The origin of Schrodinger equation

$$Total\ energy = Kinetic\ Energy(KE) + Potential\ energy\ (V)$$

$$E = KE + V \ldots\ldots..(3.1)$$

$$KE = \frac{1}{2}\ mv^2 = \frac{m^2v^2}{2m} = \frac{P^2}{2m}$$

$$\therefore E = \frac{P^2}{2m} + V\ and\ E\psi = \frac{P^2}{2m}\psi + V\psi \ldots\ldots(3.2)$$

- $\psi = e^{\frac{i}{t}(xp_x - Et)}$
- $\frac{\partial \psi}{\partial x} = \frac{i}{\hbar}\, p_x \; e^{\frac{i}{\hbar}(xp_x - Et)}$
- $\frac{\partial \psi}{\partial x} = \frac{i}{\hbar}\; p_x\, \psi$
- $\frac{\partial^2 \psi}{\partial x^2} = \left(\frac{i}{\hbar} p_x\right)^2 \psi = \; -\frac{p_x^2}{\hbar^2}\psi$
- $\therefore\, p_x^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}\; sub.in \ldots.(3.2)$

$E\psi = \frac{-\hbar^2}{2m}\,\frac{\partial^2}{\partial x^2} + V\psi$ Time Independent Schrodinger Equation (TISE)

- $\psi = e^{\frac{i}{\hbar}(xp_x - Et)}$ $wavefunction$
- $\frac{\partial \psi}{\partial t} = \frac{-i}{\hbar} E \; e^{\frac{i}{\hbar}(xp_x - Et)}$
- $\frac{\partial \psi}{\partial t} = \frac{-i}{\hbar} E \; \psi$
- $-\frac{\hbar}{i} \; \frac{\partial \psi}{\partial t} = E \; \psi$
- $\therefore \; E_{op} = -\frac{\hbar}{i} \; \frac{\partial}{dt}$
- $E_{op} = i\hbar \; \frac{\partial}{d\hbar}$
- $\therefore -\frac{\hbar^2}{2m} \; \frac{\partial^2 \psi}{\partial x^2} + V\psi = \; i\hbar \; \frac{\partial \psi}{\partial \hbar}$ ……(3.3)
- Time Dependent Schrodinger equation (TDSE)

- ***Test your understanding**
- start with $E = \hbar\omega \ \ to\ find\ TDSE$
- Then we have different formula of Schrodinger equation(S.E.)
- $-\frac{\hbar^2}{2m}\ \frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi\ (TISE)\ in\ one\ dimension\ (IDTISE)$
- $-\frac{\hbar^2}{2m}\ \frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\ \frac{\partial\psi}{dt}(TDSE)\ in\ on\ dimension$
- $-\frac{\hbar^2}{2m}\nabla\ \psi^2_{(x,y,z)} + V\psi(x,y,z) = E\psi(x,y,z)\ (3DTISE)$
- $-\frac{\hbar^2}{2m}\nabla^2\ (x,y,z,t) + V\psi(x,y,z,t) = i\hbar\frac{\partial\psi}{\partial t}\ (3D.TDSE)$
- The simplest formula of S.E. is:
- $H\psi = E\psi$
- $H = T = KE + V$
- $H{:}\,Hamiltonian$

- **3.2 Probability current Density**
- The idea of probability current comes from the multiplication (product)
- of $\dot{\psi}^*(x,t)\psi\,(x,t)$
- This relation leads to the term of probability current, if we examine the principles of classical physics, we can find the concepts of density and of current.
- The change in the electrical charge density with time within a certain is zone leading to transfer the electrical current outside the surrounding surface and the same concept is applied on material, heat,.....etc.

