University of Technology
Laser & Optoelectronics Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

Lec. 1: Fundamentals of Digital Signal Processing: 2024-Sep-30

Lecture Outline

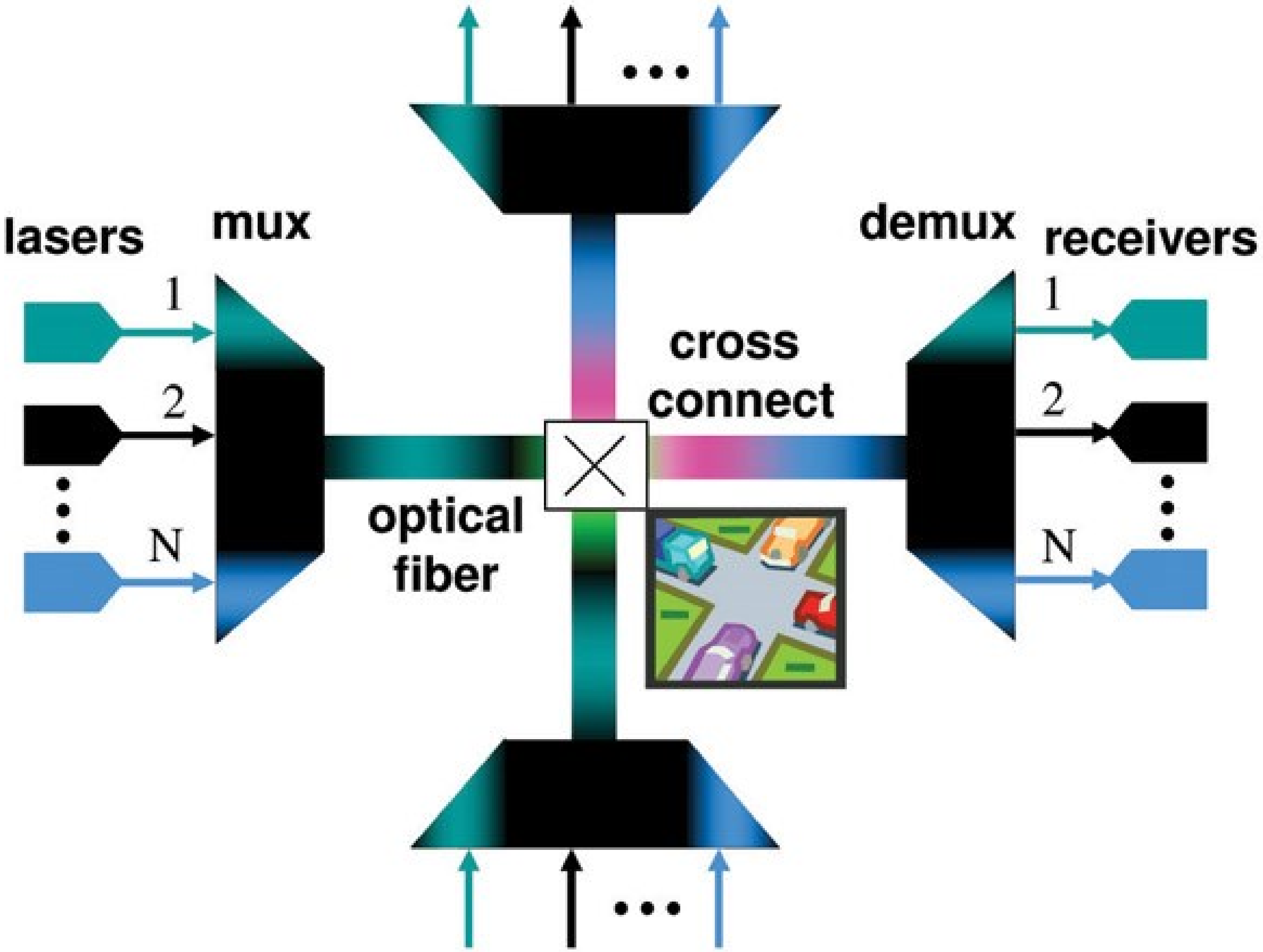
1. Aim of the course
2. General Introduction
3. Signal Processing Applications
4. Linear VS Nonlinear Signals
5. Continuous VS Discrete Signals
6. Course Requirements and References
7. HW

Aim of the Course

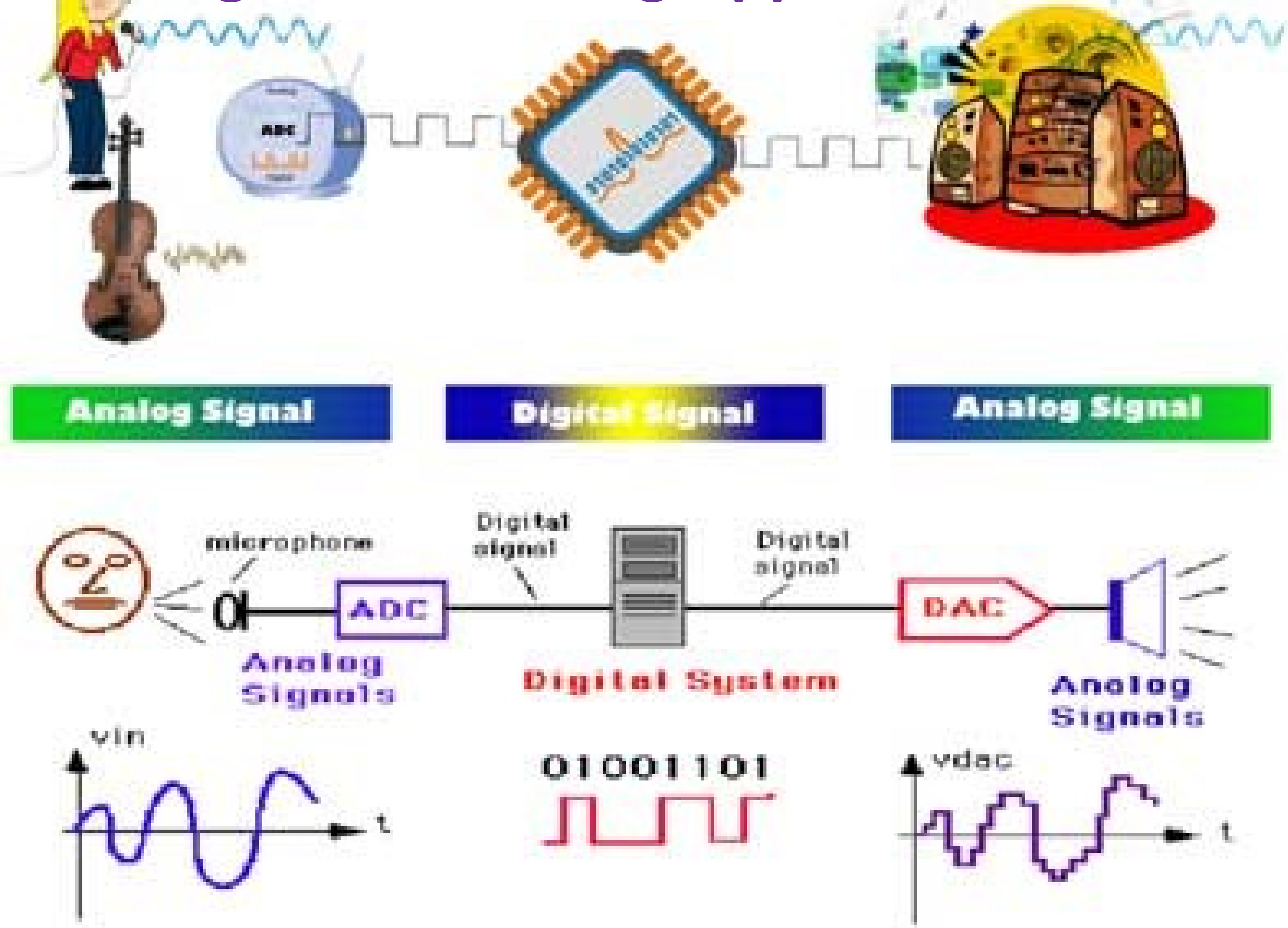
Study the following general topics

1. **Signal analysis**
2. **Linear system analysis**
3. **Filter design**
4. **Discrete Fourier transform**
5. **Sampling theorem**
6. **Z-transform and discrete Fourier transform**
7. **Design and implementation of digital filters**
8. **MATLAB Signal Processing Tools**

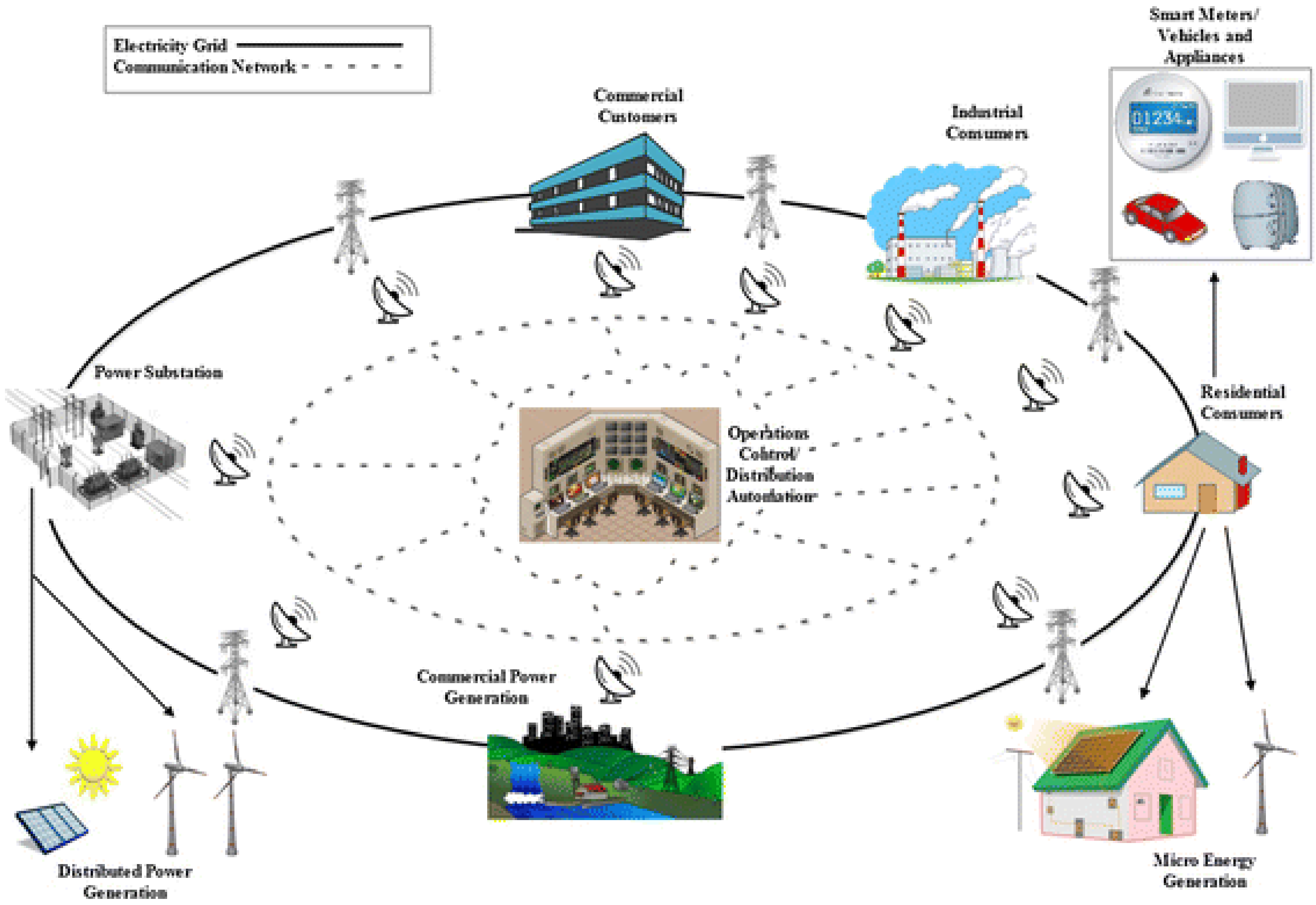
General Introduction



Signal Processing Applications



Smart grid architecture highlighting communication, control and signal processing



A **signal** is defined as **any physical quantity** that varies with **time**, **space**, or any other **independent variable** or variables. Mathematically, we describe a signal as a **function** of one or more **independent Variables**.

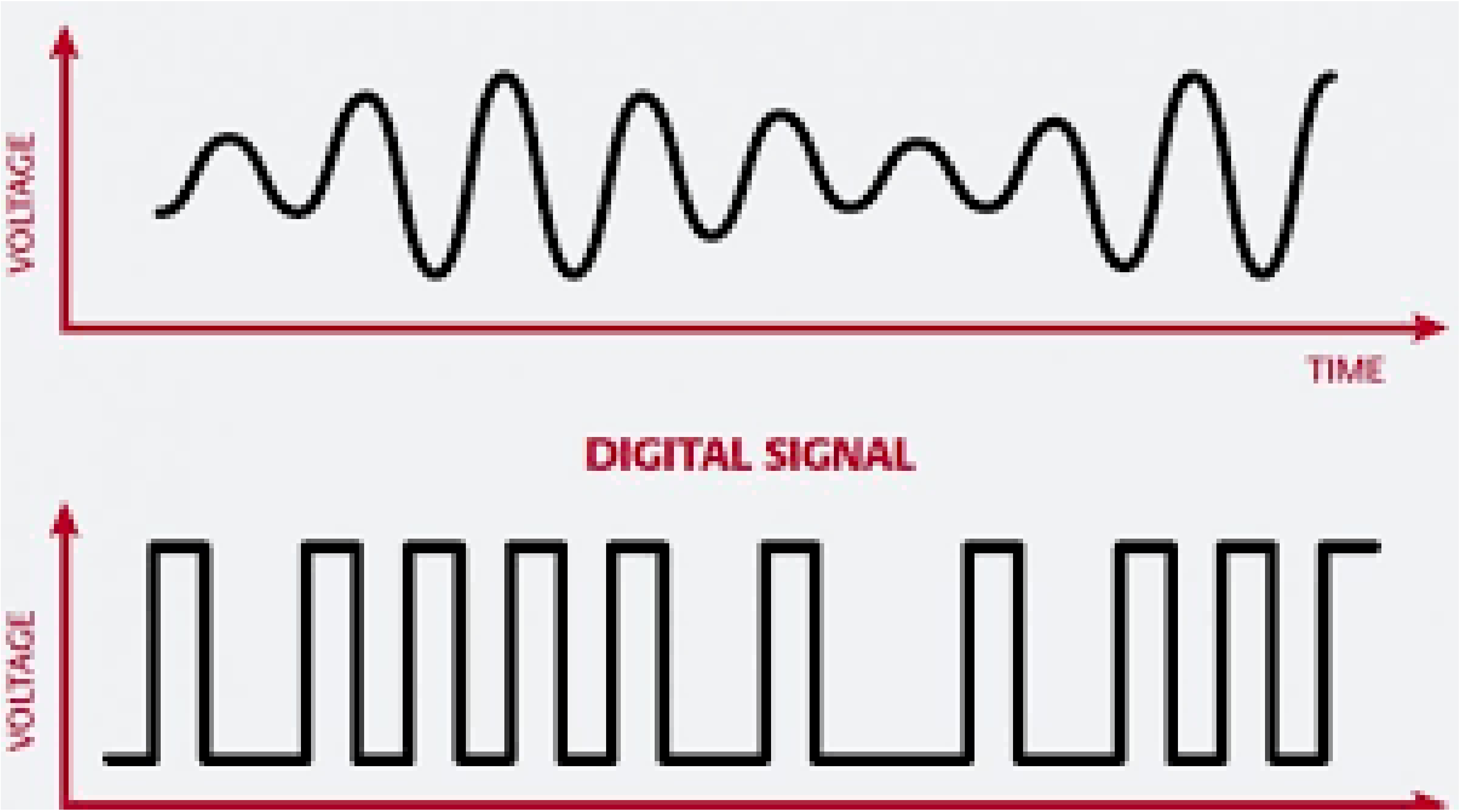
$$\{s_1(t) = 5t\}$$

Linear

$$\{s_2(t) = e^{6t}\}$$

Non-Linear

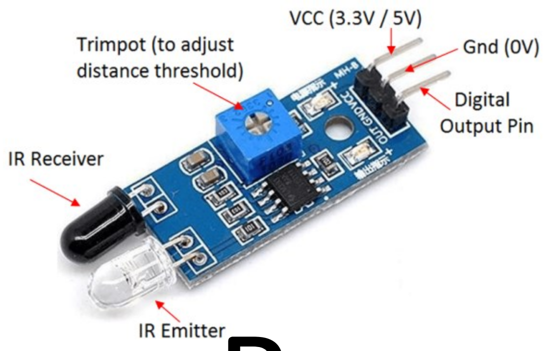
Analog VS Digital Signal



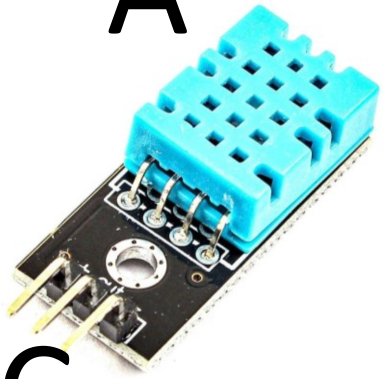
Sensors

A. **PIR Sensor**: The PIR **Motion** Sensor Module allows you to sense motion. It is almost always used to detect the motion of a human body within the sensor's range.
B. **IR Sensor**: The IR **Proximity** Sensor module has great adaptive capability of the ambient light
C. **DHT11 Temperature Sensor**: The module can detect the surrounding environment of the **humidity** and **temperature** and gives the output from the digital output.
D. **HCSR04 Ultrasonic Sensor**: This HC-SR04 Ultrasonic **Range Finde**r is a very popular sensor that is found in many applications
E. **MQ2 Gas Sensor**: The MQ-2 Smoke LPG Butane Hydrogen Gas Sensor Detector Module is useful for **gas leakage detection**.
F. **LDR sensor module**: The LDR sensor Module is used to detect the **presence of light** and measure the **intensity of light.**

A

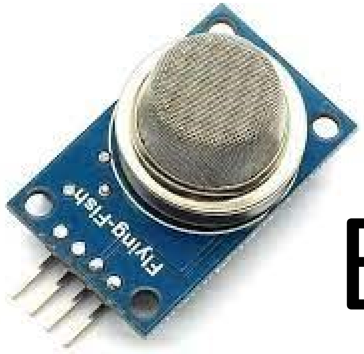


B

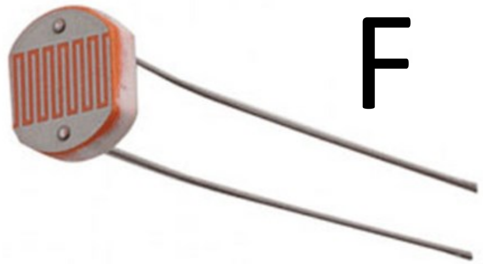


C

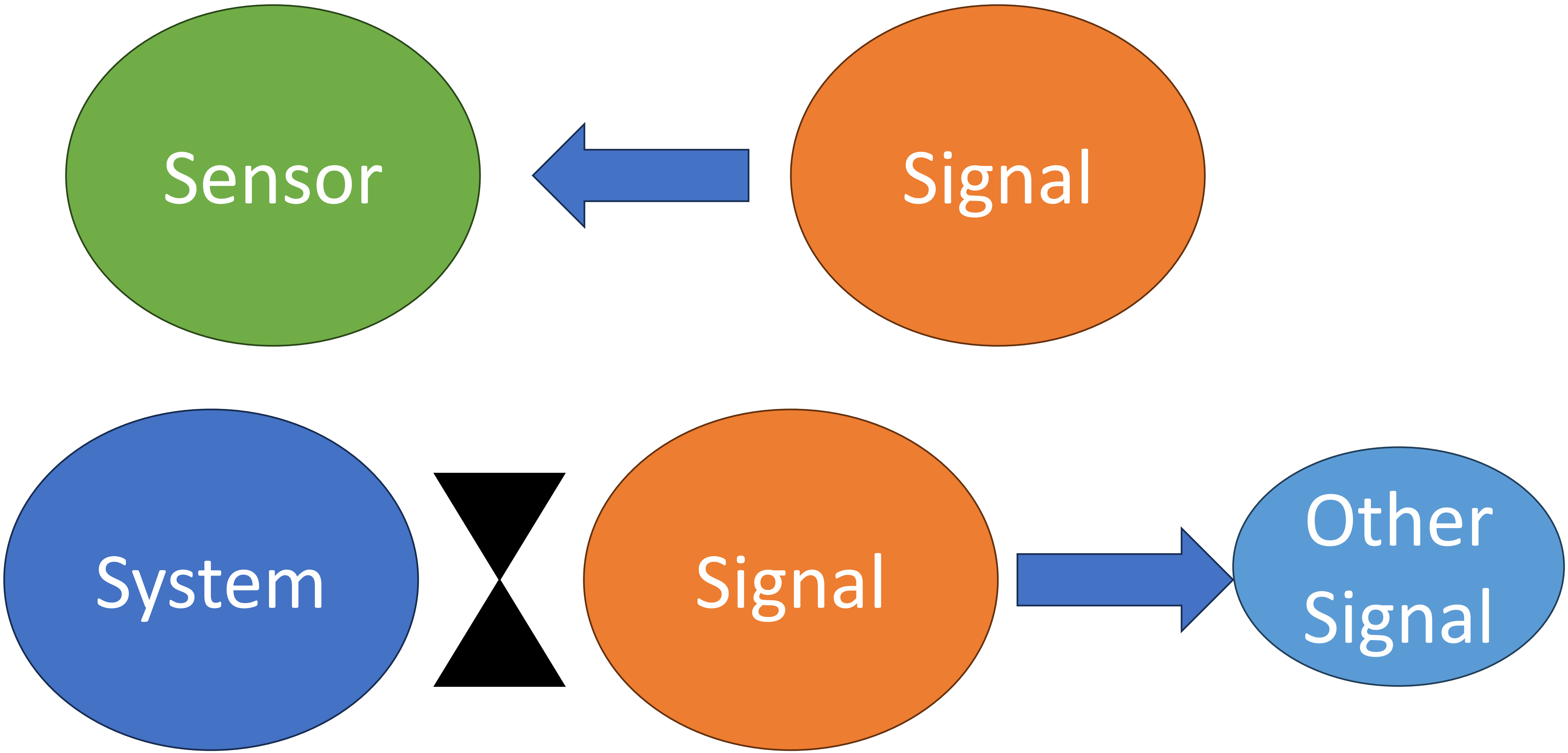
D

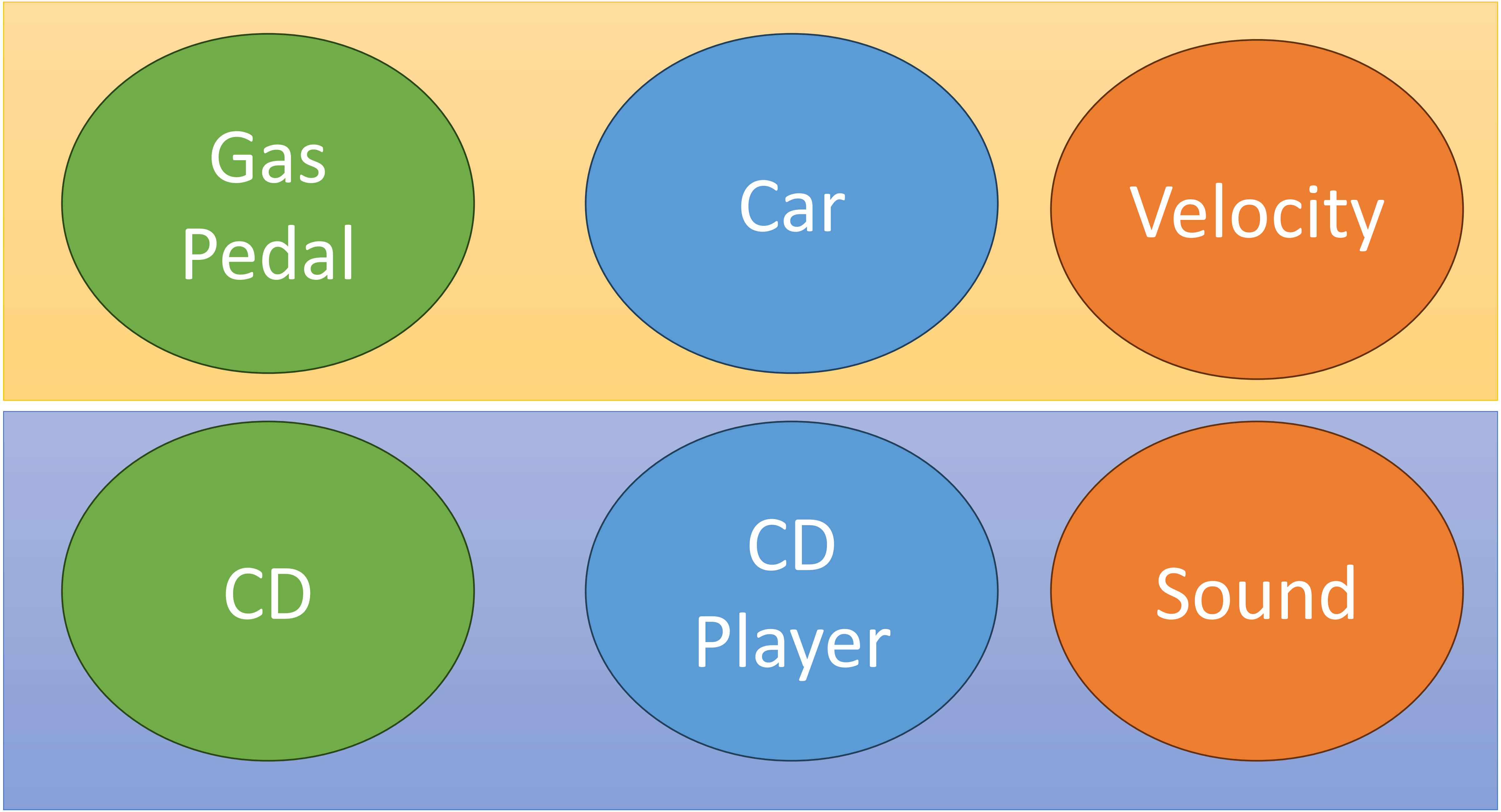


E



F


Sensor
Signal
System
Signal
Other
Signal


Gas Pedal
Car
Velocity
CD
CD Player
Sound

Natural signals are found, for example, in:

- Acoustics, e.g., speech signals, sounds made by dolphins and whales
- Astronomy, e.g., cosmic signals originating in galaxies and pulsars, astronomical images
- Biology, e.g., signals produced by the brain and heart
- Seismology, e.g., signals produced by earthquakes and volcanoes
- Physical sciences, e.g., signals produced by lightnings, the room temperature, the atmospheric pressure

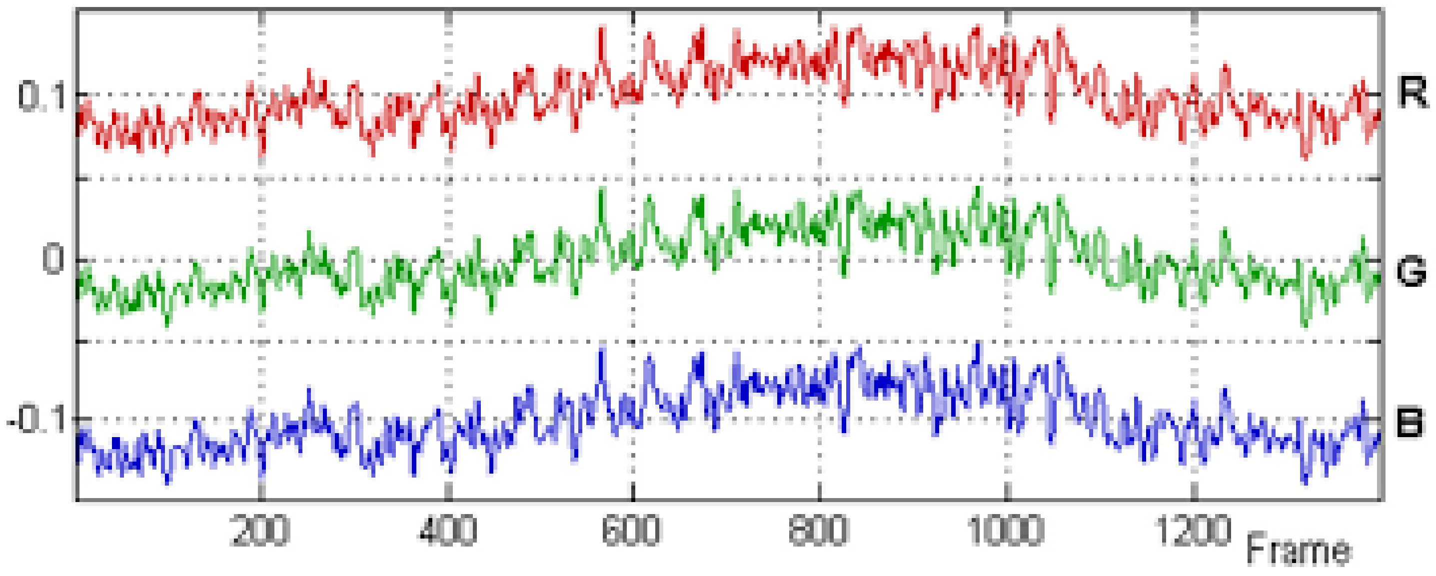
Spatial VS Temporal & 1 Var VS 2 Var

$$s_1(x, y) = 5x + 6y$$

$$s_2(x, y) = 5\cos(x) + 6\sin(y)$$

$$\boldsymbol{s_3(t) = \sum_{i=1}^{N} A_i\,(t)\sin([2\pi F_i(t)t + \theta_i(t)])}$$

RGB Color Image Signal



$$\boldsymbol{I}(x,y,t) = \begin{matrix} \boldsymbol{I}_r(x,y,t) \\ \boldsymbol{I}_g(x,y,t) \\ \boldsymbol{I}_b(x,y,t) \end{matrix}$$

- A *continuous-time signal* is a signal that is defined at each and every instant of time.
- Typical examples are:
  - An electromagnetic wave originating from a distant galaxy
  - The sound wave produced by a dolphin
  - The ambient temperature
  - The light intensity along the x and y axes in a photograph
- A continuous-time signal can be represented by a function

$$x(t) \quad \text{where} \quad -\infty < t < \infty$$

- A *discrete-time signal* is a signal that is defined at discrete instants of time.
- Typical examples are:
  - The closing price of a particular commodity on the stock exchange
  - The daily precipitation
  - The daily temperature of a patient as recorded by a nurse

- A discrete-time signal can be represented as a function

$$x(nT) \quad \text{where} \quad -\infty < n < \infty$$

  and T is a constant.
- The quantity $x(nT)$ can represent a voltage or current level or any other quantity.
- In DSP, $x(nT)$ always represents a series of numbers.
- Constant T usually represents time but it could be any other physical quantity depending on the application.

