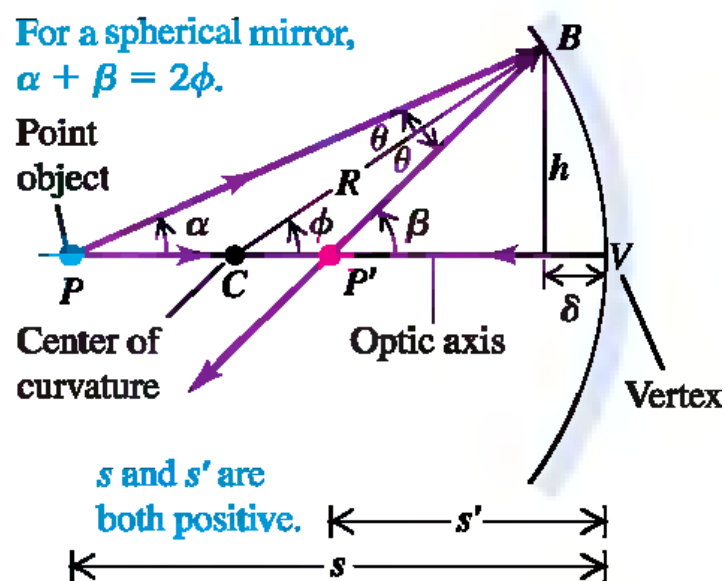
Thus for a plane mirror the lateral magnification m is unity. When you look at yourself in a plane mirror, your image is the same size as the real you. The image arrow points in the same direction as the object arrow; we say that the image is **erect**. In this case, y and y' have the same sign, and the lateral magnification m is positive. The y and y' have both the same magnitude and the same sign; the lateral magnification of a plane mirror is always $m = + 1$. Later we will encounter situations in which the image is **inverted**; that is, the image arrow points in the direction opposite to that of the object arrow. For an **inverted** image, y and y' have opposite signs, and the lateral magnification m is negative.



Reflection at a Spherical Surface

Image of a Point Object: Spherical Mirror

Figure shows a spherical mirror with radius of curvature R , with its concave side facing the incident light. The center of curvature of the surface (the center of the sphere of which the surface is a part) is at C , and the vertex of the mirror (the center of the mirror surface) is at V . The line CV is called the **optic axis**. Point P is an object point that lies on the optic axis; for the moment, we assume that the distance from P to V is greater than R . Ray PV , passing through C , strikes the mirror normally and is reflected back on itself. Ray PB , at an angle α with the axis, strikes the mirror at B , where the angles of incidence and reflection are θ . The reflected ray intersects the axis at point P' . We will show shortly that all rays from P intersect the axis at the same point P' . Point P' therefore the image of object point. The reflected rays in Fig. actually do intersect at point P' , then diverge from P' as if they had originated at this point. Thus P' is a real image.



The object distance, measured from the vertex **V**, is **s**; the image distance, also measured from **V**, is **s'**. The signs of **s**, **s'**, and the radius of curvature **R** are determined by the sign rules. The object point **P** is on the same side as the incident light, so according to sign rule 1, **s** is positive. The image point **P'** is on the same side as the reflected light, so according to sign rule 2, the image distance **s'** is also positive. The center of curvature **C** is on the same side as the reflected light, so according to sign rule 3, **R**, too, is positive; **R** is always positive when reflection occurs at the concave side of a surface.

We now use the following theorem from plane geometry: An exterior angle of a triangle equals the sum of the two opposite interior angles. Applying this theorem to triangles **PBC** and **P'BC** in Fig., we have

$$\phi = \alpha + \theta \quad \beta = \phi + \theta$$

Eliminating θ between these equations gives :

$$\beta + \alpha = 2\phi$$

We may now compute the image distance **s'**. Let **h** represent the height of point **B** above the optic axis, and let δ represent the short distance from **V** to the foot of this vertical line. We now write expressions for the tangents of β , α and ϕ , remembering that **s**, **s'**, and **R** are all positive quantities:

$$\tan\alpha = \frac{h}{s - \delta} \quad \tan\beta = \frac{h}{s' - \delta} \quad \tan\phi = \frac{h}{R - \delta}$$

if the all angle is small, then:

$$\alpha = \frac{h}{s} \quad \beta = \frac{h}{s'} \quad \phi = \frac{h}{R}$$

□

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \quad (\text{object-image relationship, spherical mirror})$$

Such rays nearly parallel to the axis and close to it, are **called paraxial rays**. Since all such reflected light rays converge on the image point, a concave mirror is also called a converging mirror.

As a result, a spherical mirror, unlike a plane mirror, does not form a precise point image of a point object; the image is "**smearred out**." This property of a spherical mirror is called **spherical aberration**. When

If the radius of curvature becomes infinite ($R = \infty$), the mirror becomes Plane.

Focal Point and Focal Length

When the object point P is very far from the spherical mirror ($s = \infty$), the incoming rays are parallel. The image distance s' in this case is given by

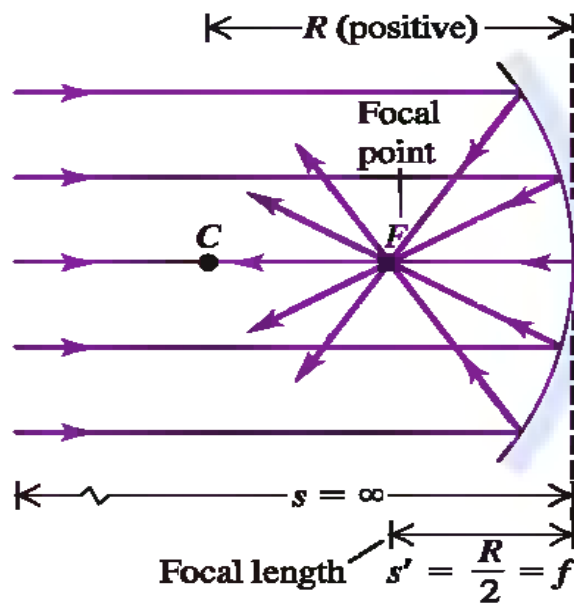
$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \quad s' = \frac{R}{2}$$

The beam of incident parallel rays converges, after reflection from the mirror, to a point F at a distance $R/2$ from the vertex of the mirror. The point **F** at which the incident parallel rays converge is called the **focal point**; we say that these rays are brought to a focus. The distance from the vertex to the focal point, denoted by **f**, is called the **focal length**. We see that **f** is related to the radius of curvature **R** by:

$$f = \frac{R}{2} \quad (\text{focal length of a spherical mirror})$$

Then

$$\frac{2}{R} + \frac{1}{s'} = \frac{2}{R} \quad \frac{1}{s'} = 0 \quad s' = \infty$$



We will usually express the relationship between object and image distances for a mirror, f :

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (\text{object-image relationship, spherical mirror})$$

Image of an Extended Object: Spherical Mirror

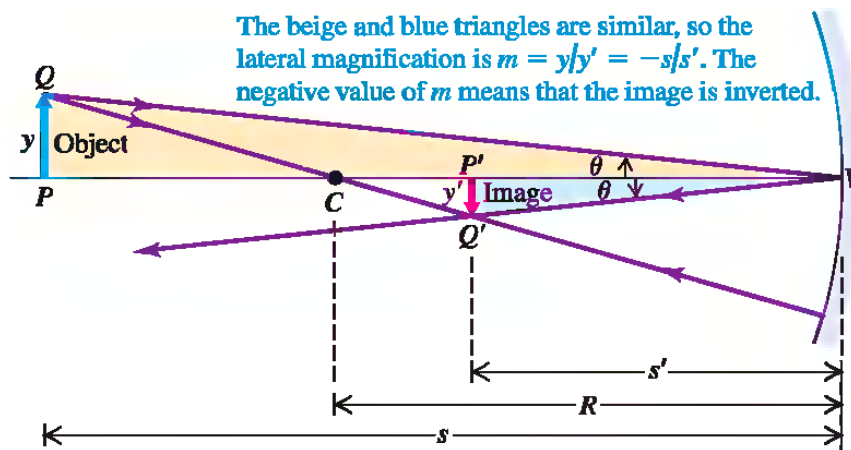
Now suppose we have an object with finite size, represented by the arrow **PQ** in Fig., perpendicular to the optic axis **CV**. The image of **P** formed by paraxial rays is at **P'**. The object distance for point **Q** is very nearly equal to that for point **P**, so the image **P' Q'** is nearly straight and perpendicular to the axis. Note that the object and image arrows have different sizes, y and y' , respectively, and that they have opposite orientation. we defined the lateral magnification m as the ratio of image size y' to object size y :

$$m = \frac{y'}{y}$$

Image of an Extended Object: Spherical Mirror

The image of **P** formed by paraxial rays is at **P'**. The object distance for point **Q** is very nearly equal to that for point **P**, so the image **P' Q'** is nearly straight and perpendicular to the axis. Note that the object and image arrows have different sizes, **y** and **y'**, respectively, and that they have opposite orientation. We defined the lateral magnification **m** as the ratio of image size **y'** to object size **y**:

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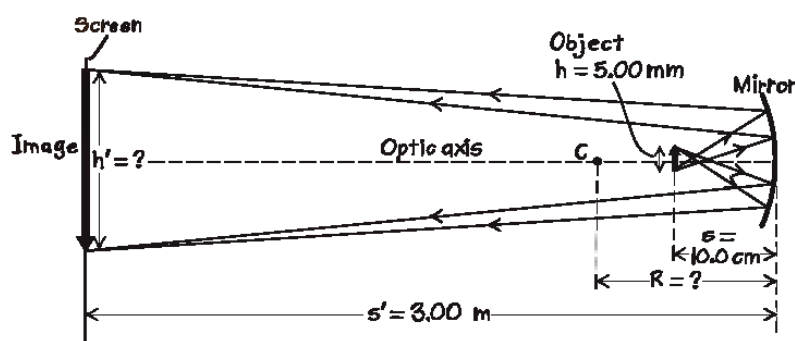
We also have the relationship $y/s = -y'/s'$. The negative sign is needed because object and image are on opposite sides of the optic axis; if **y** is positive, **y'** is negative. Therefore

$$m = \frac{y'}{y} = -\frac{s'}{s} \quad \text{(lateral magnification, spherical mirror)}$$

If **m** is positive, the image is **erect** in comparison to the object; if **m** is negative, the image is **inverted** relative to the object.

Example:

□ A concave mirror forms an image, on a wall **3.00 m** from the mirror, of the filament of a headlight lamp **10.0 cm** in front of the mirror. (a) What are the radius of curvature and focal length of the mirror? (b) What is the height of the image if the height of the object is **5.00 mm**?



$s = 10.0 \text{ cm}$ and $s' = 3.00 \text{ cm}$.

$$\frac{1}{10.0 \text{ cm}} + \frac{1}{300 \text{ cm}} = \frac{2}{R}$$
$$R = \frac{2}{0.100 \text{ cm}^{-1} + 3.33 \times 10^{-3} \text{ cm}^{-1}} = 19.4 \text{ cm}$$

The focal length of the mirror is $f = R/2 = 9.7 \text{ cm}$.

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{300 \text{ cm}}{10.0 \text{ cm}} = -30.0$$

Because m is negative, the image is inverted. The height of the image is 30.0 times the height of the object, or $(30.0)(5.00 \text{ mm}) = 150 \text{ mm}$.

Convex Mirrors

In Fig. the convex side of a spherical mirror faces the incident light. The center of curvature is on the side opposite to the outgoing rays; R is negative. Ray **PB** is reflected, with the angles of incidence and reflection both equal to θ . The reflected ray, projected backward, intersects the axis

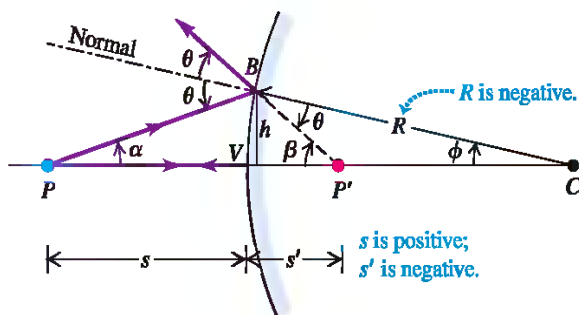
at P' . As with a concave mirror, all rays from P that are reflected by the mirror diverge from the same point P' , provided that the angle α is small. Therefore P' is the image of P . The object distance s is positive, the image distance s' is negative, and the radius of curvature R is negative for a convex mirror.

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

and the lateral magnification is

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

(a) Construction for finding the position of an image formed by a convex mirror



(b) Construction for finding the magnification of an image formed by a convex mirror

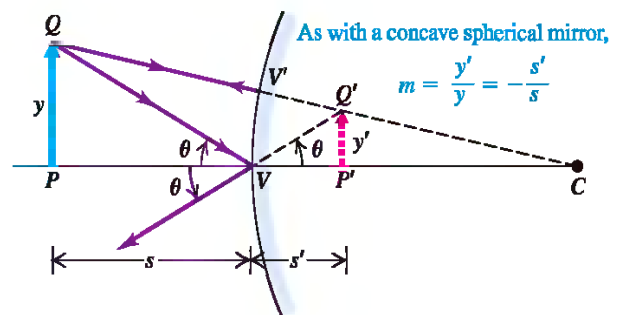


Image of a Point Object: Spherical Refracting Surface

In Fig. a spherical surface with radius **R** forms an interface between two materials with different indexes of refraction **n_a** and **n_b**. The surface forms an image **P'** of an object point **P**.

We are going to prove that if the angle **α** is small; all rays from **P** intersect at the same point **P'**, so **P'** is the real image of **P**. We again use the theorem that an exterior angle of a triangle equals the sum of the two opposite interior angles; applying this to the triangles **PBC** and **P'BC** gives:

$$\theta_a = \alpha + \phi \quad \phi = \beta + \theta_b$$

From the law of refraction,

$$n_a \sin \theta_a = n_b \sin \theta_b$$

Also, the tangents of **α**, **β**, and **φ** are

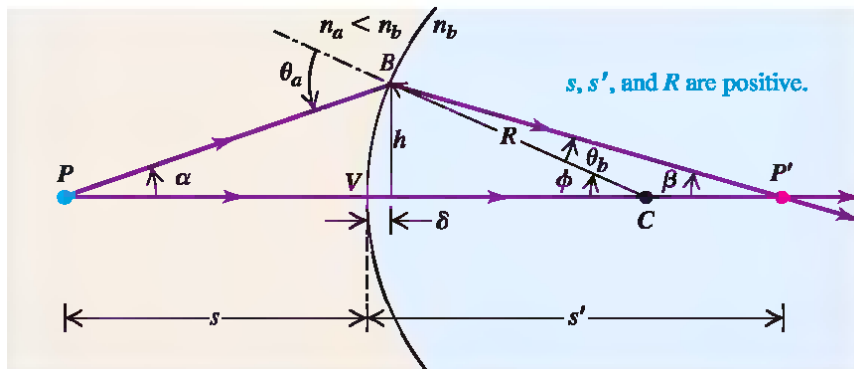
$$\tan \alpha = \frac{h}{s + \delta} \quad \tan \beta = \frac{h}{s' - \delta} \quad \tan \phi = \frac{h}{R - \delta}$$

$$n_a \theta_a = n_b \theta_b$$

$$\theta_b = \frac{n_a}{n_b} (\alpha + \phi)$$

$$n_a \alpha + n_b \beta = (n_b - n_a) \phi$$

$$\alpha = \frac{h}{s} \quad \beta = \frac{h}{s'} \quad \phi = \frac{h}{R}$$



Finally, we substitute these into and divide out the common factor h .

