Second lecture

1.5Energy quantization

1.5.1The Atomic models

Thomson model (1856- 1940 England)

The atoms are positively charged, uniform spheres as shown in figure (1.6)

The electrons are embedded

He thought that the atoms spheres with negative electrons are embedded in them.

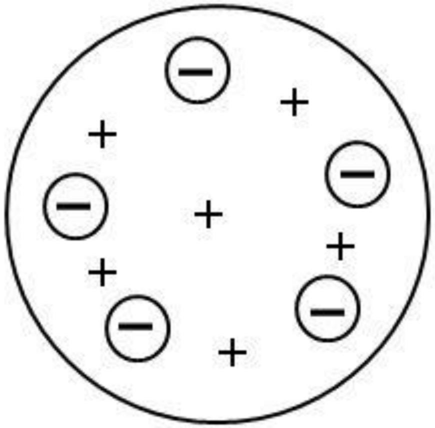

Figure 1.6 Thomson atomic model

Thomson models failed in interpretation the following points.
*Electrons distribution
*Nucleus presence
*Energy levels and sub-levels
*Electron spin, angular momentum

- **Rutherford model (1871- 1937)**
- **This model describes:**
- The atom as a tiny, dense and positively charged nucleus
- Most of the atom's mass is concentrated in the nucleus. The validity of this conclusion has been proven experimental work as shown in figure (1.7)
- The electrons, which are the light negative charge, are surrounding the nucleus and circulating at some distances like plants revolving around the sun.
- However, Rutherford model can also be called nuclear model.
- **Failing points: -**
- - Electrons distribution
- - Electrons levels and sub-level.
- - Electrons energy and momentum.