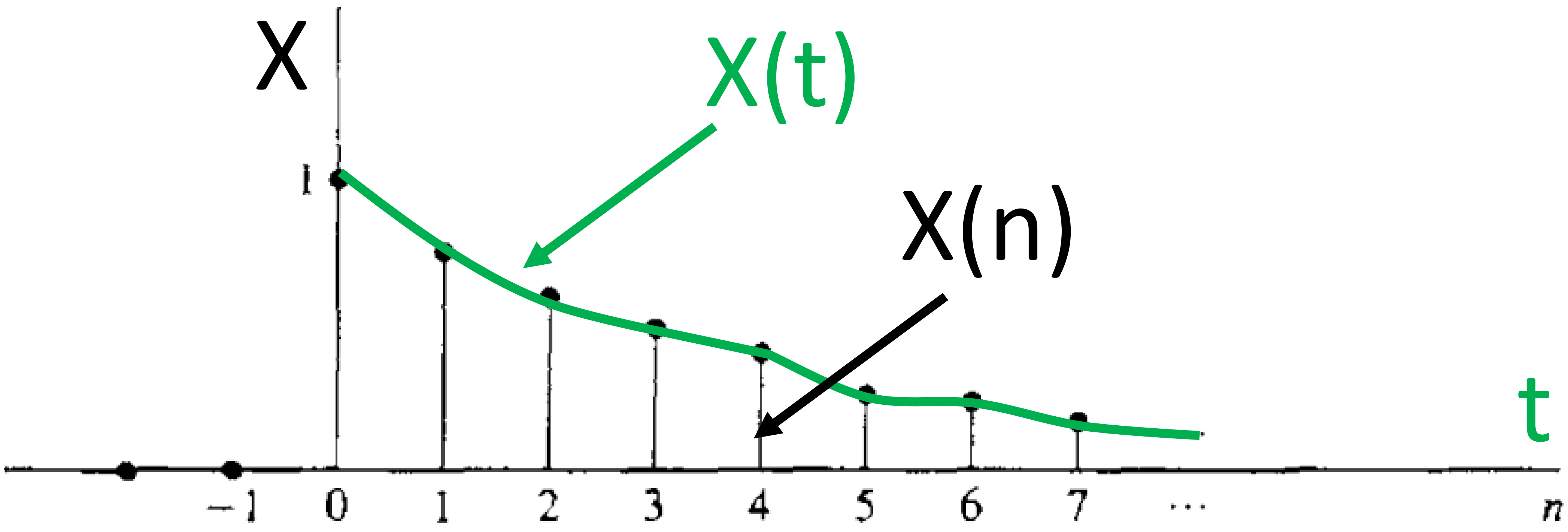
Discrete Signal

$$x(n) = \begin{cases} 0.8^n & if\ n \geq 0 \\ 0 & otherwise \end{cases}$$

Continuous Signal

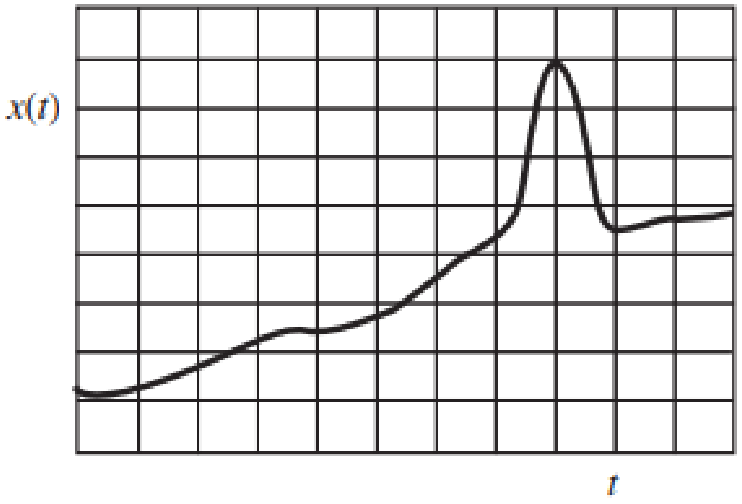
$$x(t) = 0.8^t$$

Continuous VS Discrete Signal

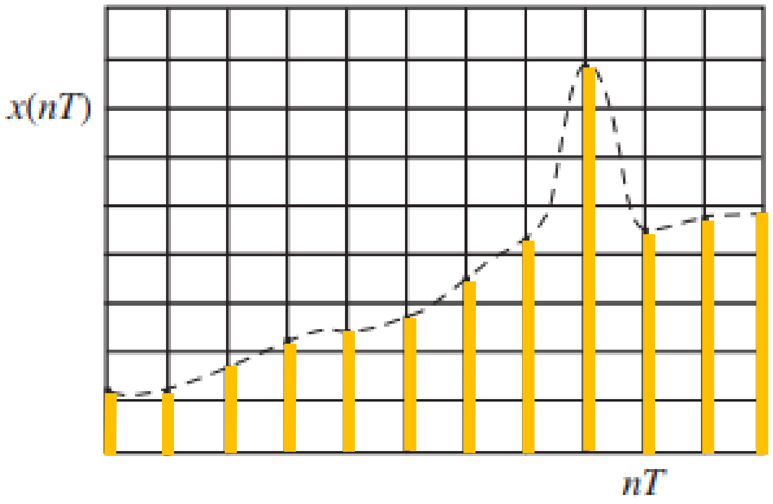


Signal Classification

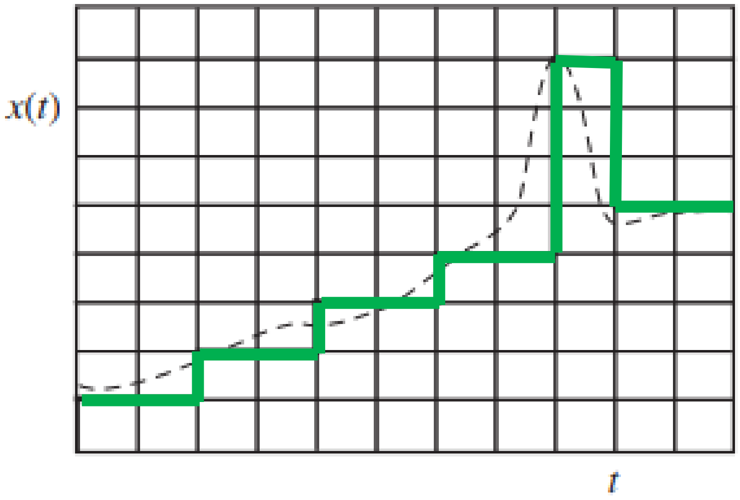
- Signals can also be classified as:
  - Nonquantized
  - Quantized
- A *nonquantized signal* is a signal that can assume any value within a given range, e.g., the ambient temperature.
- A *quantized signal* is a signal that can assume only a finite number of discrete values, e.g., the ambient temperature as measured by a digital thermometer.



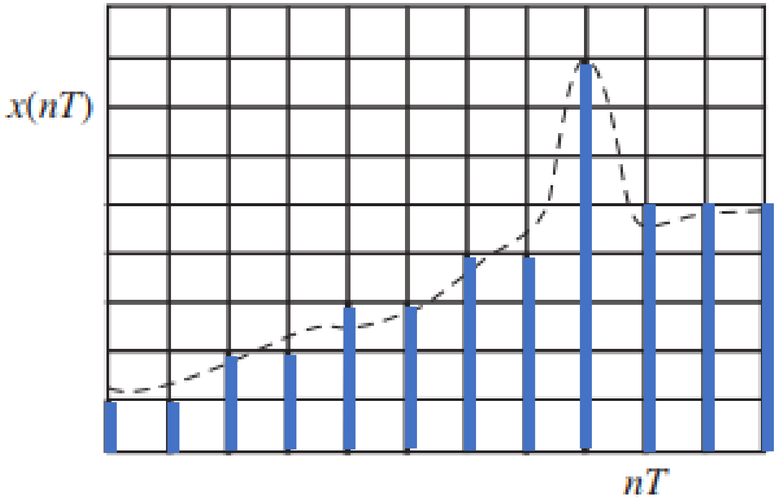
(a) Continuous-time, nonquantized



(b) Discrete-time, nonquantized

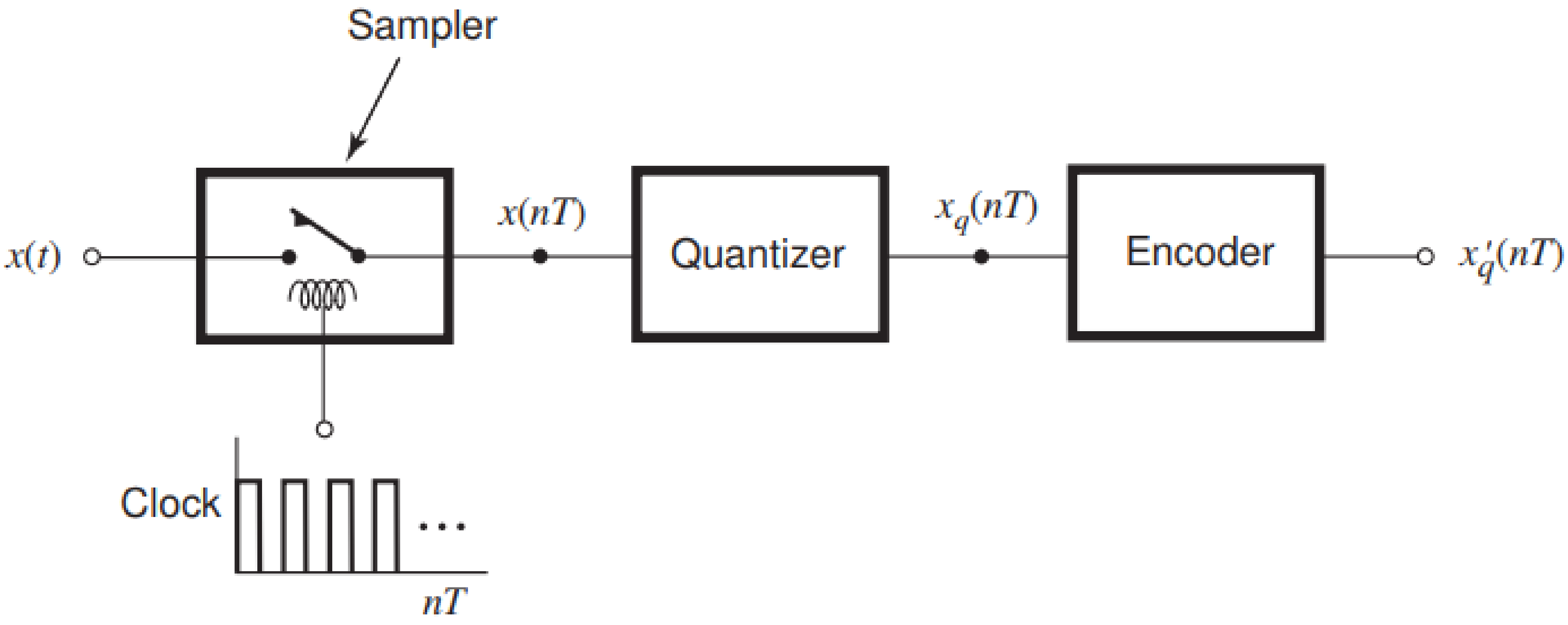


(c) Continuous-time, quantized



(d) Discrete-time, quantized

- To be able to process a nonquantized continuous-time signal by a digital system, we must first sample it to generate a discrete-time signal.
- We must then quantize it to get a quantized discrete-time signal.
- That way, we can generate a numerical representation of the signal that entails a finite amount of information.



Sampling system

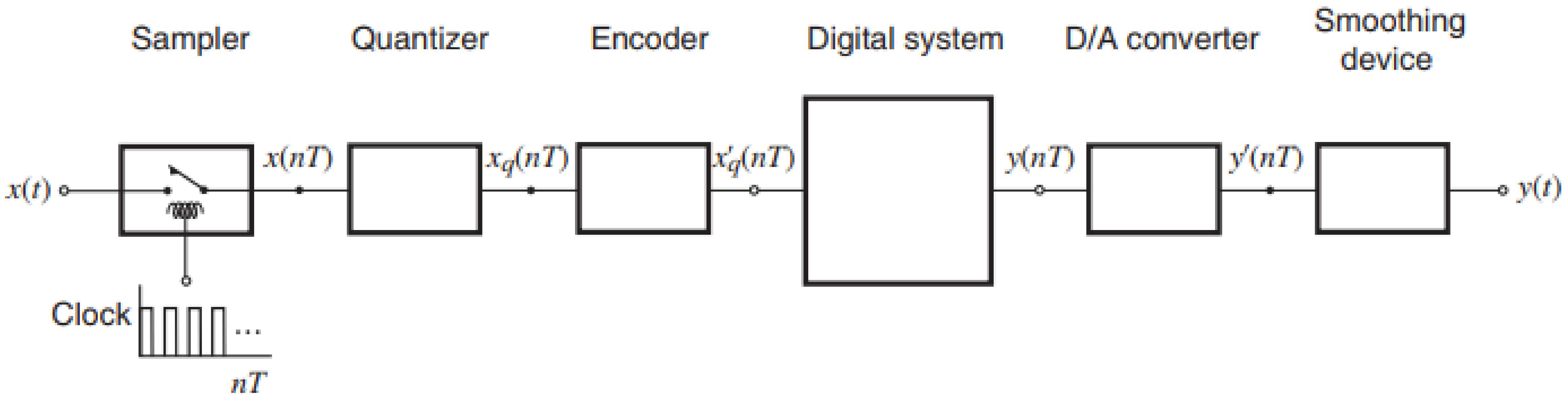
- A *sampler* in its bare essentials is a switch controlled by a clock signal which closes momentarily every T seconds thereby transmitting the level of the input signal $x(t)$ at instant nT, i.e., $x(nT)$, to its output.
- Parameter T is called the *sampling period*.

- A *quantizer* is a device that will sense the level of its input and produce as output the nearest available level, say, $x_q(nT)$, from a set of allowed levels, i.e., a quantizer will produce a quantized continuous-time signal.

- An *encoder* is essentially a digital device that will sense the voltage or current level of its input and produce a corresponding binary number at its output, i.e., it will convert a quantized continuous-time signal into a corresponding discrete-time signal in binary form.

- A quantized discrete-time signal produced by an A/D converter is, of course, an approximation of the original nonquantized continuous-time signal.
- The accuracy of the representation can be improved by increasing
  - the sampling rate, and/or
  - the number of allowable quantization levels in the quantizer
- The sampling rate is simply $1/T = f_s$ in Hz or $2\pi/T = \omega_s$ in radians per second (rad/s).

Complete DSP system



- Signal processing is the science of analyzing, synthesizing, sampling, encoding, transforming, decoding, enhancing, transporting, archiving, and generally manipulating signals in some way or another.
- These presentations are concerned primarily with the branch of signal processing that entails the manipulation of the *spectral characteristics of signals.*
- If the processing of a signal involves modifying, reshaping, or transforming the spectrum of the signal in some way, then the processing involved is usually referred to as *filtering*.

List of References for More information

1. https://hcis-journal.springeropen.com/articles/10.1186/s13673-018-0126-9
2. https://community.arm.com/arm-community-blogs/b/embedded-blog/posts/signal-processing-capabilities-of-cortex-m-devices
3. https://blogs.mathworks.com/deep-learning/2019/05/13/deep-learning-for-signal-processing-applications/
4. https://spie.org/news/0592-ultrafast-all-optical-signal-processing-and-packet-switching?SSO=1

Course References Can Be Downloaded Here

Ref 1

Ref 2

Home Work (Due date 14-10-2024)

1. Prepare a report on DSP applications in Laser Engineering (3 Pages)
2. Install MATLAB and Use the link below to simulate the same example https://www.mathworks.com/videos/how-to-process-signals-as-frames-in-simulink-1605770729615.html

HW Form

Telegram Group

University of Technology Laser & Optoelectronics Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

Lec. 2: Discrete-Time Signals and Systems: 2024-Oct-06

Lecture Outline

1. Basic Signals
2. Periodicity and Symmetry
3. Transformation of Time
4. Addition, Multiplication and Scaling
5. Input-Output Description
6. Accumulator
7. Block Diagram Representation
8. Classifications of Discrete-Time Systems
9. Signal Representation

Basic Signals

The *unit sample sequence* is denoted as $\delta(n)$ and is defined as

$$\delta(n) \equiv \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases}$$

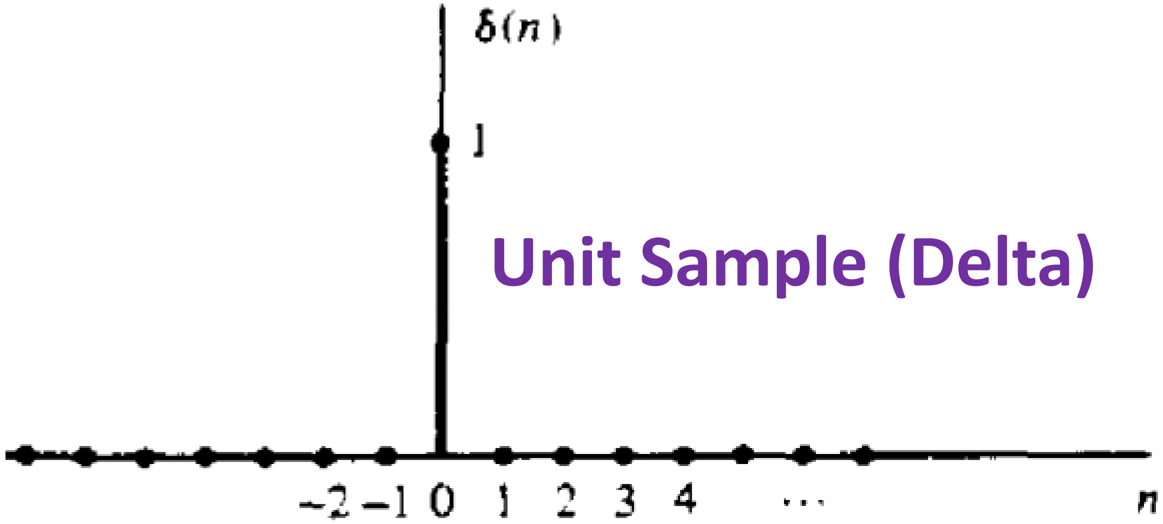
The *unit step signal* is denoted as $u(n)$ and is defined as

$$u(n) \equiv \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$$

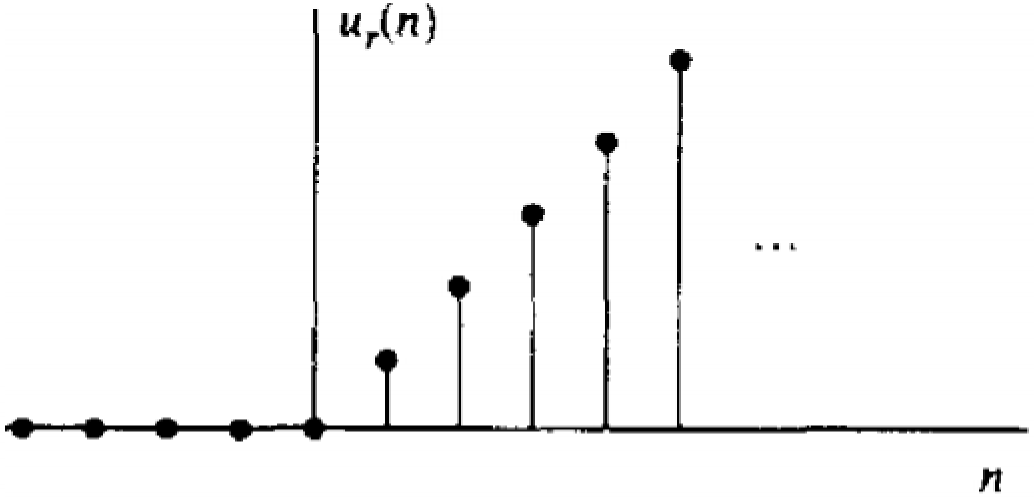
The *unit ramp signal* is denoted as $u_r(n)$ and is defined as

$$u_r(n) \equiv \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$$

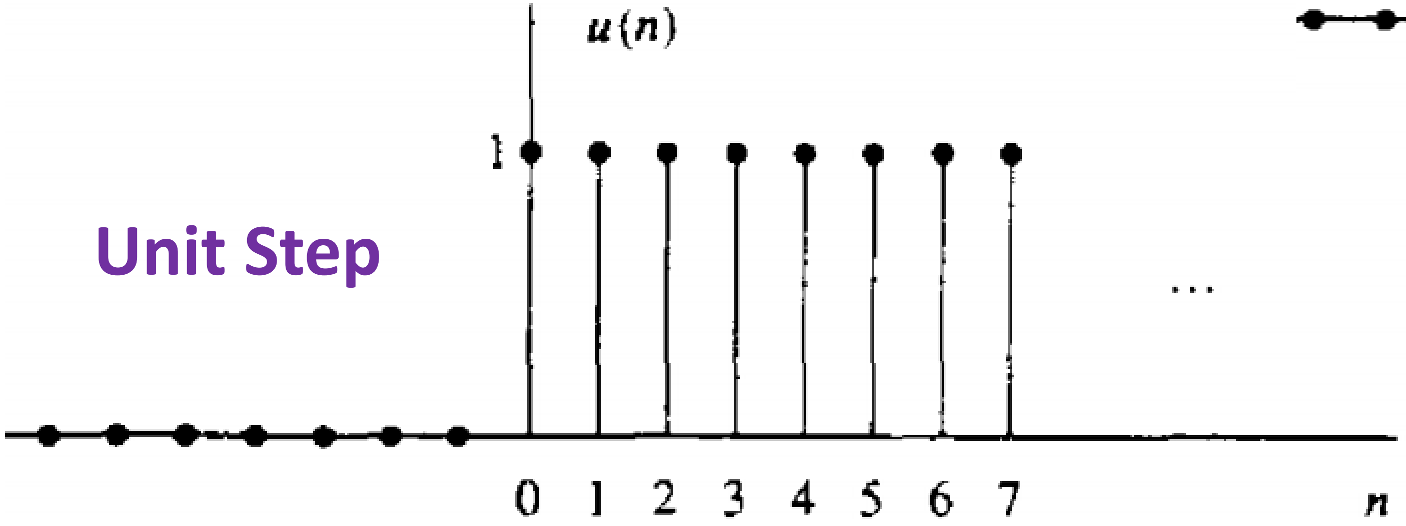
Unit Sample (Delta)



Unit Ramp



Unit Step

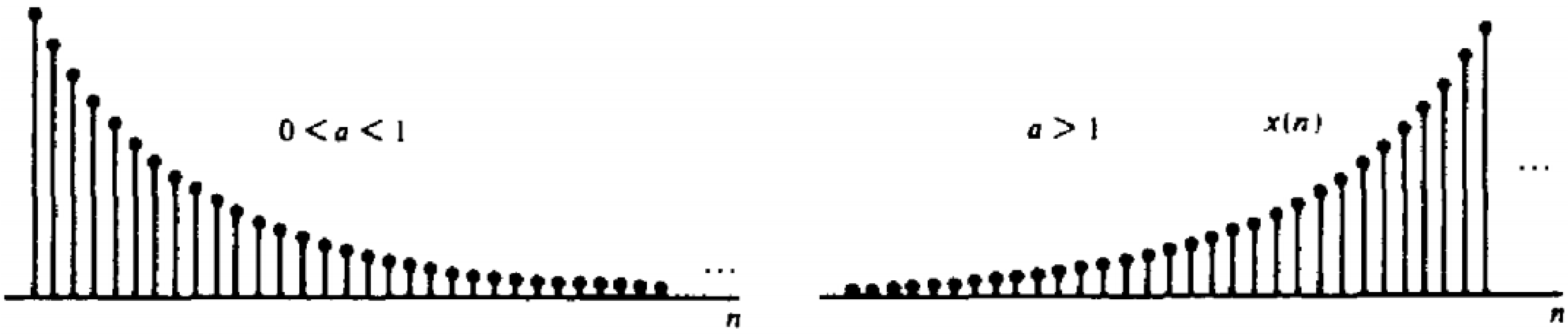


The *exponential signal* is a sequence of the form

$$x(n) = a^n \qquad \text{for all } n \qquad a = re^{j\theta}$$

$$x(n) = r^n e^{j\theta n}$$

$$= r^n (\cos\theta n + j \sin\theta n)$$



$$|x(n)| = A(n) \equiv r^n$$

Amplitude

$$x_R(n) \equiv r^n \cos \theta n$$

Real

$$\angle x(n) = \phi(n) \equiv \theta n$$

Phase

$$x_I(n) \equiv r^n \sin \theta n$$

Imaginary

Energy signals and power signals.

$$E \equiv \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} u^2(n)$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1} = \lim_{N \to \infty} \frac{1+1/N}{2+1/N} = \frac{1}{2}$$

The power of unit step is 1\2

Periodic signals and aperiodic signals.

$$x(n+N) = x(n) \text{ for all } n$$

$$x(n) = A \sin 2\pi f_0 n$$

$$f_0 = \frac{k}{N}$$

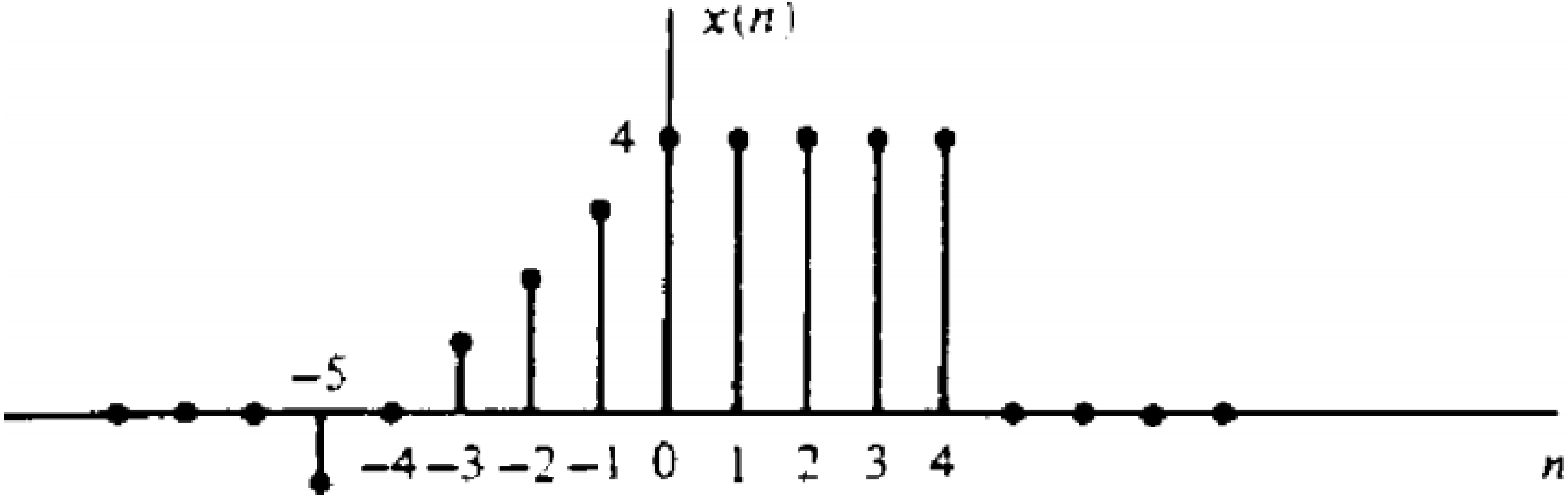
$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

Symmetric (even) and antisymmetric (odd) signals.

$$x(-n) = x(n) \qquad x(-n) = -x(n)$$

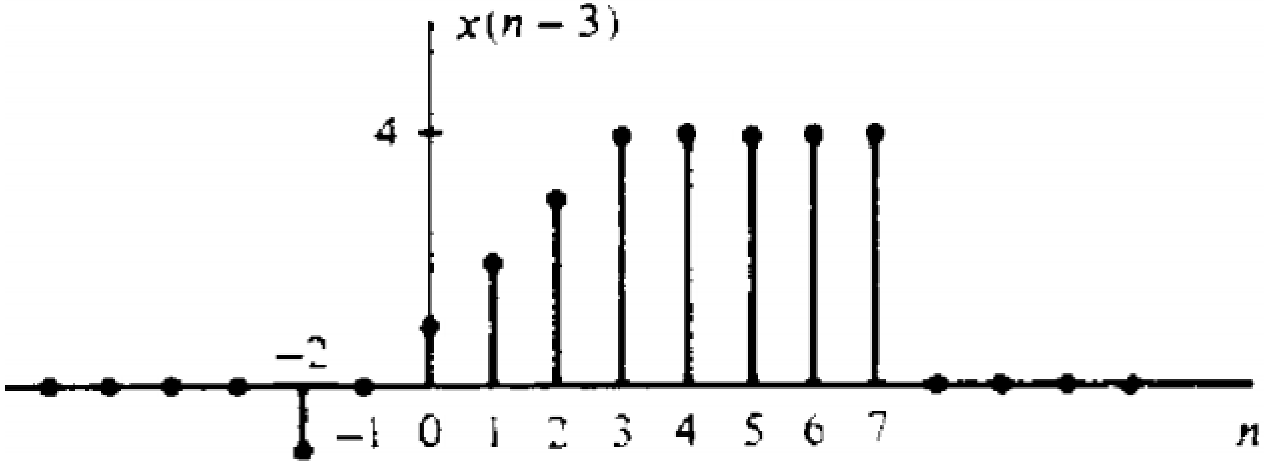
$$x(n) = x_e(n) + x_o(n)$$

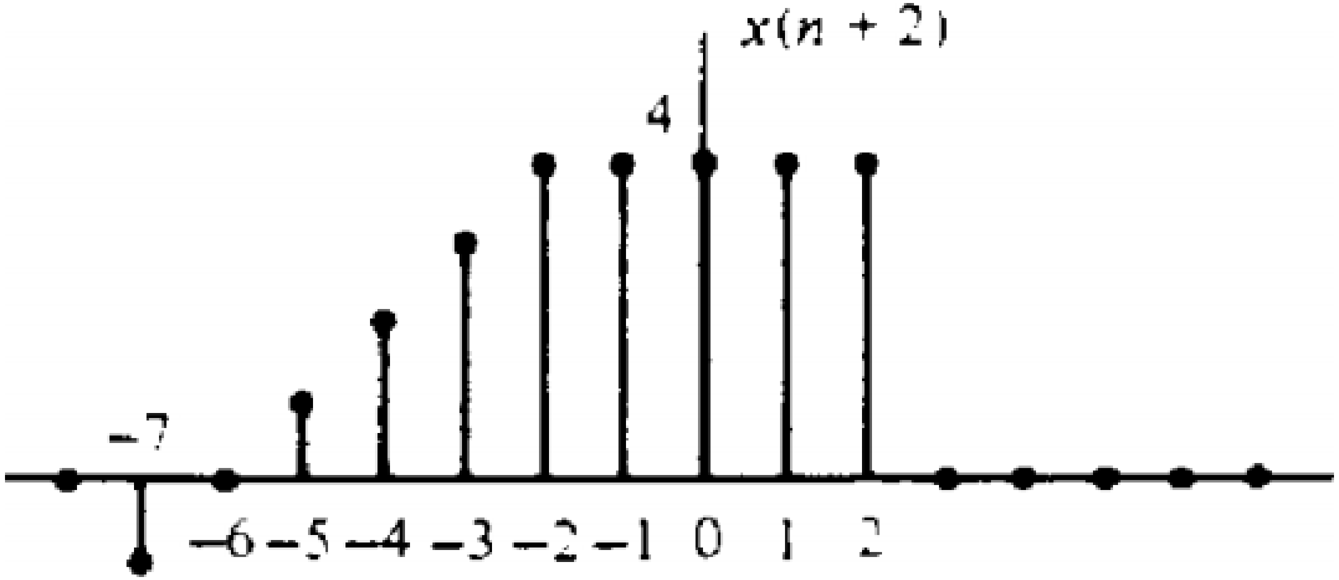
Transformation of the independent variable (time).



The signal $x(n-3)$ is obtained by delaying $x(n)$ by three units in time.

