Third Lecture

1.7 Electron Diffraction

According to de Broglie, the particles have a wave nature with their wavelength given by

$$\lambda = \frac{h}{p}$$

This can be demonstrated by observing the electron diffraction. (figure1.9) Hence, the electron momentum can be calculated using

$$P = \sqrt{2mE_k}$$

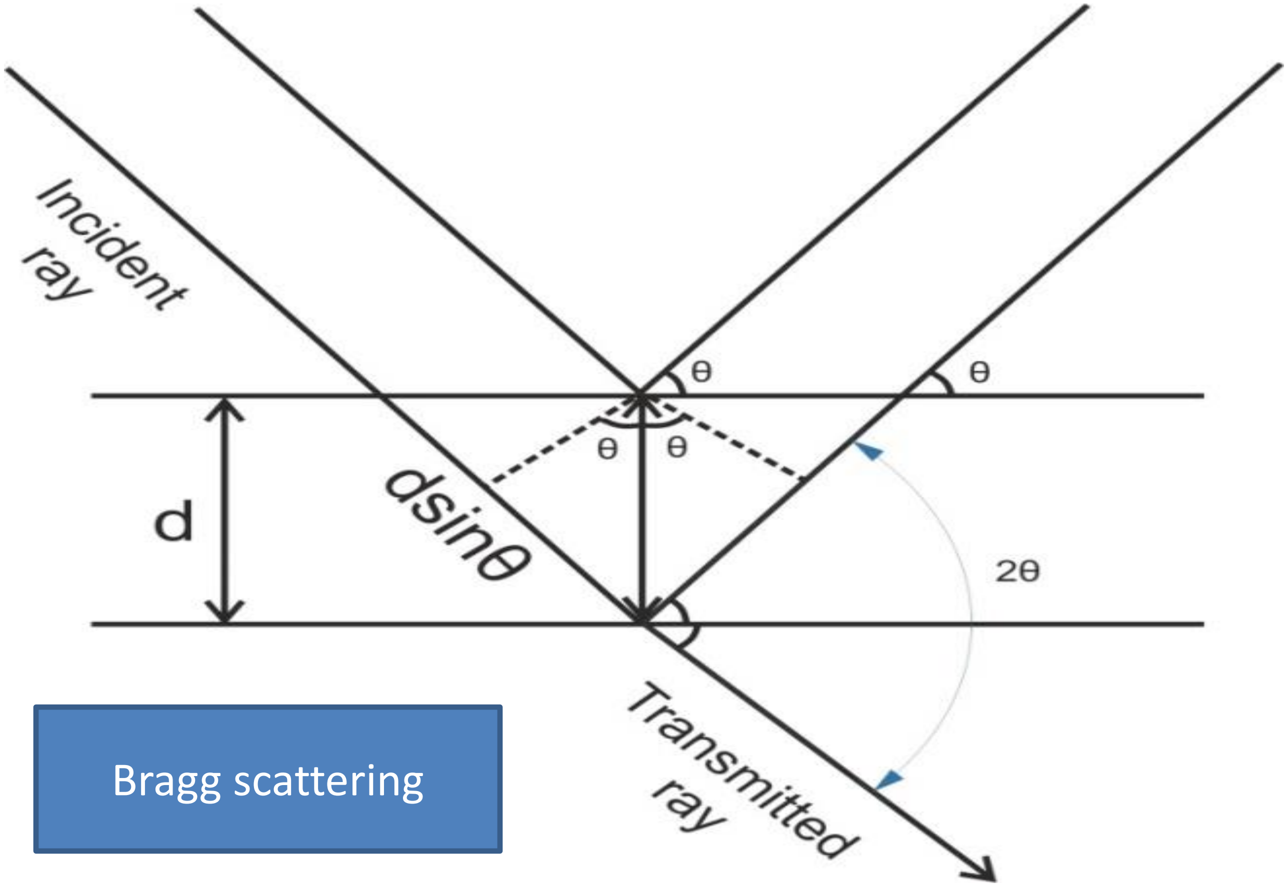
where m is the electron rest mass and E_k is the kinetic energy of electron. E_k is given by accelerating them through a measurable electron Potential.

$$E_k = ev$$

$$e\ ischarge\ \ of\ electron\ = 1.6 \times 10^{-19}\ C$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2mev}} \text{........(1.51)}$$

V: electric potential



$$2d \ \sin\theta = n\lambda \quad .....1.52$$

d: The spacing between Braggs planes
n= integer

- **1.8 Heisenberg Uuncertainty Principle**
- Consider a measurement of position of an atomic particle of mass. If it has to be located within a distance Δx then the light with a wavelength of the size of particle should be used to illuminate it for the particle to be seen. A photon must be collide in some ways with a particle, then, the photon will pass right through and the particle will appear transparent.