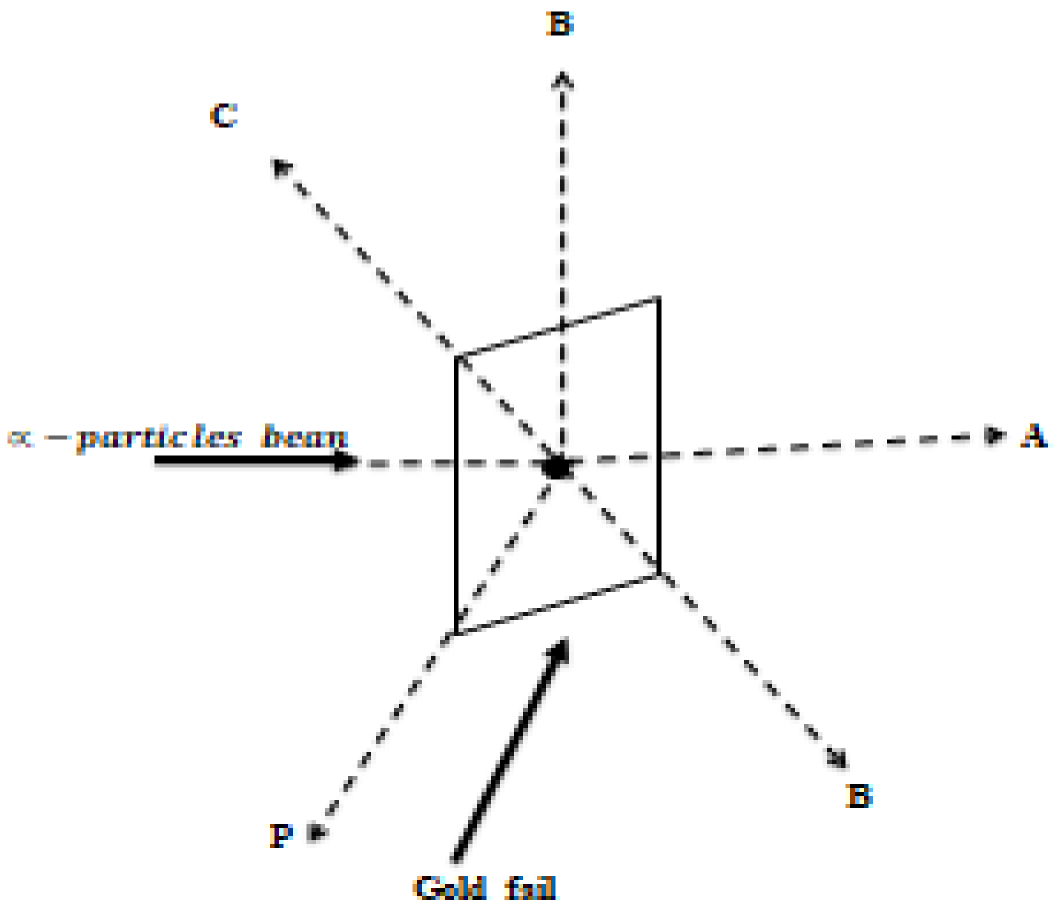

Figure 1.7 Rutherford Experiment

A: transmitted beam.
B: small deflation scattered beam.
C: Large deflection scattered beam

- **3. Bohr model for the hydrogen atom**
- Bohr depends on the idea of energy quantization of Planck model and can be summarized by the following points:
- Radiant energy is emitted or absorbed discontinuously in a form of tiny energy beams known as quanta.
- Each quanta are associated with definite amount of energy (E) depending upon the frequency of radiation according to the equation
- $E = h\nu$
- Where E is the Energy in Joules, h is Planck constant $(6.6 \times 10^{-34}\ J.S)$ and ν is the frequency in Hz.
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- A body can emit or absorb energy in the whole number of quantum, $1hv, 2hv, 3hv, \ldots \ldots \ldots etc$.

- **Notes:**
- **The Energy in the fraction of quantum cannot be lost or absorbed. Bohr adopted these ideas for H-atom**
- - An atom consists of nucleus (proton+ neutrons), surrounded by revolving electrons, the electrons moved around the nucleus in circular paths. These paths are called orbitals and they are numbered as 1, 2, 3,……etc. These numbers have named then as the base quantum number.
- - The Columbic force of attraction between the electrons and nucleus holds the atoms together.
- - Each orbit has discrete energy and is called a stationary state.
- − Mathematical condition determines the size of orbital. The angular momentum of an electron is the multiple of $\left(\frac{h}{2\pi}\right)$ Angular momentum= $mvr = \frac{nh}{2\pi} \ldots \ldots (1.31)$

- Where n=1,2, 3,……….and is called the principle quantum number.
- - Emission or absorption of radiation energy takes place if an electron jumps from one stationary state of energy E_1 to another state of energy E_2. The frequency is given by the relation below.
- $\upsilon = (E_2 - E_1)/h$-----1.33
- $\Delta E = h\upsilon$
- $\upsilon = \frac{\Delta E}{h}$

- Radius of an atoms orbit.
- $The\ centrifugal\ force = \frac{mv^2}{r}$.....(1.٣٤)
- $Electro\ state\ force = \frac{Ze^2}{4\pi Eor^2}$......(1.٣5)
- $\therefore \frac{mv^2}{r} = \frac{ze^2}{4\pi E_O}$
- multiple by r
- $mv^2 = \frac{ze^2}{4\pi E_O}$.......1.36
- $\frac{1}{2}mv^2 = \frac{1}{2}\frac{ze^2}{4\pi Eor}$.......1.37

- The angular momentum of electron is
- $\frac{h}{2\pi}$
- $mvr = \frac{nh}{2\pi}$.....1.38
- $Velocity\ \ V = \frac{nh}{4\pi\ mr}$.....1.39
- Substitute Equation 1.37 in Equation 1.35 and for H-atom Z =1
- $radius\ \ \longrightarrow\ \ r = \frac{n^2 h^2 \epsilon_o}{Ze^2 m\pi}$....1.40
- For H-atom $Z = 1$, $n = 1$
- $\therefore\ \ r = \frac{\left(6.6\times10^{-34}\right)^2 \left(8.85\times10^{-12}\ cn\ m^{-2}\right)}{(1.6\times10^{-19}\ C)^2\ (9.1\times10^{-31}\ kg)(\ 3.14)}$