Example 1





Addition, multiplication, and scaling of sequences.

$$y(n) = Ax(n) \qquad -\infty < n < \infty$$

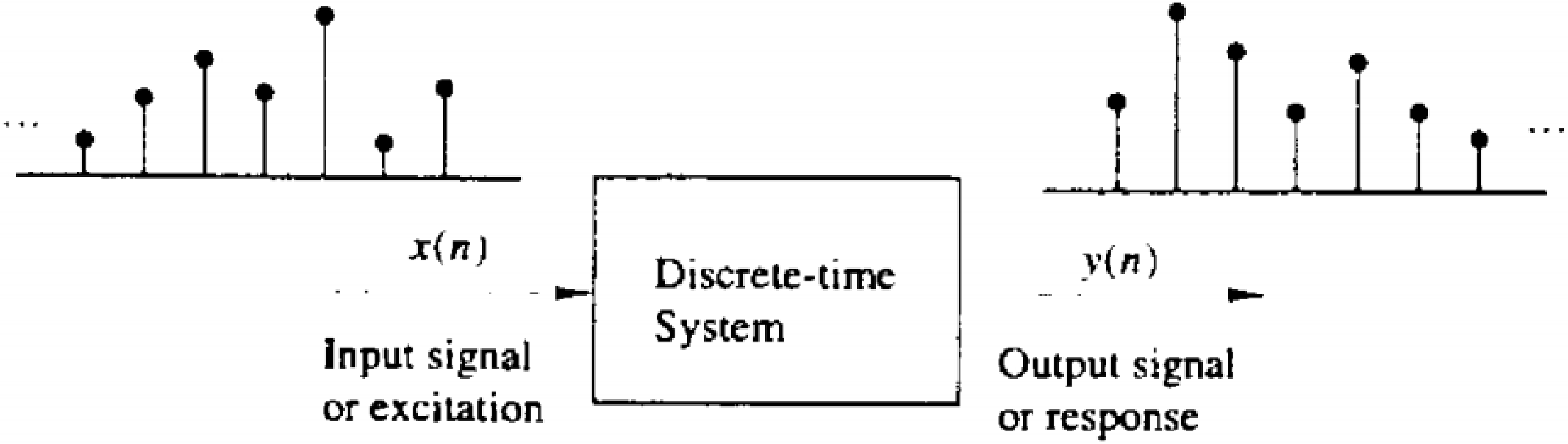
$$y(n) = x_1(n) + x_2(n) \qquad -\infty < n < \infty$$

$$y(n) = x_1(n)x_2(n) \qquad -\infty < n < \infty$$

$$y(n) \equiv \mathcal{T}[x(n)]$$

$\mathcal{T}$ denotes the transformation (also called an operator)

Input–Output Description of Systems



$$x(n) \xrightarrow{\mathcal{T}} y(n)$$

Example 2

Determine the response of the following sytems to the input signal

$$x(n) = \begin{cases} |n|, & -3 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) $y(n) = x(n)$

(b) $y(n) = x(n-1)$

(c) $y(n) = x(n+1)$

(d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

(e) $y(n) = max\{x(n+1), x(n), x(n-1)\}$

(f) $y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \cdots$

$$x(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}$$

(a) In this case the output is exactly the same as the input signal. Such a system is known as the *identity* system.

(b) $$x(n) = \{\ldots, 0, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 0, \ldots\}$$

(c) $$x(n) = \{\ldots, 0, 3, 2, 1, 0, \underset{\uparrow}{1}, 2, 3, 0, \ldots\}$$

(d) $$y(0) = \tfrac{1}{3}[x(-1) + x(0) + x(1)] = \tfrac{1}{3}[1 + 0 + 1] = \tfrac{2}{3}$$

$$y(n) = \{\ldots, 0, 1, \tfrac{5}{3}, 2, 1, \underset{\uparrow}{\tfrac{2}{3}}, 1, 2, \tfrac{5}{3}, 1, 0, \ldots\}$$

(e) $$y(n) = \{0, 3, 3, 3, 2, \underset{\uparrow}{1}, 2, 3, 3, 3, 0, \ldots\}$$

(f) $$y(n) = \{\ldots, 0, 3, 5, 6, \underset{\uparrow}{6}, 7, 9, 12, 0, \ldots\}$$

$$y(n) = \sum_{k=-\infty}^{n} x(k) = \sum_{k=-\infty}^{n-1} x(k) + x(n) \qquad \textit{accumulator.}$$

$$= y(n-1) + x(n)$$

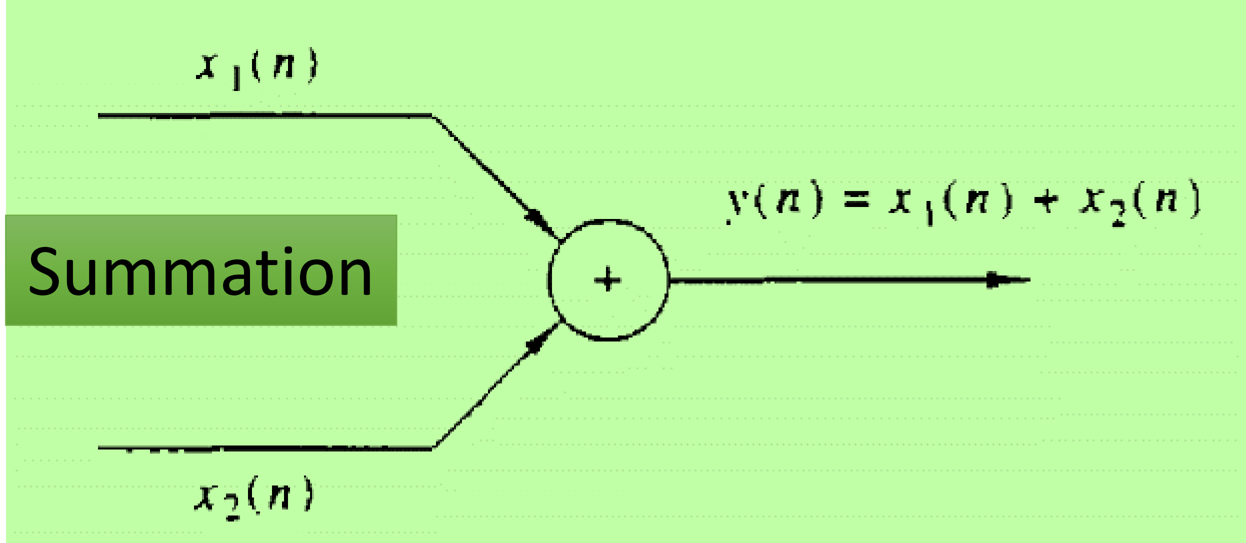
$$y(n_0) = y(n_0 - 1) + x(n_0)$$

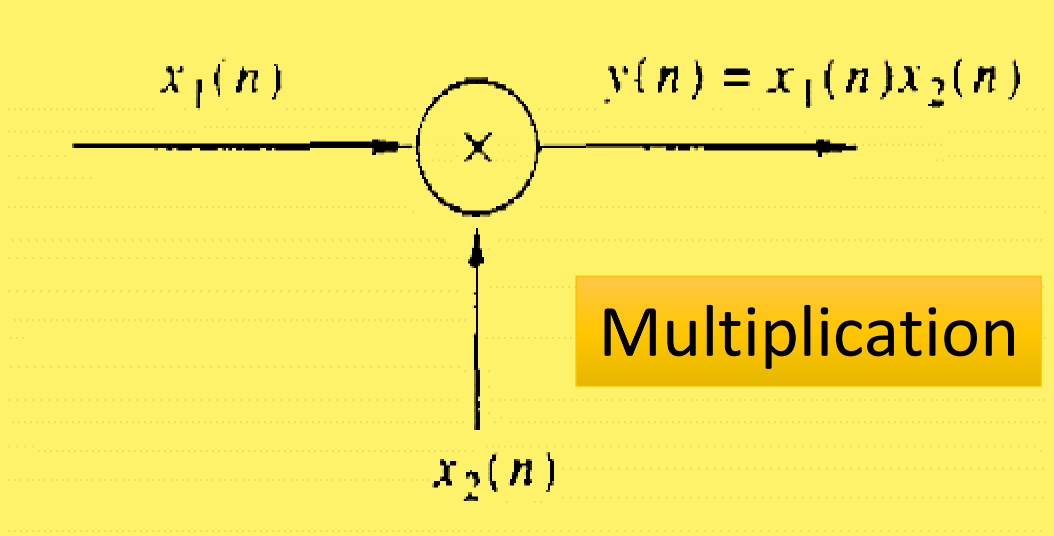
$$y(n_0 + 1) = y(n_0) + x(n_0 + 1)$$

$$y(n_0 - 1) = \sum_{k=-\infty}^{n_0 - 1} x(k)$$

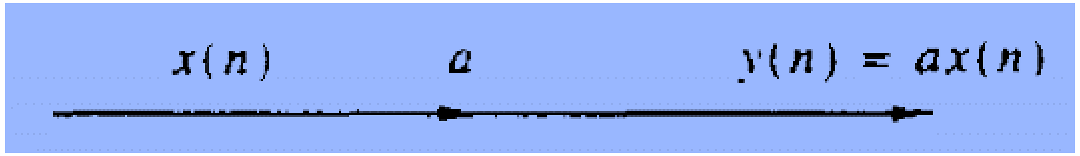
Block Diagram Representation

Summation

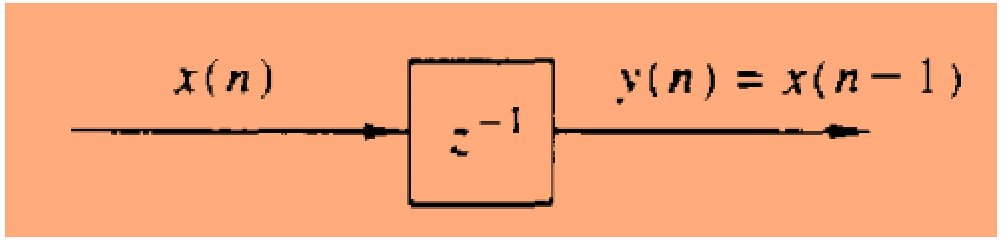




Multiplication

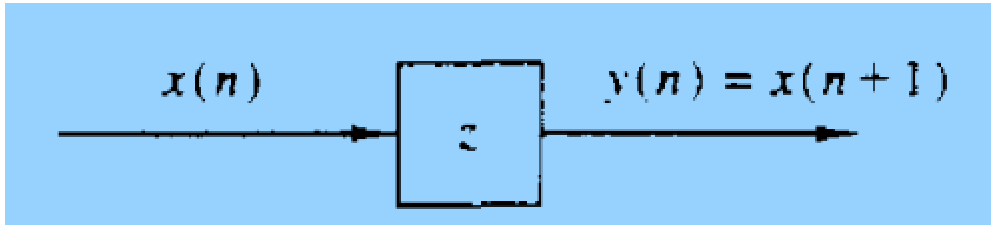


Integer Multiplication



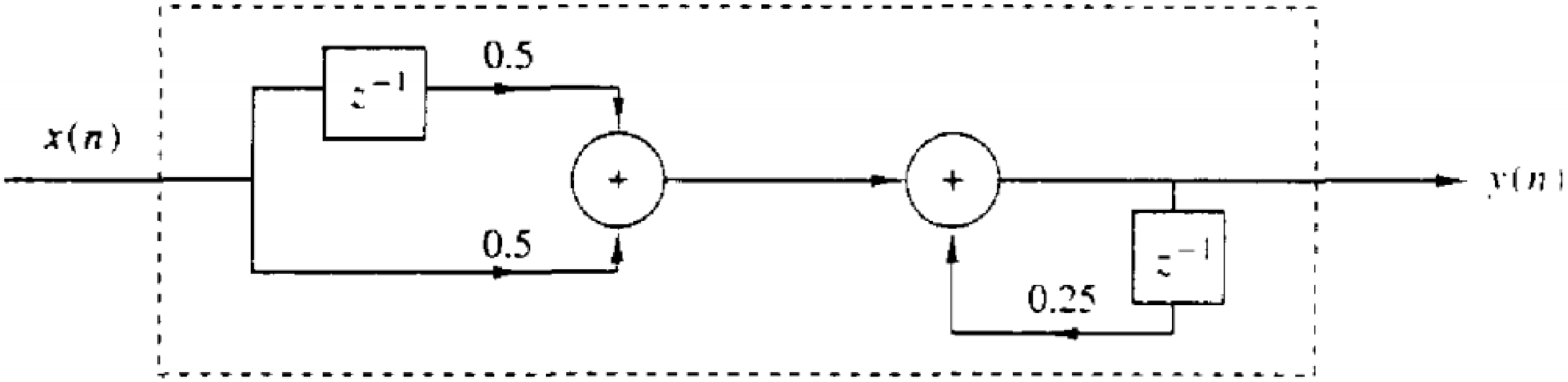
Negative Shifting (Delay)

Positive Shifting (Advancing)

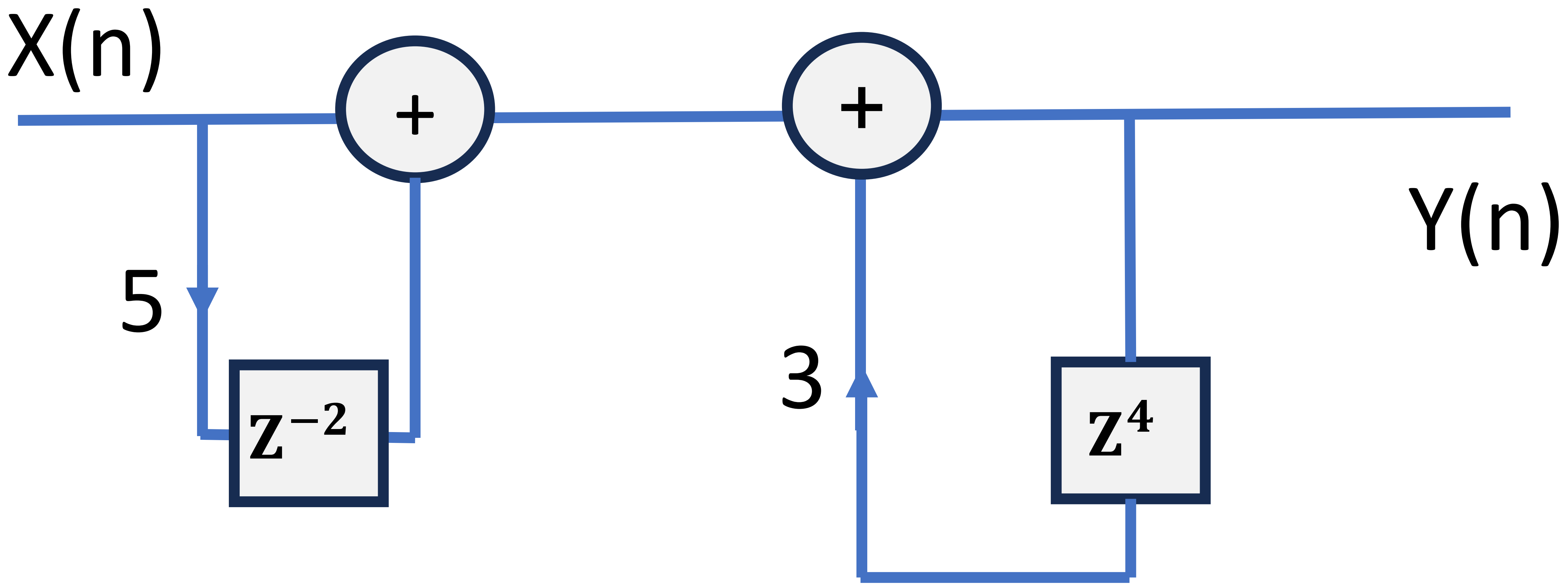


Example 3

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

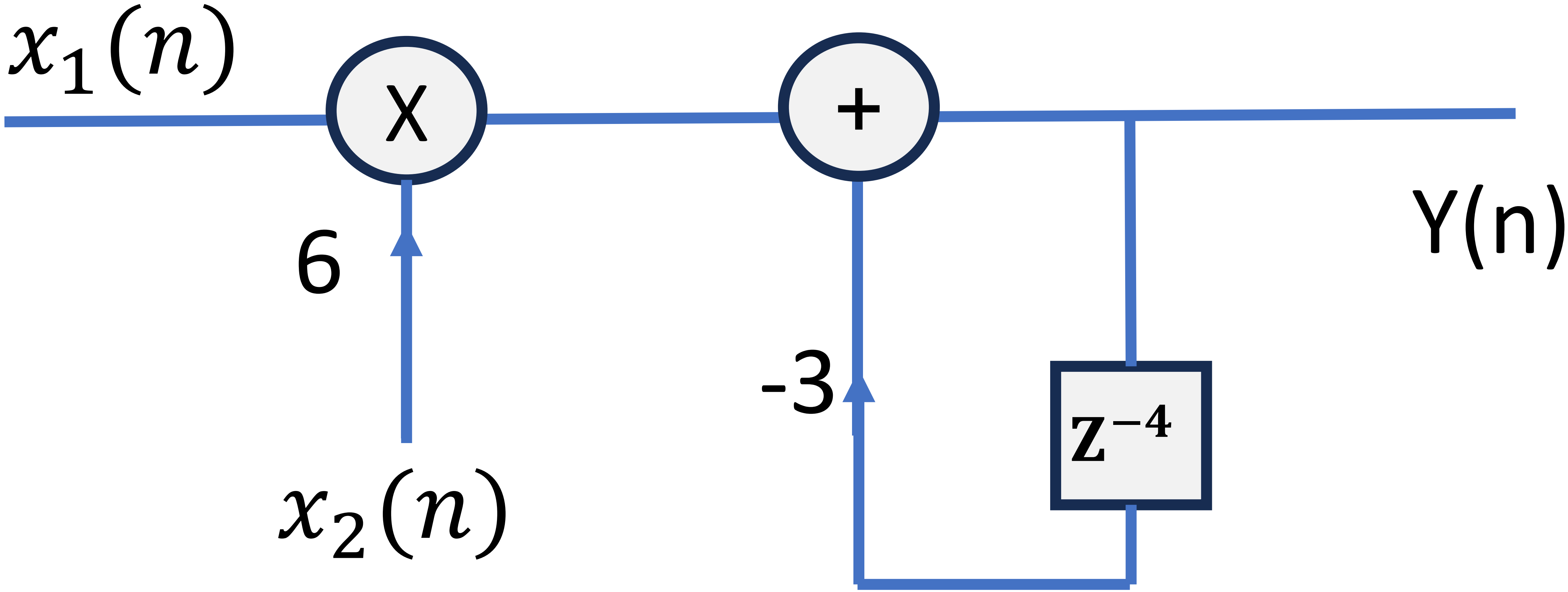


Example 4

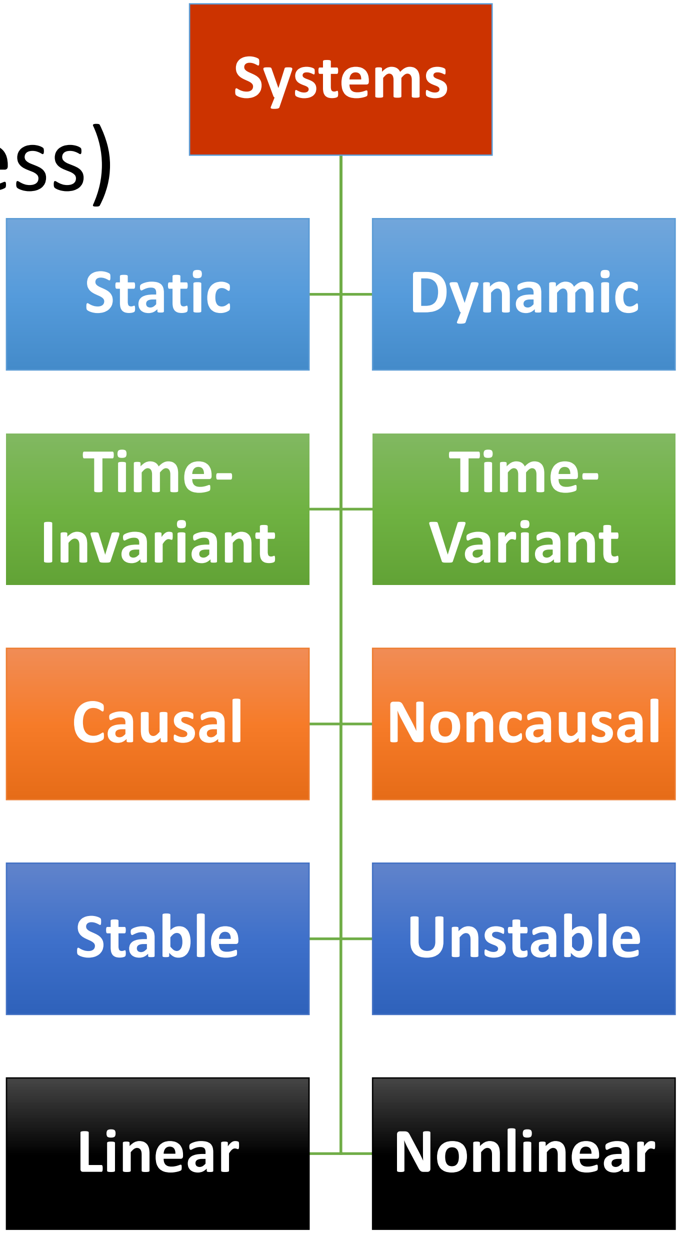


Y(n) - 3*Y(n+4) = 5*X(n-2) +X(n)

Example 5



$$y(n) = x_1(n) * 6x_2(n) - 3y(n-4)$$



No Delay (Memoryless)

$$y(n) = \mathcal{T}[x(n), n]$$

Delay (Memory)

$$y(n) = x(n) + 3x(n-1)$$

Differentiator

$$y(n, k) = y(n-k)$$

Modulator

$$y(n, k) \neq y(n-k)$$

Present and Past

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots]$$

Present, Past and Future

$$y(n) = x(n) + 3x(n+4)$$

BIBO

$$y(n) = y^2(n-1) + x(n)$$

Superposition

$$\mathcal{T}[a_1x_1(n) + a_2x_2(n)] = a_1\mathcal{T}[x_1(n)] + a_2\mathcal{T}[x_2(n)]$$

$$y(n) = e^{x(n)}$$

Signal Representation

Consider the special case of a finite-duration sequence given as

$$x(n) = \{2, \underset{\uparrow}{4}, 0, 3\}$$

Resolve the sequence $x(n)$ into a sum of weighted impulse sequences.

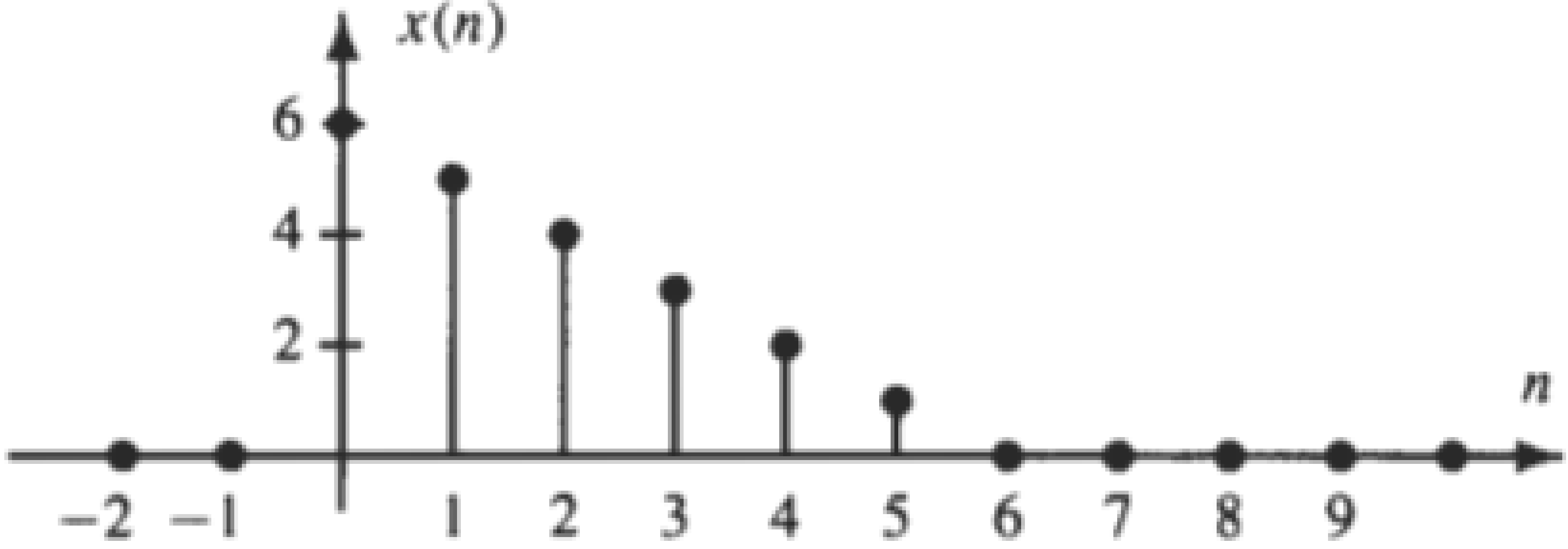
Solution Since the sequence $x(n)$ is nonzero for the time instants $n = -1, 0, 2$, we need three impulses at delays $k = -1, 0, 2$. Following (2.3.10) we find that

$$x(n) = 2\delta(n+1) + 4\delta(n) + 3\delta(n-2)$$

Home Work

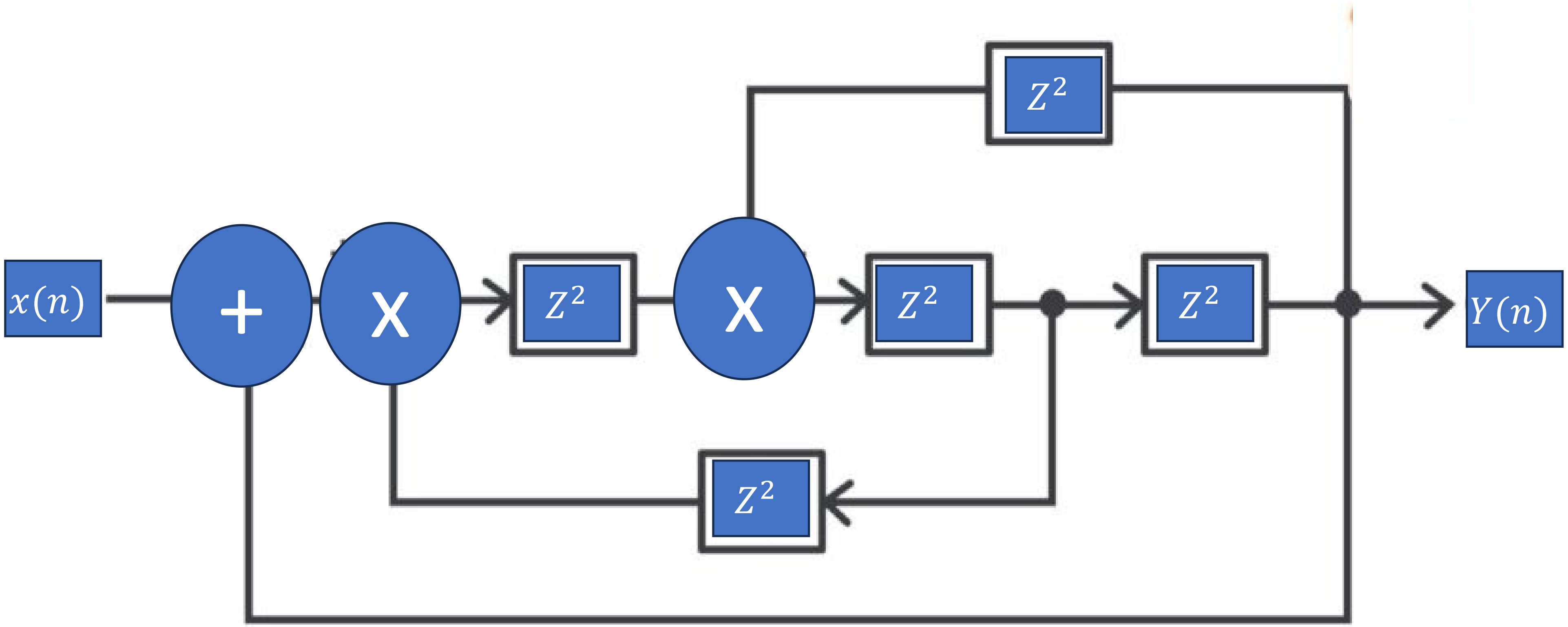
1 Given the sequence x(n) make a sketch of the following

$$(a)\ y_1(n) = x(6+n)$$
$$(b)\ y_2(n) = x(n-7)$$



Home Work

2 Write the equation of the system below



Home Work

3 Write the equations below as a sum of weighted impulse responses

$$x_1(n) = \{6\ 5\ -1\ \underset{\uparrow}{0}\ 2\ 2\}$$

$$x_2(n) = \{8\ \underset{\uparrow}{-5}\ 0\ 0\ 0\ 4\}$$

University of Technology
Laser & Optoelectronics Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

Lec. 3: Discrete-Time Signals and Systems 2: 2024-Oct-13

Lecture Outline

1. Convolution
2. Properties
3. Impulse response of convoluted signals
4. HW
5. Additional Questions

Convolution

$$y(n,k) \equiv h(n,k) = \mathcal{T}[\delta(n-k)]$$

$$c_k h(n,k) = x(k)h(n,k)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

$$y(n) = \mathcal{T}[x(n)] = \mathcal{T}\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\mathcal{T}[\delta(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} x(k)h(n,k)$$

$$h(n) \equiv \mathcal{T}[\delta(n)]$$

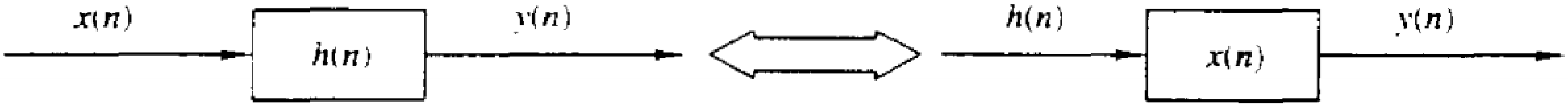
$$h(n-k) \equiv \mathcal{T}[\delta(n-k)]$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Interchange

$$y(n) = x(n) * h(n) \equiv \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n) = h(n) * x(n) \equiv \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$



Properties

$$x(n) * h(n) = h(n) * x(n)$$

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

$$y(n_0) = \sum_{k=-\infty}^{\infty} x(k)h(n_0 - k)$$

1. *Folding*. Fold $h(k)$ about $k = 0$ to obtain $h(-k)$.
2. *Shifting*. Shift $h(-k)$ by n_0 to the right (left) if n_0 is positive (negative), to obtain $h(n_0 - k)$.
3. *Multiplication*. Multiply $x(k)$ by $h(n_0 - k)$ to obtain the product sequence $v_{n_0}(k) \equiv x(k)h(n_0 - k)$.
4. *Summation*. Sum all the values of the product sequence $v_{n_0}(k)$ to obtain the value of the output at time $n = n_0$.