We obtain:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \quad (\text{object-image relationship, spherical refracting surface})$$

$$\tan \theta_a = \frac{y}{s} \quad \tan \theta_b = \frac{-y'}{s'}$$

and from the law of refraction,

$$n_a \sin \theta_a = n_b \sin \theta_b$$

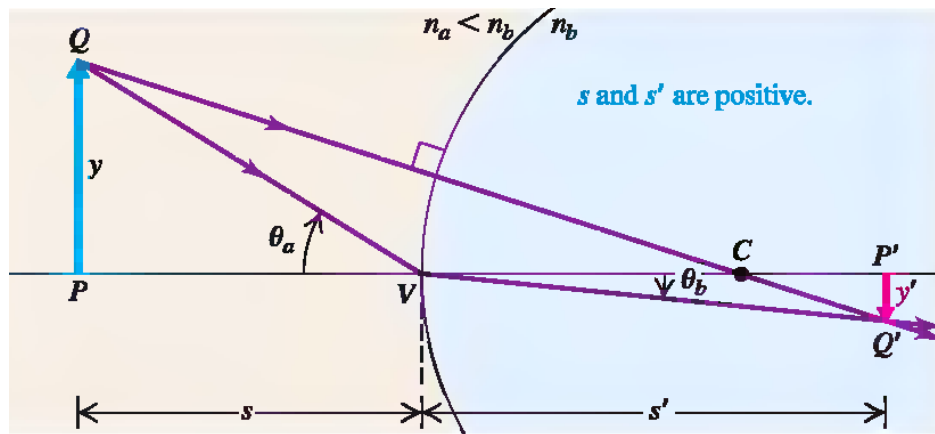
For small angles,

$$\tan \theta_a = \sin \theta_a \quad \tan \theta_b = \sin \theta_b$$

so finally

$$\frac{n_a y}{s} = -\frac{n_b y'}{s'} \quad \text{or}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \quad (\text{lateral magnification, spherical refracting surface})$$



$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \quad (\text{plane refracting surface})$$

$$m = 1$$

Thin Lenses

The most familiar and widely used optical device (after the plane mirror) is the lens. A lens is an optical system with two refracting surfaces. The simplest lens has two spherical surfaces close enough together that we can neglect the distance between them (the thickness of the lens); we call this a **thin lens**.

Properties of a Lens

When a beam of rays parallel to the axis passes through the lens, the rays converge to a point **F2** and form a real image at that point. Such a lens is called a **converging lens**. Similarly, rays passing through point **F1** emerge from the lens as a beam of parallel rays. The points **F1** and **F2** are called the first and second focal points, and the distance **f** (measured from the center of the lens) is called the **focal length**. As for a concave mirror, the focal length of a converging lens is defined to be a **positive** quantity, and such a lens is also called a **positive** lens. The centers of curvature of the two spherical surfaces lie on and define the optic axis. The two focal

lengths both labeled f , are always equal for a thin lens, even when the two sides have different curvatures.

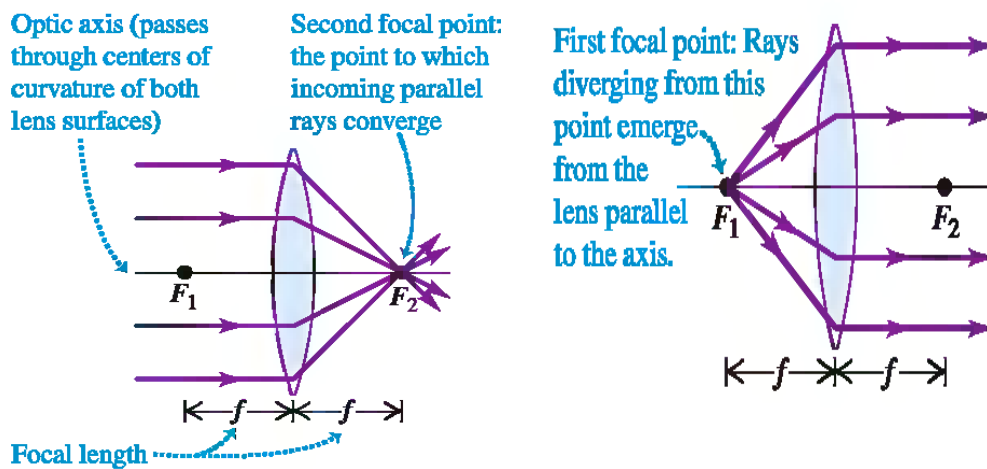


Image of an Extended Object: Converging Lens

We let s and s' be the object and image distances, respectively, and let y and y' be the object and image heights. Ray **QA**, parallel to the optic axis before refraction, passes through the second focal point **F2** after refraction. The two angles labeled α are equal. Therefore the two right triangles **PQO** and **P'Q'O** are similar, and ratios of corresponding sides are equal.

$$\frac{y}{s} = -\frac{y'}{s'} \quad \text{or} \quad \frac{y'}{y} = -\frac{s'}{s}$$

(The reason for the negative sign is that the image is below the optic axis and y' is negative.) Also, the two angles labeled β are equal, and the two right triangles **OAF2** and **P'Q'F2** are similar, so

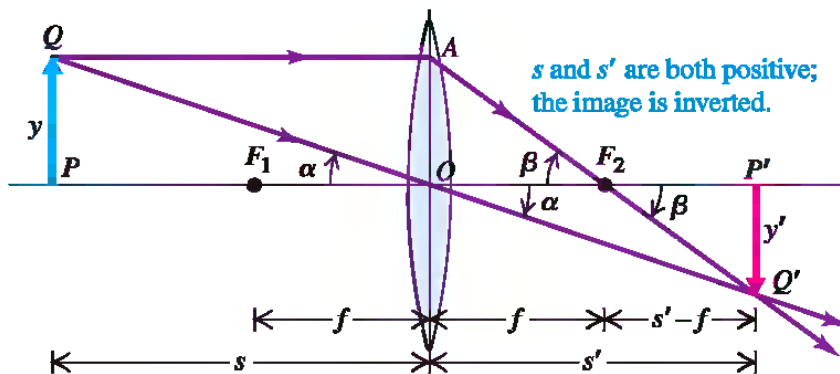
$$\frac{y}{f} = -\frac{y'}{s' - f} \quad \text{or}$$

$$\frac{y'}{y} = -\frac{s' - f}{f}$$

Divide by s' :

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (\text{object-image relationship, thin lens})$$

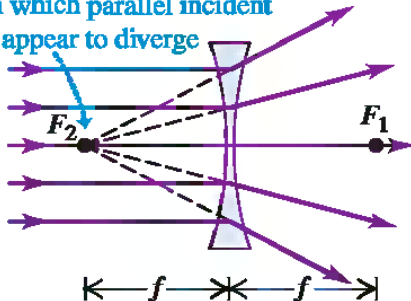
$$m = -\frac{s'}{s} \quad (\text{lateral magnification, thin lens})$$



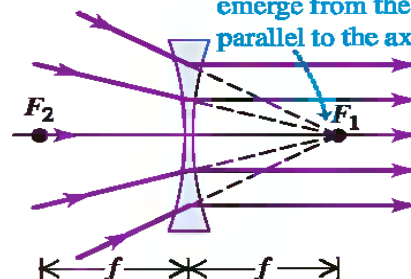
Diverging Lenses

Diverging lens; the beam of parallel rays incident on this lens diverges after refraction. The focal length of a diverging lens is a negative quantity, and the lens is also called a **negative lens**. The focal points of a negative lens are reversed, relative to those of a positive lens.

Second focal point: The point from which parallel incident rays appear to diverge

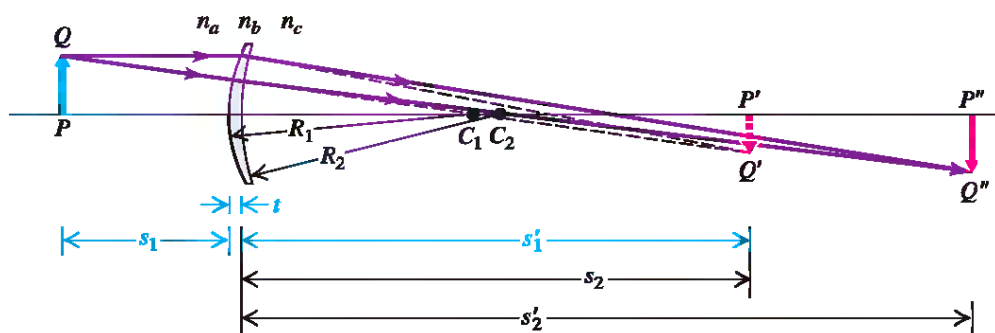


First focal point: Rays converging on this point emerge from the lens parallel to the axis.



The Lensmaker's Equation

Lensmaker's equation, which is a relationship among the focal length f , the index of refraction n of the lens, and the radii of curvature R_1 and R_2 of the lens surfaces. We use the principle that an image formed by one reflecting or refracting surface can serve as the object for a second reflecting or refracting surface. Two spherical interfaces separating three materials with indexes of refraction n_a , n_b and n_c , as shown in Fig. The object and image distances for the first surface are s_1 and s'_1 and those for the second surface are s_2 and s'_2 . We assume that the lens is thin, so that the distance t between the two surfaces is small in comparison with the object and



$$\frac{n_a}{s_1} + \frac{n_b}{s'_1} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s'_2} = \frac{n_c - n_b}{R_2}$$

Ordinarily, the first and third materials are air or vacuum, so we set $n_a = n_c = 1$. The second index n_b is that of the lens, which we can call simply n . Substituting these values and the relationship $s_2 = -s'_1$ we get:

$$\frac{1}{s_1} + \frac{n}{s'_1} = \frac{n-1}{R_1}$$

$$-\frac{n}{s'_1} + \frac{1}{s'_2} = \frac{1-n}{R_2}$$

$$\frac{1}{s_1} + \frac{1}{s'_2} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{s} + \frac{1}{s'} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (\text{lensmaker's equation for a thin lens})$$

Cameras

The basic elements of a camera are a light-tight box, a converging lens, a shutter to open the lens for a prescribed length of time, and a light-sensitive recording medium. In a digital camera this is an electronic detector called a chargecoupled device (CCD) array; in an older camera, this is photographic film. The lens forms an inverted real image on the recording medium of the object being photographed. High-quality camera lenses have several elements, permitting partial correction of various aberrations, including the dependence of index of refraction on wavelength and the limitations imposed by the paraxial approximation.

Camera Lenses: Focal Length

The choice of the focal length f for a camera lens depends on the film size and the desired angle of view. A lens of long focal length, called a **telephoto lens**, gives a small angle of view and a large image of a distant object. A lens of short focal length gives a small image and a wide angle

of view and is called a **wide-angle lens**. The ratio of the image height y' to the object height y (the lateral magnification) is equal in absolute value to the ratio of image distance s' to the object distance s .

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Camera Lenses -Number

The effective area of the lens is controlled by means of an adjustable lens aperture, or diaphragm, a nearly circular hole with variable diameter D ; hence the effective area is proportional to D^2 . Putting these factors together, we see that the intensity of light reaching the film with a particular lens is proportional to D^2 / f^2 . The light-gathering capability of a lens is commonly expressed by photographers in terms of the ratio f/D , called the **f-number** of the lens:

$$\text{f-number} = \frac{\text{Focal length}}{\text{Aperture diameter}} = \frac{f}{D}$$

The Magnifier

The apparent size of an object is determined by the size of its image on the retina. If the eye is unaided, this size depends on the angle θ subtended by the object at the eye, called its **angular size**.

converging lens can be used to form a virtual image that is larger and farther from the eye than the object itself, object can be moved closer to the eye, and the angular size of the image may be substantially larger than the angular size of the object at 25 cm without the lens. A lens used in this way is called a **magnifier**, otherwise known as a magnifying glass or a **simple magnifier**.

$$M = \frac{\theta'}{\theta} \quad (\text{angular magnification})$$

$$\theta = \frac{y}{25 \text{ cm}} \quad \theta' = \frac{y}{f}$$

$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25 \text{ cm}} = \frac{25 \text{ cm}}{f} \quad (\text{angular magnification for a simple magnifier})$$

Microscopes

The object **O** to be viewed is placed just beyond the first focal point **F1** of the objective, a converging lens that forms a real and enlarged image. In a properly designed instrument this image lies just inside the first focal point **F'1** of a second converging lens called the **eyepiece** or **ocular**. The eyepiece acts as a simple magnifier, and forms a final virtual image **I'** of **I**. The position of **I'** may be anywhere between the near and far points of the eye.

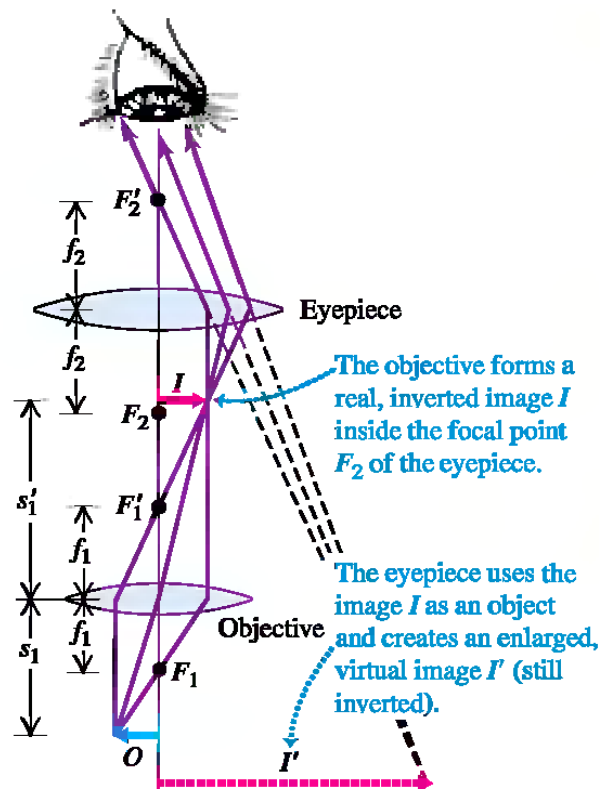
The overall angular magnification of the compound microscope is the product of two factors. The first factor is the lateral magnification **m1** of the objective, which determines the linear size of the real image **I**; the second factor is the angular magnification **M2** of the eyepiece, which relates the angular size of the virtual image seen through the eyepiece to the angular size that the real image **I** would have if you viewed it without the eyepiece. The first of these factors is given by

$$m_1 = -\frac{s'_1}{s_1}$$

s'_1 is very great in comparison to the focal length f_1 of the objective lens. Thus S_1 is approximately equal to f_1 and we can write $m_1 = -s'_1/f_1$.

$$M_2 = (25 \text{ cm})/f_2.$$

$$M = m_1 M_2 = \frac{(25 \text{ cm})s'_1}{f_1 f_2} \quad (\text{angular magnification for a microscope})$$



Telescopes

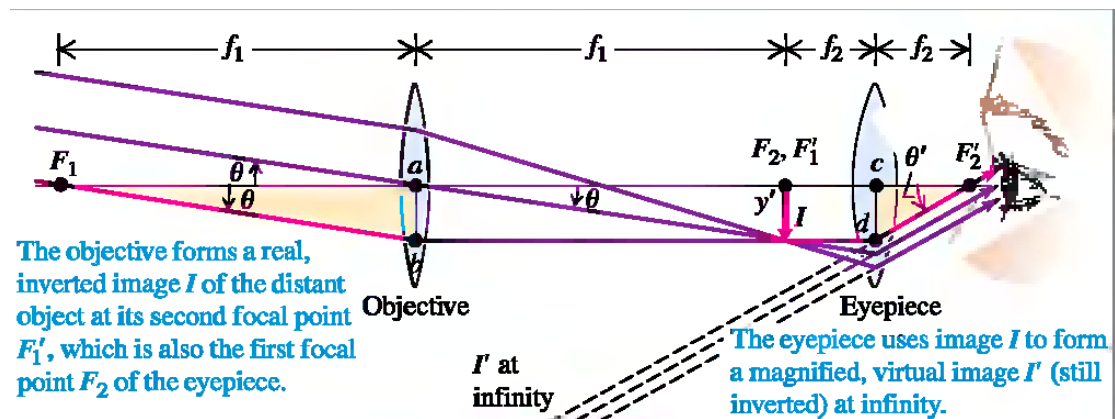
The optical system of a telescope is similar to that of a compound microscope. In both instruments the image formed by an objective is viewed through an eyepiece. The key difference is that the telescope is used to view large objects at large distances and the microscope is used to view small objects close at hand. Another difference is that many telescopes use a curved mirror, not a lens, as an objective. The objective

lens forms a real, reduced image I of the object. This image is the object for the eyepiece lens, which forms an enlarged, virtual image of I .

$$\theta = \frac{-y'}{f_1} \quad \theta' = \frac{y'}{f_2}$$

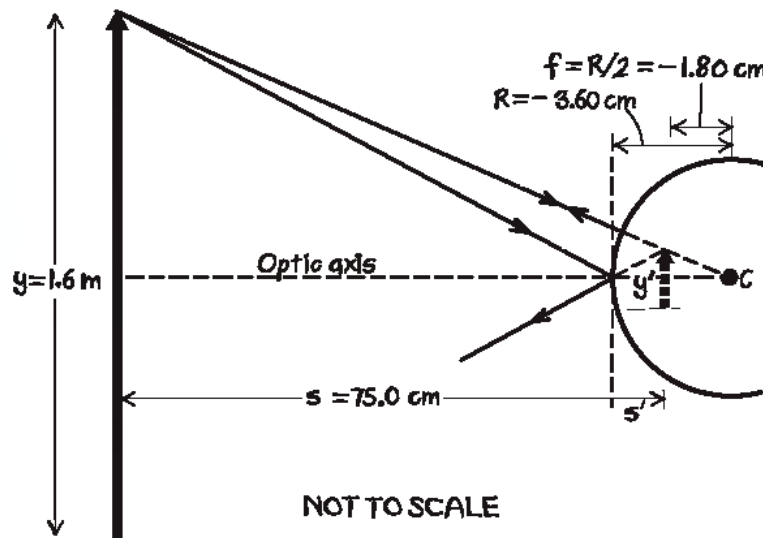
and the angular magnification M is

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2} \quad \text{(angular magnification for a telescope)}$$



Example:

Santa checks himself for soot, using his reflection in a shiny silvered Christmas tree ornament 0.750 m away. The diameter of the ornament is 7.20 cm. Standard reference works state that he is a "right jolly old elf," so we estimate his height to be 1.6 m. Where and how tall is the image of Santa formed by the ornament? Is it erect or inverted?