- $r = 5.29 \times 10^{-11} \qquad m = 52.9\, pm$.
- *Pm: pico meter =10^{-12} meter*
- This result is in a good agreement with the experimental value. From Equation (1.40) the radius of the n^{th}orbit can also be calculated as
- $\gamma_n = n^2 r = 52.9\, n^2\, pm$........(1.41)
- To derive the electron velocity, the value of *r* from Equation (1.40) in Equation (1.39)
- $V^2 = \frac{Ze}{4n^2h^2\epsilon_0^2} \Longrightarrow V = \frac{Ze^2}{2nh\, \epsilon_o}$....1.42
- Velocity of *e* in H-atom in the ground state is
- $V = 2.15 \times 10^6\, m/s$
- **Note:** This high speed of electron gives the atom appears as a spherical shape.

- **1.5.2Energy states:**
- The total energy of the electron in a stationary state is equal to the sum of its kinetic and potential energies .
- $=\frac{1}{2}mv^2-\frac{Ze^2}{4\pi\epsilon_o r}$....1.43
- Substituting the value of kinetic energy from Equation (1.35) in Equation (1.36) will get
- $E=\frac{1}{2}\ \frac{Ze^2}{4\pi\epsilon_o r}-\frac{Ze}{4\pi\epsilon_o r}=-\frac{1}{2}\ \frac{Ze^2}{4\pi\epsilon_o r}$.......1.44
- $r=\frac{n^2h^2\epsilon_0}{Ze^2m\pi}$

- Substituting the value of *r* from Equation 1.38 in Equation 1.42
- $E = -\frac{me^4Z^2}{8\epsilon_0 n^2 h^2}$........(1.45)
- Then
- $E_1 = -\frac{me^4Z^2}{8\epsilon_o^2\ h^2}\ \frac{1}{n_1^2 2}$
- $E_2 = -\frac{me^4Z^2}{8\epsilon_o^2\ h^2}\ \frac{1}{n_2 2}$
- $h\upsilon = E_2 - E_1 = \frac{me^4Z^2}{8\epsilon_o^2\ h^2}\left[\frac{1}{n_1 2} - \frac{1}{n_2 2}\right]$......(1.46)
- For $H, Z = 1$
- $\upsilon = \frac{E_2 - E_1}{h} = \frac{me^4}{8\epsilon_o^2 h^3}\ \left(\frac{1}{n_1 2} - \frac{1}{n_2 2}\right)$
- $\upsilon = R_H\ \left(\frac{1}{n_1 2} - \frac{1}{n_2 2}\right)$...1.47
- Where R_H is the Rydberg constant which equals $3.289 \times 10^{15}\ \ Hz$
- $\frac{R_H}{C} = 109724\ cm^{-1}$.....(1.48)
- The values of υ is given a series of Hydrogen atom Spectrum.

- **1.6 De Broglie's Hypothesis**
- The light behaves as waves when it undergoes interference or diffraction,……etc., and it described by Maxwell's Equation. However, the light behaves as particles when it undergoes photoelectric effect, Compton scattering or blackbody Radiation.
- The Duality of Light led de Brogli to the belief that the probability of particles duality according to de Broglie idea the particles behaves as a waves and as a particles.
- Let the maps of photon $m\gamma$ associated with his energy and frequency $h\nu$

- $E = h\upsilon$
- $E = mc^2$...1.49
- $mc^2 = h\upsilon$
- $m_\gamma c^2 = h\upsilon$
- $m_\gamma = \frac{h\upsilon}{C^2}$
- $P_\gamma = m_\gamma c = \frac{h\upsilon}{C^2}.C$
- $P_\gamma = \frac{h\upsilon}{C} \quad \left(\frac{\upsilon}{c} = \frac{1}{\lambda}\right)$
- $P_\gamma = \frac{h}{\lambda}$ $de\ Broglie\ equation$ 1.50