- $-\frac{d\rho}{dt} = \frac{ds}{dx} \dots\dots\dots\dots\dots\dots\dots\dots (3.4)$
- Where ρ is the density of the electrical charge, *t* is time, *S* is the output current and *x* is the surrounding surface. However the negative (-) sign means decreasing the density inside the enclosure volume.
- This idea con be generalized to the principle of probability density of quantum mechanics.
- Let $\psi(x, t)$ is the variation with time and the probability density $-\frac{d\rho}{dt}$ variation with time, the probability density reducing with time, this agrees

- with equation(3.4) then the leakage current is found and to describe this leakage starting by Schrodinger equation.
- $-\frac{\hbar^2}{2m}\ \frac{d^2\psi}{dx^2} + v_{(x)}\psi = -\frac{\hbar\, d\psi}{i\, dt}$ … … … … . … … … (3.5)
- And the conjugate equation
- $-\frac{\hbar^2}{2m}\ \frac{d^2\psi^*}{dx^2} + v_{(x)}\psi^* = \frac{\hbar\, d\psi^*}{i\, dt}$ … … … … … … … (3.6)
- Multiple equation (3.5) by ψ^* & equation (3.6) by ψ and Subtract them
- we will get the following
- $-\frac{\hbar^2}{2m}\left(\psi^*\ \frac{d^2\psi}{dx^2} - \psi\,\frac{d^2\psi^*}{dx^2}\right) = \frac{\hbar}{i}\ \left(\dot{\psi}^*\,\frac{d\,\psi}{dt} + \psi\,\frac{d\,\psi}{dt}\right)^*$
- $-\frac{\hbar}{2}\left[mi\,\frac{d}{dx}\ \left(\psi^*\,\frac{d\psi}{dx} - \psi\,\frac{d\psi^*}{dx}\right)\right] = \frac{d}{dt}\ \left(\dot{\psi}^*\ \psi\right)$ … . (3.7)

- By comparing equation(3.7) and equation (3.4) we can conclude that the term $\psi^{*}\psi$ representing the probability density and corresponding to ρ.
- Thus, the probability current density equation can be written in the following form .$S = -\frac{\hbar}{2m\lambda}\left[\dot{\psi}^{*}\frac{d\,\psi}{dx} - \psi\frac{d\,\psi}{dx}\right]^{*} \ldots\ldots\ldots\ldots\ldots..(3.8)$
- Equation (3.8) is called continuous equation ; by using the above equation can be determine the current probability density for any wave equation or wave function.

- **3.3 Applications of Schrodinger equation**
- **3.3.1 Free particle**
- Free partial means no force effect on particle then the potential energy is equal to =zero.
- $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$ TISE….(A)
- $V = -\int_{x_1}^{x_2} F dx = 0\ ,\ \text{since } F = 0$
- The TISE
- $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \ldots\ldots\ldots\ldots\ldots\ldots(B)$

- $\frac{d^2\psi}{dx^2} + \frac{2m\,E}{\hbar^2}\psi = 0 \ldots\ldots\ldots\ldots.(C)$
- $E = T + V \;\&\; T = \frac{P^2}{2m}, v = 0$
- Then
- $p^2 = 2mE\;,\; P = \hbar k$
- $\hbar^2 k^2 = 2mE$
- $\therefore\; k^2 = \frac{2mE}{\hbar^2}\; sub\ in\ eq\ (C)$
- $\frac{d^2\psi}{dx^2} + k^2\,\psi = 0$

- $\frac{d^2\psi}{dx^2} + k^2\,\psi = 0$
- The solution :
- $\psi = Ae^{ikx}$ represents the particle that is move to the right side, with momentum equals $P = \hbar k$.
- $\psi_{(x)} = Be^{-ikx}$represents the particle that is moved to the left side, with momentum equals $P = \hbar k$.
- The probability Density of free particle at any point equals
- $\psi^{*}{}_{(x)}\psi_{(x)} = A^{*}e^{ikx}\,Ae^{-ikx} = A^{*}A = Constant$
- The probability density of free particle in any point equals to the P.D. in any other points this means the uncertainty $\Delta x = \infty$. The result agrees with the Heisenberg uncertainty principle.

- The particle moved with the momentum of $P = hk$ means $\Delta P = 0$
- $\Delta x.\Delta P \geq \hbar$
- $\Delta x = \infty \; when \; \Delta P_x = 0$

Thank You For Listening