The radius of the convex mirror (half the diameter) is $R = -(7.20 \text{ cm})/2 = -3.60 \text{ cm}$, and the focal length is $f = R/2 = -1.80 \text{ cm}$.

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-1.80 \text{ cm}} - \frac{1}{75.0 \text{ cm}}$$

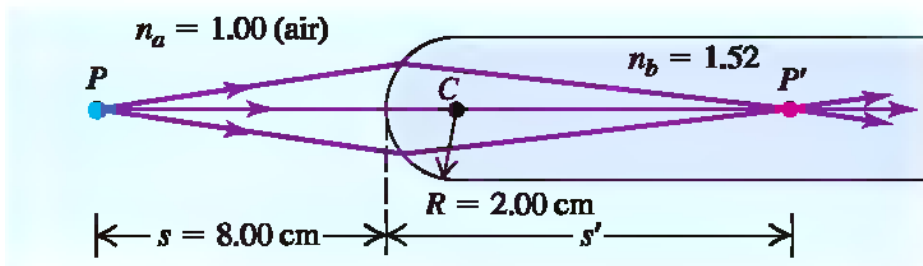
$$s' = -1.76 \text{ cm}$$

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{-1.76 \text{ cm}}{75.0 \text{ cm}} = 0.0234$$

$$y' = my = (0.0234)(1.6 \text{ m}) = 3.8 \times 10^{-2} \text{ m} = 3.8 \text{ cm}$$

Example

A cylindrical glass rod in air has index of refraction 1.52. One end is ground to a hemispherical surface with radius $R = 200$ cm. (a) Find the image distance of a small object on the axis of the rod, 8.00 cm to the left of the vertex. (b) Find the lateral magnification.



$$\frac{1.00}{8.00 \text{ cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.00}{+2.00 \text{ cm}}$$
$$s' = +11.3 \text{ cm}$$

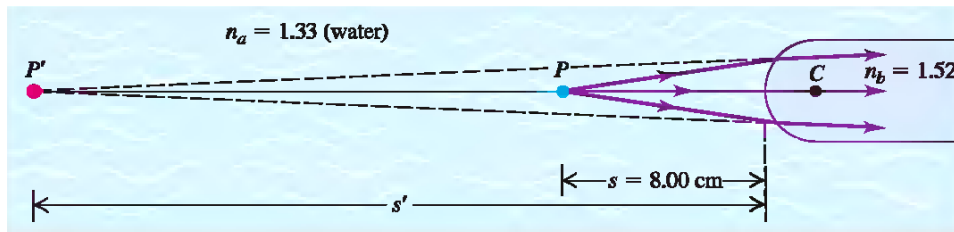
$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(11.3 \text{ cm})}{(1.52)(8.00 \text{ cm})} = -0.929$$

Example

The glass rod in Example above is immersed in water (index of refraction $n = 1.33$), as shown in Fig. The other quantities have the same values as before. Find the image distance and lateral magnification.

$$\frac{1.33}{8.00 \text{ cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.33}{+2.00 \text{ cm}}$$
$$s' = -21.3 \text{ cm}$$

$$m = -\frac{(1.33)(-21.3 \text{ cm})}{(1.52)(8.00 \text{ cm})} = +2.33$$



Example □ Swimming pool owners know that the pool always looks shallower than it really is and that it is important to identify the deep parts conspicuously so that people who can't swim won't jump into water that's over their heads. If a nonswimmer looks straight down into water that is actually 2.00 m (about 6 ft, 7 in.) deep, how deep does it appear to be?

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{1.33}{2.00 \text{ m}} + \frac{1.00}{s'} = 0$$

$$s' = -1.50 \text{ m}$$

Example

- (a) Suppose the absolute values of the radii of curvature of the lens surfaces are both equal to 10 cm and the index of refraction is $n = 1.52$. What is the focal length f of the lens?
- (b) Suppose the lens also has $n = 1.52$, and the absolute values of the radii of curvature of its lens surfaces are also both equal to 10 cm. What is the focal length of this lens?

$$\frac{1}{f} = (1.52 - 1) \left(\frac{1}{+10 \text{ cm}} - \frac{1}{-10 \text{ cm}} \right)$$

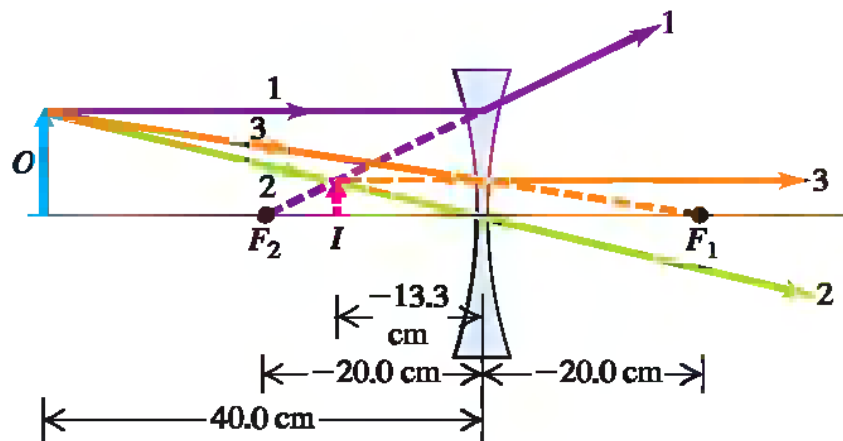
$$f = 9.6 \text{ cm}$$

$$\frac{1}{f} = (1.52 - 1) \left(\frac{1}{-10 \text{ cm}} - \frac{1}{+10 \text{ cm}} \right)$$

$$f = -9.6 \text{ cm}$$

Example

You are given a thin diverging lens. You find that a beam of parallel rays spreads out after passing through the lens, as though all the rays came from a point 20.0 cm from the center of the lens. You want to use this lens to form an erect virtual image that is 1/3 the height of the object. (a) Where should the object be placed? (b) Draw a principal-ray diagram.



$$\frac{1}{s} + \frac{1}{-s/3} = \frac{1}{-20.0 \text{ cm}}$$

$$s = 40.0 \text{ cm}$$

$$s' = -\frac{s}{3} = -\frac{40.0 \text{ cm}}{3} = -13.3 \text{ cm}$$

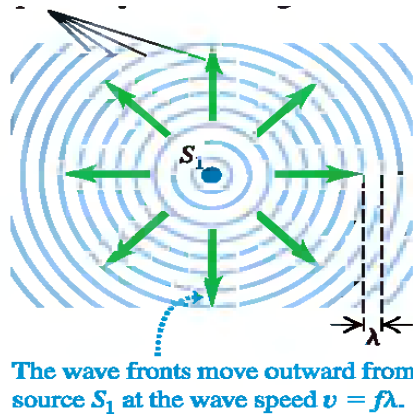
INTERFERENCE

Interference and Coherent Sources

The term interference refers to any situation in which two or more waves overlap in space or the initial and reflected waves overlap in the same region of the medium. This overlapping of waves is called interference.

Interference in two or Three Dimensions

Interference effects are most easily seen when we combine sinusoidal waves with a single frequency f and wavelength λ . Figure shows a "snapshot" or "freeze-frame" of a single source **S1** of sinusoidal waves and some of the wave fronts produced by this source. The figure shows only the wave fronts corresponding to wave crests, so the spacing between successive wave fronts is one wavelength. The material surrounding **S1** is uniform, so the wave speed is the same in all directions, and there is no refraction (and hence no bending of the wave fronts). If the waves are two-dimensional, like waves on the surface of a liquid, the circles in Fig. represent circular wave fronts; if the waves propagate in three dimensions, the circles represent spherical wave fronts spreading away from **S1**. In optics, sinusoidal waves are characteristic of **monochromatic** light (light of a single color). common sources of light do not emit monochromatic. However, there are several ways to produce approximately monochromatic light. For example, some filters block all but a very narrow range of wavelengths. By far the most nearly **monochromatic** source that is available at present is the laser. An example is the helium-neon laser, which emits red light at 632.8 nm.



Constructive and Destructive Interference

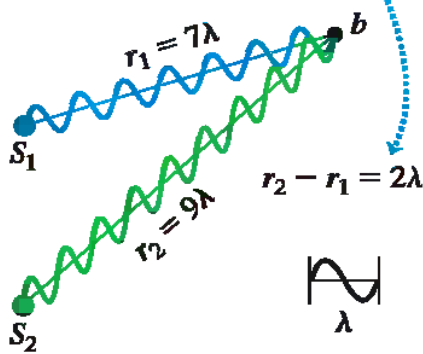
In general, when waves from two or more sources arrive at a point in phase, the amplitude of the resultant wave is the sum of the amplitudes of the individual waves; the individual waves reinforce each other. This is called **constructive interference**. For constructive interference to occur at **P**, the path difference $r_2 - r_1$ for the two sources must be an integral multiple of the wavelength λ :

$$r_2 - r_1 = m\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \dots) \quad \text{(Constructive Interference)}$$

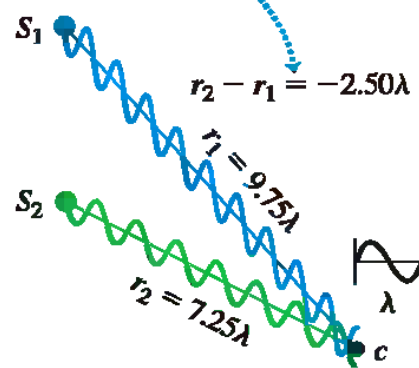
The path difference $r_2 - r_1 = -2.50 \lambda$, which is a half-integral number of wavelengths. This cancellation or partial cancellation of the individual waves called **destructive interference**, the condition for destructive interference in the situation shown in Fig. □

$$r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \dots) \quad \text{(Destructive Interference)}$$

Waves interfere constructively if their path lengths differ by an integral number of wavelengths: $r_2 - r_1 = m\lambda$.



Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.

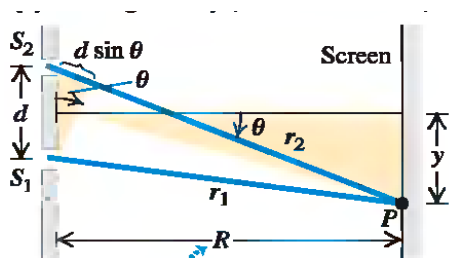


Two-Source Interference of Light

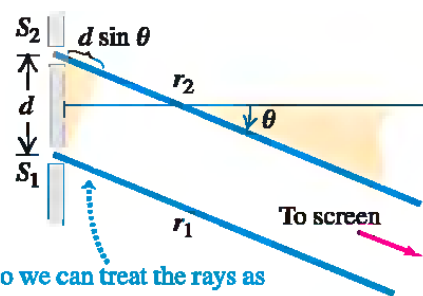
To visualize the interference pattern, a screen is placed so that the light from **S1** and **S2** falls on it. The screen will be most brightly illuminated at points **P**, where the light waves from the slits interfere constructively, and will be darkest at points where the interference is destructive.

To simplify the analysis of Young's experiment, we assume that the distance R from the slits to the screen is so large in comparison to the distance d between the slits that the lines from **S1** and **S2** to **P** are very nearly parallel, as in Fig. This is usually the case for experiments with light; the slit separation is typically a few millimeters, while the screen may be a meter or more away. The difference in path length is then given by:

$$r_2 - r_1 = d \sin \theta$$



In real situations, the distance R to the screen is usually very much greater than the distance d between the slits ...



... so we can treat the rays as parallel, in which case the path-length difference is simply $r_2 - r_1 = d \sin \theta$.

Constructive and Destructive Two-Slit Interference

$$d \sin \theta = m \lambda \quad (m = 0, \pm 1, \pm 2, \dots) \quad (\text{constructive interference, two slits})$$

$$d \sin \theta = \left(m + \frac{1}{2}\right) \lambda \quad (m = 0, \pm 1, \pm 2, \dots) \quad (\text{destructive interference, two slits})$$

Thus the pattern on the screen of Figs. is a succession of bright and dark bands, or interference fringes, Let y_m the distance from the center of the pattern ($\theta = 0$) to the center of the m th bright band. Let θ_m be the corresponding value of θ ; then

$$y_m = R \tan \theta_m$$

$$y_m = R \sin \theta_m$$

$$y_m = R \frac{m \lambda}{d} \quad (\text{constructive interference in Young's experiment})$$

Amplitude in Two-Source Interference

In Fig. **E1** is the horizontal component of the phasor representing the wave from source **S1** and **E2** is the horizontal component of the phasor for the wave from **S2**. As shown in the diagram, both phasors have the same magnitude **E**, but **E1** is ahead of **E2** in phase by an angle ϕ . Both phasors rotate counterclockwise with constant angular speed ω , and the sum of the projections on the horizontal axis at any time gives the instantaneous value of the total **E** field at point **P**. Thus the amplitude **E_p** of the resultant sinusoidal wave at **P** is the magnitude of the dark red

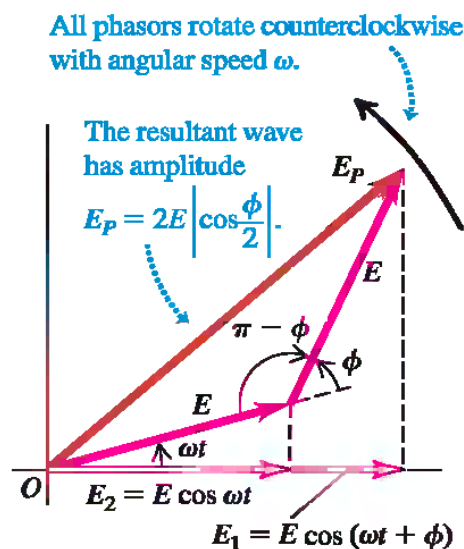
phasor in the diagram (labeled \mathbf{E}_p); this is the vector sum of the other two phasors. To find \mathbf{E}_p , we use the law of cosines and the trigonometric identity $\cos(\pi - \phi) = -\cos \phi$:

$$\begin{aligned} E_p^2 &= E^2 + E^2 - 2E^2 \cos(\pi - \phi) \\ &= E^2 + E^2 + 2E^2 \cos \phi \end{aligned}$$

Then, using the identity $1 + \cos \phi = 2 \cos^2(\phi/2)$, we obtain

$$E_p^2 = 2E^2(1 + \cos \phi) = 4E^2 \cos^2\left(\frac{\phi}{2}\right)$$

$$E_p = 2E \left| \cos \frac{\phi}{2} \right| \quad (\text{amplitude in two-source interference})$$



Intensity in Two-Source Interference

To obtain the intensity \mathbf{I} at point \mathbf{P} , that \mathbf{I} is equal to the average magnitude of the Poynting vector, \mathbf{S}_{av} . For a sinusoidal wave with electric-field amplitude \mathbf{E}_p , this is given by with \mathbf{E}_{max} replaced by \mathbf{E}_p . Thus we can express the intensity in any of the following equivalent forms:

$$I = S_{\text{av}} = \frac{E_P^2}{2\mu_0 c} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_P^2 = \frac{1}{2} \epsilon_0 c E_P^2$$

$$I = \frac{1}{2} \epsilon_0 c E^2 = 2\epsilon_0 c E^2 \cos^2 \frac{\phi}{2}$$

In particular, the maximum intensity I_0 , which occurs at points where the phase difference is zero ($\phi = 0$), is

$$I_0 = 2\epsilon_0 c E^2$$

$$I = I_0 \cos^2 \frac{\phi}{2} \quad (\text{intensity in two-source interference})$$

If we average over all possible phase differences, the result is $I_0/2 = \epsilon_0 c E^2$ (the average of $\cos^2(\phi/2)$ is $1/2$).

