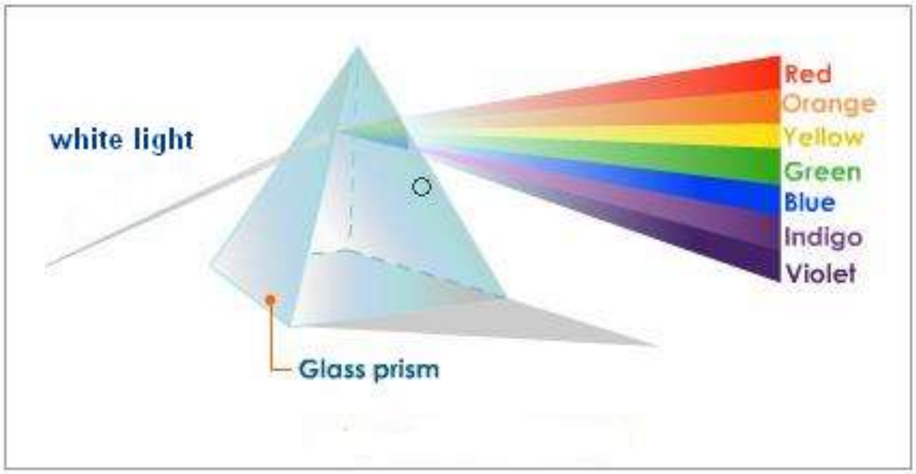
Line Spectra



The visible light spectrum.

White light ⟹ continuous spectrum (all wavelengths present).

Discharge in gas ⟹ few colors appear. (Isolated lines).

This is a line spectrum.

EXAMPLE

Discharge in :

Hydrogen

Sodium

Iron

Energy Levels

Every element has a characteristic line spectrum ⟹ result from the structure of atoms of the element.

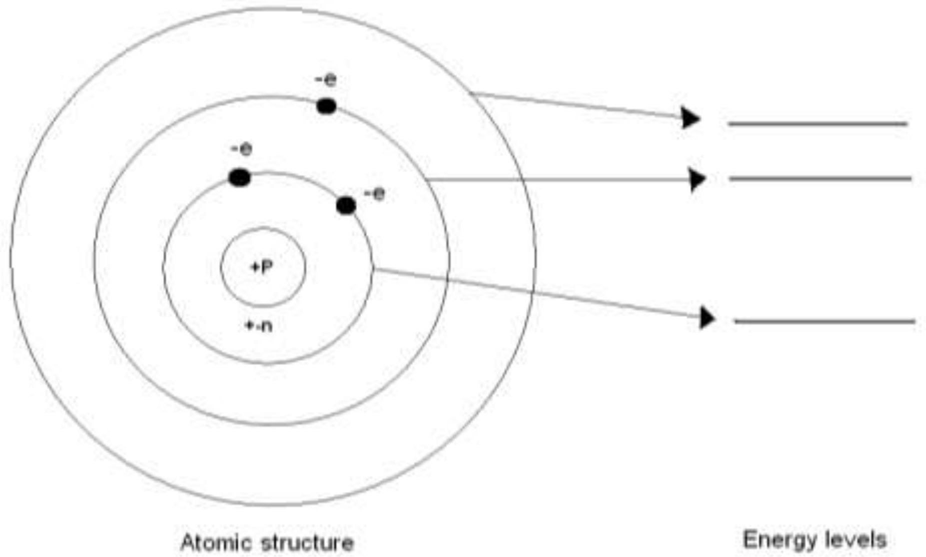


Figure (6): Atomic structure.

Bohr model of the atom:

The spectrum of H-atoms is explained by Bohr using 3 basic postulates:

1. **The electron in H-atom can rotate about the nucleus in certain fixed orbit of radius (r),** where orbital angular momentum L is a multiple of $\frac{h}{2\pi}(\hbar)$ ➔i.e. Angular momentum is quantized.