- De Broglie assumed that there is a material wave accompany each particle animations and the frequency of this wave (ν_m) is linked with its energy (E) and wave-length λ
- $P = mv$
- $E = hv_m \Longrightarrow v_m = \frac{E}{h} = \frac{mc^2}{h}$
- $\lambda_m = \frac{h}{p} = \frac{h}{mv}$
- The hypothesis was proved to be correct when de Broglie was passed a band of electrons accelerated through the crystal as shown in figure (1.8).
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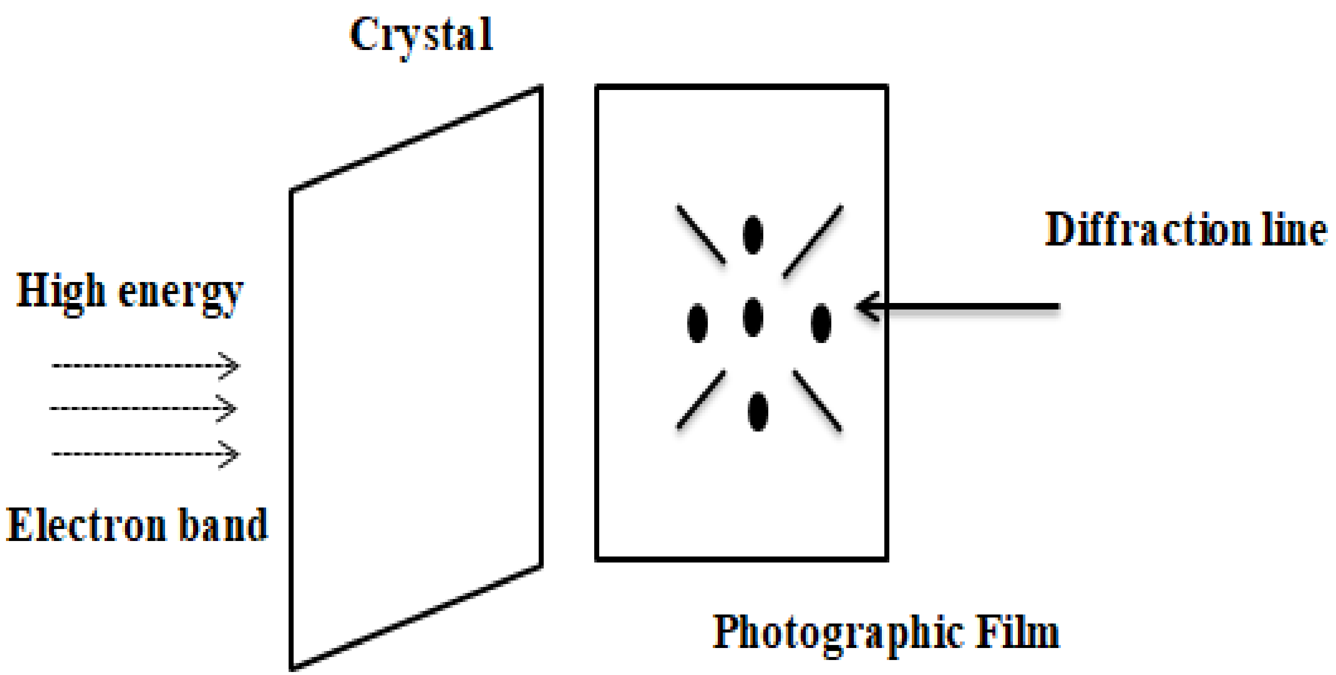

Figure 1.8 de Brogli experiment

The velocity of the material wave V_m is

$$V_m \Longrightarrow \upsilon_m \lambda_m = \frac{mc^2}{h} \lambda_m$$

- $\frac{mc^2}{h} = \frac{h}{mv} = \frac{C^2}{V} \quad \left(\lambda = \frac{h}{p}\right)$
- $If \quad V = C \quad then \quad V_m = C$
- Then the velocity of the material wave is greater than the velocity of the particle but this conclusion cannot assume that the wave leave the particle because of there are many wave accompany the particle, due to instability of its momentum according to uncertainty principle of Heisenberg and also the union of waves will generate a wave beam surrounding the particle and its speed equals the speed of the particle.
  - Test your understanding, prove that $P = \sqrt{2mE_k}$