Phase Difference and Path Difference

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

$$\phi = \frac{2\pi}{\lambda} (r_2 - r_1) = k(r_2 - r_1) \quad (\text{phase difference related to path difference})$$

$$\lambda = \frac{\lambda_0}{n} \quad \text{and} \quad k = nk_0$$

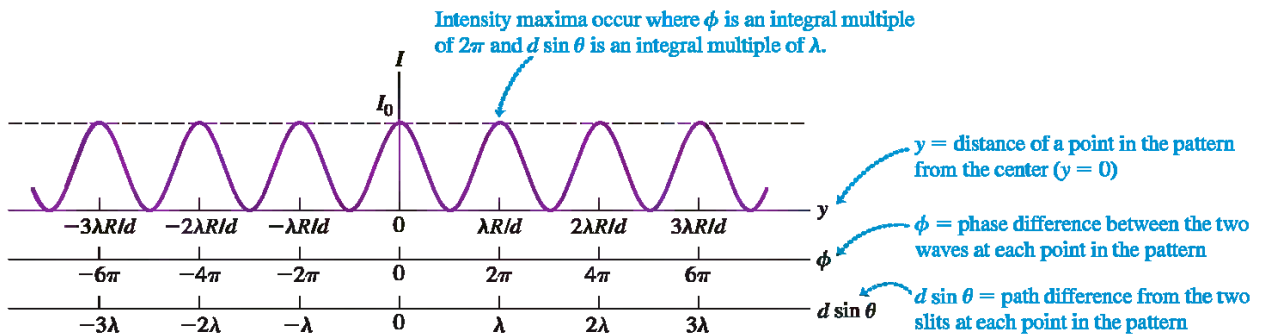
$$r_2 - r_1 = d \sin \theta$$

$$\phi = k(r_2 - r_1) = kd \sin \theta = \frac{2\pi d}{\lambda} \sin \theta$$

$$I = I_0 \cos^2\left(\frac{1}{2}k d \sin\theta\right) = I_0 \cos^2\left(\frac{\pi d}{\lambda} \sin\theta\right) \quad (\text{intensity far from two sources})$$

$$\frac{\pi d}{\lambda} \sin\theta = m\pi \quad (m = 0, \pm 1, \pm 2, \dots)$$

$$d \sin\theta = m\lambda$$



$$I = I_0 \cos^2\left(\frac{kdy}{2R}\right) = I_0 \cos^2\left(\frac{\pi dy}{\lambda R}\right) \quad (\text{intensity in two-slit interference})$$

Thin-Film Interference and Phase Shifts During Reflection

Suppose a light wave with electric-field amplitude E_i is traveling in an optical material with index of refraction n_a . It strikes, at normal incidence, an interface with another optical material with index n_b . The amplitude E_r of the wave reflected from the interface is proportional to the amplitude E_i of the incident wave and is given by

$$E_r = \frac{n_a - n_b}{n_a + n_b} E_i \quad (\text{normal incidence})$$

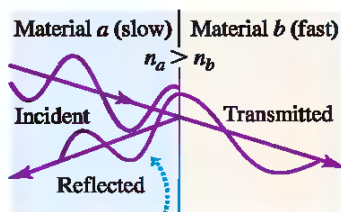
This result shows that the incident and reflected amplitudes have the same sign when n_a is larger than n_b and opposite sign when n_b is larger than n_a . We can distinguish three cases, as shown in Fig.:

Figure a: When $n_a > n_b$, light travels more slowly in the first material than in the second. In this case, E_r and E_i have the same sign, and the phase shift of the reflected wave relative to the incident wave is zero.

Figure b: When $n_a = n_b$, the amplitude E_r of the reflected wave is zero. The incident light wave can't "see" the interface, and there is no reflected wave.

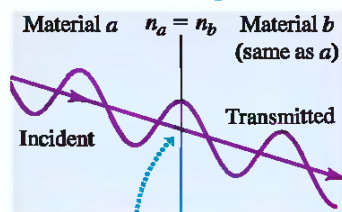
Figure c: When $n_a < n_b$ light travels more slowly in the second material than in the first. In this case, E_r and E_i have opposite signs, and the phase shift of the reflected wave relative to the incident wave is π rad (180° or a half-cycle).

(a) If the transmitted wave moves faster than the incident wave ...



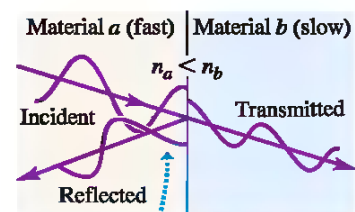
... the reflected wave undergoes no phase change.

(b) If the incident and transmitted waves have the same speed ...



... there is no reflection.

(c) If the transmitted wave moves slower than the incident wave ...



... the reflected wave undergoes a half-cycle phase shift.

If the film has thickness t , the light is at normal incidence and has wavelength λ in the film; if neither or both of the reflected waves from the two surfaces have a half-cycle reflection phase shift, the conditions for constructive and destructive interference are

$$2t = m\lambda \quad (m = 0, 1, 2, \dots)$$

(constructive reflection from thin film, no relative phase shift)

$$2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \dots)$$

(destructive reflection from thin film, no relative phase shift)

$$2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \dots)$$

(constructive reflection from thin film, half-cycle relative phase shift)

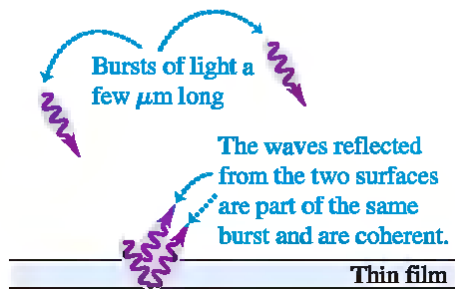
$$2t = m\lambda \quad (m = 0, 1, 2, \dots)$$

(destructive reflection from thin film, half-cycle relative phase shift)

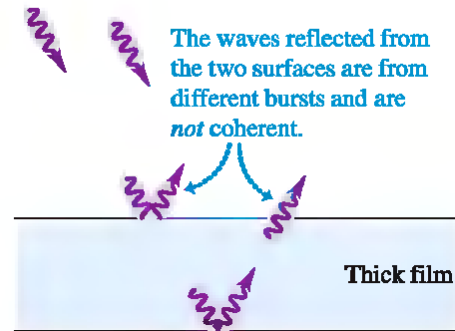
Thin and Thick Films

If light reflects from the two surfaces of a thin film, the two reflected waves are part of the same burst (Fig. a). Hence these waves are coherent and interference occurs as we have described. If the film is too thick, however, the two reflected waves will belong to different bursts (Fig. b).

(a) Light reflecting from a thin film



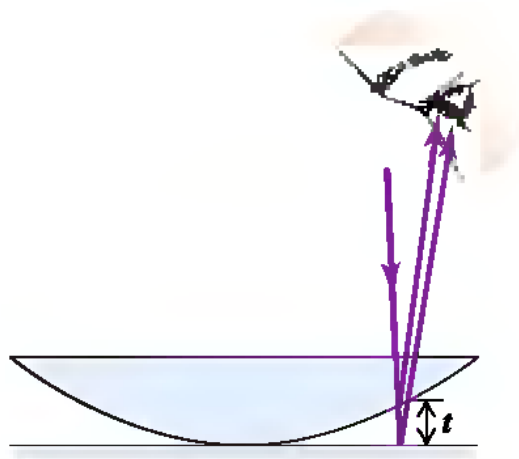
(b) Light reflecting from a thick film



Newton's Rings

Figure 35.17a shows the convex surface of a lens in contact with a plane glass plate. A thin film of air is formed between the two surfaces. When you view the setup with monochromatic light, you see circular interference fringes (Fig.b). These were studied by Newton and are called **Newton's rings**.

(a) A convex lens in contact with a glass plane



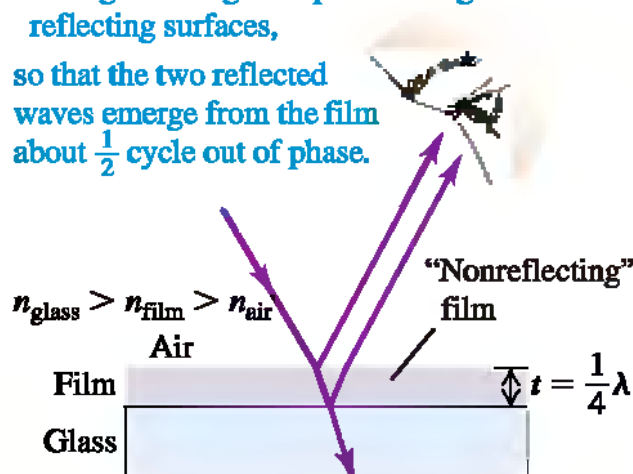
Nonreflective and Reflective Coatings

Nonreflective coatings for lens surfaces make use of thin-film interference. A thin layer or film of hard transparent material with an index of refraction smaller than that of the glass is deposited on the lens surface, as in Fig.. Light is reflected from both surfaces of the layer. In both reflections the light is reflected from a medium of greater index than that in which it is traveling, so the same phase change occurs in both reflections. If the film thickness is a quarter (one-fourth) of the wavelength in the film (assuming normal incidence), the total path difference is a half-wavelength. Light reflected from the first surface is then a half-cycle out of phase with light reflected from the second, and there is destructive interference. The thickness of the nonreflective coating can be a quarter-wavelength for only one particular wavelength. If a quarter-wavelength thickness of a material with an index of refraction greater than that of glass is deposited on glass, then the reflectivity is increased and the deposited material is called a **reflective coating**.

Destructive interference occurs when

- the film is about $\frac{1}{4}\lambda$ thick and
- the light undergoes a phase change at both reflecting surfaces,

so that the two reflected waves emerge from the film about $\frac{1}{2}$ cycle out of phase.



The Michelson Interferometer

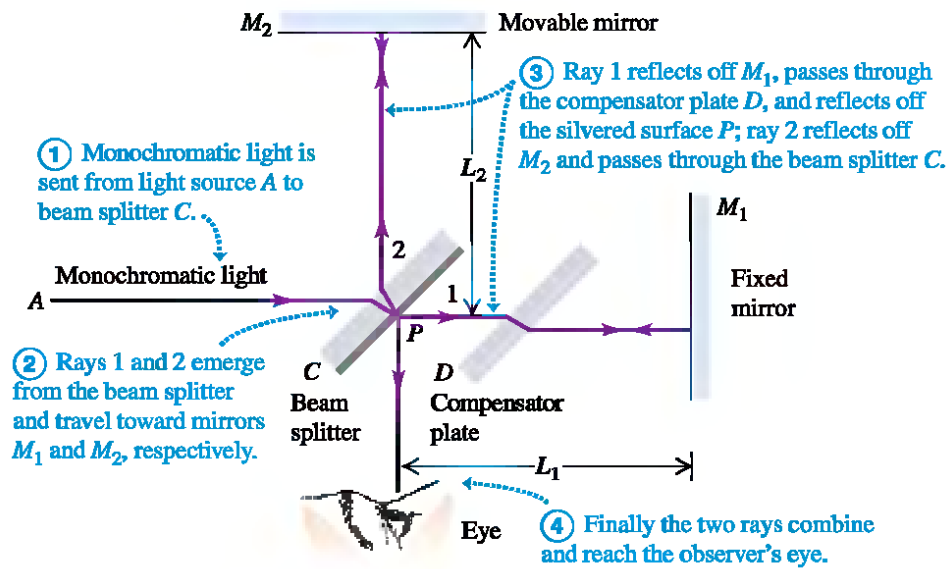
Like the Young two-slit experiment, a Michelson interferometer takes monochromatic light from a single source and divides it into two waves that follow different paths. In Young's experiment this is done by sending part of the light through one slit and part through another in a Michelson interferometer a device called a **beam splitter** is used. Interference occurs in both experiments when the two light waves are recombined.

How a Michelson Interferometer Works

The principal components of a Michelson interferometer are shown schematically in Fig.. A ray of light from a monochromatic source **A** strikes the beam splitter **C**, which is a glass plate with a thin coating of silver on its right side. Part of the light (**ray 1**) passes through the silvered surface and the compensator plate **D** and is reflected from mirror **M1**. It then returns through **D** and is reflected from the silvered surface of **C** to the observer. The remainder of the light (**ray 2**) is reflected from the silvered surface at point **P** to the mirror **M2** and back through **C** to the observer's eye. The purpose of the compensator plate **D** is to ensure that rays **1** and **2** pass through the same thickness of glass; plate **D** is cut from the same piece of glass as plate **C**, so their thicknesses are identical to within a fraction of a wavelength. If the distances **L1** and **L2** are exactly equal and the mirrors **M1** and **M2** are exactly at right angles, the virtual image of **M1** formed by reflection at the silvered surface of plate **C** coincides with mirror **M2**. If **L1** and **L2** are not exactly equal, the image of **M1** is displaced slightly from **M2**. If we observe the fringe positions through a telescope with a crosshair eyepiece and **m** fringes cross the crosshairs when we move the mirror a distance **y**, then:

□

$$y = m \frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{2y}{m}$$



DIFFRACTION

That the behavior of waves after they pass through an aperture or around an edge.

Fresnel and Fraunhofer Diffraction

When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called **Fraunhofer diffraction**. When the source or the observer is relatively close to the obstructing surface, it is **Fresnel diffraction**.

Diffraction and Huygens's Principle

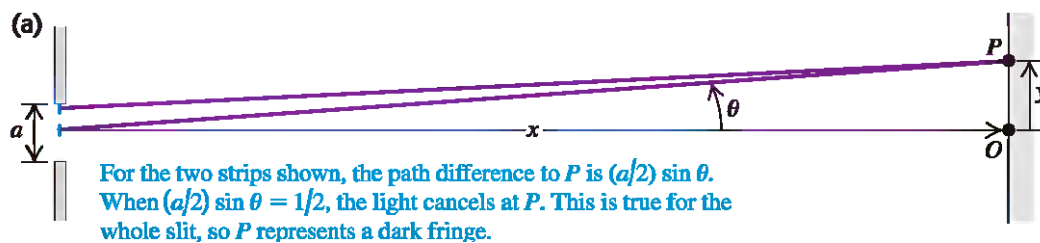
Diffraction usually involves a continuous distribution of Huygens's wavelets across the area of an aperture, or a very large number of sources or apertures. But both categories of phenomena are governed by the same basic physics of superposition and Huygens's principle.

□ *Diffraction from a Single Slit*

in this section we'll discuss the diffraction pattern formed by plane-wave (parallel ray) monochromatic light when it emerges from a long, narrow slit. We call the narrow dimension the width, the diffraction pattern consists of a central bright band, which may be much broader than the width of the slit, bordered by alternating dark and bright bands with rapidly decreasing intensity. About 85% of the power in the transmitted beam is in the central bright band, whose width is found to be inversely proportional to the width of the slit. In general, the smaller the width of the slit, the broader the entire diffraction pattern.

