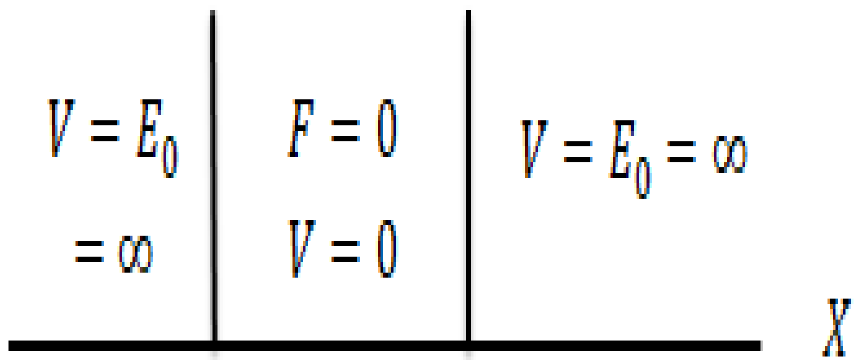
4.1 Potential Box in one dimension

Region of dimension -a- , the potential energy in this region equals zero [v=0 inside the box]. The potential in outside equals to $[V = E_0 = \infty]$. sometimes the potential box is called Infinite potential well. In side potential Box no Force effected on any particle $F = 0 \Longrightarrow V = 0$.



- Figure(4.1) Potential well in one dimension
- To find the wave Function of particle in potential box we can solve Schrodinger equation.

- $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+v\psi=E\psi\ \ (ODTISE)$
- $V=0\ (inside\ box)$
- $\therefore -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=E\psi$ ……………. (4.1)
- $E=T+V=\frac{p^2}{2m}$
- $\therefore k^2=\frac{2mE}{\hbar^2}$
- $\frac{\partial^2\psi}{\partial x^2}+k^2\psi=0$ ……..(4.2)
- The solution of equation (4.2) is:
- $\therefore \psi_{(x)}=Ae^{ikx}+Be^{-ikx}$……… (4.3)

- The boundary conditions
- $0 \geq \times \geq a$
- When $x = 0 \overset{and}{\Longrightarrow} \psi_{(x)} = 0$
- $0 = Ae^{0} + Be^{0}$
- $\therefore A + B = 0 \quad A = -B, \quad B = -A$
- $\therefore \psi_{(x)} = Ae^{ikx} - A\acute{e}^{ikx}$
- $[Acoskx + i\, Asinkx] - [Acoskx - i\, Asinkx] = 2\, i\, Asinkx$
- $Let = c = 2\, i\, A$
- $\psi_{(x)} = C\ sinkx, x = a$
- $\psi_{(0)} = 0 = Csinka$
- $C\ \neq 0$ Because $\psi \neq 0$

- $\therefore sinka = 0 \Longrightarrow ka = n\pi \ \ (n = 1, 2, 3, \dots \dots .)$
- $k = \frac{n\,\pi}{a}$
- $\therefore \psi_{(x)} = C\ sin\ \frac{n\pi x}{a}$
- To find C. we can apply the normalized condition
- $\int_{all\ space} \psi^{*}\psi\ \partial x = 1$
- $\psi_{(x)} = Csin\ \frac{n\pi x}{a}\ ,\ \psi^{*}_{(x)} = C^{*}sin\ \frac{n\pi x}{a}$
- $\int Csin\ \frac{n\pi x}{a}\ ,\ C^{*}sin\ \frac{n\pi x}{a} = 1$
- $|C|^{2} \int_{all\ space} sin^{2}\ \frac{n\pi x}{a}\ = 1$

- $|C|^2 \int_0^a \frac{1}{2}\left(1-\cos\frac{2n\pi x}{a}\right) = 1$
- $\frac{C^2}{2}\left[x - \frac{\sin\frac{2n\pi x}{a}}{\frac{2n\pi}{a}}\right]_0^a = 1$
- $\frac{C^2}{2}[a-0] = 1$
- $\therefore c = \sqrt{\frac{2}{a}}$
- $\psi_{(n)} = \sqrt{\frac{2}{a}}\, sin\, \frac{n\pi x}{a}$ ……(4.4)
- The general form of the wave function for the particle in the potential Box of one dimension.

- **Example**(1) :Find the wave Function of particle in potential box in one dimension at ground state, first excited state and second excited state solution:
- Ground state n=1
- $\therefore \psi_{n(x)} = \sqrt{\frac{2}{a}} \; sin \; \frac{n\pi x}{a}$
- $\therefore \psi_{1(x)} = \sqrt{\frac{2}{a}} \; sin \; \frac{\pi x}{a}$
- First excited state n=2
- $\therefore \psi_{n(x)} = \sqrt{\frac{2}{a}} \; sin \; \frac{n\pi x}{a}$
- $\therefore \psi_{2(x)} = \sqrt{\frac{2}{a}} \; sin \; \frac{2\pi x}{a}$
- Second excited state n=3
- $\therefore \psi_{n(0)} = \sqrt{\frac{2}{a}} \; sin \; \frac{n\pi x}{a}$
- $\therefore \psi_{3(0)} = \sqrt{\frac{2}{a}} \; sin \; \frac{3\pi x}{a}$

- The energy of particle in the potential Box.
- To calculate the energy of particle in P.B. of one dimension
- $k = \frac{n\pi}{a}$, $n = 1, 2,3, \dots \dots \dots$ ….(4.5)
- $\therefore k = \frac{\pi}{a}, \frac{2\pi}{a}, \frac{3\pi}{a}, \dots \dots \dots$
- $P = \hbar k \Longrightarrow P_n = \frac{n\pi\hbar}{a} = \quad n = 1, 2, 3, 4, \dots \dots \dots$
- $P_1 = \frac{\pi\hbar}{a}$., $P_2 = \frac{2\pi\hbar}{a}$, $P_3 = \frac{3\pi\hbar}{a}, \dots \dots \dots \dots$
- $V = 0$
- $\therefore E = T + V = T = \frac{P^2}{2m}$
- $\therefore E_n = \frac{P^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}$ …….(4.6)
- $$\boxed{\therefore E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad n = 1, 2, 3, \dots \dots \dots}$$ ….(4.7)

The relationship of Energy for particle in the P.B. of one dimension

Particle state	n	The wave function ψ	The energy E
Ground state	1	$\psi_1 = \sqrt{\frac{2}{a}}\, sin \frac{\pi x}{n}$	$E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$
1st excited state	2	$\psi_2 = \sqrt{\frac{2}{a}}\, sin \frac{2\pi x}{n}$	$E_2 = \frac{4\pi^2 \hbar^2}{2ma^1}$
2nd excited state	3	$\psi_3 = \sqrt{\frac{2}{n}}\, sin \frac{3\pi x}{n}$	$E_3 = \frac{9\pi^2 \hbar^2}{2ma^2}$
3nd excited state	4	$\psi_4 = \sqrt{\frac{2}{n}}\, sin \frac{4\pi x}{n}$	$E_4 = \frac{16\, \pi^2 \hbar^2}{2ma^2}$
4th excited state	5	$\psi_5 = \sqrt{\frac{2}{n}}\, sin \frac{5\pi x}{n}$	$E_5 = \frac{25\pi^2 \hbar^2}{2ma^2}$

- **Test your understanding why $n \neq 0$:Prove mathematically**

- *Thank you for your listening*