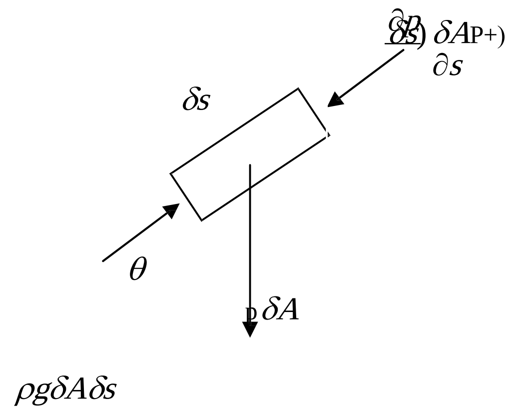
Fluid and fluid flow

Definitions

Fluid, laminar flow, turbulent flow, Boundary layer, Reynolds Number, adiabatic flow, steady flow , unsteady flow, Uniform flow, non-uniform flow, rotational flow, Irrotational flow, one dimension flow , two dimension flow, three dimension flow , stream line

Derivation of Eulers equation along a stream line

) assume a particle in a stream line, steady and frictionless flow the a force diagram 6Q can be written as



Cos $\theta = \frac{\delta z}{\delta s}$

$\sum F_s = ma_s$

$$p\delta A - (p + \frac{\partial p}{\partial s}\delta s)\delta A - \gamma \delta A \delta s \cos\theta = \rho \delta A \delta s a_s$$

$p\delta A - p\delta A - \frac{\partial p}{\partial s}\delta s \delta A - \gamma \delta A \delta s \cos\theta = \rho \delta A \delta s a_s$ dividing by $\delta A \delta s$ then

$$-\frac{\partial p}{\partial s} = \gamma \cos\theta + \rho a_s$$

But $a_s = v\frac{\partial v}{\partial s}$ for steady state flow

$$\frac{\partial p}{\partial s} = -\rho g \cos\theta - \rho a_s$$

$$\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{\partial z}{\partial s} + v\frac{\partial v}{\partial s} = 0$$

Since s is the only dependent variable then

$$\frac{dp}{\rho} + g\,dz + v\,dv = 0$$

This is Euler's equation and for incompressible flow (i.e ρ = constant) then

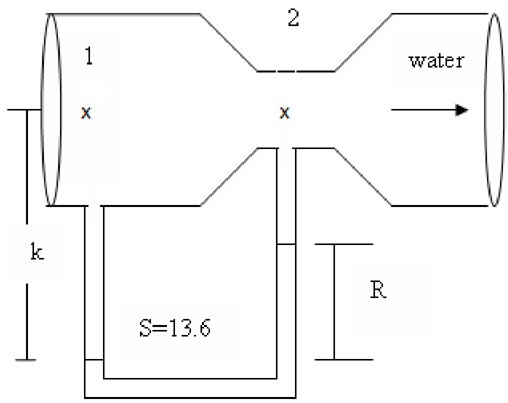
$$\frac{p}{\rho} + \frac{v^2}{2} + gz = 0$$

this is Bernoulli equation.

٢) continuity equation and its application

٣) Application of Bernoulli equation

Ex.1 For the venturi meter shown in fig, derive an equation that relate the distance R and the velocity at point 2 .If R=2 cm and D_1 = 20 cm, D_2 = 10 cm, what is the volume flow rate in m^3/s



Bernoulli equation between 1&2 then neglect loss,

$$\frac{P_1}{\gamma_w} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma_w} + z_2 + \frac{V_2^2}{2g} \qquad \text{(eq1)}$$

then eq(1) becomes ($z_1 = z_2$ and

$$\frac{P_1 - P_2}{\gamma_w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad \text{(eq2)}$$

Pressure equation 1&2

$$P_2 + k\gamma_w - RS\gamma_w - (k+R)\gamma_w = p_1 \qquad \text{(eq3)}$$

Then

$$\frac{P_1 - P_2}{\gamma_w} = R(S-1) \qquad \text{(eq4)}$$

But from continuity equation

$$\dot m_1 = \dot m_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad \text{(eq5)}$$

assume for incompressible flow $\rho_1 = \rho_2$

also $A_1 = \pi \frac{D_1^2}{4}$, $A_2 = \pi \frac{D_2^2}{4}$ then into eq(5)

$$V_1 = V_2 \left(\frac{D_2}{D_1}\right)^2 \qquad \text{(eq6)}$$

eq(2)=eq(4)

