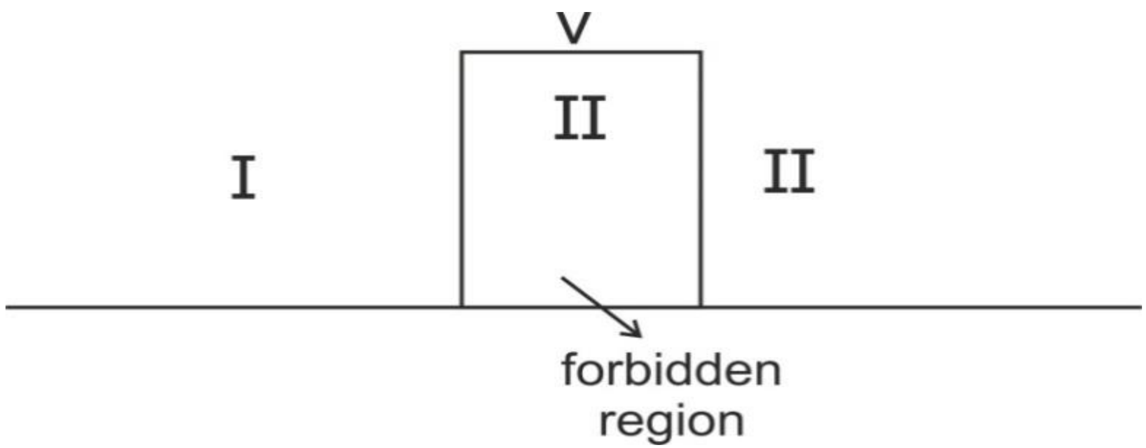
Tunneling effect or tunneling barrier

In classical physics the kinetic energy cannot be negative or the total energy E cannot be less than the potential energy V Schrodinger equation can be solved for this case.

Solution of the equation in the classically forbidden regions as shown in figure (4.2)



Figure(4.2) Tunneling barrier

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \mathrm{V}\,\psi = \mathrm{E}\,\psi$$

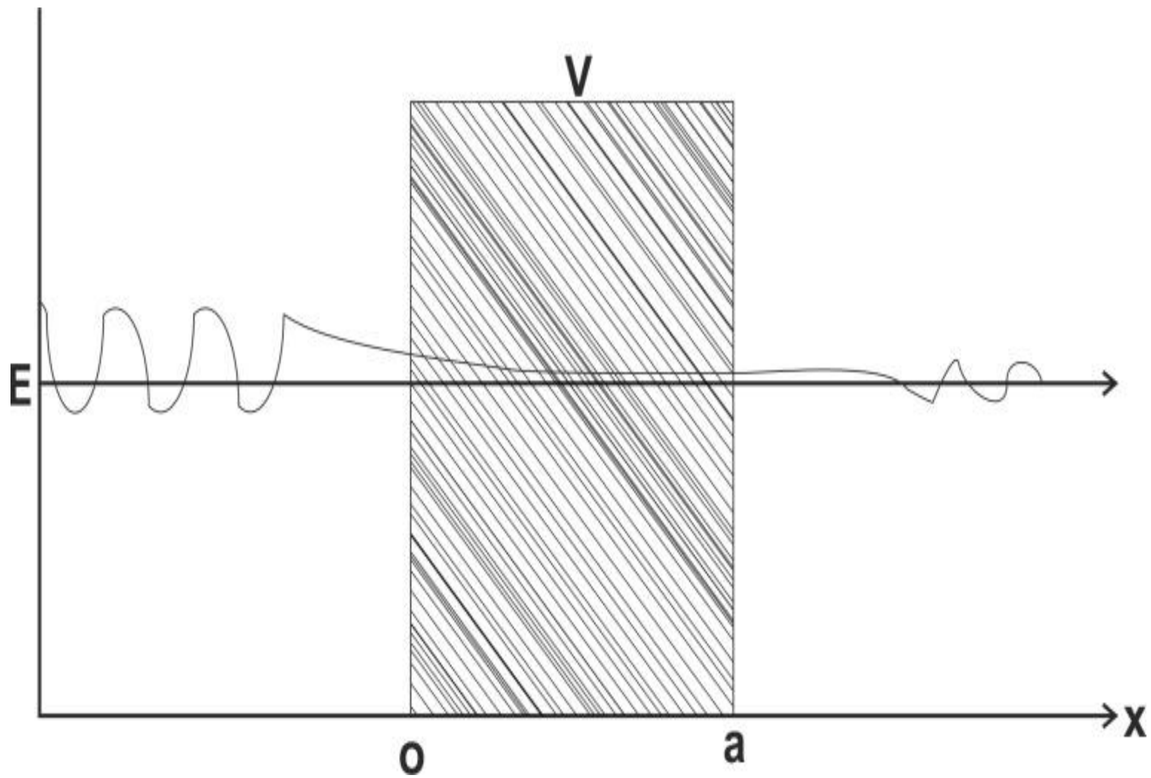
$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + (\mathrm{E}-\mathrm{V})\,\psi = 0$$

V: constant, but $E - V < 0$

- Let $\psi = Ae^{kx} \implies \frac{\partial^2\psi}{\partial x^2} = Ak^2\, e^{kx}$
- $-\frac{\hbar^2}{2m}\, Ak^2\, e^{kx} + (\mathrm{E} - \mathrm{V})Ae^{kx} = 0$
- $K^2 + \frac{\hbar^2}{2m}\,(\mathrm{E} - \mathrm{V}) = 0$
- $\mathrm{K} = \pm\sqrt{\frac{2m(V-E)}{\hbar^2}}$
- Both positive and negative values of K are positive, we can write a general solution as

$$\psi = A \exp\left[+\sqrt{\frac{2m(V-E)}{\hbar^2}}\right] x + B\ exp\left[-\sqrt{\frac{2m(V-E)}{\hbar^2}}\right]\ x$$

- Where B and A are constants
- The first exponential term $+\infty$ when $x \longrightarrow \infty$
- The 2nd exponential term $-\infty$ no solution
- Then the boundary conditions on the wave function over all space regions.



Figure(4.3) The wave across barrier

$E - V < 0 \; for \; 0 < x < a$

$V = 0 \; for \; x < 0$

$and \; x > a$

- Rough approximation to the tunneling probability can be simply estimated by taking B=1 and finding the value of ψ at x=a. Squaring this according to Born interpretation.
- P: probability of tunneling

$$p = exp\left[-2a\sqrt{\frac{2m(V-E)}{\hbar^2}}\right] \quad ......(4.13)$$

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- This equation shows that the tunneling probability decreases with:
- The width of the barrier.
- The mass of the particle.
- (V-E) the energy compared with the value required for a classical particle pass the barrier.