- The photon has a momentum of $\left(P = \frac{h}{\lambda} \right)$ and during the collision some of its momentum will be transferred to the particle. The particle location leads to change in its momentum. If the particle is located more accurately, the light leads to change the momentum of particle and becomes greater. A careful analysis of this process was carried out by Heisenberg, who showed that it is not possible to exactly determine how much momentum is transferred to electron.

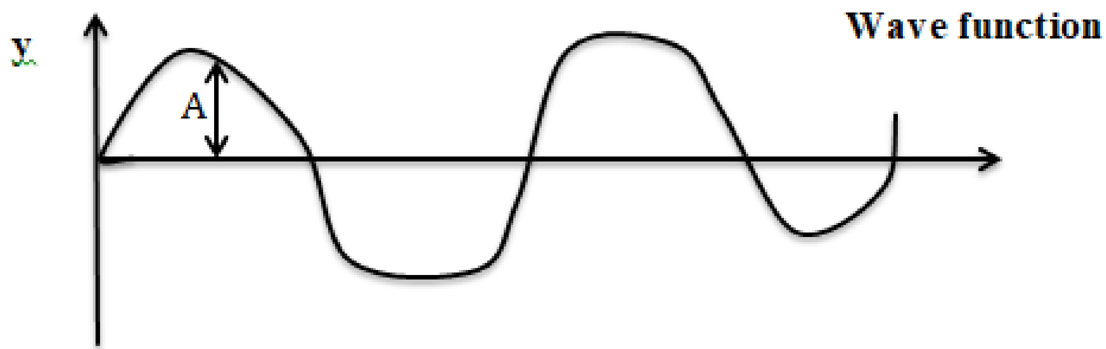
- This means that if a particle has to be located within a region Δx , then this confirms uncertainty in the momentum of particle. Heisenberg was able to show that the ΔP is the uncertainty in the momentum as follow.
- $\Delta x.\ \Delta P \geq \frac{h}{4\pi}\ or\ \frac{h}{2\pi}\ or\ \hbar$ ....(1.53)
- $if\quad \Delta x\ is\ smaller \implies \Delta P\ \ greater$
- $\Delta x\ is\ \ greater \implies \Delta P\ \ smaller$

- **Solved problem**
- - Calculate the uncertainty of position of a body of mass 500 kg moving with a speed of $50 + \ 0.001\ km/hr$
- **Solution:**
- $\Delta x.\ \Delta P\ \geq \frac{h}{4\pi} \Longrightarrow\ \Delta x = \frac{h}{4\pi\Delta p} = \frac{h}{4\pi\ m\ \Delta V}$
- $= \frac{6.6\times10^{-34}\ J.S}{4\times3.14\times5\times10^{-2}\times2.77\times10^{-4}\ m/s} = 3.779 \times 10^{-2}\ m$
- This value is very small distance, then, it is negligible comparing with the mass and velocity.

- **Test your understanding**: What is the uncertainty of momentum of an electron in an atom in order that Δx becomes 52.9 pm, and discuss the result with electron speed in H-atom.

- **1.9 State Function**
- Suppose that the wave moves in x direction according to the following equation

$$y = A\ sin\ (kx - \omega t)$$



where

$k = \frac{2\pi}{\lambda}$ $(wave\ Number)$ …….(1.54)

$\omega = 2\pi v$ $(angylar\ freq\ )$……..(1.55)

$\lambda\ is\ thewavelenght$

- The 2nd derivative of the above equation with respect to *x* is.
- $\frac{\partial y}{\partial x} = kA\cos(kx - wt)$
- $\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(kx - wt)$
- $\therefore \frac{\partial^2 y}{\partial x^2} = -k^2 y$.....A
- The 2nd derivative with respect to *t* is
- $\frac{\partial y}{\partial t} = wA\cos(kx - wt)$
- $\frac{\partial^2 y}{\partial x^2} = -w^2 A\sin(kx - wt)$
- $\therefore \frac{\partial^2 y}{\partial x^2} = -w^2 y$....B