Single-Slit Diffraction: Locating the Dark Fringes

In fact, the light from every strip in the top half of the slit cancels out the light from a corresponding strip in the bottom half. The result is complete cancellation at **P** for the combined light from the entire slit, giving a dark fringe in the interference pattern. That is, a dark fringe occurs whenever



$$\frac{a}{2} \sin \theta = \pm \frac{\lambda}{2} \quad \text{or} \quad \sin \theta = \pm \frac{\lambda}{a}$$

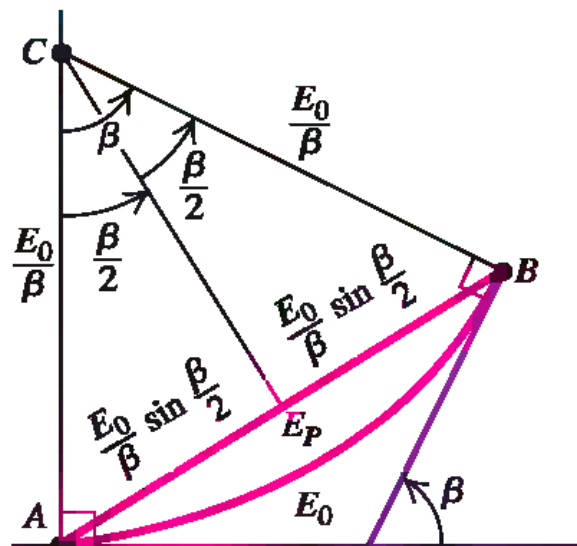
The plus-or-minus (+ -) sign in Eq says that there are symmetrical dark fringes above and below point **O** in. The upper fringe ($\theta > 0$) occurs at a point **P** where light from the bottom half of the slit travels $\lambda/2$ farther to **P** than does light from the top half; the lower fringe ($\theta < 0$) occurs where light from the top half travels $\lambda/2$ farther than light from the bottom half.

$$\sin \theta = \frac{m\lambda}{a} \quad (m = \pm 1, \pm 2, \pm 3, \dots) \quad \text{(dark fringes in single-slit diffraction)}$$

$$\theta = \frac{m\lambda}{a} \quad (m = \pm 1, \pm 2, \pm 3, \dots) \quad \text{(for small angles } \theta \text{)}$$

$$y_m = x \frac{m\lambda}{a} \quad (\text{for } y_m \ll x)$$

Intensity in the Single-Slit Pattern



$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2} \quad (\text{amplitude in single-slit diffraction})$$

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 \quad (\text{intensity in single-slit diffraction})$$

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

$$I = I_0 \left\{ \frac{\sin[\pi a (\sin\theta)/\lambda]}{\pi a (\sin\theta)/\lambda} \right\}^2 \quad (\text{intensity in single-slit diffraction})$$

$$\frac{a \sin \theta}{\lambda} = m \quad (m = \pm 1, \pm 2, \dots)$$

$$\sin \theta = \frac{m \lambda}{a} \quad (m = \pm 1, \pm 2, \dots)$$

Width of the Single-Slit Pattern

$$\theta_1 = \frac{\lambda}{a}$$

Two Slits of Finite Width

$$I = I_0 \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2 \quad (\text{two slits of finite width})$$

$$\phi = \frac{2\pi d}{\lambda} \sin \theta \quad \beta = \frac{2\pi a}{\lambda} \sin \theta$$

The Diffraction Grating

An array of a large number of parallel slits, all with the same width **a** and spaced equal distances **d** between centers, is called a **diffraction grating**.

$$d \sin \theta = m \lambda \quad (m = 0, \pm 1, \pm 2, \pm 3, \dots) \quad (\text{intensity maxima, multiple slits})$$

Grating Spectrographs

Diffraction gratings are widely used to measure the spectrum of light emitted by a source, a process called spectroscopy or spectrometry. Light incident on a grating of known spacing is dispersed into a spectrum. The angles of deviation of the maxima are then measured, and Eq. above is used to compute the wavelength.

Resolution of a Grating Spectrograph

In spectroscopy it is often important to distinguish slightly differing wavelengths. The minimum wavelength difference $\Delta\lambda$ that can be distinguished by a spectrograph is described by the chromatic resolving power R , defined as:

$$R = \frac{\lambda}{\Delta\lambda} \quad (\text{chromatic resolving power})$$

The phase difference is also given by $\phi = (2\pi d \sin\theta)/\lambda$, so the angular interval $d\theta$ corresponding to a small increment $d\phi$ in the phase shift can be obtained from the differential of this equation:

$$d\phi = \frac{2\pi d \cos\theta d\theta}{\lambda}$$
$$\frac{2\pi}{N} = \frac{2\pi d \cos\theta d\theta}{\lambda} \quad \text{or} \quad d \cos\theta d\theta = \frac{\lambda}{N}$$

X-Ray Diffraction

A crystal serves as a three-dimensional diffraction grating for x rays with wavelengths of the same order of magnitude as the spacing between atoms in the crystal. For a set of crystal planes spaced a distance d apart, constructive interference occurs when the angles of incidence and scattering (measured from the crystal planes) are equal and when the Bragg condition Therefore the conditions for radiation from the entire array to reach the observer in phase are (1) the angle of incidence θ_a must equal the angle of scattering θ_b and (2) the path difference for adjacent

rows must equal $m\lambda$, where m is an integer. We can express the second condition as

$$2d\sin\theta = m\lambda \quad (m = 1, 2, 3, \dots) \quad \text{(Bragg condition for constructive interference from an array)}$$

Circular Apertures and Resolving Power

The diffraction pattern from a circular aperture of diameter D consists of a central bright spot, called the Airy disk, and a series of concentric dark and bright rings. Equation gives the angular radius θ_1 , of the first dark ring, equal to the angular size of the Airy disk. Diffraction sets the ultimate limit on resolution (image sharpness) of optical instruments. According to Rayleigh's criterion, two point objects are just barely resolved when their angular separation θ is given by :

$$\sin\theta_1 = 1.22 \frac{\lambda}{D} \quad \text{(diffraction by a circular aperture)}$$

The angular radii of the next two dark rings are given by

$$\sin\theta_2 = 2.23 \frac{\lambda}{D} \quad \sin\theta_3 = 3.24 \frac{\lambda}{D}$$

Between these are bright rings with angular radii given by

$$\sin\theta = 1.63 \frac{\lambda}{D}, \quad 2.68 \frac{\lambda}{D}, \quad 3.70 \frac{\lambda}{D}$$

Holography

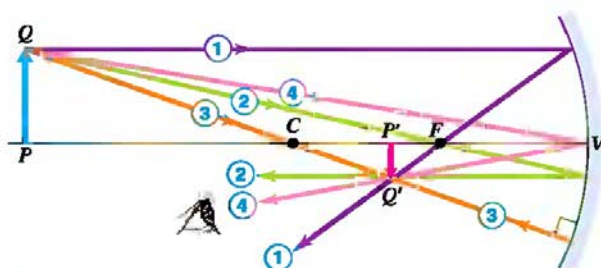
Holography is a technique for recording and reproducing an image of an object through the use of interference effects.

Graphical Methods for Mirrors

We can determine the properties of the image by a simple graphical method. This method consists of finding the point of intersection of a few particular rays that diverge from a point of the object (such as point Q in Fig.) and are reflected by the mirror. Then (neglecting aberrations) all rays from this object point that strike the mirror will intersect at the same point. For this construction we always choose an object point that is not on the optic axis. Four rays that we can usually draw easily are shown in Fig. These are called **principal rays**.

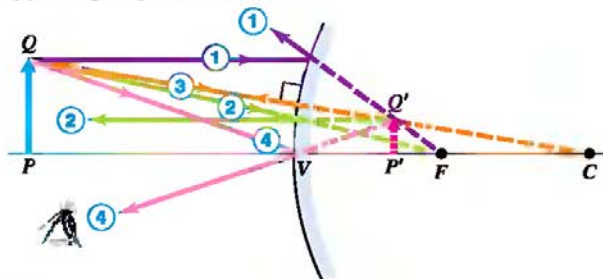
1. A ray parallel to the axis, after reflection, passes through the focal point F of a concave mirror or appears to come from the (virtual) focal point of a **convex mirror**.
2. A ray through (or proceeding toward) the focal point F is reflected parallel to the axis.
3. A ray along the radius through or away from the center of curvature C intersects the surface normally and is reflected back along its original path.
4. A ray to the vertex V is reflected forming equal angles with the optic axis.

(a) Principal rays for concave mirror



- ① Ray parallel to axis reflects through focal point.
- ② Ray through focal point reflects parallel to axis.
- ③ Ray through center of curvature intersects the surface normally and reflects along its original path.
- ④ Ray to vertex reflects symmetrically around optic axis.

(b) Principal rays for convex mirror



- ① Reflected parallel ray appears to come from focal point.
- ② Ray toward focal point reflects parallel to axis.
- ② As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
- ④ As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

Once we have found the position of the image point by means of the intersection of any two of these **principal rays (1, 2, 3, 4)**, we can draw the path of any other ray from the object point to the same image point.

Example :

□ A concave mirror has a radius of curvature with absolute value 20 cm. Find graphically the image of an object in the form of an arrow perpendicular to the axis of the mirror at each of the following object distances: (a) 30 cm, (b) 20 cm, (c) 10 cm, and (d) 5 cm. Check the construction by computing the size and lateral magnification of each image.

Solution: This problem asks us to use both graphical methods and calculations to find the image made by a mirror. This is a good practice to follow in all problems that involve image formation. We are given the radius of curvature $\mathbf{R} = 20\text{cm}$ (positive since the mirror is concave) and hence the focal length $\mathbf{f} = \mathbf{R}/2 = 10$ cm. In each case we are told the object distance s and are asked to find the image distance s' and the lateral magnification $\mathbf{m} = -s'/s$

Figure shows the **principal-ray diagrams** for the four cases. Several points are worth noting. First, in (b) the object and image distances are equal. Ray3 cannot be drawn in this case because a ray from Q through the center of curvature C does not strike the mirror. Ray2 cannot be drawn in (c) because a ray from Q toward F also does not strike the mirror. In this case the outgoing rays are parallel, corresponding to an infinite image distance. In (d) the outgoing rays have no real intersection point; they must be extended backward to find the point from which they appear to diverge---that is, from the virtual image point Q'. The case shown in (d) illustrates the general observation that an object placed

inside the focal point of a concave mirror produces a virtual image. Measurements of the figures, with appropriate scaling, give the following approximate image distances: (a) **15 cm**; (b) **20 cm**; (c) ∞ or $-\infty$ (because the outgoing rays are parallel and do not converge at any finite distance); (d) **-10 cm**. To compute these distances,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad f = 10 \text{ cm}$$

$$(a) \frac{1}{30 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}} \quad s' = 15 \text{ cm}$$

$$(b) \frac{1}{20 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}} \quad s' = 20 \text{ cm}$$

$$(c) \frac{1}{10 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}} \quad s' = \infty \text{ (or } -\infty \text{)}$$

$$(d) \frac{1}{5 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}} \quad s' = -10 \text{ cm}$$

In (a) and (b) the image is real; in (d) it is virtual. In (c) the image is formed at infinity. The lateral magnifications measured from the figures are approximately (a) $1/2$; (b) **-1**; (c) ∞ or ∞ (because the image distance is infinite); (d) **+2** Computing the magnifications:

$$(a) m = -\frac{15 \text{ cm}}{30 \text{ cm}} = -\frac{1}{2}$$

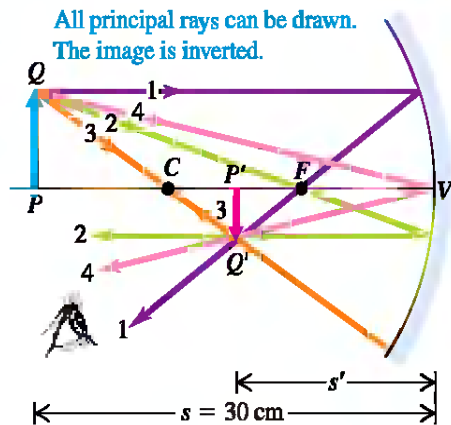
$$(b) m = -\frac{20 \text{ cm}}{20 \text{ cm}} = -1$$

$$(c) m = -\frac{\infty \text{ cm}}{10 \text{ cm}} = -\infty \text{ (or } +\infty \text{)}$$

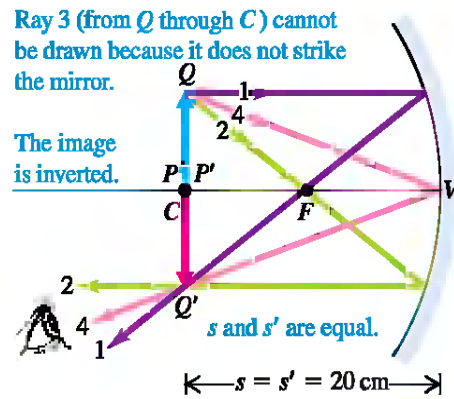
$$(d) m = -\frac{-10 \text{ cm}}{5 \text{ cm}} = +2$$

In (a) and (b) the image is **inverted**; in (d) it is **erect**.

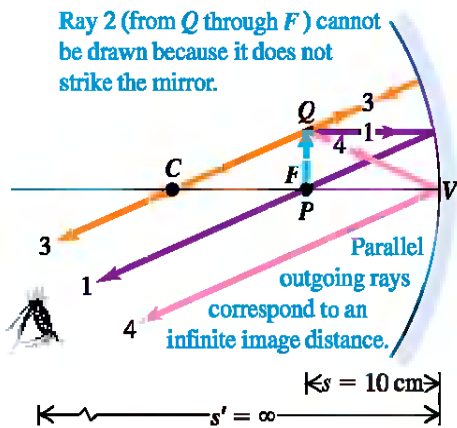
(a) Construction for $s = 30\text{ cm}$



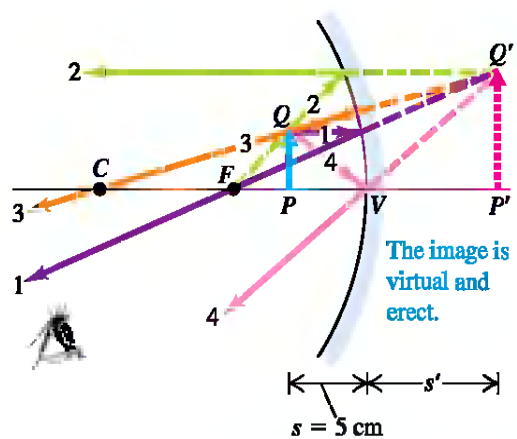
(b) Construction for $s = 20\text{ cm}$



(c) Construction for $s = 10\text{ cm}$



(d) Construction for $s = 5\text{ cm}$



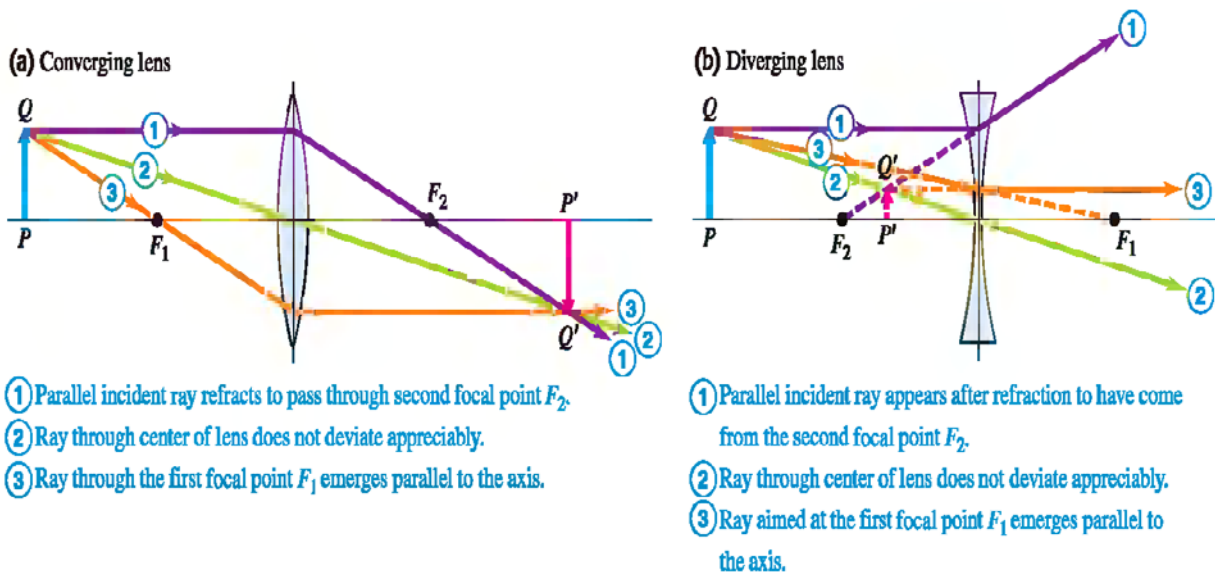
Graphical Methods for Lenses

We can determine the position and size of an image formed by a thin lens by using a graphical method. Again we draw a few special rays called **principal rays** that diverge from a point of the object that is not on the optic axis. The intersection of these rays, after they pass through the lens, determines the position and size of the image. In using this graphical method, we will consider the entire deviation of a ray as occurring at the midplane of the lens, as shown in Fig. This is consistent with the assumption that the distance between the lens surfaces is negligible.

The three principal rays whose paths are usually easy to trace for lenses are shown in Fig.:

1. A ray parallel to the axis emerges from the lens in a direction that passes through the second focal point **F₂** of a converging lens, or appears to come from the second focal point of a diverging lens.
2. A ray through the center of the lens is not appreciably deviated; at the center of the lens the two surfaces are parallel, so this ray emerges at essentially the same angle at which it enters and along essentially the same line.
3. A ray through (or proceeding toward) the first focal point **F₁** emerges parallel to the axis.

When the image is real, the position of the image point is determined by the intersection of any two rays 1,2, and 3 (Fig.a). When the image is virtual, we extend the diverging outgoing rays backward to their intersection point to find the image point (Fig.b).



Example

A converging lens has a focal length of 20 cm. Find graphically the image location for an object at each of the following distances from the lens: (a) 50 cm; (b) 20 cm; (c) 15 cm; (d) -40 cm. Determine the magnification in each case.

Solution

In each case we are given the focal length $f = 20$ cm and the value of the object distance s . Our target variables are the image distance s' and the lateral magnification $m = -s'/s$.

