- **Quantization of the angular momentum**
- The physical quantities that do not change during motion called the motion constants.
- $\ell = r \times P$ ....(6.21)
- But in quantum value
- $l^2 = \ell(\ell + 1)\hbar^2$ ......(6.22)
- Where $\ell = 0,1,2,3, \dots \dots, n - 1$
- ℓ: $orbital\ quautum\ Number$.
- $for\ example\ if$

$$n = 1 \Longrightarrow \ell = 0$$
$$n = 2 \Longrightarrow \ell = 0,1$$
$$n = 3 \Longrightarrow \ell = 0,1,2$$

n	**ℓ**			
1	0			
2	0	1		
	S	P		
3	0	1	2	
	S	P	d	
4	0	1	2	**3**
	S	p	d	**f**

The selection rule of transition are:

$$\left.\begin{aligned}\Delta\ell &= \pm 1\\ \Delta m_\ell &= 0, \pm 1\end{aligned}\right\}$$

m_ℓ: magnetic quantum number.

The transition occurs if the difference in (ℓ) between the two cases equals to ± 1.

Such as $2p \rightarrow 3d$ (transition)

The transition cannot happen from $2p \rightarrow 4p$ because of $\Delta\ell = 0$

No transition from	$\left.\begin{aligned} s &\rightarrow s \\ p &\rightarrow p \\ d &\rightarrow d \end{aligned}\right\} = \Delta\ell = 0$
No transition from	$\left.\begin{aligned} s &\rightarrow d \\ d &\rightarrow s \\ p &\rightarrow f \\ f &\rightarrow p \end{aligned}\right\} \Delta\ell = \pm 2$
No transition from	$\left.\begin{aligned} s &\rightarrow f \\ f &\rightarrow s \end{aligned}\right\} \Delta\ell = \pm 3$

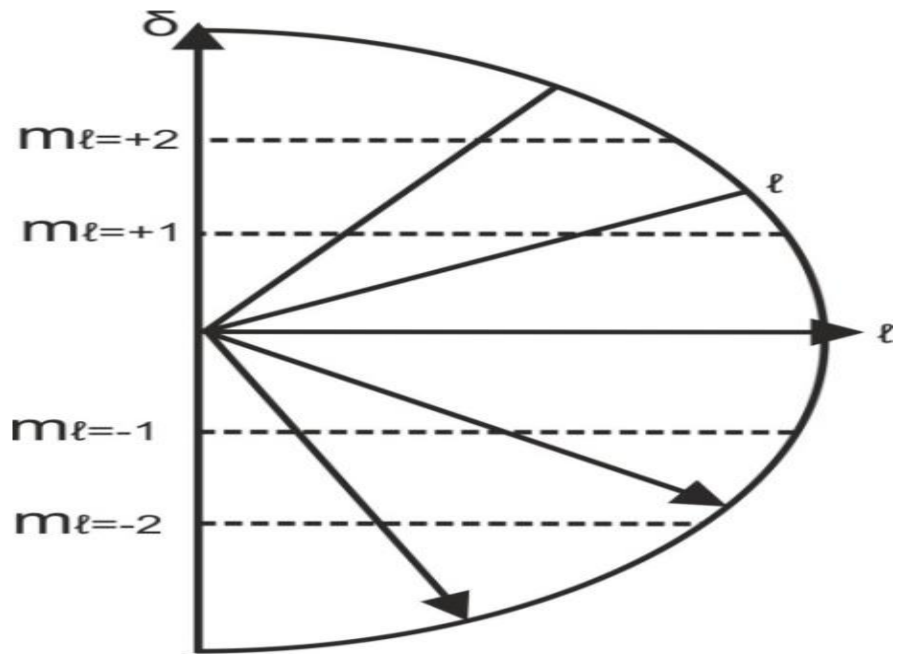
- The transition is happed If $\Delta\ell = \pm 1$
- $s \to p, p \to s, p \to d, d \to p, d \to f$

$$f \to d$$

n	E	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
5	E_5	$5S$	$5P$	$5d$	$5F$	$5g$
4	E_4	$4S$	$4P$	$4d$	$4F$	
3	E_3	$3S$	$3P$	$3d$		
2	E_2	$2S$	$2P$			
1	E_1	$1S$				