- By dividing Equation B on Equation A will get
- $\frac{\frac{\partial^2 y}{\partial t^2}}{\frac{\partial^2 y}{\partial x^2}} = \frac{w^2 y}{-k^2 y} = \frac{w^2}{k^2}$
- $\therefore \frac{w^2}{k^2} = \frac{(2\pi\upsilon)^2}{\left(\frac{2\pi}{\lambda}\right)^2} = \upsilon^2\lambda^2 = V^2$
- $\therefore \frac{\frac{\partial^2 y}{\partial t^2}}{\frac{\partial^2 y}{\partial x^2}} = V^2$

- $\therefore \frac{\partial^2 y}{\partial t^2} = V^2 \frac{\partial^2 y}{\partial x^2}$…1.56
- Where V is the wave velocity.
- Equation (1.56), however, is a 2nd order differential equation and its solution can be:
- $y = A\, sin(kx - wt)$
- $y = B\, cos(kx - wt) \quad Or$
- $y = A\, sin(kx - wt) + \; B\, cos(kx - wt)$
- $\therefore y = Ce^{\,i}\,(kx - wt)$…..1.57

- Equation (1.57) represents the exponential solution of Equation 3. *K* and ω can be modified to be in terms of particle momentum *P* and energy *E* respectively, by applying de Broglie hypothesis.
- $\frac{\lambda}{1} = \frac{h}{p}$
- $\frac{\lambda}{2\pi} = \frac{h}{2\pi p} \qquad , where\ \hbar = \frac{h}{2\pi} and\ k = \frac{2\pi}{\lambda}$
- $\therefore \frac{1}{k} = \frac{\hbar}{p} \Longrightarrow k = \frac{P}{\hbar}$....c
- $\upsilon = \frac{E}{h}$
- $2\pi\upsilon = \frac{2\pi E}{h}$

- $\therefore w = \frac{E}{\hbar}$ ....D
- Substitute Equations C and D in Equation (1.57) we get.
- $\therefore y = Ce^{\frac{i}{\hbar}}\left(x\frac{p_x}{\hbar} - \frac{E}{\hbar}t\right)$
- $let\ y\ = \psi(x,t)\ \ wave\ funtion$.
- $\psi(x,t) = ce^{\frac{i}{\hbar}}\,(xP_x - Et)$
- Where $\psi(x,t)$ is the wave function or state function because of described the motion of material wave in speed of in $(V = v\lambda) in\ x$ direction and accompanying with particle has momentum P_x and energy E.
- $\psi(x,t)$: can be called probability density because of it can be used to determine the probability of particle Presence in any point or in any region.

– **Hypothesis of quantum Mechanic**

- ***Wave Function**: represents the dynamic state of moving particle in terms of wave function or probability density ψ and often $\psi(x,t)$ is a complex function and represents the complex conjugate ψ^{x}.
- Then if ψ_1, ψ_2, ψ_3 are a wave function then the linear sum represents other dynamic state.
- $\psi(x,t) = C_1\psi_1 + C_2\psi_2 + C_3\psi_3 + \cdots$
- Where C_1, C_2, C_3… **Probability density** of particle
- in state of $\psi(x,t)$ 1,2,3, … and can be called weight state.
- The Properties of wave function has single value.

- **Example:** if
- $y = x^2 + x - 3$
- $IF \;\; x = 1 \Longrightarrow \; y = -1, and \; if \; x = 2 \Longrightarrow y = 3$
- Any value of (x) Give us one value off function(y).
- The state function is specific not infinite
- $\psi \neq \; \infty$
- **Continuous**: The function called eign value of single value and Continuous at point $X = a$ when
- $\lim_{x \to a} \psi_{(x)} \; exist \; finite$
- $\psi_{(a)} \; finite$
- $\lim_{x \to a} \psi_{(x)} = \psi_{(a)}$

- **Example:**
- $\psi_{(x)} = y = x^2 + x_1$
- $\lim_{x \to 1} \psi_{(x)} = \frac{\partial f}{\partial x} = 2x + 1 = 3$
- $\psi_{(1)} = 3$
- $\lim_{x \to 1} \psi_{(x)} = \psi_{(x)}$
- $3 = 3$
- $\psi_{(x)} = x^2 + xH$ is a continuous function
- Differential acceptable.