System Response

The impulse response of a linear time-invariant system is

$$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$$

Determine the response of the system to the input signal

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$$

Finding the h(n) signals

$h(-k)$ → Folding

$h(n-k)$ → Shifting Plot Right

$h(-n-k)$ → Shifting Plot Left

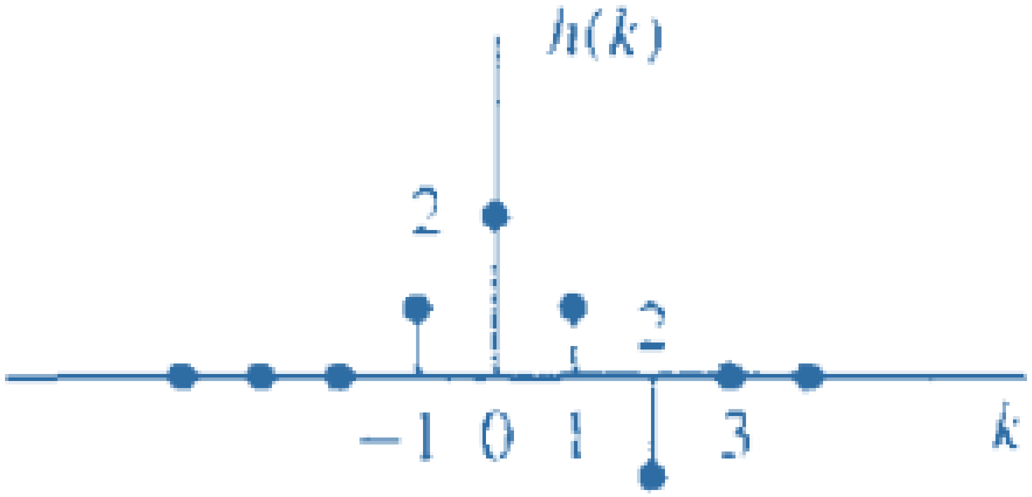
Finding the y(n) Response

$x(k)h(k)$ → **Multiplication**

$$y(n_0) = \sum_{k=-\infty}^{\infty} x(k)h(n_0 - k)$$ → **Summation**

$y(n) = \{ \quad \}$ → **Writing**

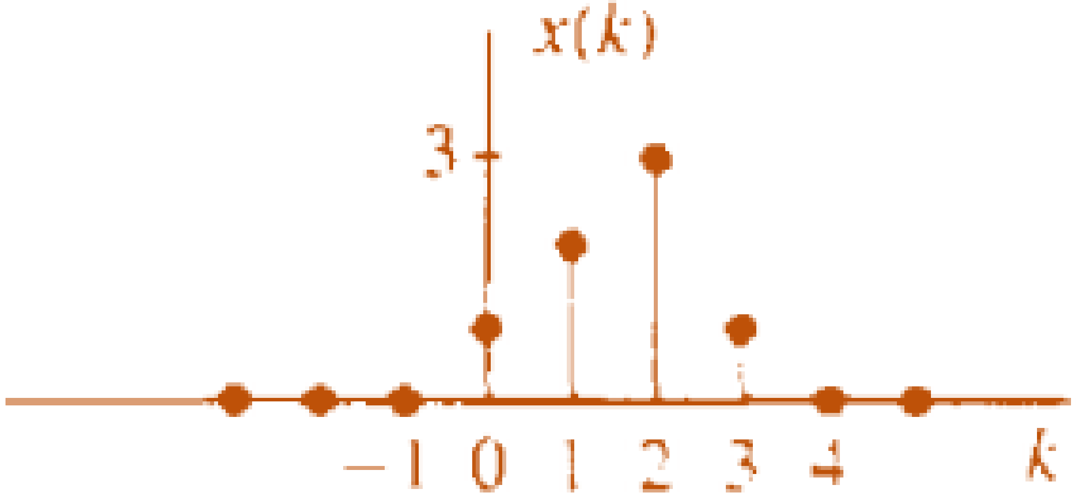
Impulse Response



$$h(n) = \{1, 2, 1, -1\}$$
$$\uparrow$$

$$y(+n) = 2 + 3 = 5$$
$$y(-n) = 1$$

Input Signal



$$x(n) = \{1, 2, 3, 1\}$$
$$\uparrow$$

$$y(+n) \& y(-n) \& y(0) = 7$$

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k)$$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k)$$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k)$$

$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k)$$

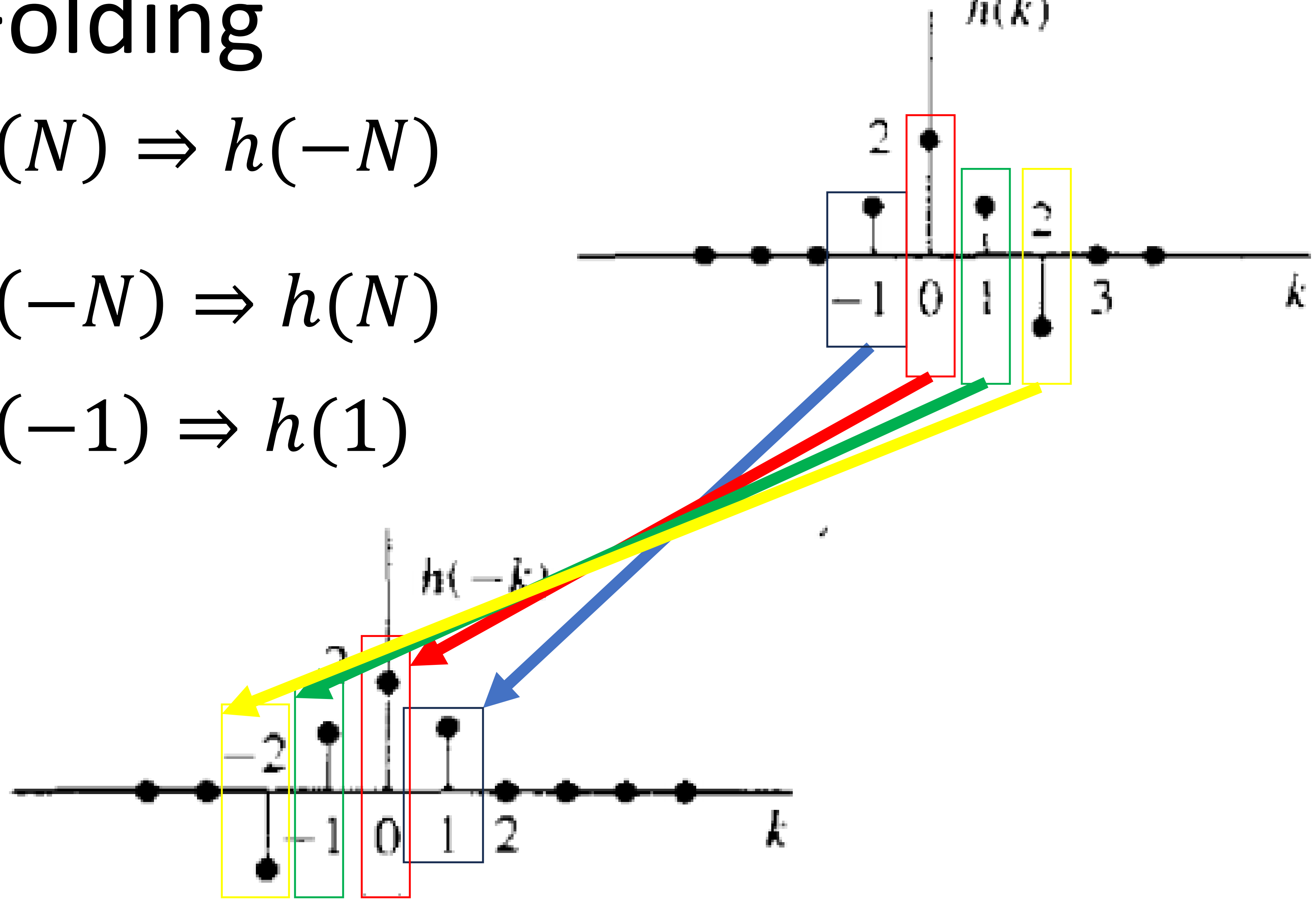
$$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k)$$

Folding

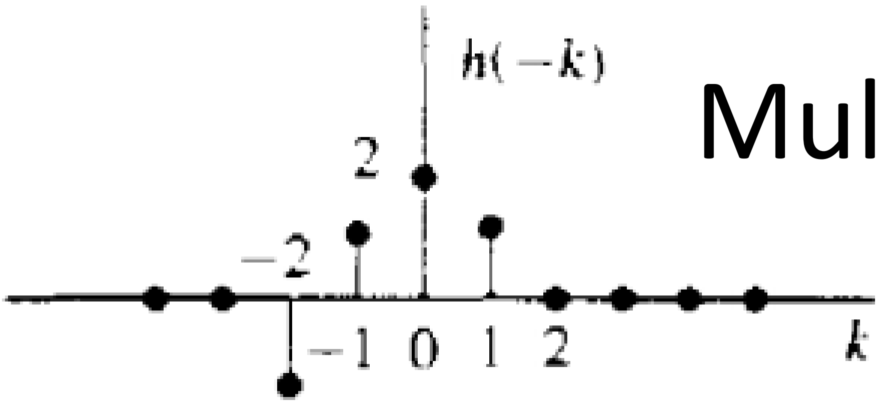
$$h(N) \Rightarrow h(-N)$$

$$h(-N) \Rightarrow h(N)$$

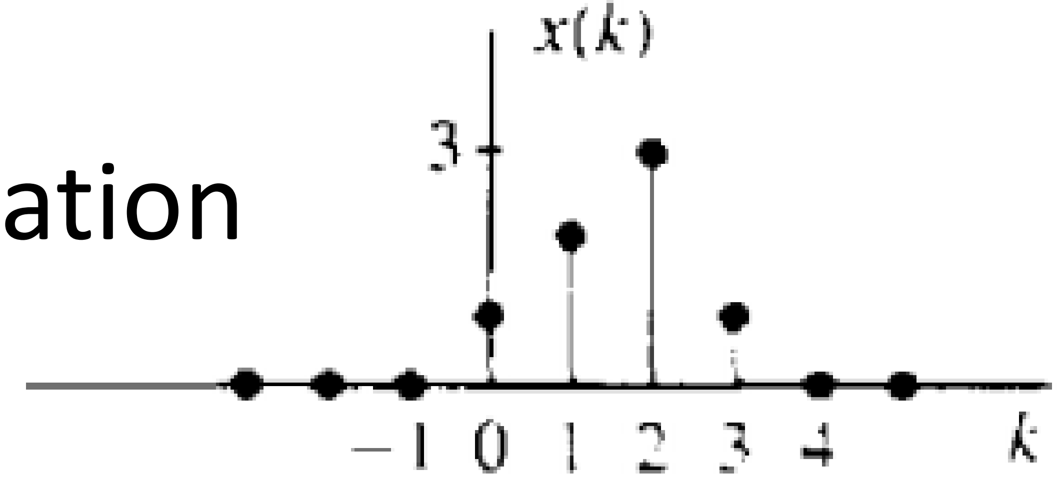
$$h(-1) \Rightarrow h(1)$$



$$v_0(k) \equiv x(k)h(-k)$$

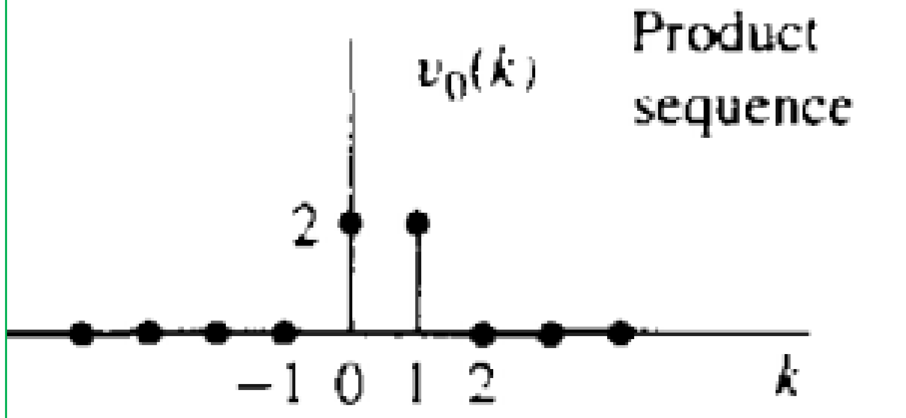


Multiplication

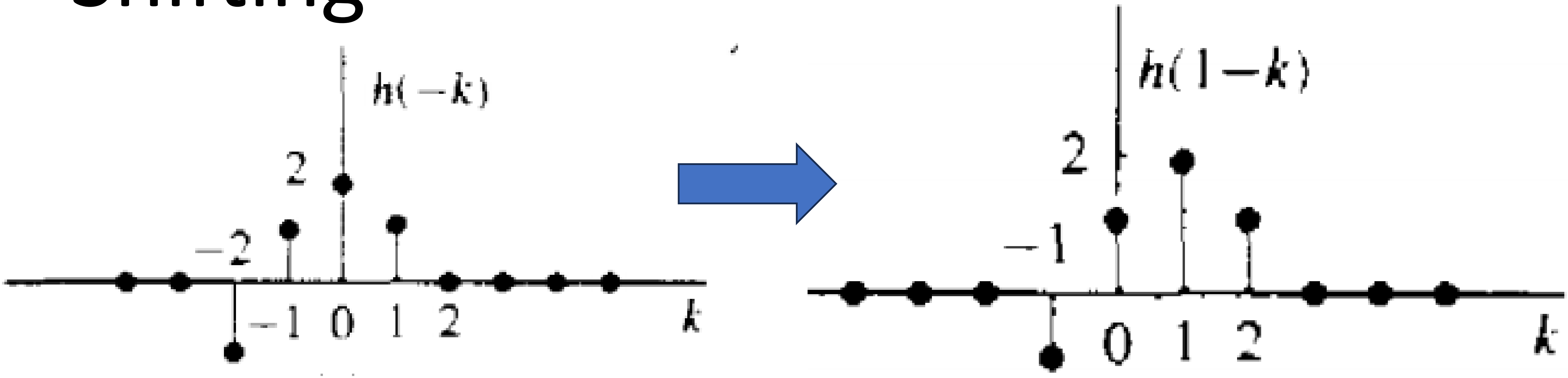


Summation

$$y(0) = \sum_{k=-\infty}^{\infty} v_0(k) = 4$$

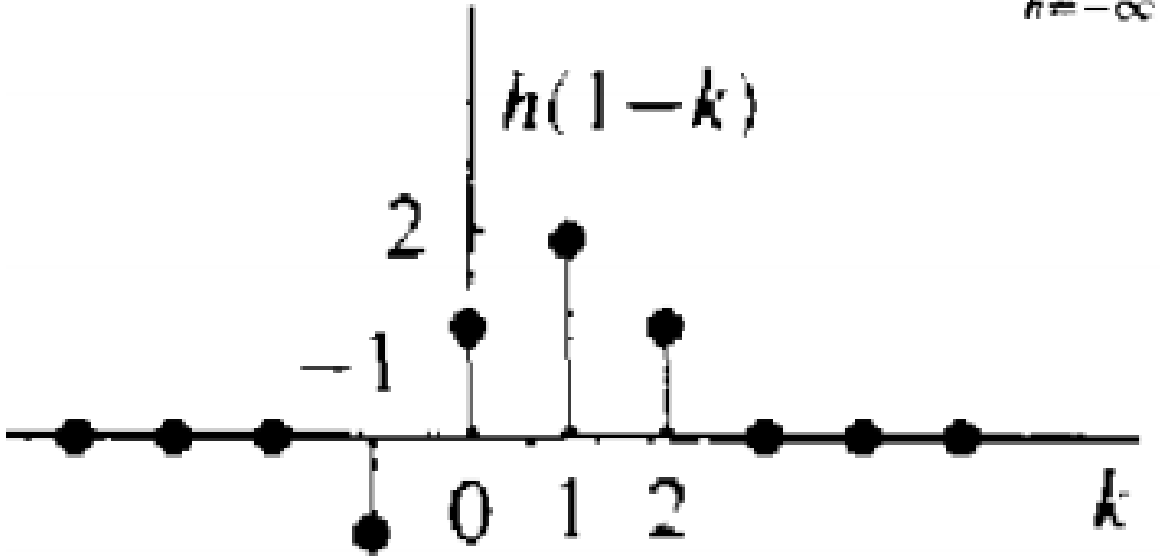


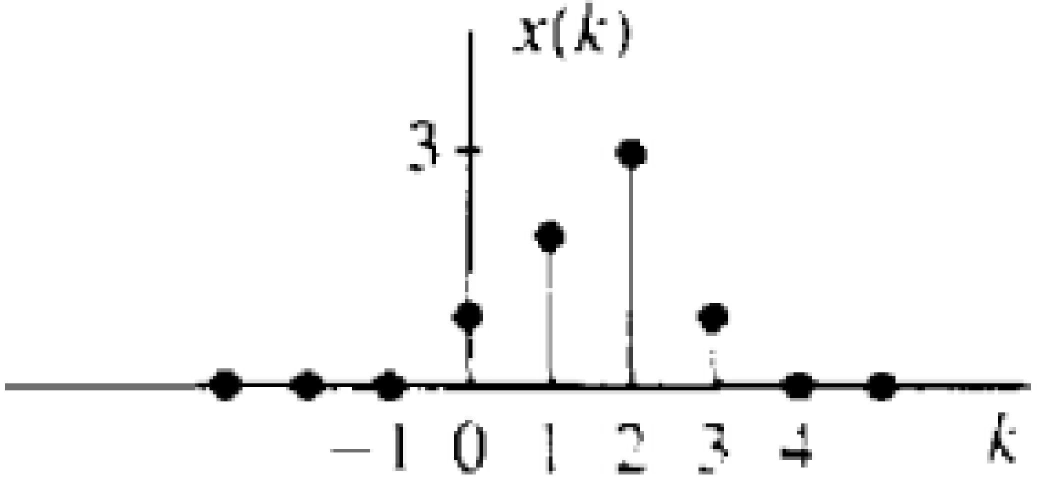
Shifting



$$h(-k) = \{-1\ 1\ 2\ 1\} \qquad h(1-k) = \{-1\ 1\ 2\ 1\}$$

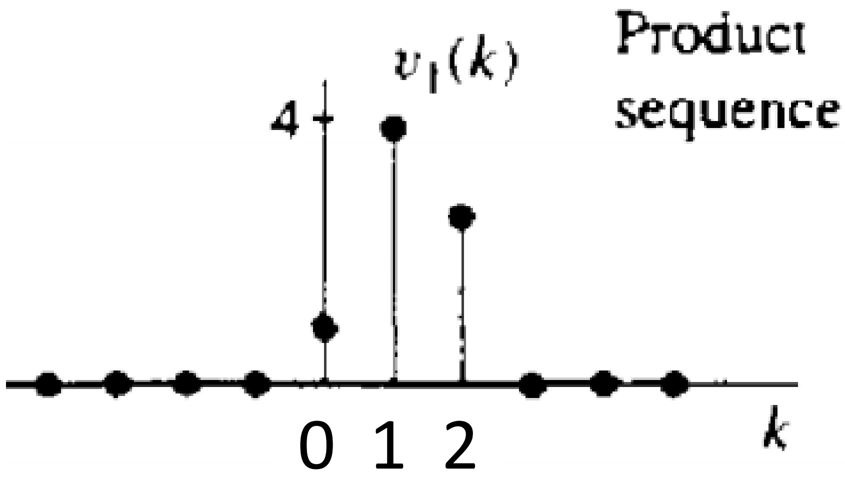
$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k)$$



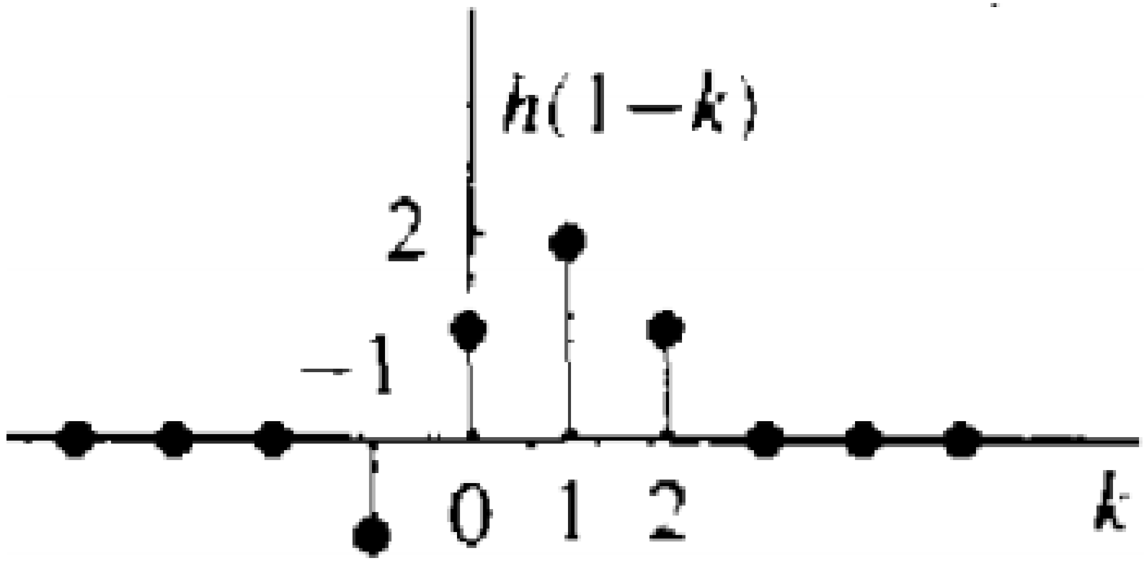


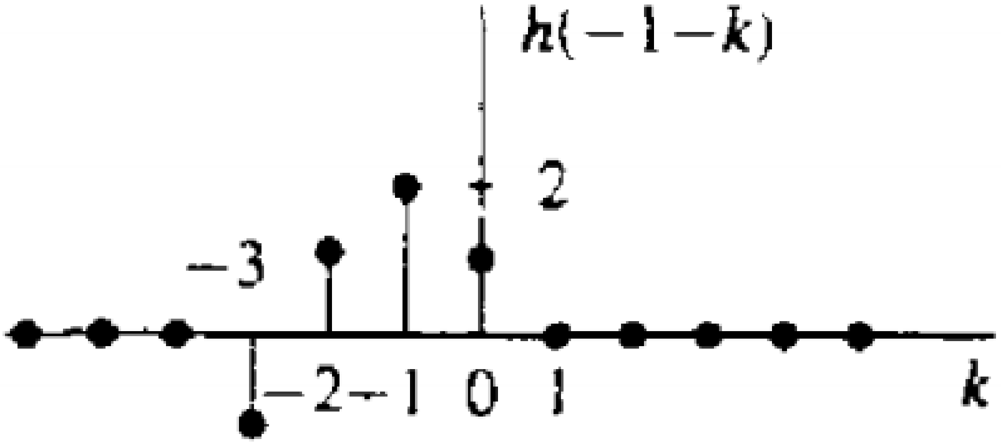
$$v_1(k) = x(k)h(1-k)$$

$$y(1) = \sum_{k=-\infty}^{\infty} v_1(k) = 8$$



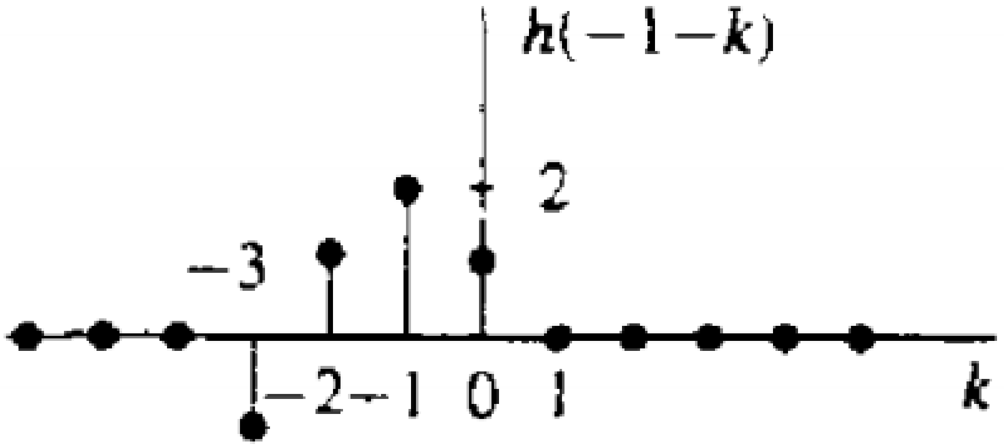
$$y(-1) = \sum^{\infty} x(k)h(-1-k) \quad ?$$

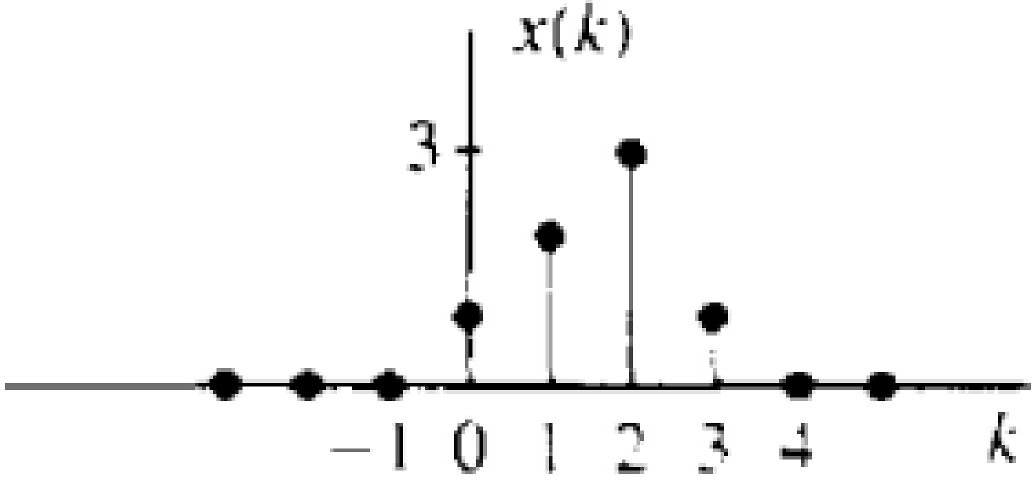


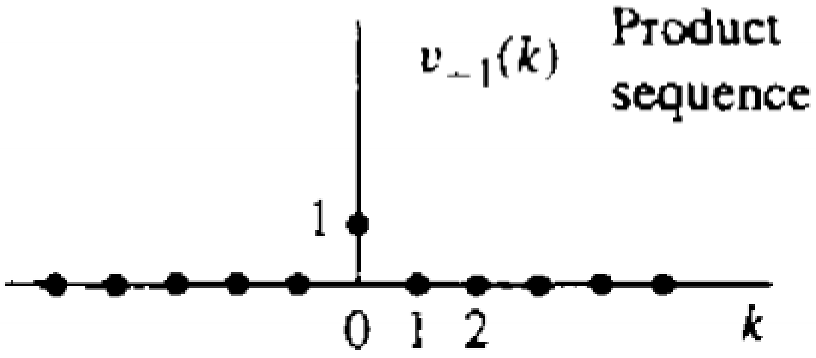


$$h(1-k) = \{-1\ 1\ 2\ 1\} \qquad h(-1-k) = \{-1\ 1\ 2\ 1\}$$

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)\,h(-1-k)$$

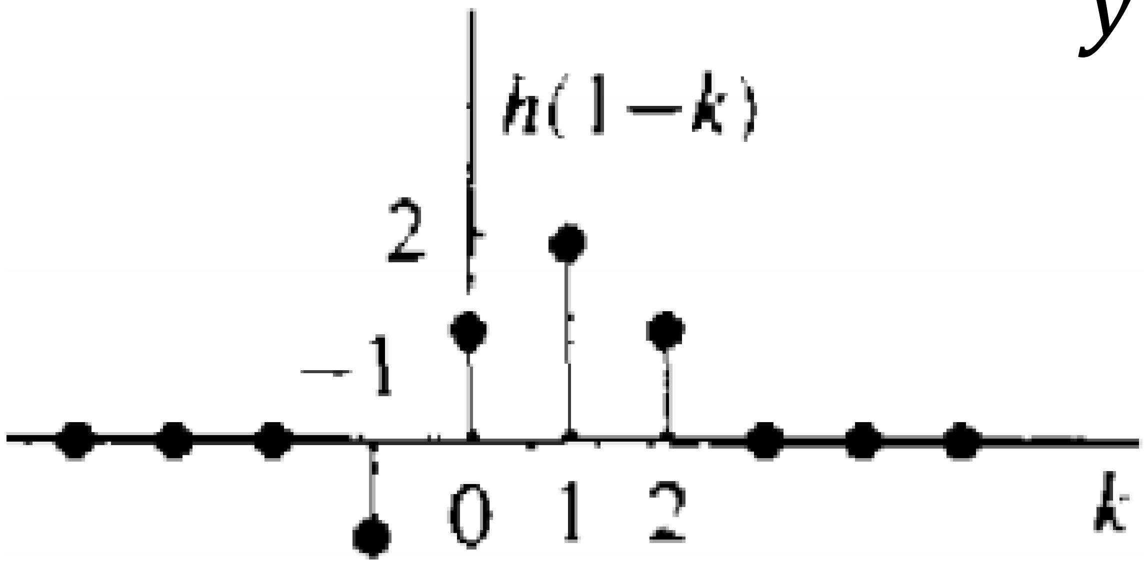




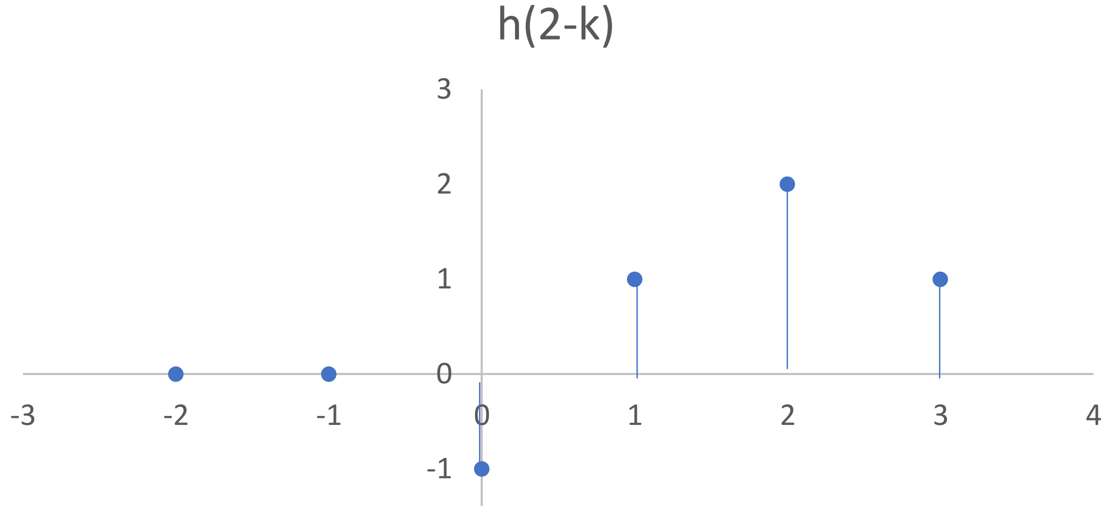


$$y(-1) = 1$$

$$v_2(k) = x(k)h(2-k)$$

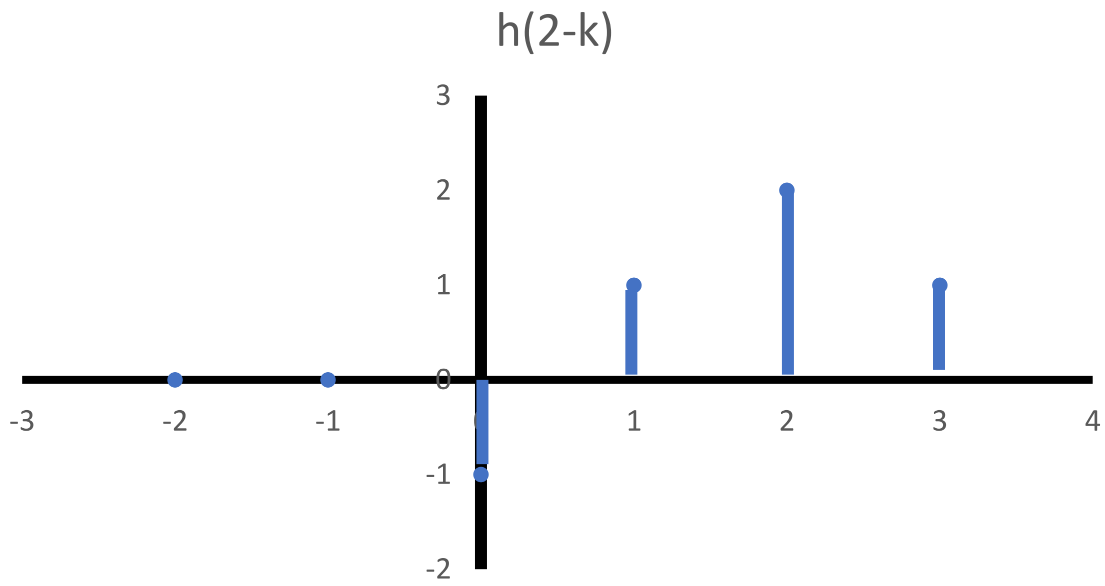


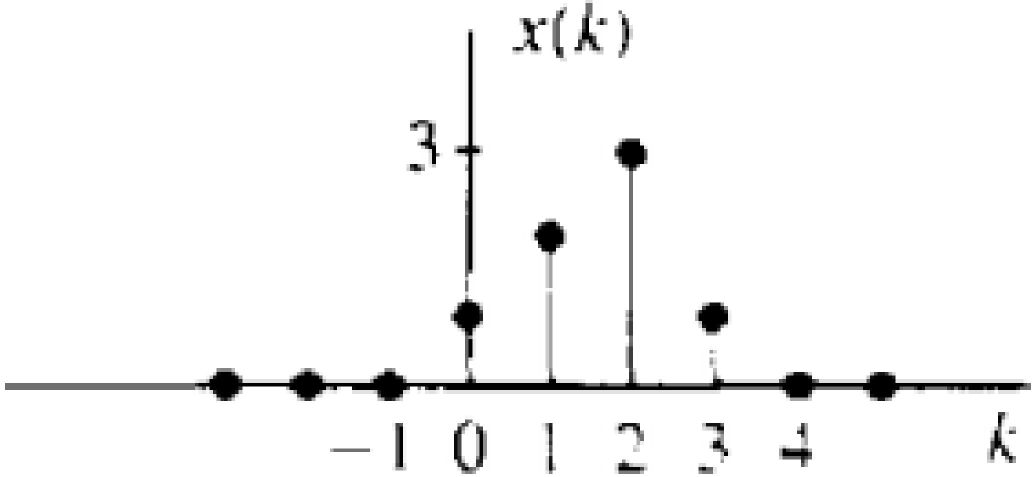
$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k)$$