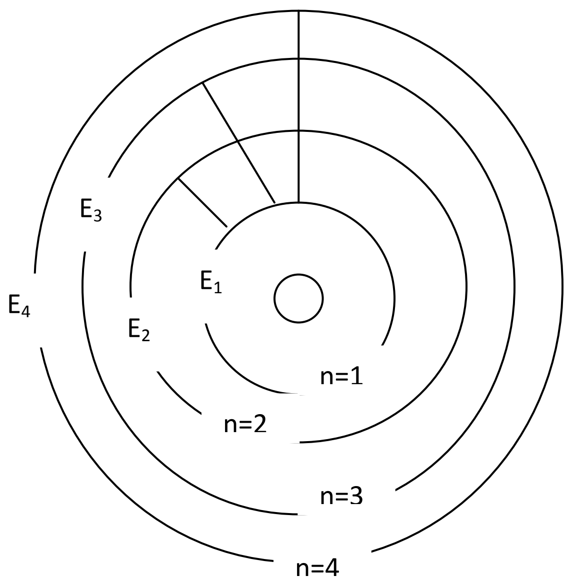
$$L = Iw = mvr = n\hbar = \frac{nh}{2\pi}$$

n = integer 1,2,3,4,…………… ∞

m = mass of the electron

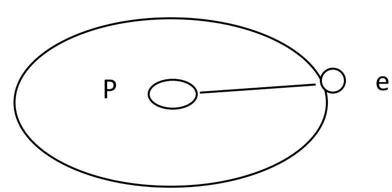
v = linear velocity

2. The electron in the **stationary orbit (or state) does not emit electromagnetic radiation**.
3. **Radiation emitted or absorbed, When an electron undergoes a transition from one orbit to another**, the energy of absorbed or emitted light photon is:
$\Delta E = E_1 - E_2 = h\nu$



Now to derive the energy equation in order to understand the energy levels:

The electrostatic force = The centripetal force



$Kq_1q_2 / r_2 = mv^2/r$

Hence:

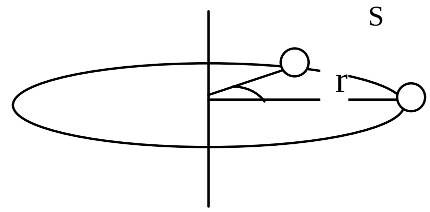
$r = kze^2/mv^2$ …………… (1)

Angular momentum $L = I\omega = n\hbar$ …………….. (2)

Since $I = mr^2$, $\omega = v/r$

This can be derived as follows:

$\theta = s/r$



$d\theta = ds/r$

$d\theta/dt = (1/r)\ ds/dt$

Hence: $\omega = v/r$

So:

$mr^2 v/r = n\hbar$

$v = n\hbar/mr$ ....(3)

Substitute (3) into (1), get:

$r = n^2\hbar^2 / kze^2 m$ ....(4)

Substitute (4) into (3):

$V = (n\hbar/m)(kze^2 m/ n^2\hbar^2)$

$V = kze^2/n\hbar$ ....(5)

Total energy:

$E_t = E_k + E_p$

$= 1/2mv^2 + E_p$

Work done $W = \int F dr$

$$= \int (kze^2/r^2)\, dr$$

$$= kze^2 \left[(1/r)\right]_{r_a}^{r_b}$$

$= -kze^2/r_b + kze^2/r_a$

If $r_b = r$, $r_a = \infty$

So :

$W = -kze^2/r = E_p$ ....(6)

So:

$$E_t = 1/2mv^2 + (-kze^2/r)$$

Substitute for the value of (v) from (5) :

$$E_t = -mz^2e^4k^2/2n^2\hbar^2$$

The negative sign is due to the connection between the nucleus and the electron.

For hydrogen atom

z =1

$$E_t = \frac{[(-9.1\times10^{-31}) \times (1.6\times10^{-19})^4 \times (9\times10^9)^2}{2 \times (6.63xx10^{-34}/6.28)^2\, n^2}$$

$= -2179.6 \times 10^{-21} / n^2$ Joule

Hence:

$$E_t = -13.6/n^2 \quad ev$$

(Dividing by the charge of the electron).