The appropriate **principal-ray** diagrams are shown in (a) Fig.a, (b) Fig.d, (c) Fig.e, and (d) Fig.f. The approximate image distances, from measurements of these diagrams, are 35 cm, $-\infty$, **-40** cm, and **15** cm, and the approximate magnifications are $-2/3$, $+\infty$, and $+3$, and $+1/3$, respectively. Calculating the image positions from Eq. (34.16), we find

$$(a) \frac{1}{50 \text{ cm}} + \frac{1}{s'} = \frac{1}{20 \text{ cm}} \quad s' = 33.3 \text{ cm}$$

$$(b) \frac{1}{20 \text{ cm}} + \frac{1}{s'} = \frac{1}{20 \text{ cm}} \quad s' = \pm \infty$$

$$(c) \frac{1}{15 \text{ cm}} + \frac{1}{s'} = \frac{1}{20 \text{ cm}} \quad s' = -60 \text{ cm}$$

$$(d) \frac{1}{-40 \text{ cm}} + \frac{1}{s'} = \frac{1}{20 \text{ cm}} \quad s' = 13.3 \text{ cm}$$

The graphical results are fairly close to these except for part (c); the accuracy of the diagram in is limited because the rays extended backward have nearly the same direction.

$$(a) m = -\frac{33.3 \text{ cm}}{50 \text{ cm}} = -\frac{2}{3}$$

$$(b) m = -\frac{\pm \infty \text{ cm}}{20 \text{ cm}} = \pm \infty$$

$$(c) m = -\frac{-60 \text{ cm}}{15 \text{ cm}} = +4$$

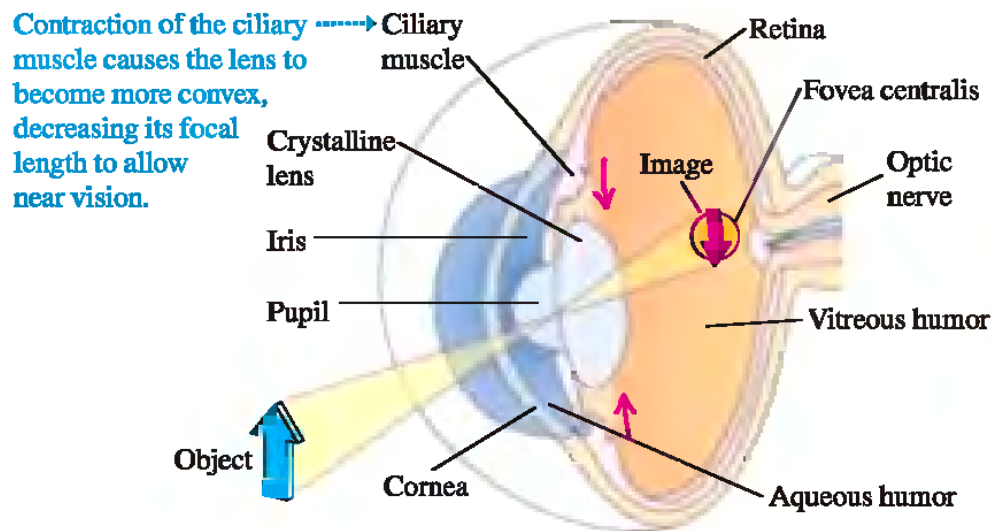
$$(d) m = -\frac{13.3 \text{ cm}}{-40 \text{ cm}} = +\frac{1}{3}$$

The Eye

The optical behavior of the eye is similar to that of a camera. The essential parts of the human eye, considered as an optical system, are shown in Fig. The eye is nearly spherical and about **2.5** cm in diameter. The front portion is somewhat more sharply curved and is covered by a tough, transparent membrane called the **cornea**. The region behind the cornea contains a liquid called the **aqueous humor**. Next comes the crystalline lens, a capsule containing a fibrous jelly, hard at the center and progressively softer at the outer portions. Behind the lens, the eye is filled with a thin watery jelly called the vitreous humor. The indexes of refraction of both the aqueous humor and the vitreous humor are about **1.336**, nearly equal to that of water. The crystalline lens, while not homogeneous, has an average index of **1.437**. This is not very different from the indexes of the aqueous and vitreous humors. As a result, most of the refraction of light entering the eye occurs at the outer surface of the cornea. Refraction at the cornea and the surfaces of the lens produces a real image of the object being viewed. This image is formed on the light-sensitive retina, lining the rear inner surface of the eye. The retina plays the same role as the film in a camera. Vision is most acute in a small central region called the **fovea centralis**, about 0.25 mm in diameter.

In front of the lens is the iris. It contains an aperture with variable diameter called the pupil, which opens and closes to adapt to changing light intensity. The receptors of the retina also have intensity adaptation mechanisms. For an object to be seen sharply, the image must be formed exactly at the location of the retina. The eye adjusts to different object distances s by changing the focal length f of its lens; the lens-to-retina distance, corresponding to s' , does not change. For the normal eye, an object at infinity is sharply focused when the ciliary muscle is relaxed.

To permit sharp imaging on the retina of closer objects, the tension in the ciliary muscle surrounding the lens increases, the ciliary muscle contracts, the lens bulges, and the radii of curvature of its surfaces decrease; this decreases the focal length. This process is called **accommodation**.



Defects of Vision

Several common defects of vision result from incorrect distance relationships in the eye. In the hyperopic (farsighted) eye, the eyeball is too short or the cornea is not curved enough, and the image of an infinitely distant object is behind the retina. The myopic eye produces too much convergence in a parallel bundle of rays for an image to be formed on the retina; the hyperopic eye, not enough convergence. All of these defects can be corrected by the use of corrective lenses (eye- glasses or contact lenses). To see clearly an object at normal reading distance (often assumed to be 25 cm), we need a lens that forms a virtual image of the object at or beyond the near point. This can be accomplished by a converging (positive) lens.

Astigmatism is a different type of defect in which the surface of the cornea is not spherical but rather more sharply curved in one plane than in another. As a result, horizontal lines may be imaged in a different plane from vertical lines. Astigmatism can be corrected by use of a lens with a cylindrical surface. Lenses for vision correction are usually described in terms of the **power**, defined as the reciprocal of the focal length expressed in meters. The unit of power is the **diopter**. Thus a lens with $f = 0.50$ m has a power of **2.0 diopters**.

Example

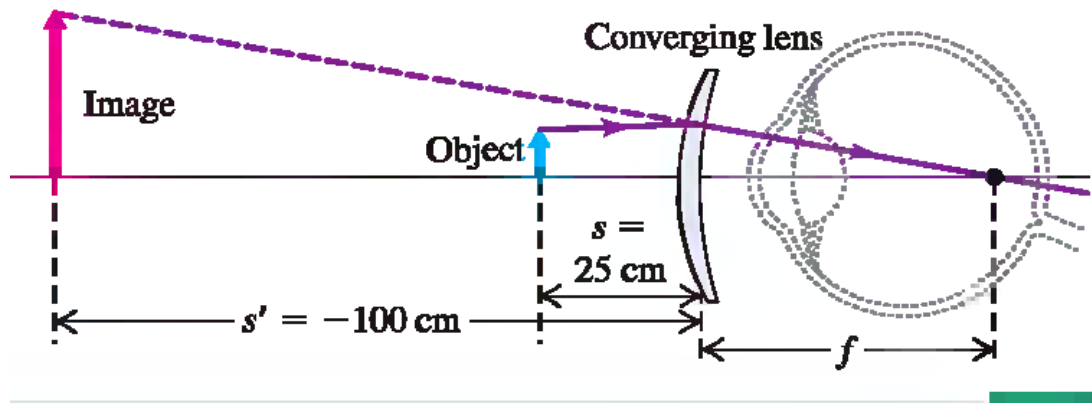
The near point of a certain hyperopic eye is 100 cm in front of the eye. To see clearly an object that is 25 cm in front of the eye, what contact lens is required?

Solution

We want the lens to form a virtual image of the object at the near point of the eye, 100 cm from it. That is, when $s = 25$ cm, we want s' to be 100 cm. We determine the required focal length of the contact lens using the object-image relationship for a thin lens,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{+25 \text{ cm}} + \frac{1}{-100 \text{ cm}}$$
$$f = +33 \text{ cm}$$

We need a converging lens with focal length $f = 33$ cm. The corresponding power is (0.33 m), or + 3.0 diopters.



INTERFERENCE

Example 1:

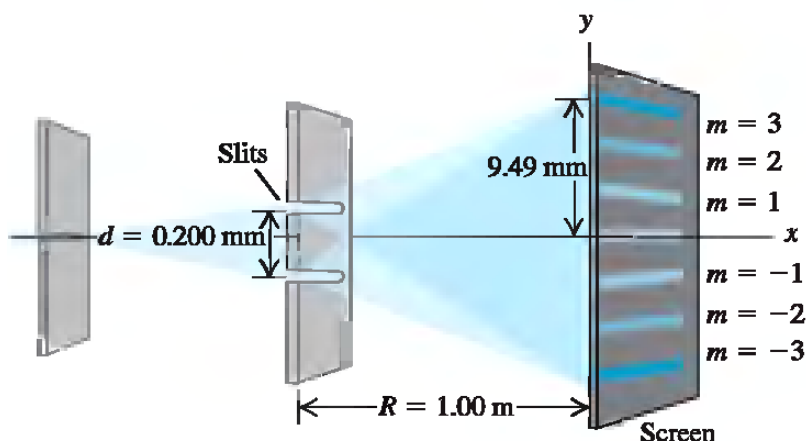
In a two-slit interference experiment, the slits are 0.200 mm apart, and the screen is at a distance of 1.00 m. The third bright fringe (not counting the central bright fringe straight ahead from the slits) is found to be displaced 9.49 mm from the central fringe. Find the wavelength of the light used.

Solution:

The third bright fringe corresponds to $m = 3$ as well as to the bright fringe labeled $m = 3$. To determine the value of the target variable λ , since $R = 1.00$ m is much greater than $d = 0.200$ mm or $y_3 = 9.49$ mm.

$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3} \text{ m})(0.200 \times 10^{-3} \text{ m})}{(3)(1.00 \text{ m})}$$

$$= 633 \times 10^{-9} \text{ m} = 633 \text{ nm}$$



Example 2:

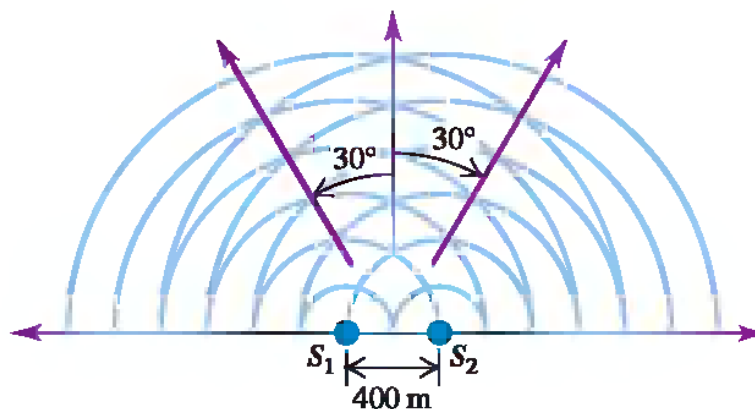
A radio station operating at a frequency of 1500 kHz = 1.5×10^6 Hz (near the top end of the AM broadcast band) has two identical vertical dipole antennas spaced 400 m apart, oscillating in phase. At distances much greater than 400 m, in what directions is the intensity greatest in the resulting radiation pattern?

Solution:

Since the resultant wave is detected at distances much greater than $d = 400$ m, we give the directions of the intensity maxima, the values of θ for which the path difference is zero or a whole number of wavelengths. The wavelength is $\lambda = c/f = 200$ m. with $m = 0, \pm 1$, and ± 2 , the intensity maxima are given by

$$\sin\theta = \frac{m\lambda}{d} = \frac{m(200 \text{ m})}{400 \text{ m}} = \frac{m}{2} \quad \theta = 0, \pm 30^\circ, \pm 90^\circ$$

$$\sin\theta = \frac{(m + \frac{1}{2})\lambda}{d} = \frac{m + \frac{1}{2}}{2} \quad \theta = \pm 14.5^\circ, \pm 48.6^\circ$$



Example 3:

Suppose the two glass plates in Fig. are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of $\lambda = \lambda_0 = 500\text{nm}$.

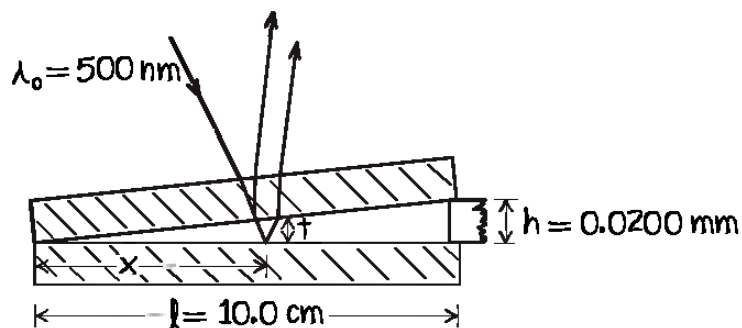
Solution:

$$2t = m\lambda_0 \quad (m = 0, 1, 2, \dots)$$

$$\frac{t}{x} = \frac{h}{l}$$

$$\frac{2xh}{l} = m\lambda_0$$

$$x = m \frac{l\lambda_0}{2h} = m \frac{(0.100 \text{ m})(500 \times 10^{-9} \text{ m})}{(2)(0.0200 \times 10^{-3} \text{ m})} = m(1.25 \text{ mm})$$



Example 4:

In Example above, suppose the glass plates have $n = 1.52$ and the space between plates contains water ($n = 1.33$) instead of air. What happens now?

Solution:

In the film of water ($n = 1.33$), the wavelength is:

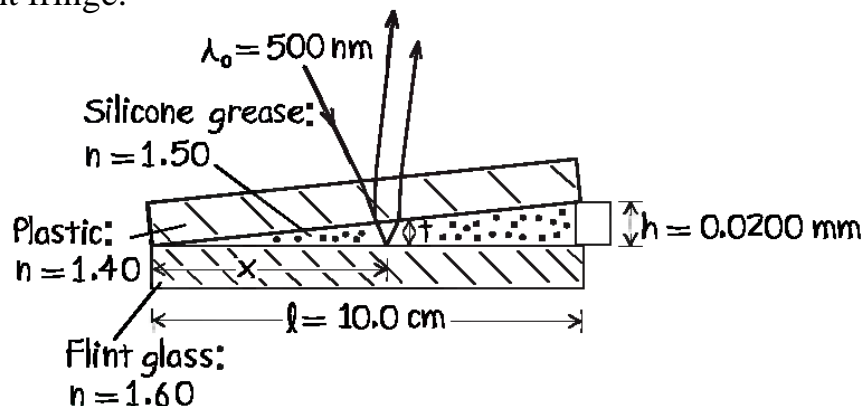
$$\lambda = \frac{\lambda_0}{n} = \frac{500 \text{ nm}}{1.33} = 376 \text{ nm}$$

Example 5:

Suppose the upper of the two plates in Example above is a plastic material with $n = 1.40$, the wedge is filled with a silicone grease having $n = 1.50$, and the bottom plate is a dense flint glass with $n = 1.60$. What happens now?

Solution:

The value of λ wavelength in the silicone grease: $\lambda = \lambda_0/n = (500\text{nm})/1.50 = 333\text{nm}$. You can readily show that the fringe spacing is 0.833 mm . Note that the two reflected waves from the line of contact are in phase (they both undergo the same phase shift), so the line of contact is at a bright fringe.



DIFFRACTION

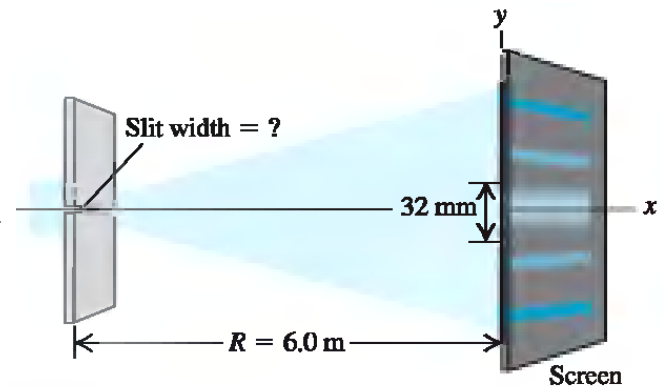
Example 1:

You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. You find that the distance on the screen between the centers of the first minima outside the central bright fringe is 32 mm. How wide is the slit?