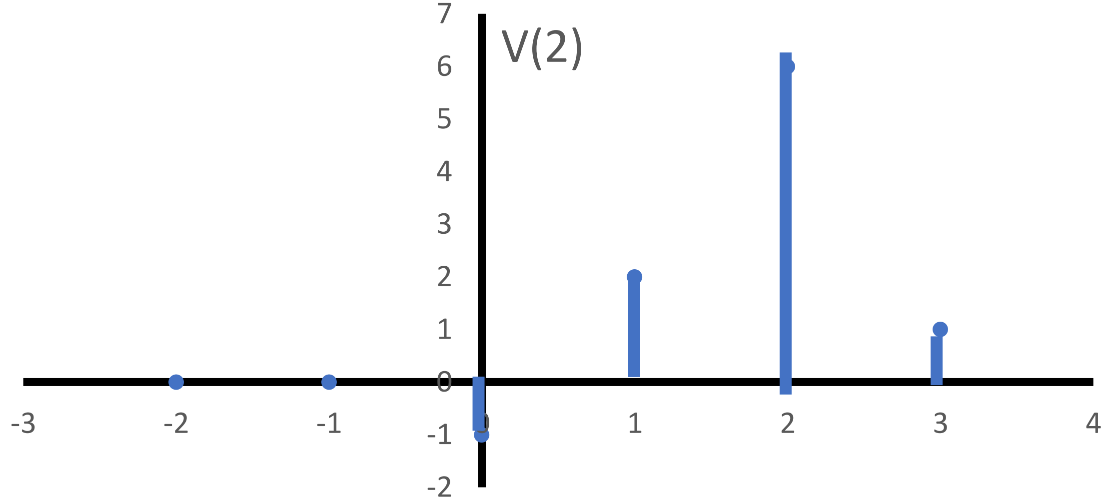


$$h(1-k) = \{-1\ 1\ 2\ 1\} \quad h(2-k) = \{-1\ 1\ 2\ 1\}$$

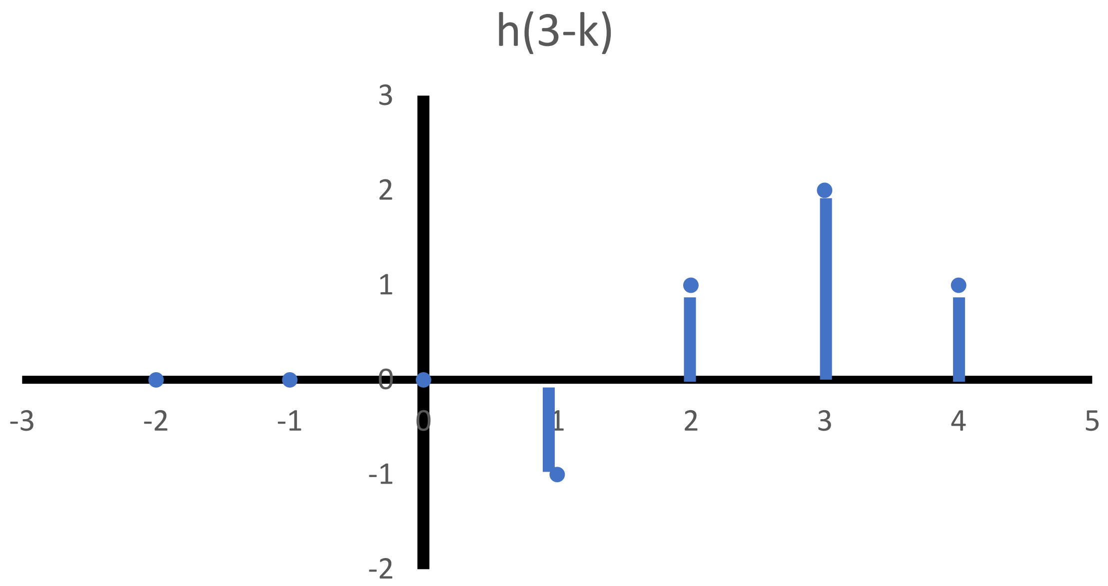
$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = -1 + 2 + 6 + 1 = 8$$

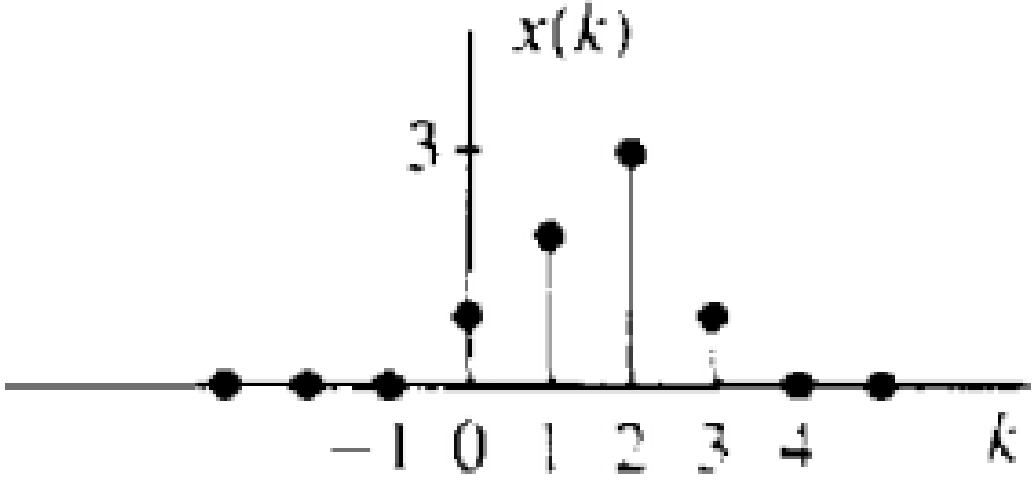


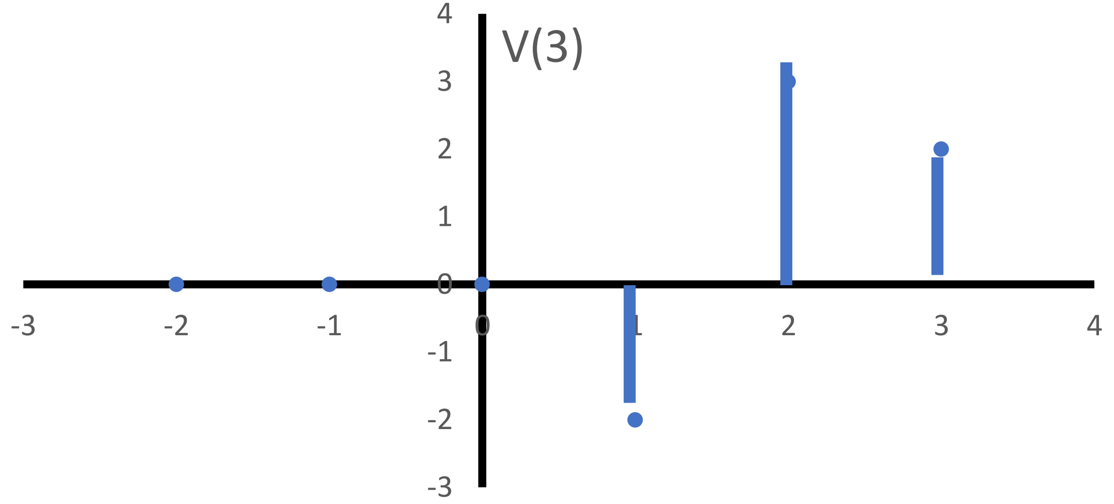




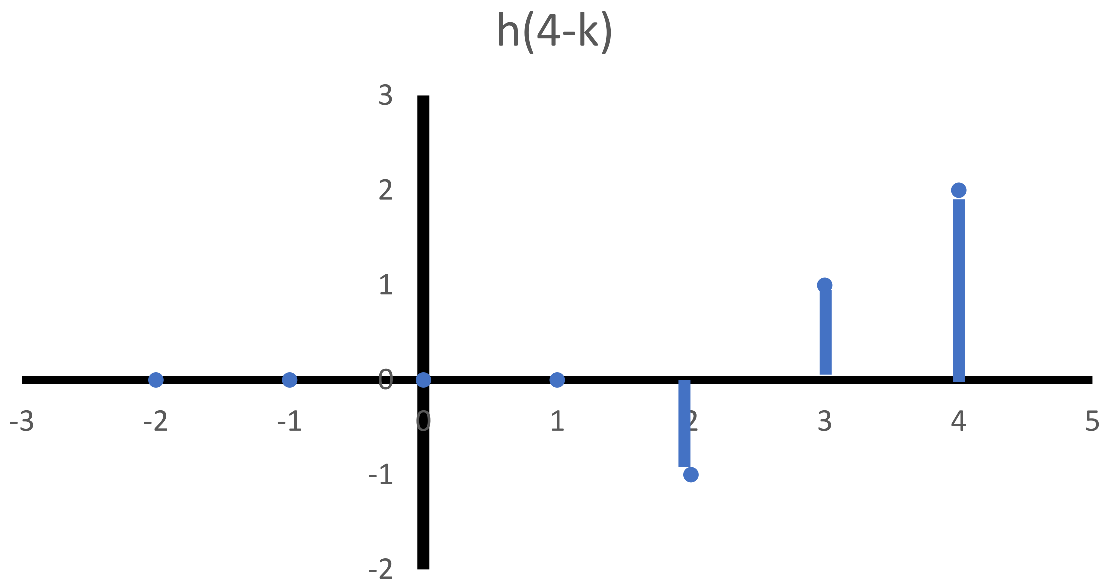
$$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = -2 + 3 + 2 = 3$$

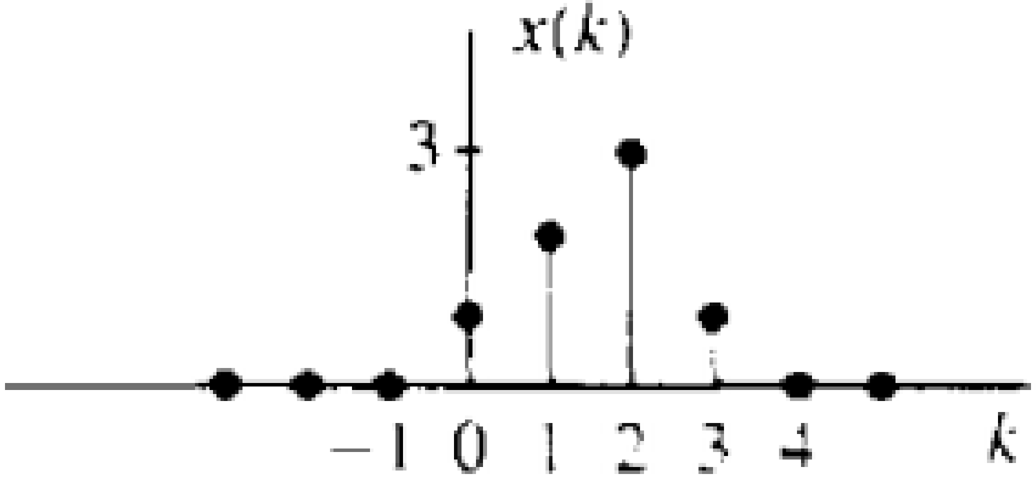


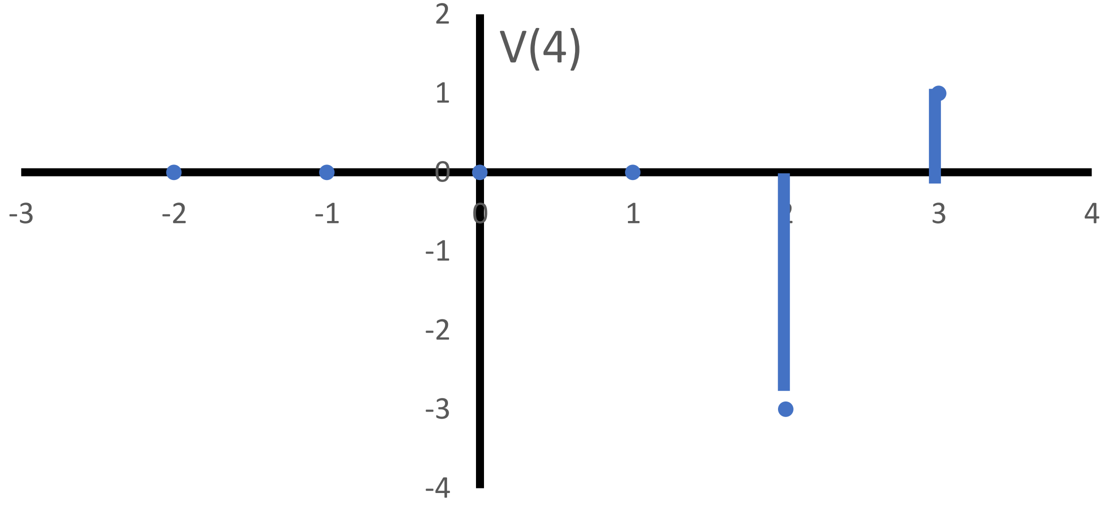




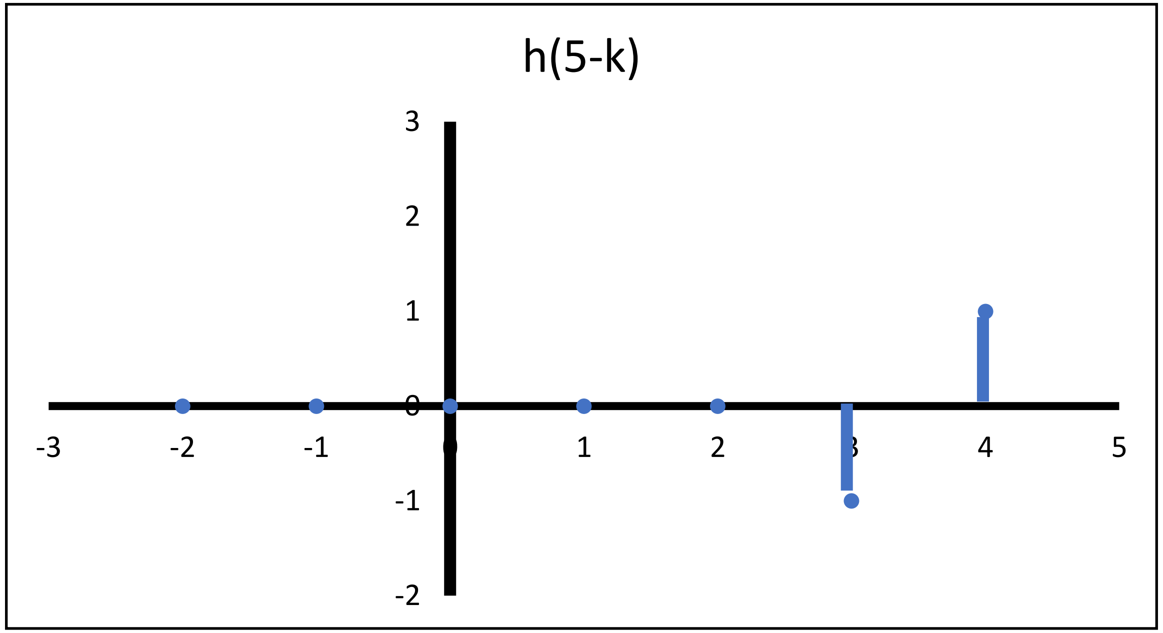
$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = -3 + 1 = -2$$

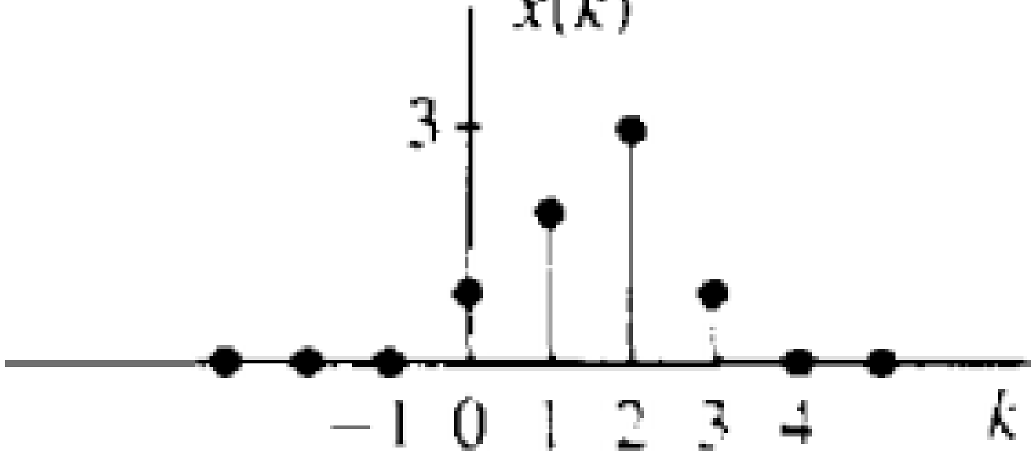


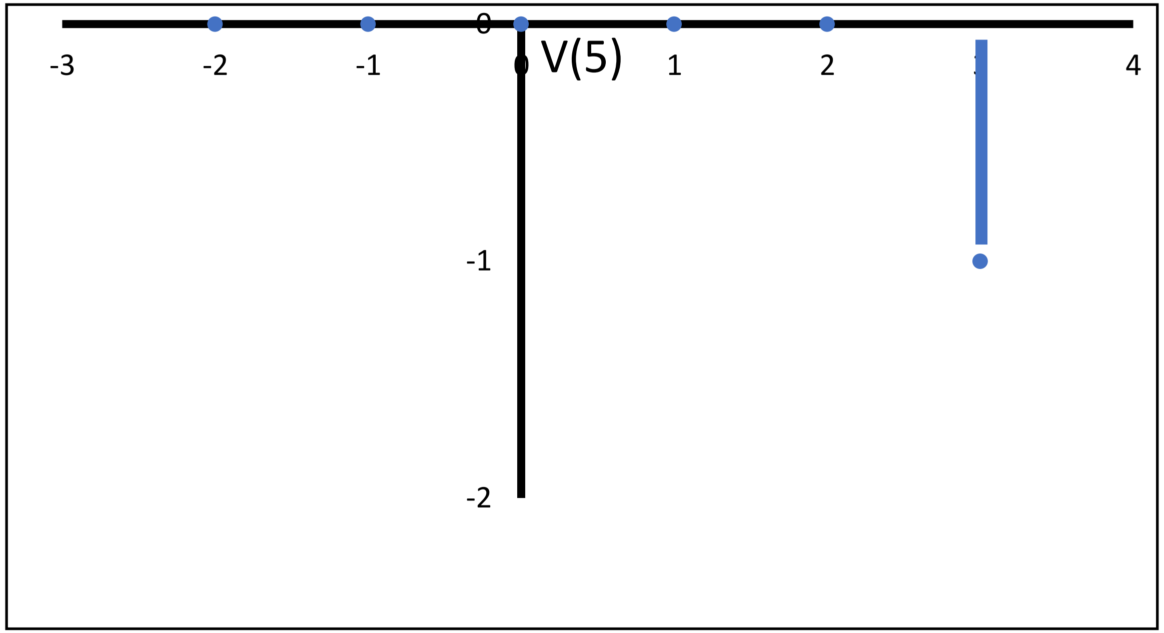




$$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k) = -1$$

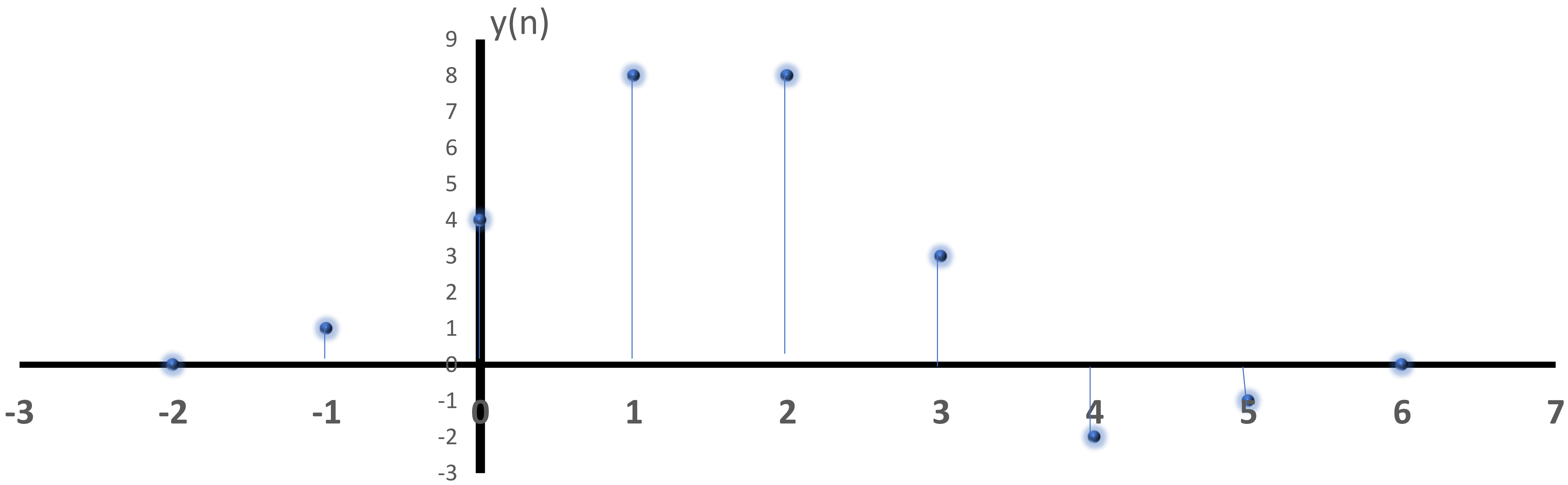






The system response y(n)

$$y(n) = \{\ldots, 0, 0, 1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1, 0, 0, \ldots\}$$

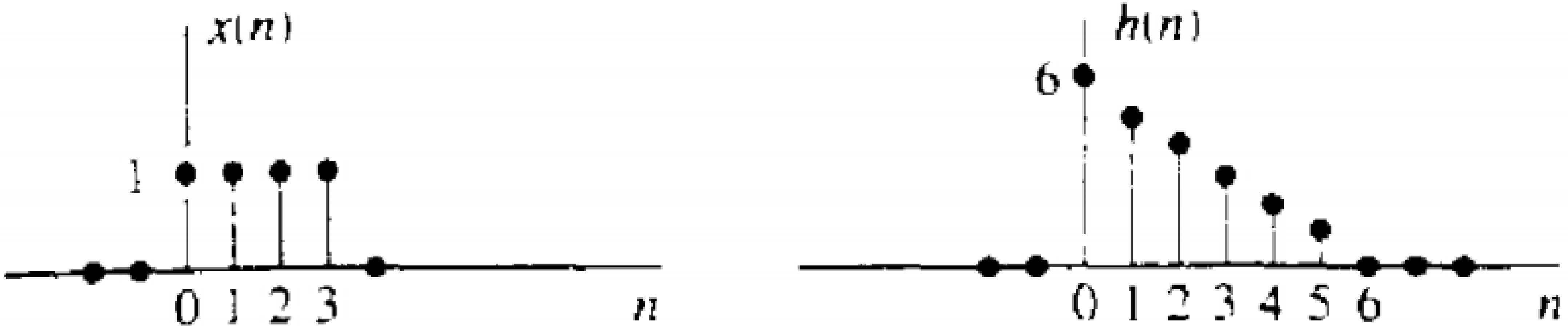


Home Work

Compute and plot the system response for input signal x(n) and the impulse response h(n)

Input Signal $x_{in}(n) = x(n-2)$

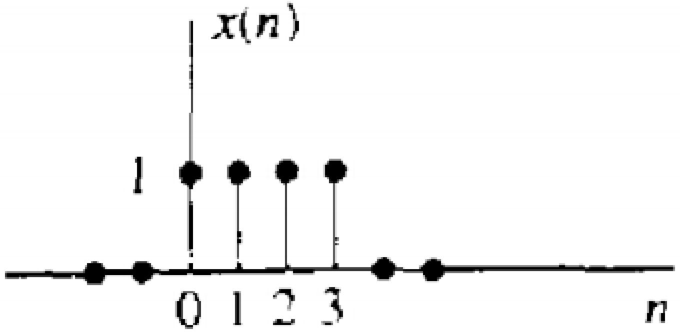
Impulse Response $h_{in}(n) = h(2n)$

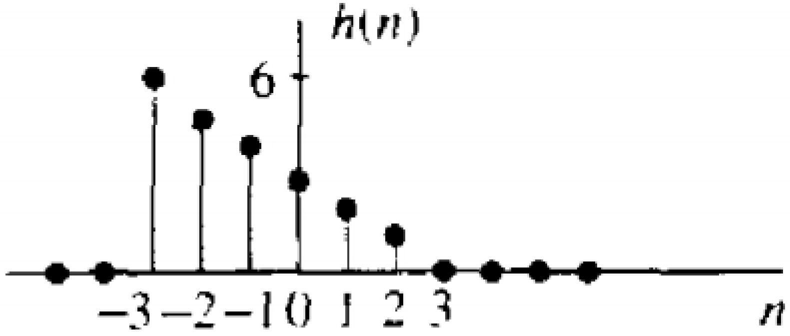


Additional Questions and Examples

To be self Studied

Q1: Compute and plot the system response for input signal x(n) and the impulse response h(n)





ANS

$$y(n) = \left\{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\right\}$$

Q2

Compute the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \alpha^n, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

ANS

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{4} h(k)x(n-k), \\
x(n) &= \left\{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots, \alpha^{5}\right\} \\
h(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1, 1\right\}
\end{aligned}
$$

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{4} x(n-k), -3 \leq n \leq 9 \\
&= 0, \text{ otherwise.}
\end{aligned}
$$

ANS

$$
\begin{aligned}
y(-3) &= \alpha^{-3}, \\
y(-2) &= x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}, \\
y(-1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1}, \\
y(0) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 \\
y(1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha, \\
y(2) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2} \\
y(3) &= \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}, \\
y(4) &= \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1 \\
y(5) &= \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(6) &= \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5} \\
y(7) &= \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(8) &= \alpha^{4} + \alpha^{5}, \\
y(9) &= \alpha^{5}
\end{aligned}
$$

University of Technology
Laser & Optoelectronics Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

Lec. 4: Discrete-Time Signals and Systems 3: 2024-Oct-20

Lecture Outline

1. **Correlation**
2. **Properties**
3. **Cross correlation**
4. **Autocorrelation**
5. **HW**

Correlation

The *correlation* of two functions or signals or waveforms is defined as the **measure of similarity** between those signals.

$$y(n) = \alpha x(n - D) + w(n)$$

Attenuation Factor

Round trip delay

Noise

Transmitted Signal

Received Signal

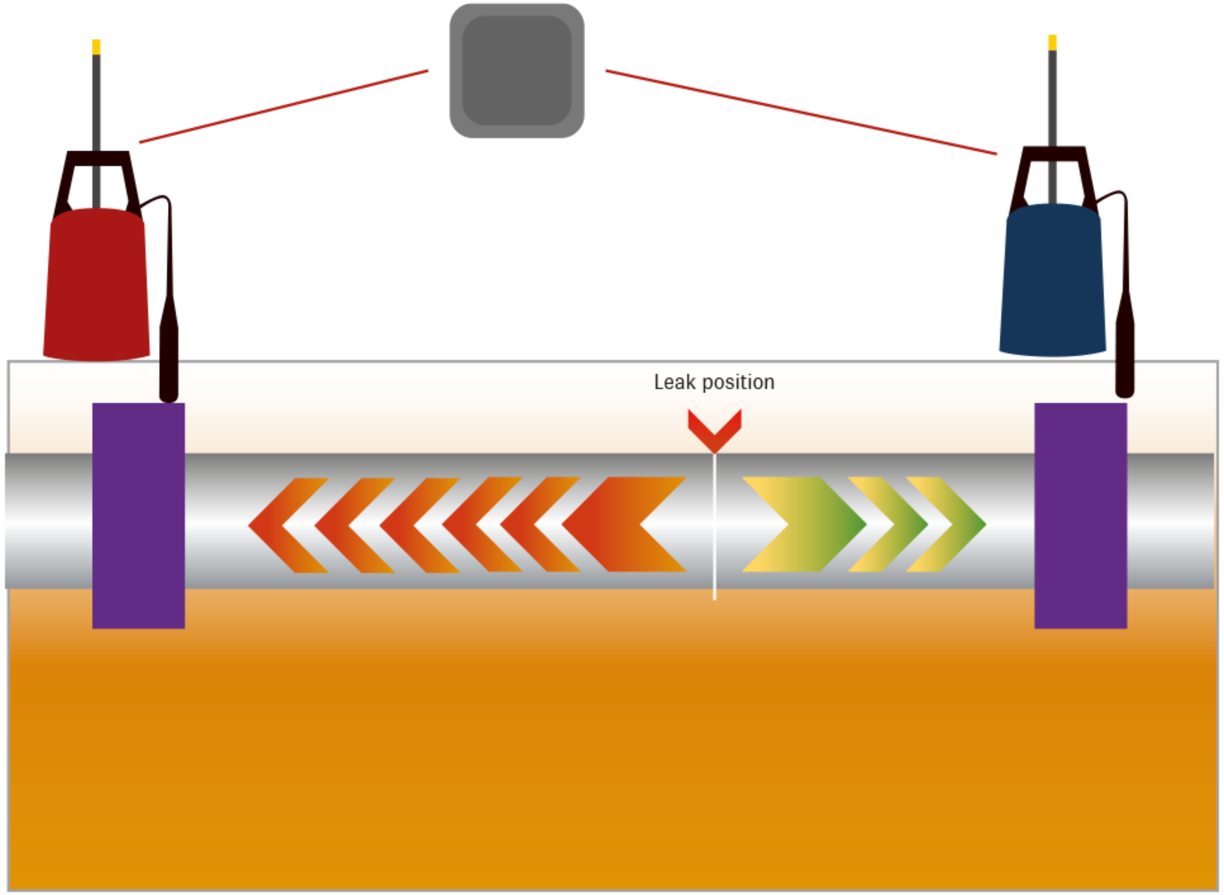
Correlator

Radar / Electronic

Optical

Applications of Correlator

A correlator works by comparing signals from two sensors positioned along the line of existing pipe work either side of the suspected leak and picks up noise created by the leak as it escapes from the pipe under pressure.



Applications of Correlator

Officials of the U.S. Air Force Research Laboratory at Wright-Patterson Air Force Base, Ohio, announced a $5.4 million contract to Epirus on Tuesday for the **Massive Cross-Correlation** (MAX) project.

