Fourth Lectuer

Linear Operators

Types and definition of linear operators

Operators: refer to the physical quantities which are if entered to other quantities turned them to new quantities or if affected on the wave function convert it to a new function.

If A: physical quantity

$$\therefore \hat{A} \text{ or } A_{op} \text{ is a operator}$$

Such as the physical quantity linear momentum $\left(\hat{P}\right)$, P_{op}

2.1.1 linear momentum operator

$$\psi = e^{\frac{i}{\hbar}(XP_x - Et)}$$

We have $\psi = e^{\frac{i}{\hbar}(XP_x - Et)}$ $\quad$ *wave function*

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} P_{x\psi} e^{\frac{i}{\hbar}(XP_x - Et)} = \frac{i}{\hbar} p_x \psi$$

- *and by the same way*

$$\widehat{p_x} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$\widehat{P_y} = \frac{\hbar}{i}\frac{\partial}{\partial y}$$

$$\widehat{P_z} = \frac{\hbar}{i}\frac{\partial}{\partial z}$$

operatots of Momentum in the x, y, and z diretion

- **2.1.2 Kinetic Energy operator**
- $K.E = \frac{1}{2}\, mv^{\,2}$
- $k.E = \frac{mv^{\,2}}{2m} = \frac{p^2}{zm} \ldots\ldots\ldots(2.1))$
- $P_{op} = \frac{\hbar}{\lambda}\frac{\partial}{\partial x} \Rightarrow (P_{op})^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$

$$\boxed{T_{op} = \frac{\hbar^2}{2m}\,\frac{\partial^2}{\partial x^2}} \quad x - direetion$$

$$\therefore\ kE_{op}\ or\ \boxed{|Top_y = \frac{\hbar^2}{2m}\,\frac{\partial^2}{\partial y^2}} \quad y - direetion$$

$$\boxed{T_{op} = \frac{\hbar^2}{2m}\,\frac{\partial^2}{\partial z^2}} \quad z - direetion$$

- **2.2 Properties of linear operators**
- Let $\alpha: linear\ operator$
- $\psi_1 , \psi_2: wave\ funetions$
- $\alpha(\psi_1 , \psi_2) = \alpha\, \psi_1 + \alpha\, \psi_2$
- For example if $\psi_1 = x^2 , \psi_2 = x^3$
- $\alpha = \frac{\partial}{\partial x}$
- $\therefore \alpha\ (\psi + \psi_2) = \alpha\, \psi_1 + \alpha\, \psi_2$
- $\frac{\partial}{\partial x}(x^2 + x^3) = \frac{\partial}{\partial x} x^2 + \frac{\partial}{\partial x} x^3$
- $2x + 3x^2 = 2x + 3x^2$
- $\therefore The\ Rule\ is\ truth$

- $cx\ \psi = C(\alpha\psi) = C\alpha\ \psi$ where Cis constant
- $if\ a, \beta\ are\ operators \Leftrightarrow [if\ \ \propto, \beta\ are\ operators]$
- $\therefore\ \ \alpha\ \beta(\psi) = \alpha(\beta\psi) = \alpha\beta\psi \Rightarrow [\propto \beta(\psi)] = \propto (\beta\psi) = \propto\ \ \beta\psi$
- $\alpha\ \beta \neq \beta\alpha$
- **The commutator**
- if α and, β are operators
- Then $\left[\alpha_, \beta\right] = \alpha\beta - \beta\alpha$
- ***Example:**
- Find the commutator $\left[\frac{d}{dx}, x\right]$
- $[a, \beta] = \alpha\beta - \beta\alpha$
- $[a, \beta]\psi = \alpha\beta\psi - \beta\alpha\psi$
- $\left[\frac{d}{dx}, x\right]\psi = \frac{d}{dx}\alpha\psi - x\frac{d}{dx}\psi$
- $\left[\frac{d}{dx}, x\right]\psi = x\frac{d}{dx} + \psi - x\frac{d}{dx}\psi$
- $\left[\frac{d}{dx}, x\right]\psi = \psi$
- $\therefore\ \ \left[\frac{d}{dx}, x\right] = 1$

- Not:
- $$x_{op} = X$$
  $$v_{op} = V$$
  $$t_{op} = t$$

- If $\psi_1(x,t)$: is a wave function that describes the system in a certain state and $\psi_2(x,t)$ is another wave function can describe the system in other state and ψ_1, ψ_2 are constant.
- $\therefore\ \psi(x,t) = \sum_n m_n\, \psi_n(x,t)$....2.2
- ***Example:** prove $P_{op}^2 = \left(P_{op}\right)^2$
- **Solution:**
- $\psi(x,t) = e^{\frac{i}{\hbar}(XP_x - Et)}$
- $\frac{\partial\psi}{\partial x} = \frac{i}{\hbar} p_x\, e^{\frac{i}{\hbar}(xp_x - Et)}$
- $\frac{\partial\psi}{\partial x} = \frac{i}{\hbar} p_x\, \psi$
- $\therefore \frac{\partial^2\psi}{\partial x^2} = \left(\frac{i}{\hbar} p_x\right)^2\ \psi = -\frac{p^2}{\hbar^2}\ \psi$

- $\therefore P_{op}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$
- We have $P_{OP} = \frac{\hbar}{i} \frac{\partial}{\partial x}$
- $\therefore (P_{OP})^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$ $\qquad \therefore P_{op}^2 = (P_{OP})^2$
- ***Test your understanding**
- Find the commutators $[P_x, x], [P_x^2 , x]$