Solution:

$$a = \frac{x\lambda}{y_1} = \frac{(6.0 \text{ m})(633 \times 10^{-9} \text{ m})}{(32 \times 10^{-3} \text{ m})/2}$$

$$= 2.4 \times 10^{-4} \text{ m} = 0.24 \text{ mm}$$



Example 2:

(a) In a single slit diffraction pattern, what is the intensity at a point where the total phase difference between wavelets from the top and bottom of the slit is 66 rad? (b) If this point is 7.0° away from the central maximum, how many wavelengths wide is the slit?

Solution:

$$I = I_0 \left[\frac{\sin(33 \text{ rad})}{33 \text{ rad}} \right]^2 = (9.2 \times 10^{-4}) I_0$$

$$a = \frac{\beta\lambda}{2\pi \sin\theta} = \frac{(66 \text{ rad})\lambda}{(2\pi \text{ rad}) \sin 7.0^\circ} = 86\lambda$$

Example 3:

In the experiment described in Example 1, what is the intensity at a point on the screen 3.0 mm from the center of the pattern? The intensity at the center of the pattern is I_0 .

Solution:

$y = 3.0$ mm and $x = 6.0$ m, so $\tan\theta = y/x = (3.0 \times 10^{-3}\text{m})/(6.0\text{m}) = 5.0 \times 10^{-4}$; since this is so small, the values of $\tan\theta$, $\sin\theta$, and θ (in radians) are all nearly the same.

$$\frac{\pi a \sin\theta}{\lambda} = \frac{\pi(2.4 \times 10^{-4}\text{ m})(5.0 \times 10^{-4})}{6.33 \times 10^{-7}\text{ m}} = 0.60$$

$$I = I_0 \left(\frac{\sin 0.60}{0.60} \right)^2 = 0.89 I_0$$

Example 4:

You direct a beam of x rays with wavelength 0.154 nm at certain planes of a silicon crystal. As you increase the angle of incidence from zero, you find the first strong interference maximum from these planes when the beam makes an angle of 34.5° with the planes. (a) How far apart are the planes? (b) Will you find other interference maxima from these planes at larger angles?

Solution:

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.154\text{ nm})}{2\sin 34.5^\circ} = 0.136\text{ nm}$$

$$\sin\theta = \frac{m\lambda}{2d} = m \frac{0.154\text{ nm}}{2(0.136\text{ nm})} = m(0.566)$$

Example 5:

A camera lens with focal length $f = 50$ mm and maximum aperture $f/2$ forms an image of an object 9.0 m away. (a) If the resolution is limited by diffraction, what is the minimum distance between two points on the object that are barely resolved, and what is the corresponding distance

between image points? (b) How does the situation change if the lens is "stopped down" to $f/16$? Assume that $\lambda = 500 \text{ nm}$ in both cases.

Solution:

The aperture diameter is $D = f / (\text{f-number}) = (50\text{mm})/2 = 25\text{mm} = 25 \times 10^{-3} \text{ m}$.

angular separation θ of two object points that are barely resolved is given by

$$\begin{aligned} \theta \approx \sin\theta &= 1.22 \frac{\lambda}{D} = 1.22 \frac{500 \times 10^{-9} \text{ m}}{25 \times 10^{-3} \text{ m}} \\ &= 2.4 \times 10^{-5} \text{ rad} \end{aligned}$$

$y/s = y' /s'$. Thus the angular separations of the object points and the corresponding image points are both equal to θ . Because the object distance s is much greater than the focal length $f = 50 \text{ mm}$, the image distance s' is approximately equal to f . Thus

$$\begin{aligned} \frac{y}{9.0 \text{ m}} &= 2.4 \times 10^{-5} & y &= 2.2 \times 10^{-4} \text{ m} = 0.22 \text{ mm} \\ \frac{y'}{50 \text{ mm}} &= 2.4 \times 10^{-5} & y' &= 1.2 \times 10^{-3} \text{ mm} \\ & & &= 0.0012 \text{ mm} \approx \frac{1}{800} \text{ mm} \end{aligned}$$

(b) The aperture diameter is now $(50\text{mm})/16$, or one-eighth as large as before. The angular separation between barely resolved points is eight times as great, and the values of y and y' are also eight times as great as before:

$$y = 1.8 \text{ mm} \quad y' = 0.0096 \text{ mm} = \frac{1}{100} \text{ mm}$$

PHOTONS, ELECTRONS, AND ATOMS

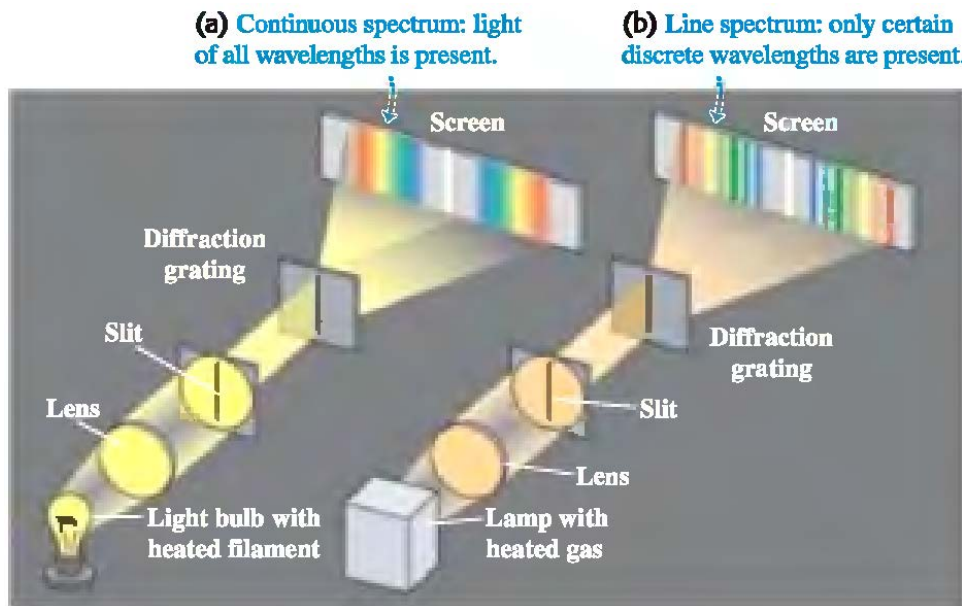
When we look more closely at the emission, absorption, and scattering of electromagnetic radiation, however, we discover a completely different aspect of light. We find that the energy of an electromagnetic wave is **quantized**; it is emitted and absorbed in particle-like packages of definite energy, called **photons** or **quanta**. The energy of a single photon is proportional to the frequency of the radiation. The internal energy of atoms is also **quantized**. For a given kind of individual atom the energy can't have just any value; only discrete values called **energy levels** are possible. The basic ideas of photons and energy levels take us a long way toward understanding a wide variety of otherwise puzzling observations. Among these are the unique sets of wavelengths emitted and absorbed by gaseous elements, **the emission of electrons from an illuminated surface, the operation of lasers, and the production and scattering of x-rays.**

Line Spectra

We can use a prism or a diffraction grating to separate the various wavelengths in a beam of light into a spectrum. If the light source is a hot solid (such as a light bulb filament) or liquid, the spectrum is **continuous**; light of all wavelengths is present (Fig.a). But if the source is a gas carrying an electric discharge (as in a neon sign) or a volatile salt heated in a flame (as when table salt is thrown into a campfire), only a few colors appear, in the form of isolated sharp parallel lines (Fig. b). (Each "line" is an image of the spectrograph slit, deviated through an angle that depends on the wavelength of the light forming that image. A spectrum of this sort is called a **line spectrum**. Each line corresponds to a definite

wavelength and frequency. The characteristic spectrum of an atom was presumably related to its internal structure, but attempts to understand this relationship solely on the basis of **classical mechanics** and **electrodynamics** the physics summarized in **Newton's three laws** and **Maxwell's four equations** were not successful.

□



Photons and Energy Levels

All these phenomena (and several others) pointed forcefully to the conclusion that classical optics, successful though it was in explaining lenses, mirrors, interference, and polarization, had its limitations. We now understand that all these phenomena result from the **quantum nature** of radiation. Electromagnetic radiation, along with its wave nature, has properties resembling those of particles. In particular, the energy in an electromagnetic wave is always emitted and absorbed in packages called **photons or quanta**, with energy proportional to the frequency of the radiation.

The Photoelectric Effect

The **photoelectric effect** is the emission of electrons when light strikes a surface. This effect has numerous practical applications (Fig.). To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of positive ions in the material of the surface. This attraction causes a **potential-energy barrier** that normally confines the electrons inside the material.

Threshold Frequency and Stopping Potential

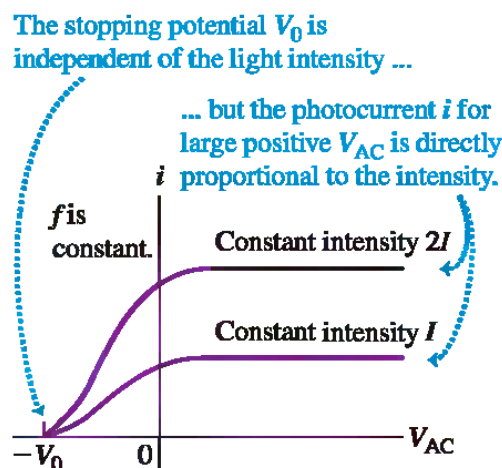
The photoelectric effect was first observed in **1887** by **Heinrich Hertz**. In **1883** **Thomas Edison** had discovered **thermionic emission**. The minimum amount of energy an individual electron has to gain to escape from a particular surface is called the **work function** for that surface, denoted by ϕ . However, the surfaces that Hertz used were not at the high temperatures needed for thermionic emission. The photoelectric effect was investigated in detail by the German physicists **Wilhelm Hallwachs** and **Philipp Lenard**. Two conducting electrodes, the **anode** and the **cathode**, are enclosed in an evacuated glass tube. The battery or other source of potential difference creates an electric field in the direction from anode to cathode in Fig.a. They studied how this **photocurrent** varies with voltage and with the frequency and intensity of the light. They found that when monochromatic light fell on the cathode, **no** photoelectrons at all were emitted unless the frequency of the light was greater than some minimum value called the **threshold frequency**. This minimum frequency depends on the material of the cathode. When the frequency **f** is **greater than** the threshold frequency, some electrons are emitted from the cathode with substantial initial speeds. This can be shown by reversing the polarity of the battery (Fig.b) so that the electric-

field force on the electrons is back toward the cathode. If the magnitude of the field is not too great, the highest-energy emitted electrons still reach the anode and there is still a current. We can determine the **maximum kinetic energy** of the emitted electrons by making the potential of the anode relative to the cathode, V_{AC} just negative enough so that the current stops. This occurs for $V_{AC} = -V_0$ where V_0 is called the **stopping potential**. As an electron moves from the cathode to the anode, the potential decreases by V_0 and negative work $-eV_0$ is done on the (negatively charged) electron; the most energetic electron leaves the cathode with kinetic energy $K_{\max} = \frac{1}{2}mv_{\max}^2$ has zero kinetic energy at the anode.

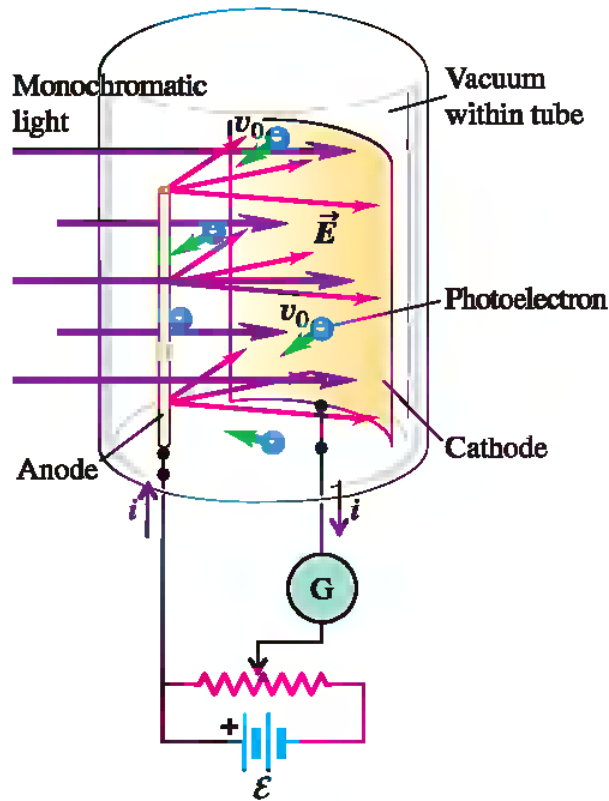
$$W_{\text{tot}} = -eV_0 = \Delta K = 0 - K_{\max} \quad \text{(maximum kinetic energy of photoelectrons)}$$

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = eV_0$$

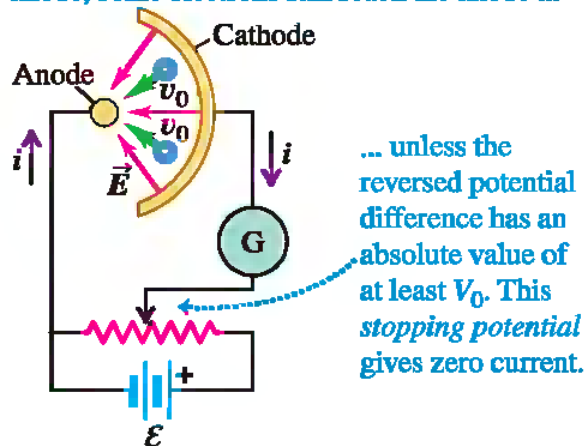
Figure shows graphs of photocurrent as a function of potential difference V_{AC} for light of constant frequency and two different intensities. When V_{AC} is sufficiently large and positive, the curves level off, showing that all the emitted electrons are being collected by the anode. The reverse potential difference $-V_0$ needed to reduce the current to zero is shown.



(a) Light causes the cathode to emit electrons, which are pushed toward the anode by the electric-field force.



(b) Overhead view with \vec{E} field reversed. Even when the direction of \vec{E} field is reversed so that the electric-field force points away from the anode, some electrons still reach the anode ...



Einstein's Photon Explanation

When the intensity (average energy per unit area per unit time) increases, electrons should be able to gain more energy, increasing the stopping potential V_0 . But V_0 was found not to depend on intensity.

The correct analysis of the photoelectric effect was developed by **Albert Einstein in 1905**. He postulated that a beam of light consists of small packages of energy called **photons** or **quanta**. The energy E of a photon is equal to a constant h times its frequency f . From $f=c/\lambda$ for electromagnetic waves in vacuum we have:

$$E = hf = \frac{hc}{\lambda} \quad (\text{energy of a photon})$$

where h is a universal constant called **Planck's constant**. The numerical value of this constant, to the accuracy known at present, is

$$\square h = 6.6260693 (11) \times 10^{-34} \text{ J} \cdot \text{S}$$

A photon arriving at the surface is absorbed by an electron; the electron gets all the photon's energy or none at all. If this energy is greater than the work function ϕ , the electron may escape from the surface.

Recall that ϕ is the **minimum** energy needed to remove an electron from the surface. Einstein applied conservation of energy to find that the **maximum** kinetic energy $K_{\max} = \frac{1}{2}m\tilde{v}_{\max}^2$ for an emitted electron is the energy hf gained from a photon minus the work function ϕ :

$$K_{\max} = \frac{1}{2}m\tilde{v}_{\max}^2 = hf - \phi$$

Substituting $K_{\max} = eV_0$ from Eq. we find

$$eV_0 = hf - \phi \quad (\text{photoelectric effect})$$

We can measure the stopping potential V_0 for each of several values of frequency f for a given cathode material. A graph of V_0 as a function of f turns out to be a straight line, verifying Eq. above and from such a graph we can determine both the work function ϕ for the material and the value of the quantity h/e .

Electron energies and work functions are usually expressed in electron volts (eV),

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

□ To this accuracy, Planck's constant is

$$\square h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$

Photon Momentum

A photon of any electromagnetic radiation with frequency f and wavelength λ has energy E given. Every particle that has energy must also have momentum, even if it has no rest mass. A photon with energy E has momentum with magnitude p given by $E = pc$. Thus the wavelength λ of a photon and the magnitude of its momentum p are related simply by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \quad (\text{momentum of a photon})$$

The direction of the photon's momentum is simply the direction in which the electromagnetic wave is moving.