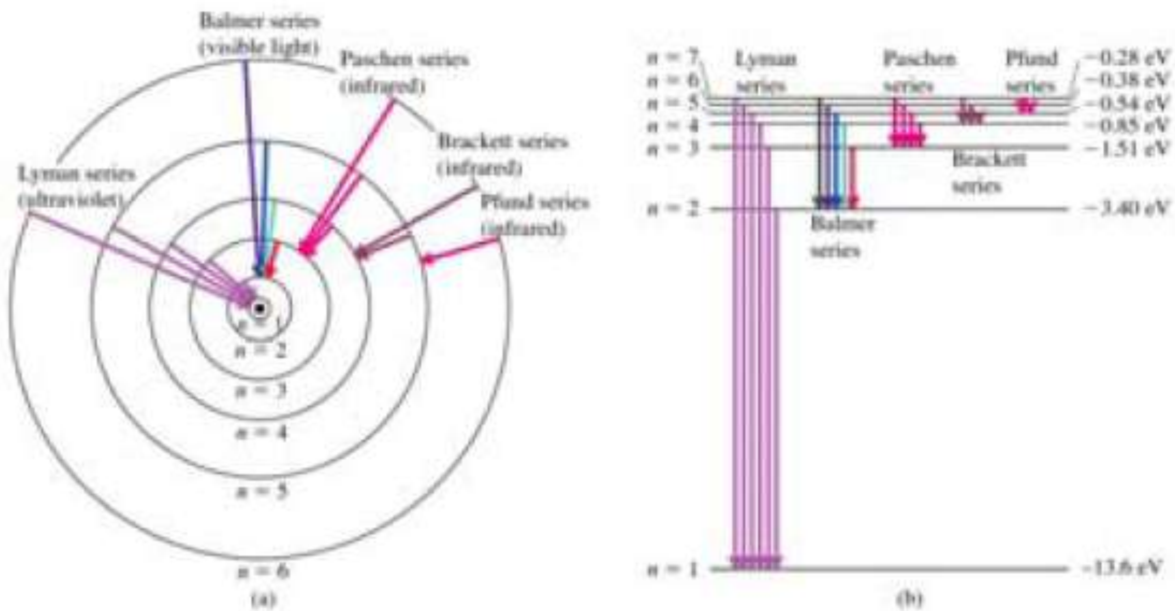
Now according to Bohr theory:

n=1 $E_1 = -13.6ev$

n=2 $E_2 = -13.6/4 = -3.4$ ev

n=3 $E_3 = -1.51$ ev

n=∞ $E_\infty = 0$



Now to calculate (**λ**) for the spectrum of the H-atom

$E = h\upsilon = hc/\lambda$

$\lambda = hc/E$

<u>1st orbit</u> For transition $1 \longrightarrow \infty$

$\lambda_{(\infty-1)}$ = $(6.63\times10^{-34} \times 3\times10^{8}) / (13.6\times1.6\times10^{-19}) = 914\times10^{-10}$m

= **91.4nm (u.v region)**

For line 2 → 1

E=13.6-3.4=10.2ev.

So $\lambda_{2\rightarrow1}$ = $(6.63\times10^{-34} \times 3\times10^{8}) / (10.2\times1.6\times10^{-19})$ = **121.8nm (u.v)**

2nd orbit

$\lambda_{(\infty-2)}$ = $(6.63\times10^{-34} \times 3\times10^{8}) / (3.4\times1.6\times10^{-19})$ = **365.6nm (u.v region)**

$\lambda_{(3-2)}$ = $(6.63\times10^{-34} \times 3\times10^{8}) / (1.9\times1.6\times10^{-19})$ = **654.2nm (visible)**

(3.4-1.5)

Finding Line Wavelength (or Frequency)

Balmer Series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

R ≡ Rydberg constant.

n ≡ $3, 4, 5, \ldots$

R ≡ $1.097 \times 10^{-7}\ m$

λ ≡ wavelength in m.

If $n = 3$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \left(\frac{1}{4} - \frac{1}{9}\right)$$

$= 1.524 \times 10^6 \ m^{-1}$

$$\therefore \lambda = 656.3 \ nm \ \Longrightarrow H_\alpha \equiv red.$$

If $n = 4 \Longrightarrow H_\beta = blue, 486.1 \ nm$

For $n = \infty$ $\quad$ $(the\ limit\ of\ the\ series).$

$$\Longrightarrow \ \lambda = 364.6 \ nm \ (shortest\ \lambda \ in\ the\ series).$$

Other Series

Lyman, Paschen, Brackett and Pfund

-Lyman: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad n = 2,3,\ldots$

-Paschen: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \qquad n = 4,5,\ldots$

-Brackett: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \qquad n = 5,6,\ldots$

-Pfund: $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) \qquad n = 6,7,\ldots$

Lyman series wavelengths **ALL U.V**

Balmer series wavelengths **U.V + Visible**

Paschen
Bracket
Pfund
} **All I.R**