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = R(S-1)$$

then6 and using eq

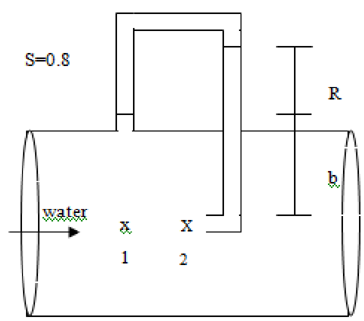
$$\frac{V_2^2}{2g}\left[1-\left(\frac{D_2}{D_1}\right)^4\right] = R(s-1)$$

$$V_2 = \sqrt{\frac{2gR(s-1)}{\left[1-\left(\frac{D_2}{D_1}\right)^4\right]}}$$

$$V_2 = \sqrt{\frac{2x10x0.02(13.6-1)}{1-\left(\frac{1}{2}\right)^4}} = 2.592\ \text{m/s}$$

$$Q = V_2 A_2 = 2.592\left(\frac{\pi D_2^2}{4}\right) = 2.592\left(\frac{\pi\, 0.1^2}{4}\right) = 0.02\ m^3/s$$

& R1 Derive an equation that relate v



then2 &1Applied Bern. Equation

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\frac{p_2 - p_1}{\gamma_{water}} = \frac{V_1^2 - V_2^2}{2g} - k \qquad (1)$$

2and 1 For pressure equation between

$$p_1 + (k+a+R)\gamma_{water} - RS\gamma_{warer} - a\gamma_{water} = p_2$$

$$\frac{p_1 - p_2}{\gamma_{water}} = R(s-1) \qquad (2)$$

) then2)=eq(1eq(

$$\frac{V_2^2 - V_1^2}{2g} = R(s-1) \qquad (3)$$

$$\text{But } V_1 = V_2\left(\frac{D_2}{D_1}\right)^2 \qquad (4)$$

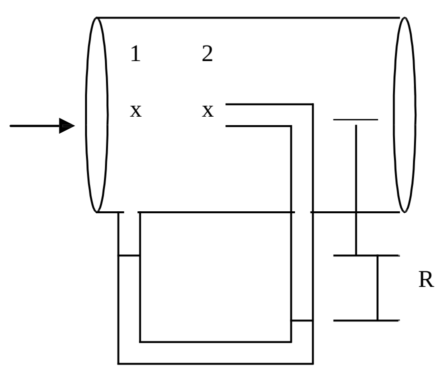
) then3INTO ((4)

$$\frac{V_2^2 - V_2^2\left(\frac{D_2}{D_1}\right)^4}{2g} = R(s-1)$$

Then

$$V_2 = \sqrt{\frac{2gR(s-1)}{\left[1-\left(\frac{D_2}{D_1}\right)^4\right]}} \qquad \text{and} \qquad \text{then } R = \frac{V_2^2\left[1-\left(\frac{D_2}{D_1}\right)^4\right]}{2g(s-1)}$$

.1m what is the velocity at point 0.2 . If R=1Derive a relation between R and v



, neglect loss then2&1Bernoulli equation between

$$\frac{P_1}{\gamma_w} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma_w} + z_2 + \frac{V_2^2}{2g} \qquad (1eq($$

$V_2 = 0$ and $z_1 = z_2$) becomes1then eq(

$$\frac{P_2 - P_1}{\gamma_w} = \frac{V_1^2}{2g} \qquad (2eq($$

2&1Pressure equation

$$P_1 - a\gamma_w - RS\gamma_w + (R + a)\gamma_w = P_2 \qquad (3eq($$

Then

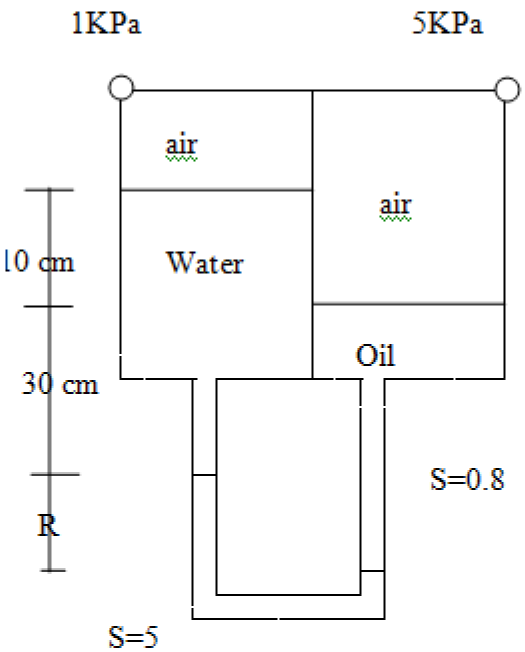
$$\frac{P_2 - P_1}{\gamma_w} = R(S - 1) \qquad (4eq($$

then)4)=eq(2eq(

$$V_1 = \sqrt{2gR(S-1)}$$

$$V_1 = \sqrt{2 x 9.8 x 0.2 (13.6 - 1)} = 7.027 \text{ m/s}$$

Ex. What is the value of R؟



$$1000 + 0.4 s \gamma_w + (R + 1)\gamma_w - s_o \gamma_w R - 0.3 = 5000$$

$$\gamma_w 0.3 x 0.8 \gamma_w + 0.4 - 1000 - 5000\ ) R \gamma_w = 0.8 - \ 5)$$

$$m0.0571 = \frac{2400 + 4000 - 4000}{42000} \ R=$$