Cross Correlation

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l)$$

$$r_{xy}(l) = r_{yx}(-l)$$

$$r_{xy}(1) = \sum_{n=-\infty}^{\infty} x(n)y(n-1)$$

Example

Determine the crosscorrelation sequence $r_{xy}(l)$ of the sequences

$$x(n) = \{\ldots, 0, 0, 2, -1, 3, 7, \underset{\uparrow}{1}, 2, -3, 0, 0, \ldots\}$$

$$y(n) = \{\ldots, 0, 0, 1, -1, 2, -2, \underset{\uparrow}{4}, 1, -2, 5, 0, 0, \ldots\}$$

Solution

Use the crosscorrelation formula until the product of X(n) . Y(n-k) =0 in both directions (-ve & +ve)

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l)$$

$$r_{xy}(0) = \sum_{n=-\infty}^{\infty} x(n)y(n)$$

$$v_0(n) = \{\ldots, 0, 0, 2, 1, 6, -14, \underset{\uparrow}{4}, 2, 6, 0, 0, \ldots\}$$

$$r_{xy}(0) = 7$$

$$x(n) = \{0\ \ 2\ \ -1\ 3\ \ \ \ 7\ 1\ \ 2\ \ -3\ 0\ \ 0\}$$

$$y(n-1) = \{0\ \ 1\ \ -1\ 2\ \ -2\ 4\ 1\ \ -2\ 5\ 0\}$$

$$v(n) = \{\ -1 - 3\ 14\ \ -2\ \ 8\ \ -3\}$$

$$r_{xy}(1) = \sum_{n=-\infty}^{\infty} x(n)y(n-1) = 13$$

$$x(n) = \{0\ \ 2\ \ -1\ 3\ \ \ \ 7\ 1\ \ 2\ \ -3\ 0\ \ 0\}$$

$$y(n-2) = \{0\ \ 1\ \ -1\ 2\ \ -2\ 4\ 1\ \ -2\ 5\ 0\}$$

$$v(n) = \{\ 3\ \ -7\ 2\ \ -4\ \ -12\ \}$$

$$r_{xy}(2) = \sum_{n=-\infty}^{\infty} x(n)y(n-2) = -18$$

$$x(n) = \{0 \;\; 2 \;\; -1 \; 3 \quad\; 7 \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n-3) = \{0 \;\; 1 \;\; -1 \; 2 \;\; -2 \; 4 \; 1 \;\; -2 \; 5 \; 0\}$$

$$v(n) = \{\; 7 \;\; -1 \; 4 \; 6 \;\}$$

$$r_{xy}(3) = \sum_{n=-\infty}^{\infty} x(n)y(n-3) = 16$$

$$x(n) = \{0 \;\; 2 \;\; -1 \; 3 \quad 7 \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n-4) = \{0 \;\; 1 \;\; -1 \; 2 \;\; -2 \; 4 \; 1 \;\; -2 \; 5 \; 0\}$$

$$v(n) = \{ \; 1 \;\; -2 \;\; -6 \; \}$$

$$r_{xy}(4) = \sum_{n=-\infty}^{\infty} x(n)y(n-4) = -7$$

$$x(n) = \{0 \;\; 2 \;\; -1 \; 3 \quad 7 \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n-5) = \{0 \;\; 1 \;\; -1 \; 2 \;\; -2 \; 4 \; 1 \;\; -2 \; 5 \; 0\}$$

$$v(n) = \{\; 2 \; 3 \;\}$$

$$r_{xy}(5) = \sum_{n=-\infty}^{\infty} x(n) y(n-5) = 5$$

$$x(n) = \{0 \;\; 2 \;\; -1 \; 3 \quad 7 \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n-6) = \{\, 0 \; 0 \;\; 1 \;\; -1 \, 2 \;\; -2 \; 4 \; 1 \;\; -2 \; 5 \; 0$$

$$v(n) = \{\; -3 \;\}$$

$$r_{xy}(6) = \sum_{n=-\infty}^{\infty} x(n) y(n-6) = -3$$

$$x(n) = \{0 \;\; 2 \;\; -1 \; 3 \quad\quad 7 \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n-7) = \{\; 0 \; 0 \; 0 \;\; 1 \;\; -1 \; 2 \;\; -2 \; 4 \; 1 \;\; -2 \; 5 \; 0\}$$

$$v(n) = \{\; 0\}$$

$$r_{xy}(7) = \sum_{n=-\infty}^{\infty} x(n)y(n-7) = 0$$

$$x(n) = \{0\ \ 2\ -1\ 3\quad 7\ 1\ \ 2\ -3\ 0\ \ 0\}$$

$$y(n+1) = \{\ 0\ 0\ 0\ \ 1\ -1\ 2\ -2\ 4\ 1\ \ -2\ 5\ 0\}$$

$$v(n) = \{-2\ \ -2\ \ -6\ 28\ 1\ -4\ \ -15\}$$

$$r_{xy}(-1) = \sum_{n=-\infty}^{\infty} x(n)y(n+1) = 0$$

$$x(n) = \{0 \;\; 2 \;\; -1 \; 3 \quad 7 \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n+2) = \{ \; 0 \; 0 \; 0 \;\; 1 \;\; -1 \; 2 \;\; -2 \; 4 \; 1 \;\; -2 \; 5 \; 0\}$$

$$v(n) = \{ \; 4 \; 2 \;\; 12 \; 7 - 2 \;\; 10\}$$

$$r_{xy}(-2) = \sum_{n=-\infty}^{\infty} x(n)y(n+2) = 33$$

$$x(n) = \{0 \;\; 2 \;\; -1 \; 3 \quad 7 \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n+3) = \{ \;\; 0 \; 0 \; 0 \;\; 1 \;\; -1 \; 2 \;\; -2 \; 4 \; 1 \;\; -2 \; 5 \; 0\}$$

$$v(n) = \{ -4 \;\; -4 \; 3 - 14 \;\; 5\}$$

$$r_{xy}(-3) = \sum_{n=-\infty}^{\infty} x(n)y(n+3) = -14$$

$$x(n) = \{0 \;\; 2 \;\; -1\, 3 \quad\; 7\, 1 \;\; 2 \;\; -3\; 0 \;\; 0\}$$

$$y(n+4) = \{\; 0\; 0\; 0 \;\; 1 \;\; -1\, 2 \;\; -2\, 4\, 1 \;\; -2\, 5\, 0\}$$

$$v(n) = \{\, 8 \;\; -1 \;\; -6\; 35 \;\; 0\}$$

$$r_{xy}(-4) = \sum_{n=-\infty}^{\infty} x(n)y(n+4) = 36$$

$$x(n) = \{0\ \ 2\ \ -1\ 3\quad 7\ 1\ \ 2\ \ -3\ 0\ \ 0\}$$

$$y(n+5) = \{\ 0\ 0\ 0\ \ 1\ \ -1\ 2\ \ -2\ 4\ 1\ \ -2\ 5\ 0\ 0\}$$

$$v(n) = \{\ 2\ 2\ 15\ 0\}$$

$$r_{xy}(-5) = \sum_{n=-\infty}^{\infty} x(n)y(n+5) = 19$$

$$r_{xy}(-6) = \sum_{n=-\infty}^{\infty} x(n)y(n+6) = -9$$

$$r_{xy}(-7) = \sum_{n=-\infty}^{\infty} x(n)y(n+7) = 10$$

$$r_{xy}(l) = \{10 \ -9\ 19\ 36\ -14\ 33\ 0\ 7\ 13\ -18\ 16\ -7\ 5\ -3\}$$

Autocorrelation

A signal with delayed version of it

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

Correlation and Convolution

$$x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\} \qquad h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$$

$$\text{convolution: } y_3(n) = \left\{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\right\}$$

$$\text{correlation: } \gamma_1(n) = \left\{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\right\}$$

Home Work

Determine the autocorrelation sequences of the following signals.

$$x(n) = \{0 \;\; 2 \; 3 \quad \underset{\uparrow}{7} \; 1 \;\; 2 \;\; -3 \; 0 \;\; 0\}$$

$$y(n) = \{0 \;\; -2 \;\; -3 \quad 1 \;\; -\underset{\uparrow}{3} \; 0 \;\; 0\}$$

University of Technology
Laser & Optoelectronics
Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

Lec. 5: Z-Transform and its Applications: 2024-Oct-27

Lecture Outline

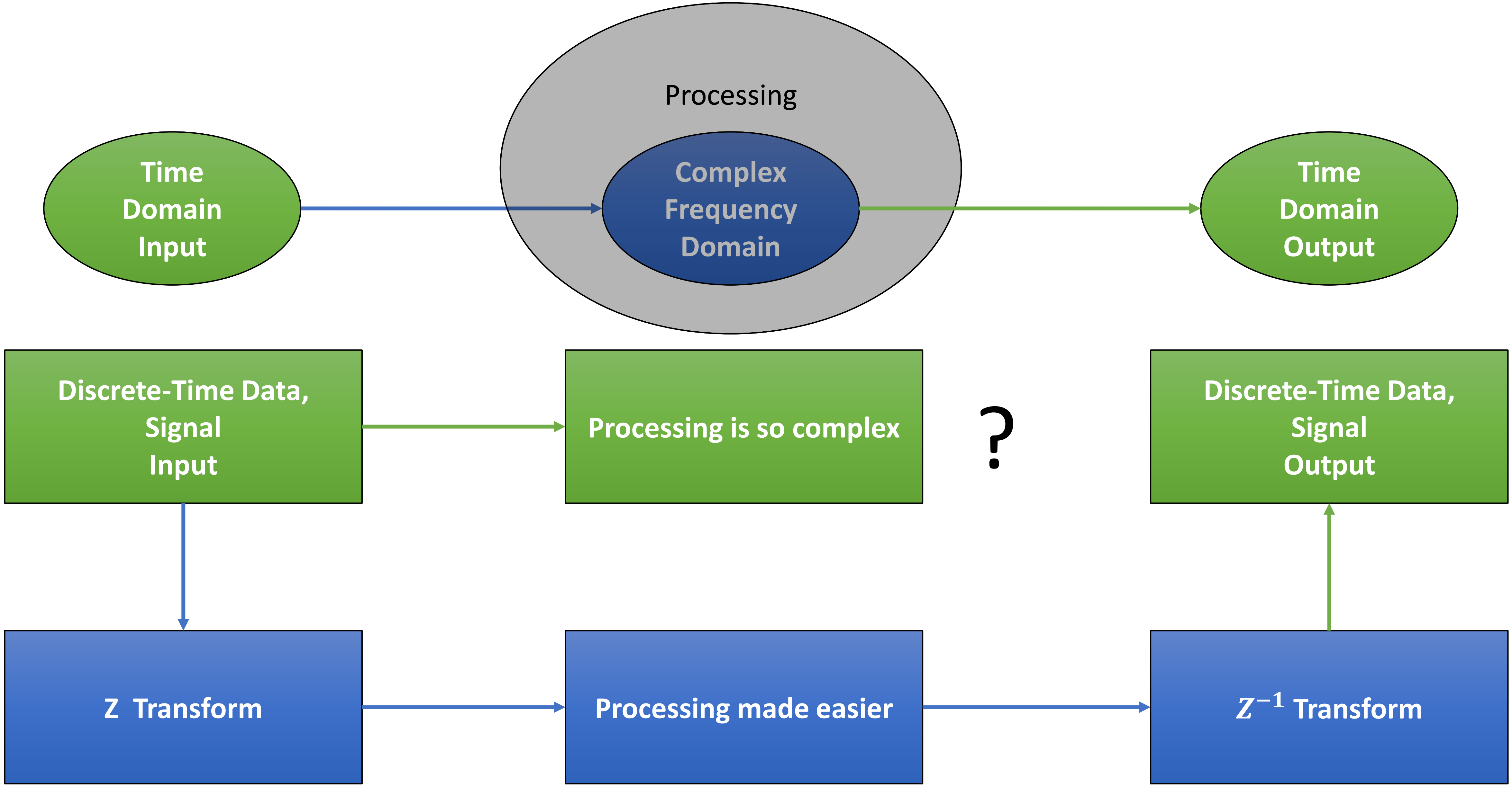
1. Z-Transform Methods
2. Table & Properties
3. Examples
4. Z-Inverse Transform Methods
5. HW

Z-Transform

The Z transform is a mathematical technique used to transform sequences of discrete-time data, from the time domain (n) to the complex frequency domain (Z). Its main purpose is to simplify the analysis and design of digital systems.

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \boldsymbol{Z}(x(n))$$

Why Z Transform?



We Have $x(n)$ **We Want** $X(z)$

Z Transform

$$X(z) = Z(x(n)) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

We Have $X(z)$ **We Want** $x(n)$

z^{-1} **Transform**

$$x(n) = z^{-1}(X(z)) = \frac{1}{2\pi j} \oint_c X(z) z^{n-1} dz$$

How do we find X(z)?

1. Direct Application of the X(z) equation
2. Power Series Expansion
3. Z-Transform Table
4. Z-Transform Properties

How do we find x(n)?

1. Power Series
2. Partial Fraction Expansion
3. Residue Theorem
4. Complex Inversion Integral

General formula

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \boldsymbol{Z}(x(n))$$

From 0 to N

$$X(z) = \sum_{n=0}^{N} x(n)z^{-n} = x(0)z^{0} + x(1)z^{-1} + x(2)z^{-2} +$$

$$x(3)z^{-3} + \cdots . + x(N)\, z^{-N}$$

From -N to N

$$X(z) = \sum_{n=-N}^{N} x(n)z^{-n} = x(-N)z^{N} + \cdots + x(-2)z^{2} + x(-1)z^{1}$$

$$+x(0)z^{0} + x(1)z^{-1}$$

$$+x(2)z^{-2} + \cdots . + x(N)\, z^{-N}$$

Example 1 Find the Z-Transform of the following signals

$$x_1(n) = u(n) \quad x_2(n) = \alpha^n u(n)$$

Solution

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} = \sum_{n=-\infty}^{\infty} u(n) z^{-n}$$

$$X_2(z) = \sum_{n=-\infty}^{\infty} x_2(n) z^{-n} = \sum_{n=-\infty}^{\infty} \alpha^n u(n) z^{-n}$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} u(n)z^{-n} = \sum_{n=0}^{\infty} (z^{-1})^n = 1 + z^{-1} + z^{-2} + \cdots$$

$$1 + r^1 + r^2 + \cdots = \frac{1}{1-r}$$

Geometric Series

$$X_1(z) = \frac{1}{1-z^{-1}} = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1}$$

Z-Transform of x1(n)

$$X_2(z) = \sum_{n=-\infty}^{\infty} \alpha^n u(n)z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n = 1 + (\alpha z^{-1})^1 + (\alpha z^{-1})^2 + \cdots$$

$$X_2(z) = \frac{1}{1-\alpha z^{-1}} = \frac{1}{1-\frac{\alpha}{z}} = \frac{z}{z-\alpha}$$

Z-Transform of x2(n)

Z-Transform Table

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n) z^{-n}$	
2	$\delta(n)$	1	$\lvert z \rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z \rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z \rvert > 1$
5	$n^2 u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z \rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z \rvert > \lvert a \rvert$
7	$e^{-na} u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z \rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z \rvert > \lvert a \rvert$

9	$\sin(an)u(n)$	$\dfrac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\dfrac{z[z - \cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\lvert z\rvert > 1$
11	$a^n \sin(bn)u(n)$	$\dfrac{[a\sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n \cos(bn)u(n)$	$\dfrac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\dfrac{[e^{-a}\sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\dfrac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z\rvert > e^{-a}$

Z-Transform Properties

Property	Sequence	z-Transform
Linearity	$ax(n) + by(n)$	$aX(z) + bY(z)$
Shift	$x(n - n_0)$	$z^{-n_0}X(z)$
Time reversal	$x(-n)$	$X(z^{-1})$
Exponentiation	$\alpha^n x(n)$	$X(\alpha^{-1}z)$
Convolution	$x(n) * y(n)$	$X(z)Y(z)$
Conjugation	$x^*(n)$	$X^*(z^*)$
Derivative	$nx(n)$	$-z\frac{dX(z)}{dz}$

Example 2 Find the Z-Transform of the following signals

$$x_1(n) = 10\sin(0.25\pi n)\,u(n)$$

$$x_2(n) = e^{-0.1n}\cos(0.25\pi n)\,u(n)$$

Solution Use the Properties and Table

$$X_1(z) = \sum_{n=-\infty}^{\infty} 10\sin(0.25\pi n)\,u(n)z^{-n}$$

$$X_2(z) = \sum_{n=-\infty}^{\infty} e^{-0.1n}\cos(0.25\pi n)\,u(n)z^{-n}$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} 10\sin(0.25\pi n)\,u(n)z^{-n}$$

$$a = 0.25\pi$$

$$= 10.\frac{z\sin a}{z^2-2z\cos a+1}$$

$$= 10.\frac{z\sin 0.25\pi}{z^2-2z\cos 0.25\pi+1} = \frac{7.07z}{z^2-1.414z+1}$$

$$X_2(z) = \sum_{n=-\infty}^{\infty} e^{-0.1n}\cos(0.25\pi n)\,u(n)z^{-n}$$

$$a = 0.1$$

$$b = 0.25\pi$$

$$= \frac{z(z-e^{-a}\cos(b))}{z^2+(2e^{-a}\cos(b))z+e^{-2a}} = \frac{z(z-e^{-0.1}\cos(0.25\pi))}{z^2+(2e^{-0.1}\cos(0.25\pi))z+e^{-0.2}}$$

$$= \frac{z(z-0.6397)}{z^2-1.2794z+0.8187}$$

Example 3 Find the Z-Transform of the following signal

$$x(n) = \{3\ 0\ 0\ 0\ 0\ \underset{\uparrow}{6}\ 1\ -4\}$$

Solution

$$X(z) = \sum_{n} x(n) z^{-n}$$

$$= 3z^{5} + 6 + z^{-1} - 4z^{-2} \ \text{ROC: } 0 < |z| < \infty$$

Example 4 — Find the Z-Transform of the following signal's Convolution

$$x_1(n) = 3\delta(n) + 2\delta(n-1)$$
$$x_2(n) = 2\delta(n) - \delta(n-1),$$

Solution

Applying z-transform on the two sequences,

$$X_1(z) = 3 + 2z^{-1}$$
$$X_2(z) = 2 - z^{-1}.$$

From the table, line 2

Therefore we get,

$$X(z) = X_1(z)X_2(z) = (3 + 2z^{-1})(2 - z^{-1})$$
$$= 6 + z^{-1} - 2z^{-2}.$$

Example 5 Prove that

$$\frac{az\sin b}{z^2-2az\cos b+a^2}=\frac{az^{-1}\sin b}{1-2az^{-1}\cos b+a^2z^{-2}}$$

Solution

$$\Rightarrow \frac{az^{-1}\sin b}{1-2az^{-1}\cos b+a^2z^{-2}}\,x\,\frac{z^2}{z^2}$$

$$=\frac{az^{-1+2}\sin b}{1z^2-2az^{-1+2}\cos b+a^2z^{-2+2}}$$

$$=\frac{az^1\sin b}{z^2-2az^1\cos b+a^2z^0}=\frac{az\sin b}{z^2-2az\cos b+a^2}$$

Inverse Z-Transform

Example 6

Find the Inverse Z-Transform of the following

$$X(z) = 2 + \frac{4z}{z-1} - \frac{z}{z-0.5}$$

Solution

We get, $x(n) = 2Z^{-1}(1) + 4Z^{-1}\left(\frac{z}{z-1}\right) - Z^{-1}\left(\frac{z}{z-0.5}\right)$

Using table, $x(n) = 2\delta(n) + 4u(n) - (0.5)^n u(n).$

Example 7

Find the Inverse Z-Transform of the following

$$X(z) = \frac{5z}{(z-1)^2} - \frac{2z}{(z-0.5)^2}$$

Solution

We get, $$x(n) = Z^{-1}\left(\frac{5z}{(z-1)^2}\right) - Z^{-1}\left(\frac{2z}{(z-0.5)^2}\right) = 5Z^{-1}\left(\frac{z}{(z-1)^2}\right) - \frac{2}{0.5}Z^{-1}\left(\frac{0.5z}{(z-0.5)^2}\right)$$

Using table, $$x(n) = 5nu(n) - 4n(0.5)^n u(n).$$

Example 8

Find inverse z-transform of $X(z) = \frac{1}{(1 - z^{-1})(1 - 0.5z^{-1})}$

Solution

First eliminate the negative power of z.

$$X(z) = \frac{z^2}{z^2(1 - z^{-1})(1 - 0.5z^{-1})} = \frac{z^2}{(z-1)(z-0.5)}$$

Dividing both sides by z: $\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{(z-1)} + \frac{B}{(z-0.5)}$

Finding the constants:

$$A = (z-1)\frac{X(z)}{z}\Big|_{z=1} = \frac{z}{(z-0.5)}\Big|_{z=1} = 2,$$

$$B = (z-0.5)\frac{X(z)}{z}\Big|_{z=0.5} = \frac{z}{(z-1)}\Big|_{z=0.5} = -1$$

$$\frac{X(z)}{z} = \frac{2}{(z-1)} + \frac{-1}{(z-0.5)}$$

$$X(z) = \frac{2z}{(z-1)} + \frac{-z}{(z-0.5)}$$

Therefore, inverse z-transform is: $x(n) = 2u(n) - (0.5)^n u(n).$

Example 9 **Find x(n)**

$$X(z) = \frac{z^2}{(z-2)(z-3)}$$

Solution

$$\frac{X(z)}{z} = \frac{z}{(z-2)(z-3)} = \frac{A}{z-2} + \frac{B}{z-3}$$

$$A = (z-2)\frac{z}{(z-2)(z-3)}\bigg|_{z=2} : B = (z-3)\frac{z}{(z-2)(z-3)}\bigg|_{z=3}$$

$$A = \frac{z}{z-3}\bigg|_{z=2} : B = \frac{z}{z-2}\bigg|_{z=3} \qquad A = -2 : B = 3$$

$$\frac{X(z)}{z} = \frac{-2}{z-2} + \frac{3}{z-3} \Rightarrow X(z) = \frac{-2z}{z-2} + \frac{3z}{z-3}$$

$$X(z) = \frac{-2z}{z-2} + \frac{3z}{z-3}$$

$$x(n) = \boldsymbol{z}^{-\mathbf{1}}\left(\frac{-2z}{z-2}\right) + \boldsymbol{z}^{-\mathbf{1}}\left(\frac{3z}{z-3}\right)$$

$$x(n) = -2\boldsymbol{z}^{-\mathbf{1}}\left(\frac{z}{z-2}\right) + 3\boldsymbol{z}^{-\mathbf{1}}\left(\frac{z}{z-3}\right)$$

$$x(n) = -2x2^{n}\boldsymbol{u}(\boldsymbol{n}) + 3x3^{n}\boldsymbol{u}(\boldsymbol{n})$$

$$x(n) = -2^{n+1}\boldsymbol{u}(\boldsymbol{n}) + 3^{n+1}\boldsymbol{u}(\boldsymbol{n})$$

Example 10

Find $y(n)$ if $Y(z) = \dfrac{z^2(z+1)}{(z-1)(z^2-z+0.5)}$.

Solution

Dividing both sides by z: $\dfrac{Y(z)}{z} = \dfrac{z(z+1)}{(z-1)(z^2-z+0.5)}$.

$$\Rightarrow \quad \frac{Y(z)}{z} = \frac{B}{z-1} + \frac{A}{(z-0.5-j0.5)} + \frac{A^*}{(z-0.5+j0.5)}$$

We first find B:

$$B = (z-1)\frac{Y(z)}{z}\bigg|_{z=1} = \frac{z(z+1)}{(z^2-z+0.5)}\bigg|_{z=1} = \frac{1\times(1+1)}{(1^2-1+0.5)} = 4.$$

Next find A:

$$A = (z-0.5-j0.5)\frac{Y(z)}{z}\bigg|_{z=0.5+j0.5} = \frac{z(z+1)}{(z-1)(z-0.5+j0.5)}\bigg|_{z=0.5+j0.5}$$

$$A = \frac{(0.5 + j0.5)(0.5 + j0.5 + 1)}{(0.5 + j0.5 - 1)(0.5 + j0.5 - 0.5 + j0.5)} = \frac{(0.5 + j0.5)(1.5 + j0.5)}{(-0.5 + j0.5)j}.$$

Using polar form $A = \frac{(0.707\angle 45^\circ)(1.58114\angle 18.43^\circ)}{(0.707\angle 135^\circ)(1\angle 90^\circ)} = 1.58114\angle -161.57^\circ$

$$A^* = \bar{A} = 1.58114\angle 161.57^\circ.$$

$$P = 0.5 + 0.5j = |P|\angle\theta = 0.707\angle 45^\circ \text{ and } P^* = |P|\angle -\theta = 0.707\angle -45^\circ.$$

Now we have: $Y(z) = \frac{4z}{z-1} + \frac{Az}{(z-P)} + \frac{A^*z}{(z-P^*)}.$

$r = \sqrt{(x^2 + y^2)}$

$\theta = \tan^{-1}(y/x)$

Therefore, the inverse z-transform is:

$$y(n) = 4Z^{-1}\left(\frac{z}{z-1}\right) + Z^{-1}\left(\frac{Az}{(z-P)} + \frac{A^*z}{(z-P^*)}\right)$$

$$\Rightarrow \begin{aligned} y(n) &= 4u(n) + 2|A|(|P|)^n \cos(n\theta + \phi)u(n) \\ &= 4u(n) + 3.1623(0.7071)^n \cos(45^\circ n - 161.57^\circ)u(n) \end{aligned}$$

Example 11

Find the Inverse Z-Transform of the following

$$X(z) = \frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}$$

Solution

$$= \frac{A}{(1+z^{-1})} + \frac{B}{(1+2z^{-1})}$$

$$A = 2, B = -1$$

$$\text{Hence}, x(n) = [2(-1)^n - (-2)^n]\,u(n)$$

Cauchy residue theorem. Let $f(z)$ be a function of the complex variable z and C be a closed path in the z-plane. If the derivative $df(z)/dz$ exists on and inside the contour C and if $f(z)$ has no poles at $z = z_0$, then

$$\frac{1}{2\pi j}\oint_C \frac{f(z)}{z - z_0}dz = \begin{cases} f(z_0), & \text{if } z_0 \text{ is inside } C \\ 0, & \text{if } z_0 \text{ is outside } C \end{cases}$$

More generally, if the $(k+1)$-order derivative of $f(z)$ exists and $f(z)$ has no poles at $z = z_0$, then

$$\frac{1}{2\pi j}\oint_C \frac{f(z)}{(z - z_0)^k}dz = \begin{cases} \dfrac{1}{(k-1)!}\dfrac{d^{k-1}f(z)}{dz^{k-1}}\bigg|_{z=z_0}, & \text{if } z_0 \text{ is inside } C \\ 0, & \text{if } z_0 \text{ is outside } C \end{cases}$$

The values on the right-hand side of (3.4.2) and (3.4.3) are called the residues of the pole at $z = z_0$. The results in (3.4.2) and (3.4.3) are two forms of the *Cauchy residue theorem*.

Multiple poles (m)

$$x(n) = \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m z^{n-1} X(z)\right)\Big|_{z=a}$$

Simple pole

$$x(n) = \sum residues\ of\ X(z)\,.\,X(z)\ \ z^{n-1}\Big|_{at\ the\ pole}$$

Example 12 **Find the x(-16) value**

$$X(z) = \frac{z^{18}}{(z-1/2)(z-1)(z-4)} \qquad ROC = |Z| = 1$$

Solution **Using the Residue Method**

$$x(n) = \sum residues\ of\ X(z)\,.\,X(z)\ z^{n-1}\Big|_{at\ the\ pole}$$

$$x(n) = \left(z-\frac{1}{2}\right)\frac{z^{18}z^{n-1}}{\left(z-\frac{1}{2}\right)(z-1)(z-4)}\Bigg|_{z=\frac{1}{2}} + (z-1)\frac{z^{18}z^{n-1}}{\left(z-\frac{1}{2}\right)(z-1)(z-4)}\Bigg|_{z=1}$$

$$+ (z-4)\,\frac{z^{18}z^{n-1}}{\left(z-\frac{1}{2}\right)(z-1)(z-4)}\Bigg|_{z=4}$$

Since $ROC = |Z| = 1$

$$x(n) = \left.\frac{z^{18}z^{n-1}}{(z-1)(z-4)}\right|_{z=\frac{1}{2}} + \left.\frac{z^{18}z^{n-1}}{\left(z-\frac{1}{2}\right)(z-4)}\right|_{z=1} + \left.\frac{z^{18}z^{n-1}}{\left(z-\frac{1}{2}\right)(z-1)}\right|_{z=4}$$

$$x(n) = \left.\frac{z^{18}z^{n-1}}{\left(z-\frac{1}{2}\right)(z-4)}\right|_{z=1} = \frac{1^{18}1^{n-1}}{(1-1/2)(1-4)} = \frac{1^{n-1}}{3/2}$$

We want x(-16)

$$x(-16) = \frac{1^{-16-1}}{3/2} = \frac{2}{3}$$

Example 13

Find x(n)

$$X(z) = \frac{2z^2 + 11z}{2z^2 - 3z - 2}$$

Solution

$$X(z) = \frac{2z^2 + 11z}{2z^2 - 3z - 2} = \frac{z(2z + 11)}{(2z + 1)(z - 2)}$$

$$x(n) = \sum residues\ of\ X(z)\ .\ X(z)\ z^{n-1}\Big|_{at\ the\ pole}$$

$$x(n) = \frac{z(2z+11)}{(2z+1)(z-2)}$$

$$x(n) = (2z+1)\frac{z(2z+11)z^{n-1}}{(2z+1)(z-2)}\bigg|_{z=-\frac{1}{2}} + (z-2)\frac{z(2z+11)z^{n-1}}{(2z+1)(z-2)}\bigg|_{z=2}$$

$$x(n) = z^n\frac{(2z+11)}{(z-2)}\bigg|_{z=-\frac{1}{2}} + z^n\frac{(2z+11)}{(2z+1)}\bigg|_{z=2}$$

$$x(n) = -1/2^n\frac{(2x-\frac{1}{2})+11}{-\frac{1}{2}-2} + 2^n\frac{2x2+11}{2.x2+1} = -1/2^n\frac{10}{-2.5} + 2^n\frac{15}{5}$$

$$x(n) = \left(3.2^n - 4\left(-\frac{1}{2}\right)^n\right)u(n)$$

Home Work (Due date 18-11-2023)

Q1: Find the Z-Transform of the Signals

1 $$x(n) = \{1\ 0\ \underset{\uparrow}{1}\ 0\ 1\ 0\}$$

2 $$x(n) = \{-1\ 0\ 1\ \underset{\uparrow}{0}\ -1\ 0\}$$

Home Work

Q2: Find the x(1)

$$X(z) = \frac{z^4}{(z-4)\,(z+3)}$$

Q3: Find the Inverse Z-Transform for

$$X(z) = \frac{z^{-6}}{1-z^{-1}} + \frac{z^{-7}}{1-z^{-1}}$$

Extra Home Work (Choose either Q4 or Q5)

Q4: Find the Inverse Z-Transform Using Partial Fraction Method

$$X(z) = \frac{z^3 + 1}{z^3 - z^2 - z - 2}$$

Q5: Compute the unit step response of the system with impulse response

$$h(n) = \begin{cases} 3^n & n < 0 \\ (\frac{2}{5})^n & n \geq 0 \end{cases}$$

University of Technology
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Engineering Department

Digital Signal Processing I

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Lec. 6: Discrete-Time Fourier Transform (DTFT): 2024-Nov-03

Lecture Outline

1. DTFT
2. Table & Properties
3. Inverse DTFT Methods
4. Frequency Response
5. Group Delay
6. HW

DTFT-Transform

General Formula $$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x(n)e^{-jn\omega}$$

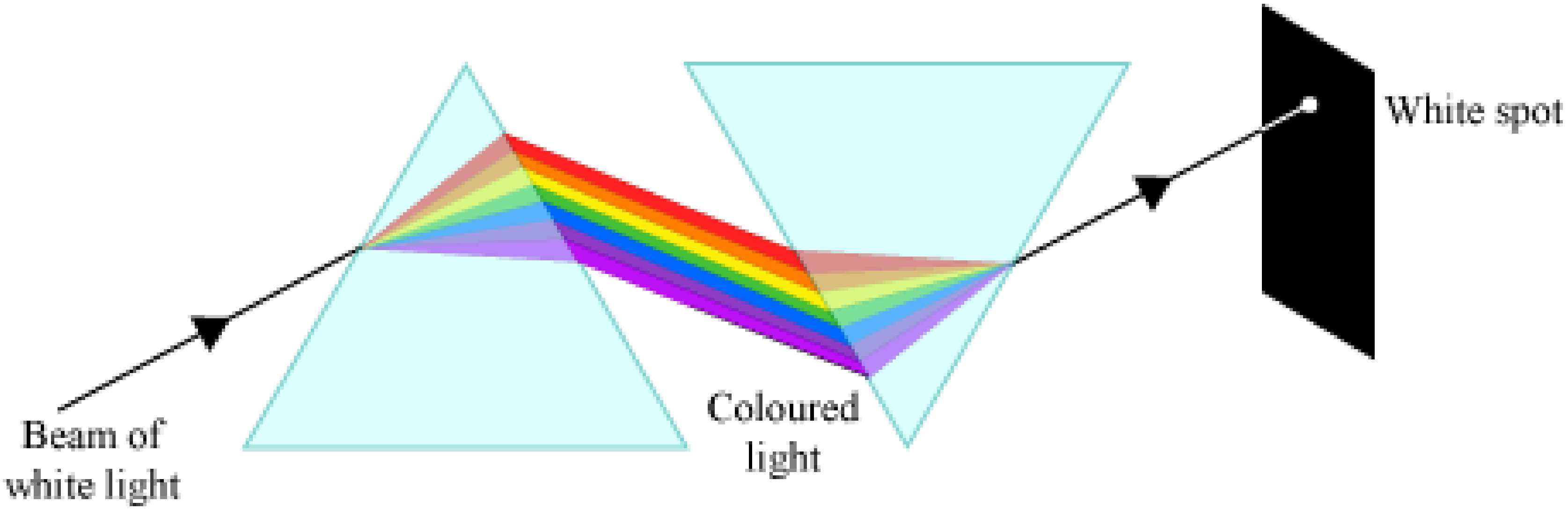
From –N to N $$X\left(e^{j\omega}\right) = \sum_{n=-N}^{N} x(n)e^{-jn\omega}$$

$$= x(-N)e^{jN\omega} + x(-(N-1))e^{-j(N-1)\omega} + \cdots + x(0)e^{0} + x(1)e^{-j\omega} + x(2)e^{-j2\omega} + \cdots + x(N)e^{-jN\omega}$$

From 0 to N

$$X(e^{j\omega}) = \sum_{n=0}^{N} x(n)e^{-jn\omega}$$

$$= x(0)e^{0} + x(1)e^{-j\omega} + x(2)e^{-j2\omega} + \cdots + x(N)e^{-jN\omega}$$



EX-1: Find the DTFT for the following signal

$$x(n) = \{\underset{\uparrow}{1} \;\; 3 \;\; -2 \;\; 5 \;\; 2\}$$

Solution

$$X(e^{j\omega}) = \sum_{n=0}^{4} x(n)e^{-jn\omega}$$

$$= x(0)e^{0} + x(1)e^{-j\omega} + x(2)e^{-j2\omega} + x(3)e^{-j3\omega} + x(4)e^{-j4\omega}$$

$$X(e^{j\omega}) = 1e^{0} + 3e^{-j\omega} - 2e^{-j2\omega} + 5e^{-j3\omega} + 2e^{-j4\omega}$$

$$\boxed{X(e^{j\omega}) = 1 + 3e^{-j\omega} - 2e^{-j2\omega} + 5e^{-j3\omega} + 2e^{-j4\omega}}$$

EX-2: Find the DTFT for the following signals

$$x_1(n) = \alpha^n u(n) \qquad |\alpha| < 1$$

Solution

$$X_1(e^{j\omega}) = \sum_{n=0}^{\infty} \alpha^n e^{-jn\omega} = \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n$$

$$X_1(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}$$

$$x_2(n) = -\alpha^n u(-n-1) \qquad |\alpha| > 1$$

Solution

$$X_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_2(n) e^{-jn\omega} = -\sum_{n=-\infty}^{-1} \alpha^n e^{-jn\omega}$$

$$X_2(e^{j\omega}) = -\sum_{n=1}^{\infty} \alpha^{-n} e^{jn\omega} = -\sum_{n=0}^{\infty} (\alpha^{-1} e^{j\omega})^n + 1$$

$$X_2(e^{j\omega}) = -\frac{1}{1 - \alpha^{-1} e^{j\omega}} + 1 = \frac{1}{1 - \alpha e^{-j\omega}}$$

Therefore, $x_1(n) = \alpha^n u(n)$ and $x_2(n) = -\alpha^n u(-n-1)$ both have the same DTFT.