- The orbital angular momentum is quantized in its direction and called space quantized this means if we have magnetic field in Z direction.
  $\vec{\ell}$ as a vector conno't have any direction according to Z $-$ direction but can be have specific directions where projection of Z direction equal to integer.
- Let $\vec{\ell}_z =$ the projection of ℓ on Z – axis (the direction of magnetic field).
- $m_{\ell}.\ell$
- $\ell_z\, m_{\ell}.\ell$
- $m_e = 0, \pm 1, \pm 2, \pm 3, \ldots \pm \ell$ m_e: magnetic quantum number.
- $m_{\ell} = 2\ell + 1$
- The no value of $m_{\ell} = 2\ell + 1$

n	ℓ	$m\ell = 2\ell + 1$	Value of $m\ell$
1	0	1	0
2	0,1	3	-1,0,+1
3	0,1,2	5	-2,-1,0,+1,+2
4	0,1,2,3	7	-3,-2,-1,0,+1,+2,+3



This figure show, for one valve of $\ell \Longrightarrow 2\ell + 1$ (directions for $\vec{\ell}$ $and\ 2\ell tl$ values of $m\ell\ 2\ell tl$ called g basic or principle degererate for orbital angular momentum state.

$g = 2\ell tl$

$g = 1\ \ when\ \ \ell = 0 \overset{called}{\Longrightarrow} s - state$

$g = 3\ \ when\ \ \ell = 1 \overset{called}{\Longrightarrow} p - state$

$g = 5\ \ when\ \ \ell = 2 \overset{called}{\Longrightarrow} d - state$

ℓ have three components $\ell_x, \ell_y\ and \ell_z$ but the theoretical analysis leads to determine one component $\ell_z\ and\ \ell_x, \ell_y$ uncertainty determined

$\Delta\ell_x . \Delta\ell_y \geq \frac{1}{2}\ \hbar\ \ell_z$

Then can be determine $\overrightarrow{|\ell|}$ and ℓ_z and can be imagine the orbital angular momentum $\vec{\ell}$ as a vector around Z: axis in specific angle (θ)

Hydrogen atom and similar atoms

Atoms with one electron

These atoms such as hydrogen, heliumlon, electron

$\mathrm{H}, \mathrm{He}^{+}, \mathrm{li}^{++}$ or Be^{+3}

${}_{1}^{1}\mathrm{H}_{0}$ = is a simplest atom in the nature which have one proton and one electron.

${}_{1}^{2}\mathrm{H}_{1}$ = deuterium, proton, neutron (nuclei)

The energy levels of H-atom

$$\boldsymbol{E_p = -\frac{ze^2}{4\pi\epsilon o r} \ldots \ldots (7.2)}$$

$$\boldsymbol{-\frac{\hbar^2}{2m}\left(\frac{d^2\psi}{dx^2}+\frac{d^2\psi}{dy^2}+\frac{d^2\psi}{dz^2}\right)-\frac{ze^2}{4\pi\epsilon o r}\psi = E\psi \ldots \ldots (7.3)}$$

The energy levels of H- atom and for atoms like H can be get from equation (7.3) which close to Bohr results (by classical mechanic) as follows.

$$E_n = -\frac{m_e e^4 z^2}{8\,6^2 h^2 n^2} = -\frac{13.6 z^2}{n^2}\,(ev) \ldots\ldots(7.4)$$

n= 1,2,3… (basic quantum No.)

The total energy is negative quaintly ≡ (attractive force between electron and nucleus).
The energy values are quantized values.

The wave function of H-atom and similar atoms

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)-\frac{ze^2}{4\pi\epsilon_o r}\,\psi = E\psi \quad \ldots..(7.7)$$