Energy Levels

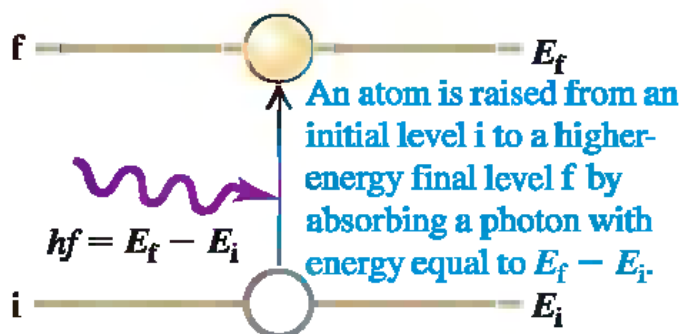
Only a few atoms and ions (such as hydrogen) have spectra whose wavelengths can be represented by a simple formula.

Every atom has a **lowest energy level** that includes the minimum internal energy state that the atom can have. This is called the **ground-state level**, or **ground level**, and all higher levels are called **excited levels**. A photon

corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level.

Photon Absorption

We pass a beam of light from a sodium-vapor lamp through a bulb containing sodium vapor. The atoms in the vapor absorb the 589.0-nm or 589.6-nm photons from the beam, reaching the lowest excited levels; after a short time they return to the ground level, emitting photons in all directions and causing the sodium vapor to glow with the characteristic yellow light. The average time spent in an excited level is called the **lifetime** of the level. More generally, a photon emitted when an atom makes a transition from an excited level to a lower level can also be absorbed by a similar atom that is initially in the lower level (Fig.). If we pass white (continuous-spectrum) light through a gas and look at the transmitted light with a spectrometer, we find a series of dark lines corresponding to the wavelengths that have been absorbed. This is called an **absorption spectrum**. A related phenomenon is **fluorescence**. An atom absorbs a photon (often in the ultraviolet region) to reach an excited level and then drops back to the ground level in steps, emitting two or more photons with lower energy and longer wavelength.



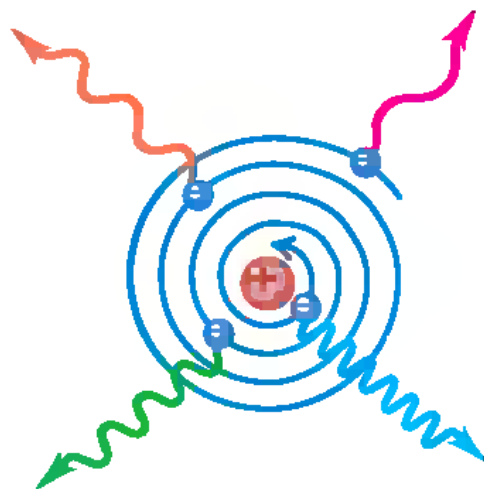
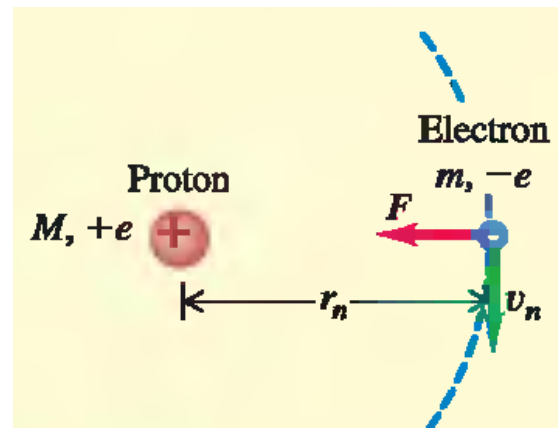
The Bohr Model

Bohr established the relationship between spectral wavelengths and energy levels, he also proposed a model of the hydrogen atom. Using this model, now known as the **Bohr model**, he was able to calculate the energy levels of hydrogen and obtain agreement with values determined from spectra. In the Bohr model of the hydrogen atom, the permitted values of angular momentum are integral multiples of $h/2\pi$. The integer multiplier n is called the **principal quantum number** for the level. The orbital radii are proportional to n^2 and the orbital speeds are proportional to $1/n$.

$$L_n = mv_n r_n = n \frac{h}{2\pi}$$
$$n = 1, 2, 3, \dots$$

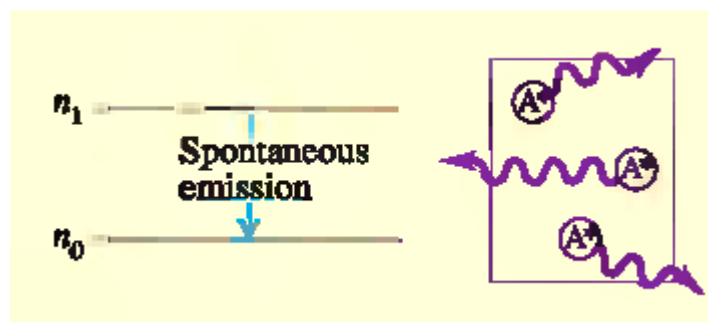
$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2} = n^2 a_0$$
$$= n^2 (5.29 \times 10^{-11} \text{ m})$$

$$v_n = \frac{1}{\epsilon_0} \frac{e^2}{2nh} = \frac{2.19 \times 10^6 \text{ m/s}}{n}$$



The Laser

The laser is a light source that produces a beam of highly coherent and very nearly monochromatic light as a result of cooperative emission from many atoms. The name "laser" is an acronym for "light amplification by stimulated emission of radiation." We can understand the principles of laser operation on the basis of photons and atomic energy levels. The laser operates on the principle of stimulated emission, by which many photons with identical wavelength and phase are emitted. Laser operation requires a nonequilibrium condition called a **population inversion**, in which more atoms are in a higher-energy state than are in a lower-energy state.



Spontaneous and Stimulated Emission

The excited atoms return to the ground level by each emitting a photon with the same frequency as the one originally absorbed. This process is called **spontaneous emission**; the direction and phase of the emitted photons are random. In **stimulated emission** each incident photon encounters a previously excited atom. A kind of resonance effect induces each atom to emit a second photon with the same frequency, direction, phase, and polarization as the incident photon, which is not changed by the process. For each atom there is one photon before a stimulated emission and two photons after thus the name light **amplification**. Because the two photons have the same phase, they emerge together as

coherent radiation. The Maxwell-Boltzmann distribution function determines the number of atoms in a given state in a gas. The function tells us that when the gas is in thermal equilibrium at absolute temperature T , the number n_i of atoms in a state with energy E_i equals:

$$Ae^{-E_i/kT}$$

where k is Boltzmann's constant and A is another constant determined by the total number of atoms in the gas. E was the kinetic energy $1/2mv^2$ of a gas molecule; here we're talking about the internal energy of an atom.) Because of the negative exponent, fewer atoms are in higher-energy states, as we should expect. If E_g is a ground-state energy and E_{ex} is the energy of an excited state, then the ratio of numbers of atoms in the two states is

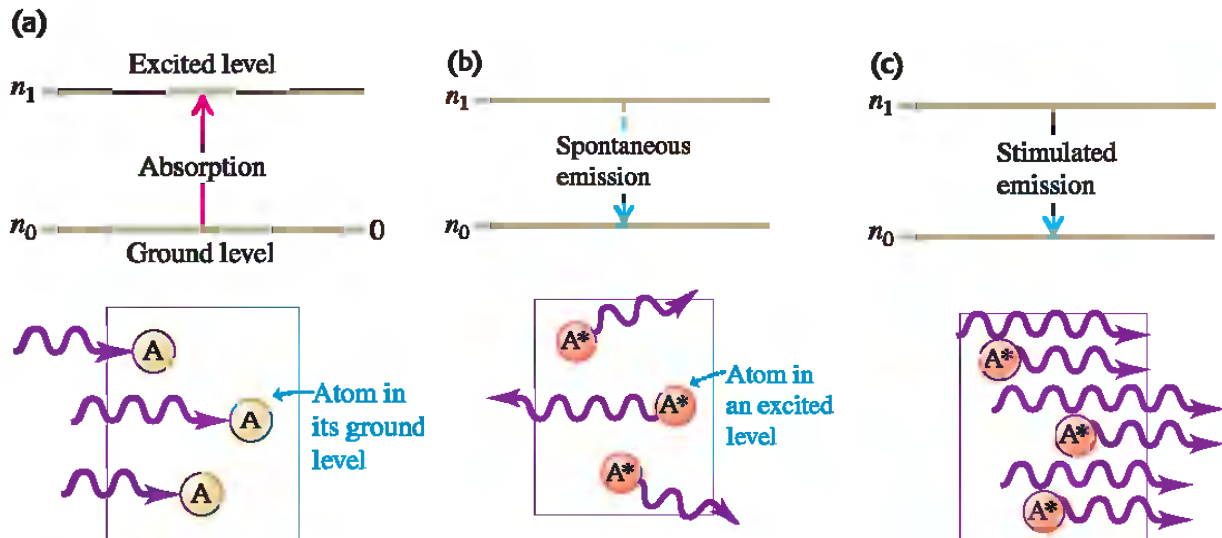
$$\frac{n_{ex}}{n_g} = \frac{Ae^{-E_{ex}/kT}}{Ae^{-E_g/kT}} = e^{-(E_{ex}-E_g)/kT}$$

For example, suppose $E_{ex} - E_g = 2.0 \text{ eV} = 3.2 \times 10^{-19} \text{ J}$, the energy of a 620 nm visible-light photon. At $T = 3000 \text{ K}$ (the temperature of the filament in an incandescent light bulb),

$$\frac{E_{ex} - E_g}{kT} = \frac{3.2 \times 10^{-19} \text{ J}}{(1.38 \times 10^{-23} \text{ J/K})(3000 \text{ K})} = 7.73$$

and

$$e^{-(E_{ex}-E_g)/kT} = e^{-7.73} = 0.00044$$



Enhancing Stimulated Emission: Population Inversions

We could try to enhance the number of atoms in excited states by sending through the container a beam of radiation with frequency $f = E/h$ corresponding to the energy difference $E = E_{ex} - E_g$. Some of the atoms absorb photons of energy E and are raised to the excited state, and the population ratio n_{ex}/n_g momentarily increases. But because n_g is originally so much larger than n_{ex} , an enormously intense beam of light would be required to momentarily increase n_{ex} to a value comparable to n_g . The rate at which energy is absorbed from the beam by the n_g ground-state atoms far exceeds the rate at which energy is added to the beam by stimulated emission from the relatively rare (n_{ex}) excited atoms. We need to create a nonequilibrium situation in which the number of atoms in a higher-energy state is greater than the number in a lower-energy state. Such a situation is called a **population inversion**. Then the rate of energy

Then the rate of energy radiation by stimulated emission can exceed the rate of absorption, and the system acts as a net source of radiation with photon energy E . Furthermore, because the photons are the result of stimulated emission, they all have the same frequency, phase, polarization, and direction of propagation. The resulting radiation is therefore very much more coherent than light from ordinary sources, in which the emissions of individual atoms are not coordinated.

The necessary population inversion can be achieved in a variety of ways. A familiar example is the **helium-neon laser**, a common and inexpensive is available in many undergraduate laboratories. A mixture of laser that typically at a pressure of the order of 10^2 Pa (10^{-3} atm), helium and neon is sealed in a glass enclosure provided with two electrodes. When a sufficiently high voltage is applied an electric discharge occurs.

Collisions between ionized atoms and electrons carrying the discharge current excite atoms to various energy states. Other types of lasers use different processes to achieve the necessary population inversion. Semiconductor laser the inversion is obtained by driving electrons and holes to a p-n junction with a steady electric field. In one type of chemical laser a chemical reaction forms molecules in metastable excited states. In a carbon dioxide gas dynamic laser the population inversion results from rapid expansion of the gas.

Lasers have found a wide variety of practical applications. A high-intensity laser beam can drill a very small hole in a diamond for use as a die in drawing very small-diameter wire. Lasers are widely used in medicine.

X-Ray Production and Scattering

X rays can be produced by electron impact on a target. If electrons are accelerated through a potential increase V_{AC} , the maximum frequency and minimum wavelength that they can produce are given by Eq:

$$eV_{AC} = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

Compton scattering is scattering of x-ray photons by electrons. For free electrons (**mass m**), the wavelengths of incident and scattered photons are related to the photon scattering angle ϕ by Eq:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

Example 1: □

Radio station WQED in Pittsburgh broadcasts at 89.3 MHz with a radiated power of 43.0 kW. (a) What is the magnitude of the momentum of each photon? (b) How many photons does WQED emit each second?

Solution:

$$p = \frac{hf}{c} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(89.3 \times 10^6 \text{ Hz})}{3.00 \times 10^8 \text{ m/s}}$$
$$= 1.97 \times 10^{-34} \text{ kg} \cdot \text{m/s}$$

$$E = pc = (1.97 \times 10^{-34} \text{ kg} \cdot \text{m/s})(3.00 \times 10^8 \text{ m/s})$$
$$= 5.92 \times 10^{-26} \text{ J}$$

$$\frac{43.0 \times 10^3 \text{ J/s}}{5.92 \times 10^{-26} \text{ J/photon}} = 7.26 \times 10^{29} \text{ photons/s}$$

Example 2:

While conducting a photoelectric-effect experiment with light of a certain frequency, you find that a reverse potential difference of 1.25 V is required to reduce the current to zero. Find (a) the maximum kinetic energy and (b) the maximum speed of the emitted photoelectrons.

Solution:

$$K_{\max} = eV_0 = (1.60 \times 10^{-19} \text{ C})(1.25 \text{ V}) = 2.00 \times 10^{-19} \text{ J}$$

Recall that 1 V = 1 J/C. In terms of electron volts,

$$K_{\max} = eV_0 = e(1.25 \text{ V}) = 1.25 \text{ eV}$$

$$\begin{aligned} v_{\max} &= \sqrt{\frac{2K_{\max}}{m}} = \sqrt{\frac{2(2.00 \times 10^{-19} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} \\ &= 6.63 \times 10^5 \text{ m/s} \end{aligned}$$

Example 3:

Electrons in an x-ray tube are accelerated by a potential difference of 10.0 kV. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting x rays?

Answer using both SI units and electron volts.

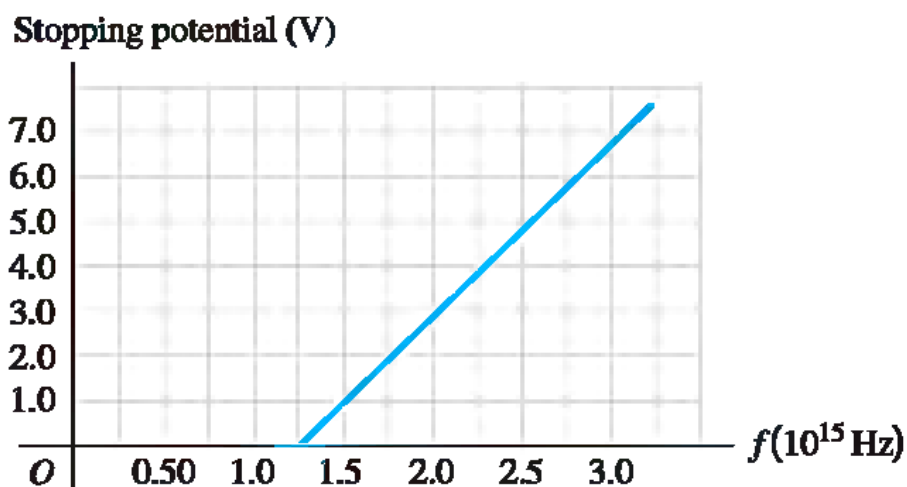
Solution:

$$\lambda_{\min} = \frac{hc}{eV_{\text{AC}}} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(10.0 \times 10^3 \text{ V})}$$

$$\begin{aligned} \lambda_{\min} &= \frac{hc}{eV_{\text{AC}}} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{e(10.0 \times 10^3 \text{ V})} \\ &= 1.24 \times 10^{-10} \text{ m} = 0.124 \text{ nm} \end{aligned}$$

H.W

1. During the photoelectric effect, light knocks electrons out of metals. So why don't the metals in your home lose their electrons when you turn on the lights?
2. In a photoelectric-effect experiment, which of the following will increase the maximum kinetic energy of the photoelectrons; (a) Use light of greater intensity; (b) use light of higher frequency; (c) use light of longer wavelength; (d) use a metal surface with a larger work function. In each case justify your answer.
3. The graph in Figure shows the stopping potential as a function of the frequency of the incident light falling on a metal surface, (a) Find the photoelectric work function for this metal. (b) What value of Planck's constant does the graph yield? (c) Why does the graph not extend below the x-axis? (d) If a different metal were used, what characteristics of the graph would you expect to be the same and which ones to be different?



4. A photon of green light has a wavelength of 520 nm. Find the photon's frequency, magnitude of momentum, and energy. Express the energy in both joules and electron volts.

5. The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 250nm falls on potassium, find **(a)** the stopping potential in volts; **(b)** the kinetic energy in electron volts of the most energetic electrons ejected; **(c)** the speed of these electrons.