DTFT TABLE

Sequence	Discrete-Time Fourier Transform
$\delta(n)$	1
$\delta(n - n_0)$	$e^{-jn_0\omega}$
1	$2\pi\,\delta(\omega)$
$e^{jn\omega_0}$	$2\pi\,\delta(\omega - \omega_0)$
$a^n u(n), \quad \lvert a\rvert < 1$	$\dfrac{1}{1 - ae^{-j\omega}}$
$-a^n u(-n-1), \quad \lvert a\rvert > 1$	$\dfrac{1}{1 - ae^{-j\omega}}$
$(n+1)\,a^n u(n), \quad \lvert a\rvert < 1$	$\dfrac{1}{(1 - ae^{-j\omega})^2}$
$\cos n\omega_0$	$\pi\,\delta(\omega + \omega_0) + \pi\,\delta(\omega - \omega_0)$

DTFT PROPERTIES

Property	Sequence	Discrete-Time Fourier Transform
Linearity	$ax(n) + by(n)$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Shift	$x(n - n_0)$	$e^{-jn_0\omega}X(e^{j\omega})$
Time-reversal	$x(-n)$	$X(e^{-j\omega})$
Modulation	$e^{jn\omega_0}x(n)$	$X(e^{j(\omega-\omega_0)})$
Convolution	$x(n) * y(n)$	$X(e^{j\omega})Y(e^{j\omega})$
Conjugation	$x^*(n)$	$X^*(e^{-j\omega})$
Derivative	$nx(n)$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Multiplication	$x(n)y(n)$	$\dfrac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$

INVERSE DTFT

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{jn\omega}d\omega$$

EX-3

Suppose $X(e^{j\omega})$ consists of an impulse at frequency $\omega = \omega_0$:

$$X(e^{j\omega}) = \delta(\omega - \omega_0)$$

Using the inverse DTFT, we have

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{jn\omega}d\omega = \frac{1}{2\pi}e^{jn\omega_0}$$

Note that although $x(n)$ is not absolutely summable, by allowing the DTFT to contain impulses, we may consider the DTFT of sequences that contain complex exponentials. As another example, if

$$X(e^{j\omega}) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

computing the inverse DTFT, we find

$$x(n) = \tfrac{1}{2}e^{jn\omega_0} + \tfrac{1}{2}e^{-jn\omega_0} = \cos(n\omega_0)$$

EX -4

Consider the linear shift-invariant system characterized by the second-order linear constant coefficient difference equation

$$y(n) = 1.3433y(n-1) - 0.9025y(n-2) + x(n) - 1.4142x(n-1) + x(n-2)$$

The frequency response may be found by inspection without solving the difference equation for $h(n)$ as follows:

$$H(e^{j\omega}) = \frac{1 - 1.4142e^{-j\omega} + e^{-2j\omega}}{1 - 1.3433e^{-j\omega} + 0.9025e^{-2j\omega}}$$

EX -5

$$H(e^{j\omega}) = \frac{1 + e^{-2j\omega}}{2 - e^{-j\omega} + 0.5e^{-2j\omega}}$$

a difference equation may be easily found that will implement this system. First, dividing numerator and denominator by 2 and rewriting the frequency response as follows,

$$H(e^{j\omega}) = \frac{0.5 + 0.5e^{-2j\omega}}{1 - 0.5e^{-j\omega} + 0.25e^{-2j\omega}}$$

we see that a difference equation for this system is

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + 0.5x(n) + 0.5x(n-2)$$

EX -6

If the unit sample response of an LSI system is

$$h(n) = \alpha^n u(n)$$

let us find the response of the system to the input $x(n) = \beta^n u(n)$, where $|\alpha| < 1$, $|\beta| < 1$, and $\alpha \neq \beta$. Because the output of the system is the convolution of $x(n)$ with $h(n)$,

$$y(n) = h(n) * x(n)$$

the DTFT of $y(n)$ is

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}} \frac{1}{1-\beta e^{-j\omega}}$$

Therefore, all that is required is to find the inverse DTFT of $Y(e^{j\omega})$. This may be done easily by expanding $Y(e^{j\omega})$ as follows:

$$Y(e^{j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})(1-\beta e^{-j\omega})} = \frac{A}{1-\alpha e^{-j\omega}} + \frac{B}{1-\beta e^{-j\omega}}$$

where A and B are constants that are to be determined. Expressing the right-hand side of this expansion over a common denominator,

SOL

$$\frac{1}{(1-\alpha e^{-j\omega})(1-\beta e^{-j\omega})} = \frac{(A+B)-(A\beta+B\alpha)e^{-j\omega}}{(1-\alpha e^{-j\omega})(1-\beta e^{-j\omega})}$$

and equating coefficients, the constants A and B may be found by solving the pair of equations

$$A+B=1$$
$$A\beta+B\alpha=0$$

The result is

$$A=\frac{\alpha}{\alpha-\beta} \qquad B=-\frac{\beta}{\alpha-\beta}$$

Therefore,

$$Y(e^{j\omega})=\frac{\alpha/(\alpha-\beta)}{1-\alpha e^{-j\omega}}-\frac{\beta/(\alpha-\beta)}{1-\beta e^{-j\omega}}$$

and it follows that the inverse DTFT is

$$y(n)=\left[\frac{\alpha}{\alpha-\beta}\alpha^n-\frac{\beta}{\alpha-\beta}\beta^n\right]u(n)$$

EX -7

Let us solve the following LCCDE for $y(n)$ assuming zero initial conditions,

$$y(n) - 0.25y(n-1) = x(n) - x(n-2)$$

for $x(n) = \delta(n)$. We begin by taking the DTFT of each term in the difference equation:

$$Y(e^{j\omega}) - 0.25e^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega}) - e^{-2j\omega}X(e^{j\omega})$$

Because the DTFT of $x(n)$ is $X(e^{j\omega}) = 1$,

$$Y(e^{j\omega}) = \frac{1 - e^{-2j\omega}}{1 - 0.25e^{-j\omega}} = \frac{1}{1 - 0.25e^{-j\omega}} - \frac{e^{-2j\omega}}{1 - 0.25e^{-j\omega}}$$

Using the DTFT pair

$$(0.25)^n u(n) \overset{DTFT}{\Longleftrightarrow} \frac{1}{1 - 0.25e^{-j\omega}}$$

the inverse DTFT of $Y(e^{j\omega})$ may be easily found using the linearity and shift properties,

$$y(n) = (0.25)^n u(n) - (0.25)^{n-2} u(n-2)$$

EX-8 Let $h(n)$ be the unit sample response of an LSI system. Find the frequency response when

(*a*) $h(n) = \delta(n) + 6\delta(n-1) + 3\delta(n-2)$

(*b*) $h(n) = \left(\frac{1}{3}\right)^{n+2} u(n-2)$.

SOL

(*a*) This system has a unit sample response that is finite in length. Therefore, the frequency response is a polynomial in $e^{j\omega}$, with the coefficients of the polynomial equal to the values of $h(n)$:

$$H(e^{j\omega}) = 1 + 6e^{-j\omega} + 3e^{-2j\omega}$$

This may be shown more formally by writing

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-jn\omega} = \sum_{n=-\infty}^{\infty} [\delta(n) + 6\delta(n-1) + 3\delta(n-2)]e^{-jn\omega}$$

Because

$$\sum_{n=-\infty}^{\infty} \delta(n-n_0)e^{-jn\omega} = e^{-jn_0\omega}$$

then

$$H(e^{j\omega}) = 1 + 6e^{-j\omega} + 3e^{-2j\omega}$$

Sol

(*b*) For the second system, the frequency response is

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-jn\omega} = \sum_{n=2}^{\infty} \left(\tfrac{1}{3}\right)^{n+2} e^{-jn\omega}$$

Changing the limits on the sum so that it begins with $n = 0$, we have

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} \left(\tfrac{1}{3}\right)^{n+4} e^{-j(n+2)\omega} = \left(\tfrac{1}{3}\right)^{4} e^{-2j\omega} \textstyle\sum_{n=0}^{\infty} \left(\tfrac{1}{3}e^{-j\omega}\right)^{n}$$

Using the geometric series, we find

$$H(e^{j\omega}) = \left(\tfrac{1}{3}\right)^{4} \frac{e^{-2j\omega}}{1-\frac{1}{3}e^{-j\omega}}$$

EX-9

An Lth-order moving average filter is a linear shift-invariant system that, for an input $x(n)$, produces the output

$$y(n) = \frac{1}{L+1}\sum_{k=0}^{L} x(n-k)$$

Find the frequency response of this system.

If the input to the moving average filter is $x(n) = \delta(n)$, the response, by definition, will be the unit sample response, $h(n)$. Therefore,

$$h(n) = \frac{1}{L+1}\sum_{k=0}^{L} \delta(n-k)$$

and

$$H(e^{j\omega}) = \frac{1}{L+1}\sum_{k=0}^{L} e^{-jk\omega}$$

Sol

Using the geometric series, we have

$$H(e^{j\omega}) = \frac{1}{L+1} \frac{1-e^{-j(L+1)\omega}}{1-e^{-j\omega}}$$

Factoring out a term $e^{-j(L+1)\omega/2}$ from the numerator, and a term $e^{-j\omega/2}$ from the denominator, we have

$$H(e^{j\omega}) = \frac{1}{L+1} e^{-jL\omega/2} \frac{e^{j(L+1)\omega/2} - e^{-j(L+1)\omega/2}}{e^{j\omega/2} - e^{-j\omega/2}}$$

or

$$H(e^{j\omega}) = \frac{1}{L+1} e^{-jL\omega/2} \frac{\sin(L+1)\omega/2}{\sin \omega/2}$$

EX-10

Find the magnitude, phase, and group delay of a system that has a unit sample response

$$h(n) = \delta(n) - \alpha\delta(n-1)$$

where α is real.

The frequency response of this system is

$$H(e^{j\omega}) = 1 - \alpha e^{-j\omega} = 1 - \alpha\cos\omega + j\alpha\sin\omega$$

Therefore, the magnitude squared is

$$|H(e^{j\omega})|^2 = H(e^{j\omega})H^*(e^{j\omega}) = (1 - \alpha e^{-j\omega})\cdot(1 - \alpha e^{j\omega}) = 1 + \alpha^2 - 2\alpha\cos\omega$$

The phase, on the other hand, is

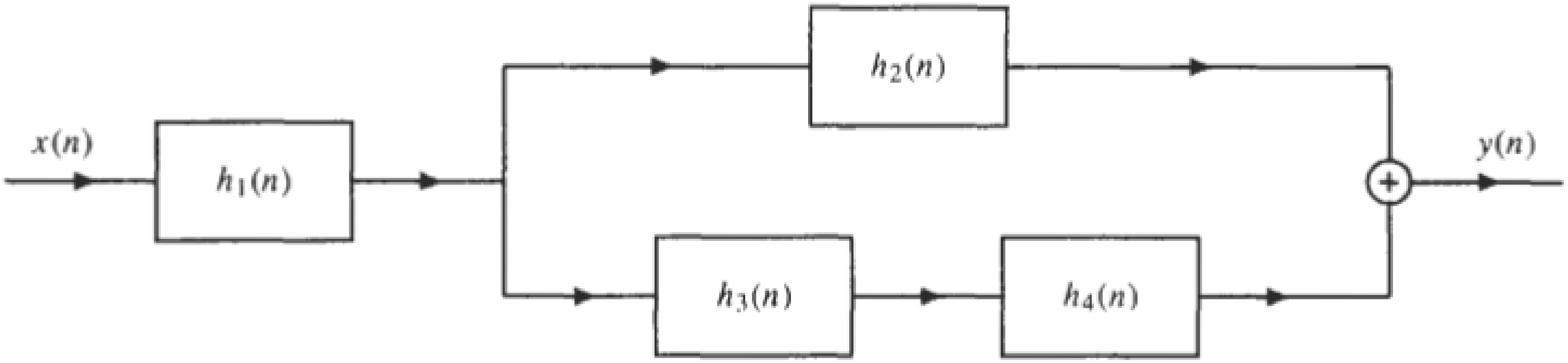
$$\phi_h(\omega) = \tan^{-1}\frac{H_I(e^{j\omega})}{H_R(e^{j\omega})} = \tan^{-1}\frac{\alpha\sin\omega}{1 - \alpha\cos\omega}$$

Finally, the group delay may be found by differentiating the phase (see Prob. 2.19). Alternatively, we may note that because this system is the inverse of the one considered in Example 2.2.2, the phase and the group delay are simply the negative of those found in the example. Therefore, we have

$$\tau_h(\omega) = \frac{\alpha^2 - \alpha\cos\omega}{1 + \alpha^2 - 2\alpha\cos\omega}$$

EX-11

Consider the interconnection of LSI systems shown in the following figure:



(a) Express the frequency response of the overall system in terms of $H_1(e^{j\omega})$, $H_2(e^{j\omega})$, $H_3(e^{j\omega})$, and $H_4(e^{j\omega})$.

(b) Find the frequency response if

$$h_1(n) = \delta(n) + 2\delta(n-2) + \delta(n-4)$$

$$h_2(n) = h_3(n) = (0.2)^n u(n)$$

$$h_4(n) = \delta(n-2)$$

Sol

(*a*) Because $h_2(n)$ is in parallel with the cascade of $h_3(n)$ and $h_4(n)$, the frequency response of the parallel network is

$$G(e^{j\omega}) = H_2(e^{j\omega}) + H_3(e^{j\omega})H_4(e^{j\omega})$$

With $h_1(n)$ being in cascade with $g(n)$, the overall frequency response becomes

$$H(e^{j\omega}) = H_1(e^{j\omega})[H_2(e^{j\omega}) + H_3(e^{j\omega})H_4(e^{j\omega})]$$

(*b*) The frequency responses of the systems in this interconnection are

$$\begin{aligned} H_1(e^{j\omega}) &= 1 + 2e^{-j2\omega} + e^{-j4\omega} = (1 + e^{-j2\omega})^2 \\ H_2(e^{j\omega}) &= H_3(e^{j\omega}) = \frac{1}{1 - 0.2e^{-j\omega}} \\ H_4(e^{j\omega}) &= e^{-j2\omega} \end{aligned}$$

Therefore,

$$\begin{aligned} H(e^{j\omega}) &= H_1(e^{j\omega})[H_2(e^{j\omega}) + H_3(e^{j\omega})H_4(e^{j\omega})] \\ &= H_1(e^{j\omega})H_2(e^{j\omega})[1 + H_4(e^{j\omega})] \\ &= \frac{(1 + e^{-j2\omega})^3}{1 - 0.2e^{-j\omega}} \end{aligned}$$

EX-12

FIND THE GROUP VELOCITY

$$H_1(e^{j\omega}) = 1 - \alpha e^{-j\omega}$$

SOL

$$H_1(e^{j\omega}) = 1 - \alpha \cos\omega + j\alpha \sin\omega$$

Therefore, the phase is

$$\phi_1(\omega) = \tan^{-1}\frac{\alpha \sin\omega}{1 - \alpha\cos\omega}$$

Because

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$$

the group delay is

$$\tau_1(\omega) = -\frac{d}{d\omega}\phi_1(\omega) = -\frac{1}{1+\left(\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right)^2}\frac{d}{d\omega}\left(\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right)$$

Therefore,

$$\tau_1(\omega) = -\frac{1}{1+\left(\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right)^2}\frac{(1-\alpha\cos\omega)\alpha\cos\omega - (\alpha\sin\omega)^2}{(1-\alpha\cos\omega)^2}$$

EX-13

FIND THE DTFT

$$x_1(n) = \left(\tfrac{1}{2}\right)^n u(n+3)$$

SOL

$$X_1(e^{j\omega}) = \sum_{n=-3}^{\infty} \left(\tfrac{1}{2}\right)^n e^{-jn\omega} = \sum_{n=-3}^{\infty} \left(\tfrac{1}{2}e^{-j\omega}\right)^n$$

$$= \left(\tfrac{1}{2}e^{-j\omega}\right)^{-3} \sum_{n=0}^{\infty} \left(\tfrac{1}{2}e^{-j\omega}\right)^n = \frac{8e^{3j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

Home Work

Q1: Find the DTFT of the Signal 3M

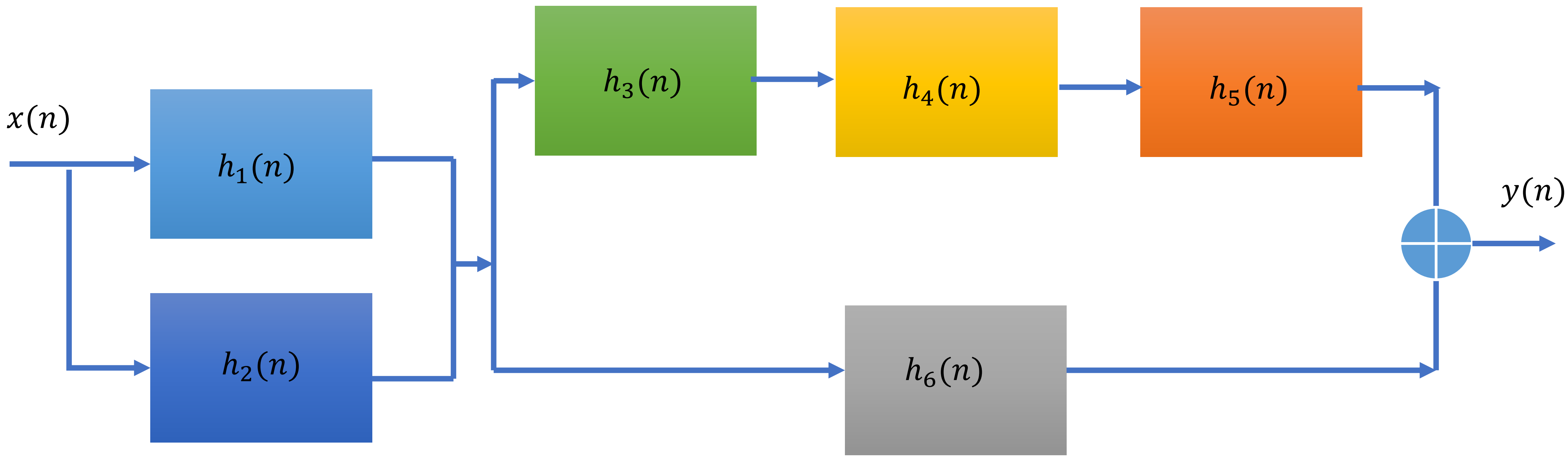
$$x(n) = \alpha^n \sin(n\omega_0)\, u(n)$$

Q2: Find the Group delay 3M

$$H(e^{j\omega}) = \frac{1}{1 - 2\alpha\cos\theta e^{-j\omega} + \alpha^2 e^{-j2\omega}}$$

Home Work

Q3: Find the frequency response for the below system 10M



$$h_1(n) = \delta(n-2) + \delta(n+4) \qquad h_3(n) = h_4(n) = (0.2)^n u(n)$$

$$h_2(n) = 4\delta(n-2) + 3\delta(n+3) \qquad h_5(n) = h_6(n) = (0.1)^n u(n)$$

University of Technology
Laser & Optoelectronics Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

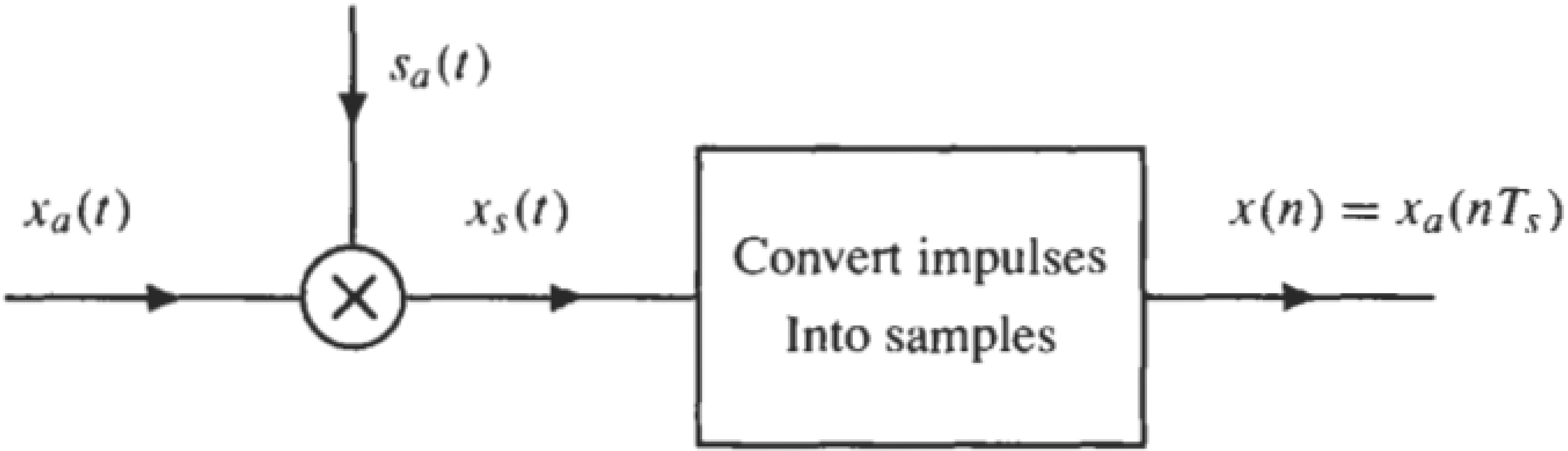
Lec. 7: Sampling and Quantization: 2024-Nov-10

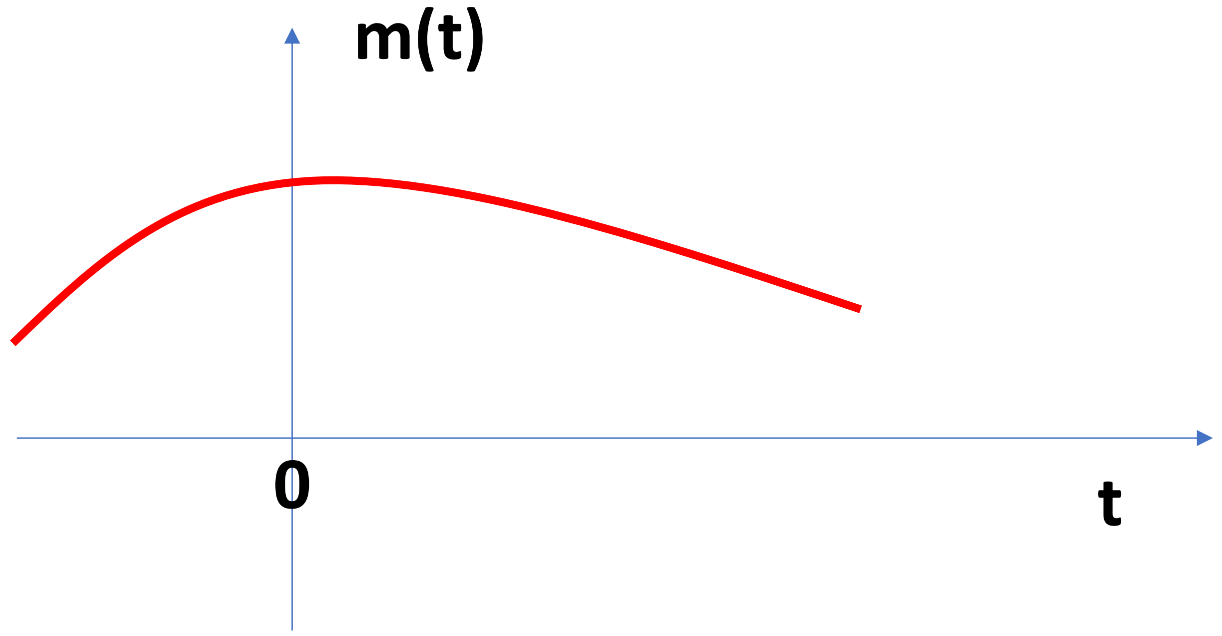
Lecture Outline

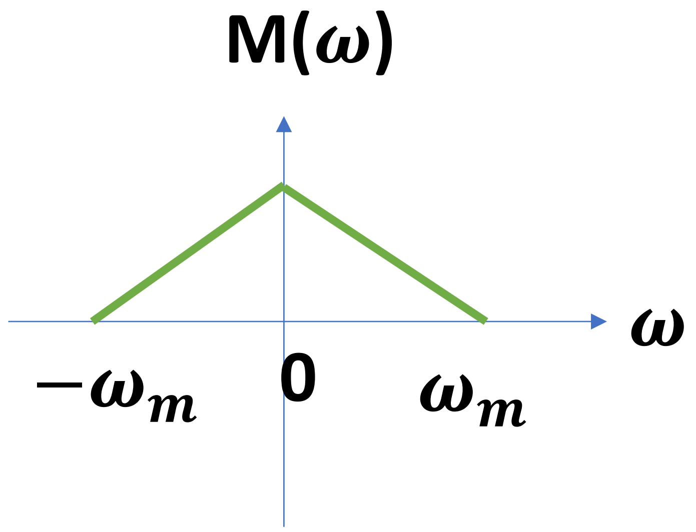
1. Sampling
2. Recovering
3. Aliasing
4. Bandpass Signal
5. Quantization
6. HW

Sampling

Is the Process of Reduction of a Continuous-Time Signal to a Discrete-Time Signal



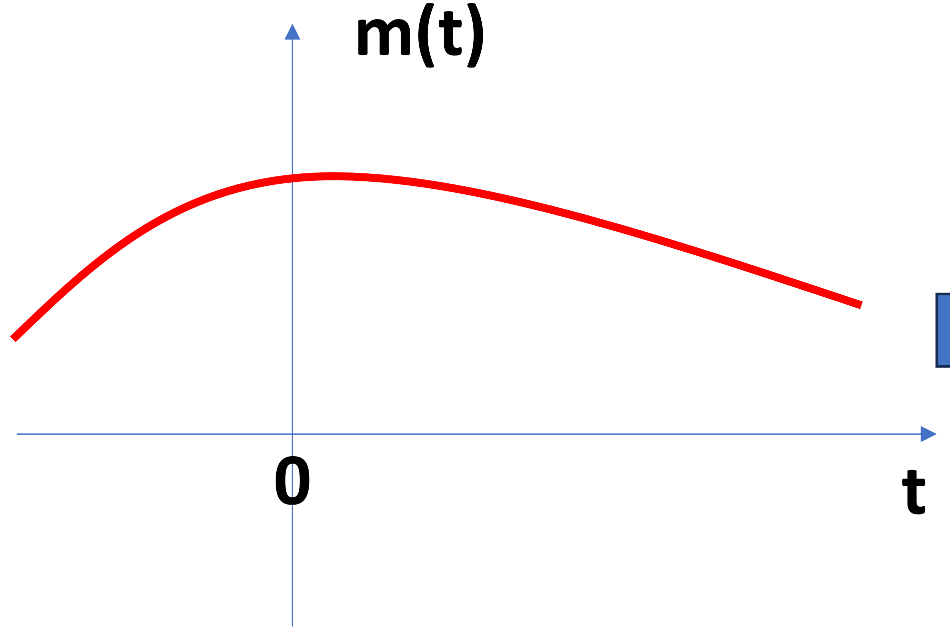




Bandlimited Signal: The Message have a Non-Zero Fourier Transform for a limited Period

ω_m: Maximum Frequency Component of the Message Signal

M(ω): Fourier Transform of the Original Message m(t)



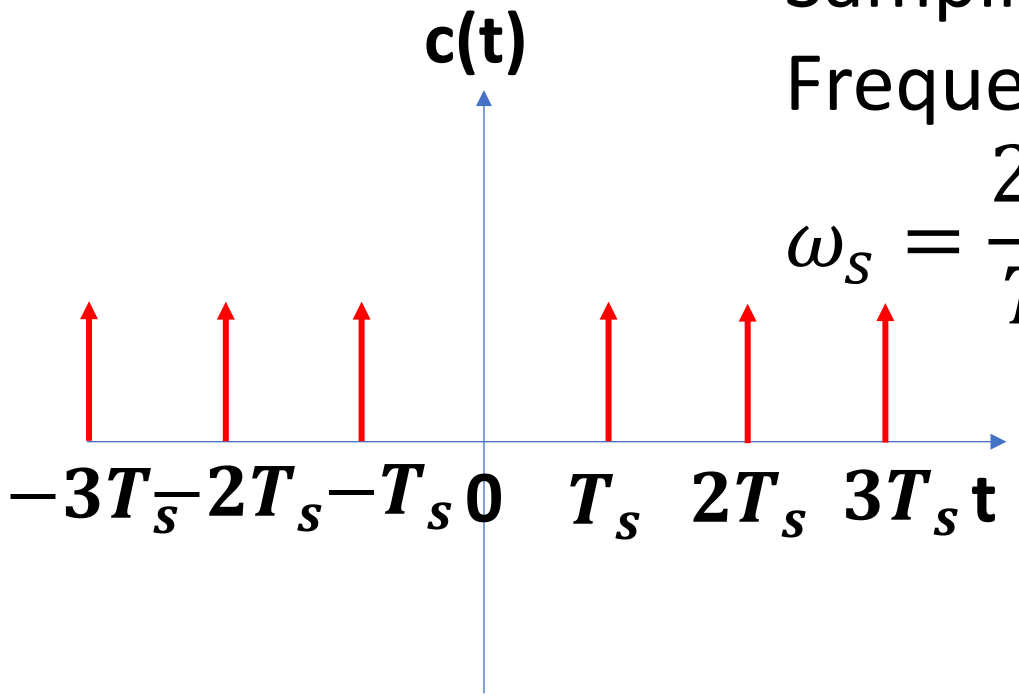
c(t): Periodic Impulse Train

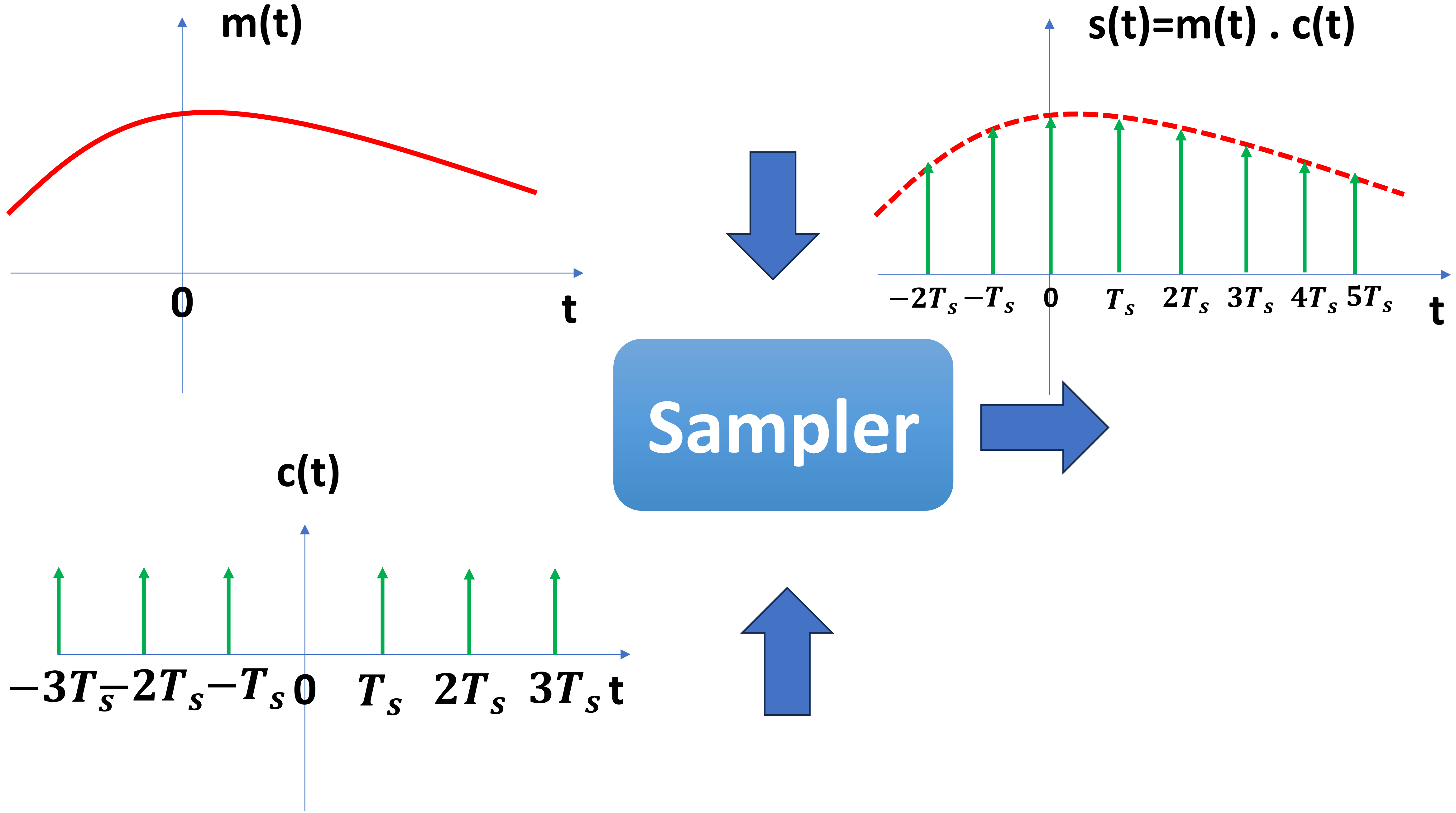
T_s: Sampling Period \ Sampling Interval

Sampling Frequency

$$\omega_s = \frac{2\pi}{T_s}$$

$$c(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$




m(t)
0
t
s(t)=m(t) . c(t)
$-2T_s$ $-T_s$ 0 T_s $2T_s$ $3T_s$ $4T_s$ $5T_s$ t
Sampler
c(t)
$-3T_s$ $-2T_s$ $-T_s$ 0 T_s $2T_s$ $3T_s$ t

We want to evaluate the Fourier Transform of the Sampled Signal

1 $$s(t) = m(t).c(t)$$

2 $$S(\omega) = \frac{1}{2\pi}(M(\omega) * C(\omega))$$

3 $$C(\omega) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)$$

4 $$S(\omega) = \frac{1}{2\pi}(M(\omega) * \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s))$$

5 $$S(\omega) = \frac{\omega_s}{2\pi}(M(\omega) * \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s))$$

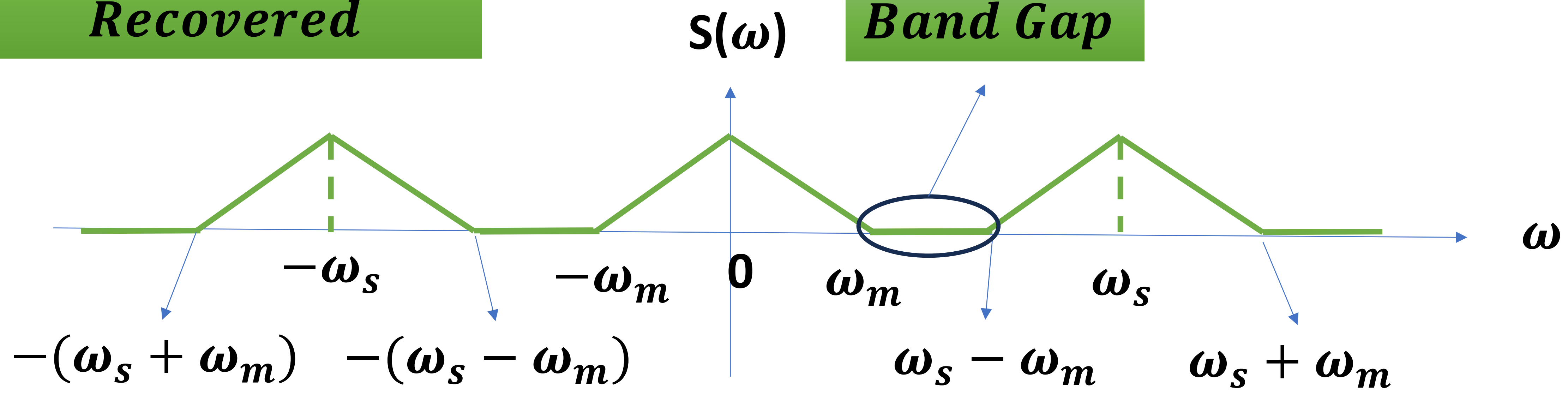
6 $$S(\omega) = \frac{1}{T_s}(\sum_{n=-\infty}^{\infty} M(\omega) * \delta(\omega - n\omega_s))$$

7 $$S(\omega) = \frac{1}{T_s}(\sum_{n=-\infty}^{\infty} M(\omega - n\omega_s))$$

The Fourier Transform of the Sampled Signal

8 $$S(\omega) = \frac{1}{T_s}(\ldots + M(\omega+\omega_s) + M(\omega) + M(\omega-\omega_s) + \cdots)$$

Signal Can be Recovered

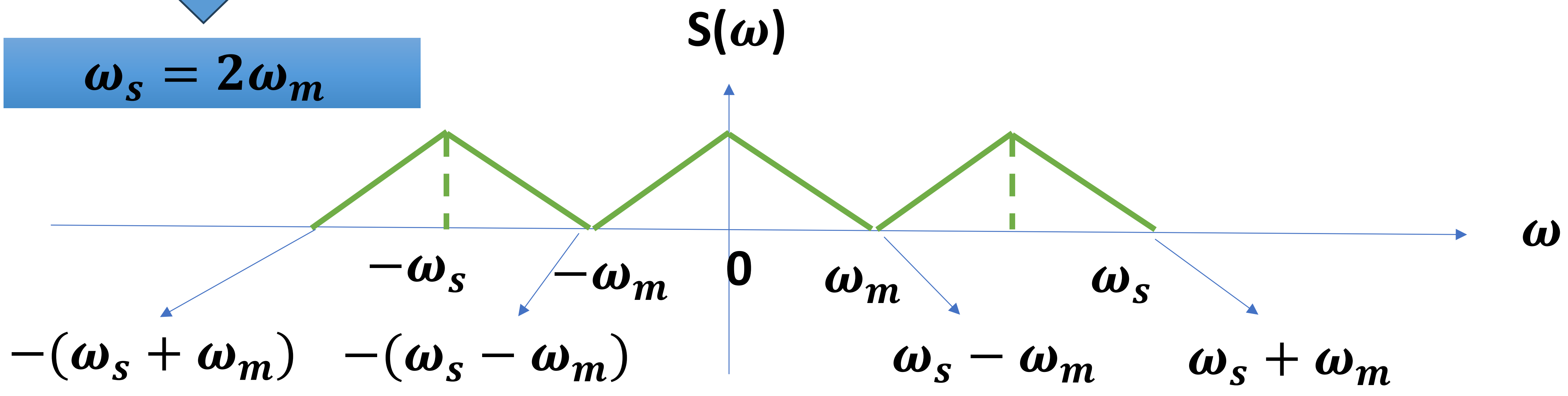


$\omega_s - \omega_m > \omega_m$ → $\omega_s > 2\omega_m$

$$\boldsymbol{\omega_s - \omega_m = \omega_m}$$

$$S(\omega) = \frac{1}{T_s}(\ldots + M(\omega + \omega_s) + M(\omega) + M(\omega - \omega_s) + \cdots)$$

$$\boldsymbol{\omega_s = 2\omega_m}$$

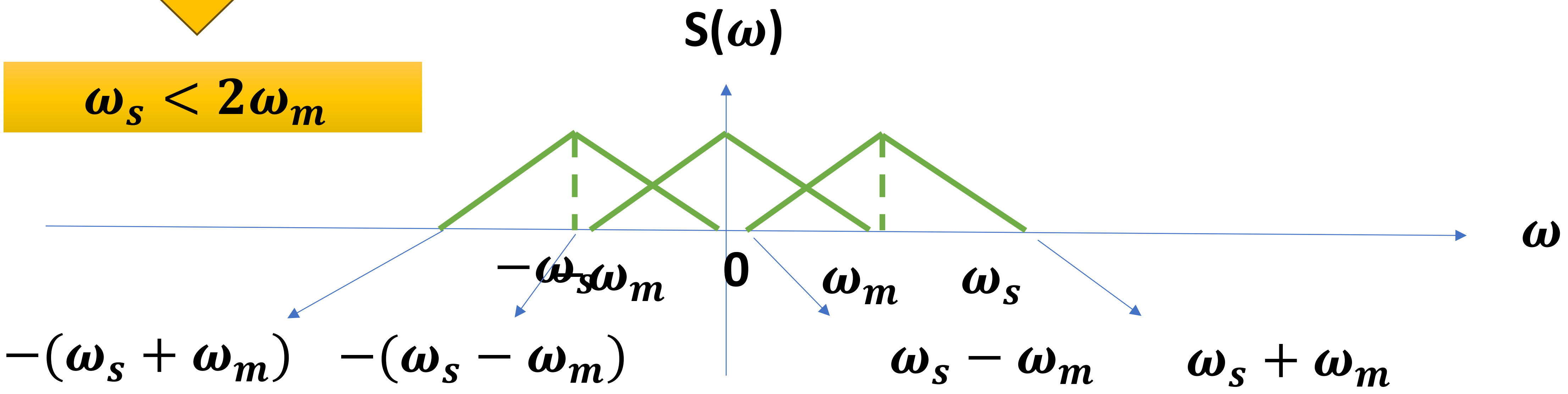


Signal Can be Recovered

$$\boldsymbol{\omega_s - \omega_m < \omega_m}$$

$$S(\omega) = \frac{1}{T_s}\left(\dots + \; M(\omega + \omega_s) + M(\omega) + M(\omega - \omega_s) + \cdots\right)$$

$$\boldsymbol{\omega_s < 2\omega_m}$$



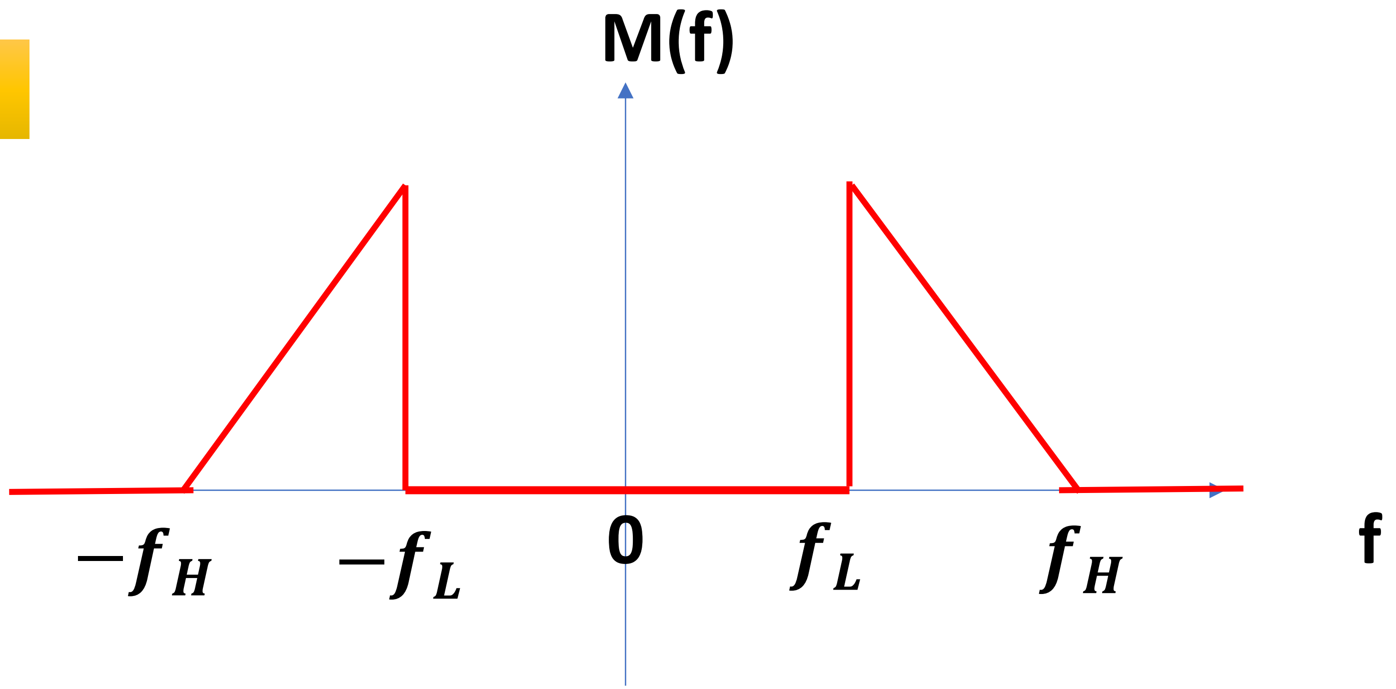
Overlapping

Aliasing

Signal Can not be Recovered

Bandpass Signal

$$BW = f_H - f_L$$



A Bandpass Signal can be recovered from its Sampled Signal if

$$f_s = \frac{2f_H}{k} \qquad k = \frac{f_H}{BW} \qquad f_s = 2BW$$

Example

A Bandpass message signal extends from 4-6 KHz. What is the smallest sampling frequency required to retain the signal completely?

Solution

$$BW = f_H - f_L = 6\ KHz - 4\ KHz = 2\ KHz$$

$$f_s = 2BW = 2x2KHz = 4KHz$$

Quantization

Is the Process of Fixating the Amplitude values of Sampled Signal for a specified number of Bit Depth

Quantization Procedure

1- Sample the Signal for Ts Period

2- Find the Corresponding X values for each selected sample

3- Define the Bit Depth (n)

4- Calculate the number of Quantized levels (L) $L = 2^n$

5- Calculate the Step-Size (Δ) $\Delta = \frac{X_{max} - X_{min}}{L}$

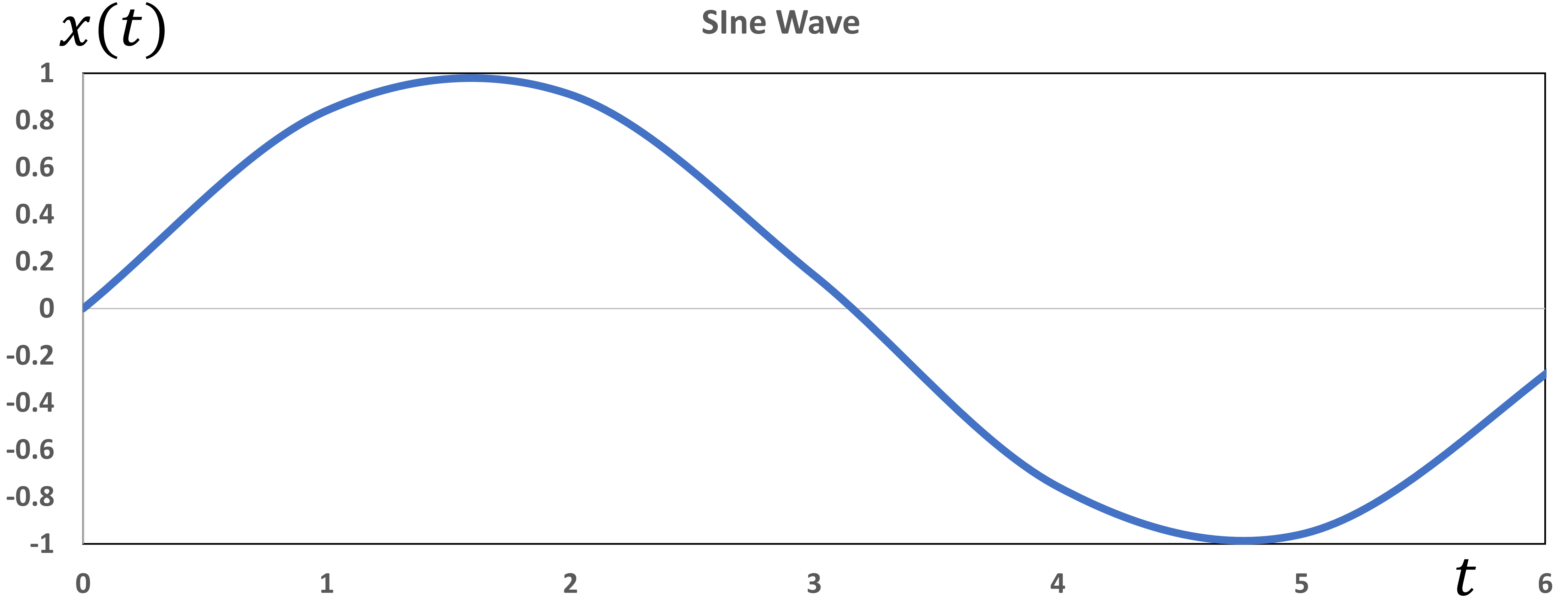
6- Evaluate the Index Value (I) $I = Round(\frac{X - X_{min}}{\Delta})$

7- Calculate the Quantized Magnitude $X_q = X_{min} + I\Delta$

8- Repeat Step 6 and 7 for each X value

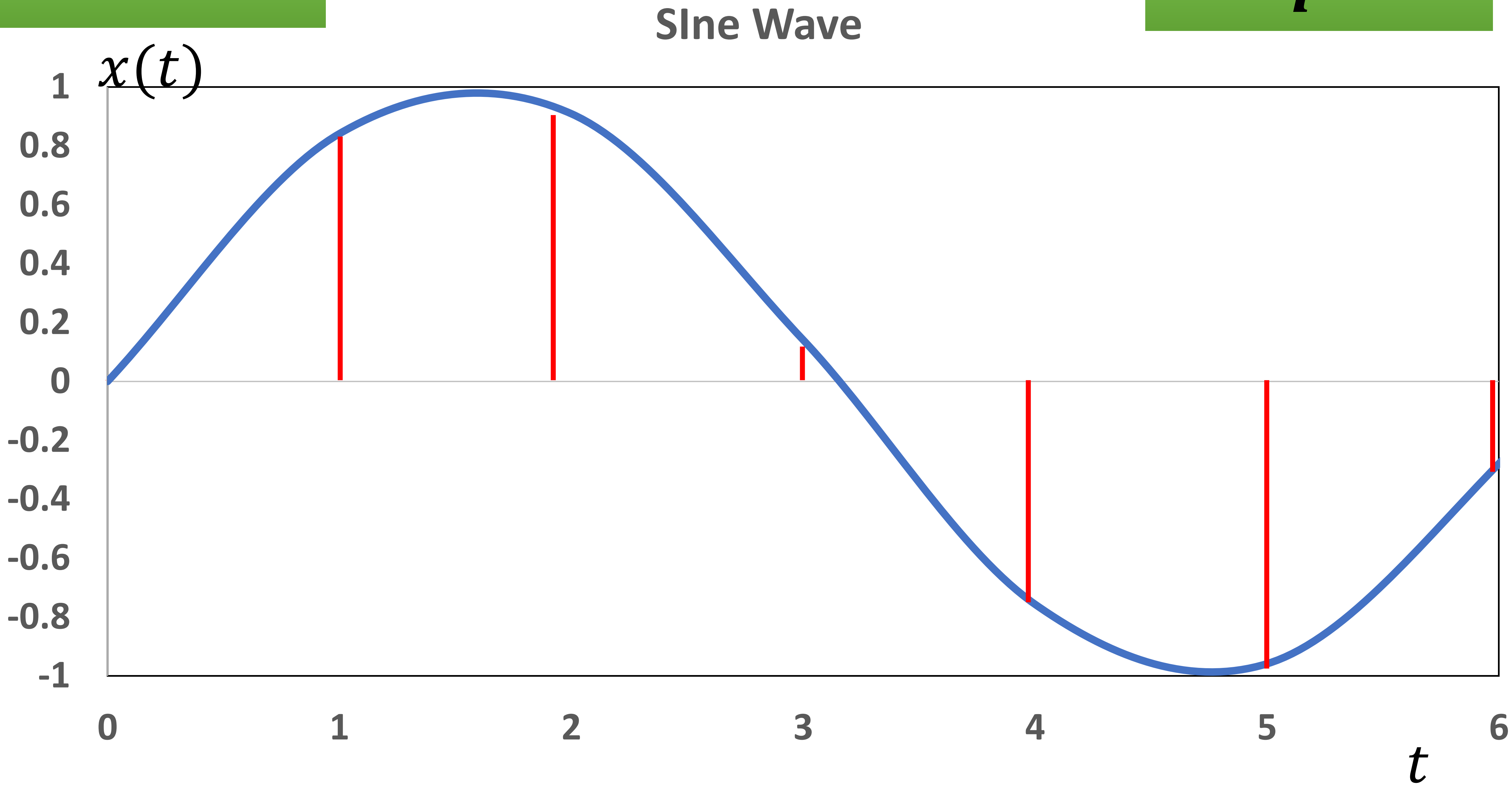
Example

For the Analog Signal X(t) find the Quantized Digital Signal for Bit depth of 2 and Sampling Time = 1 Sec



Solution

Step #1



Solution

Step #2

t	x(t)
0	0
1	0.84
2	0.90
3	0.14
4	-0.75
5	-0.95
6	-0.27

Solution $for\ t = 0;\ x = 0$

1- $n = 2$

2- $L = 2^n = 2^2 = 4$

3- $\Delta = \frac{X_{max} - X_{min}}{L} = \frac{1-(-1)}{4} = 0.5$

4- $I = Round\left(\frac{X - X_{min}}{\Delta}\right) = Round\left(\frac{0-(-1)}{0.5}\right)$
$= Round(2) = 2$

5- $X_q = X_{min} + I\Delta = -1 + 0.5 * 2 = 0$

Solution

$for\ t = 1;\ x = 0.84$

$$I = Round\left(\frac{X - X_{min}}{\Delta}\right) = Round\left(\frac{X+1}{0.5}\right)$$

$$= Round(3.68) = 4$$

$$X_q = X_{min} + I\Delta = -1 + 0.5I = 1$$

$for\ t = 2;\ x = 0.9$

$$I = Round\left(\frac{X - X_{min}}{\Delta}\right) = Round(2(x+1))$$

$$= Round(3.8) = 4$$

$$X_q = X_{min} + I\Delta = -1 + 0.5I = 1$$

Solution

$for\ t = 3;\ x = 0.14$

$$I = Round\left(\frac{X - X_{min}}{\Delta}\right) = Round(2(x+1))$$

$$= Round(2.28) = 2$$

$$X_q = X_{min} + I\Delta = -1 + 0.5I = 0$$

$for\ t = 4;\ x = -0.75$

$$I = Round\left(\frac{X - X_{min}}{\Delta}\right) = Round(2(x+1))$$

$$= Round(0.5) = 1$$

$$X_q = X_{min} + I\Delta = -1 + 0.5I = -0.5$$

Solution

$$for\ t = 5;\ x = -0.95$$

$$I = Round\left(\frac{X - X_{min}}{\Delta}\right) = Round(2(x+1))$$

$$= Round(0.1) = 0$$

$$X_q = X_{min} + I\Delta = -1 + 0.5I = -1$$

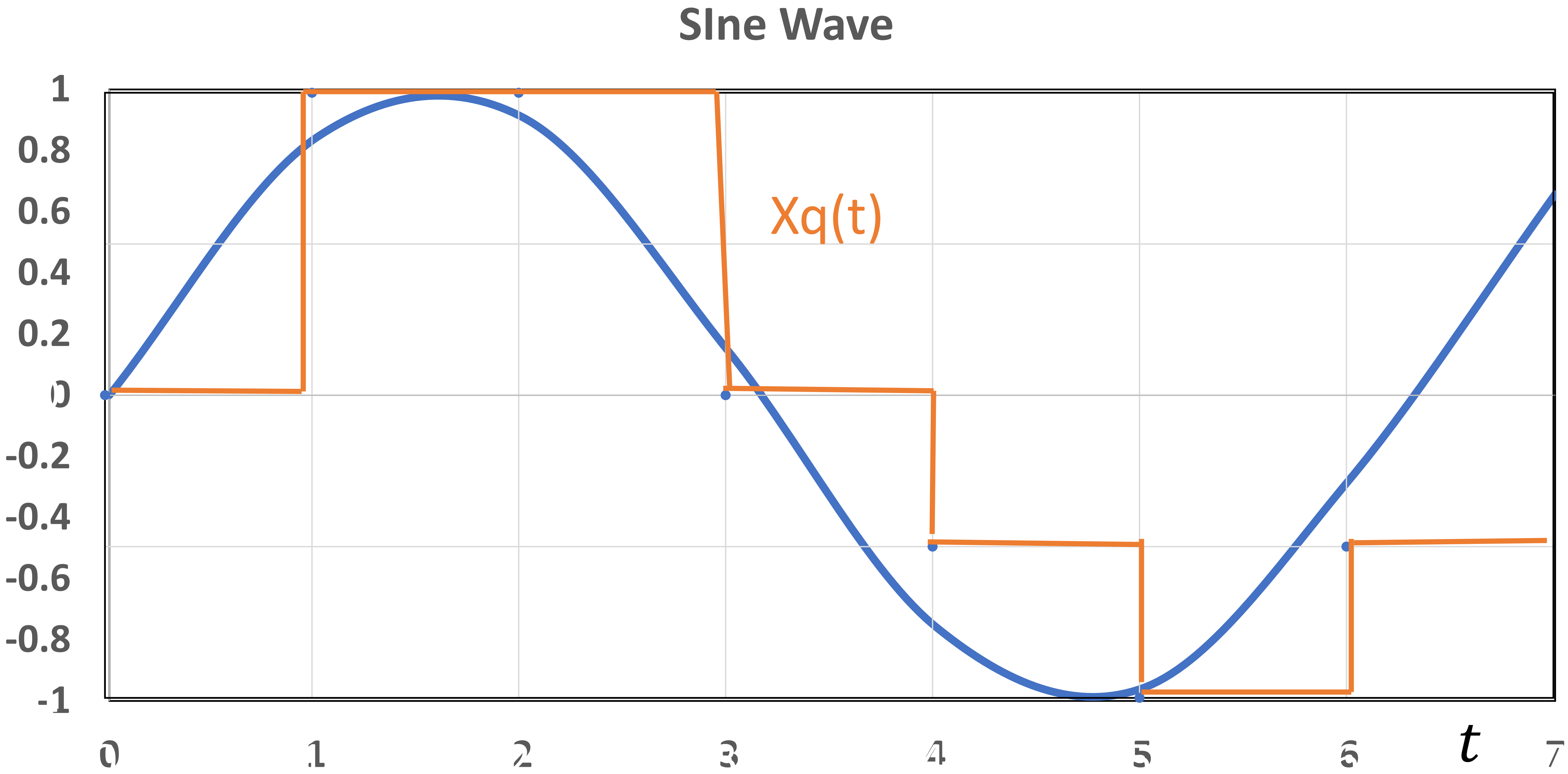
$$for\ t = 6;\ x = -0.27$$

$$I = Round\left(\frac{X - X_{min}}{\Delta}\right) = Round(2(x+1))$$

$$= Round(1.46) = 1$$

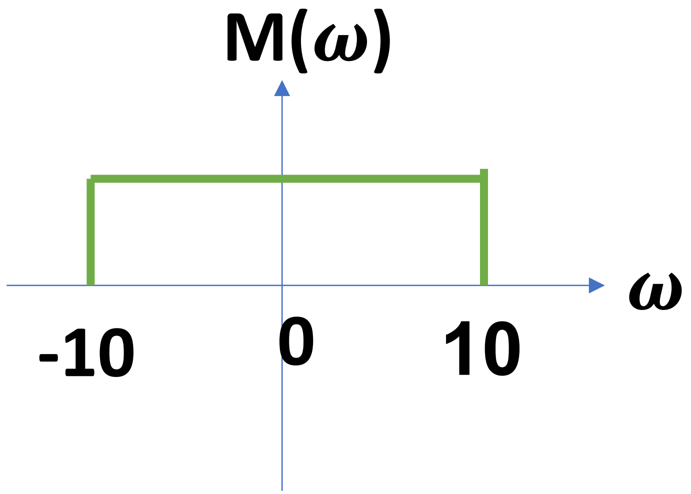
$$X_q = X_{min} + I\Delta = -1 + 0.5I = -0.5$$

Solution



Home Work

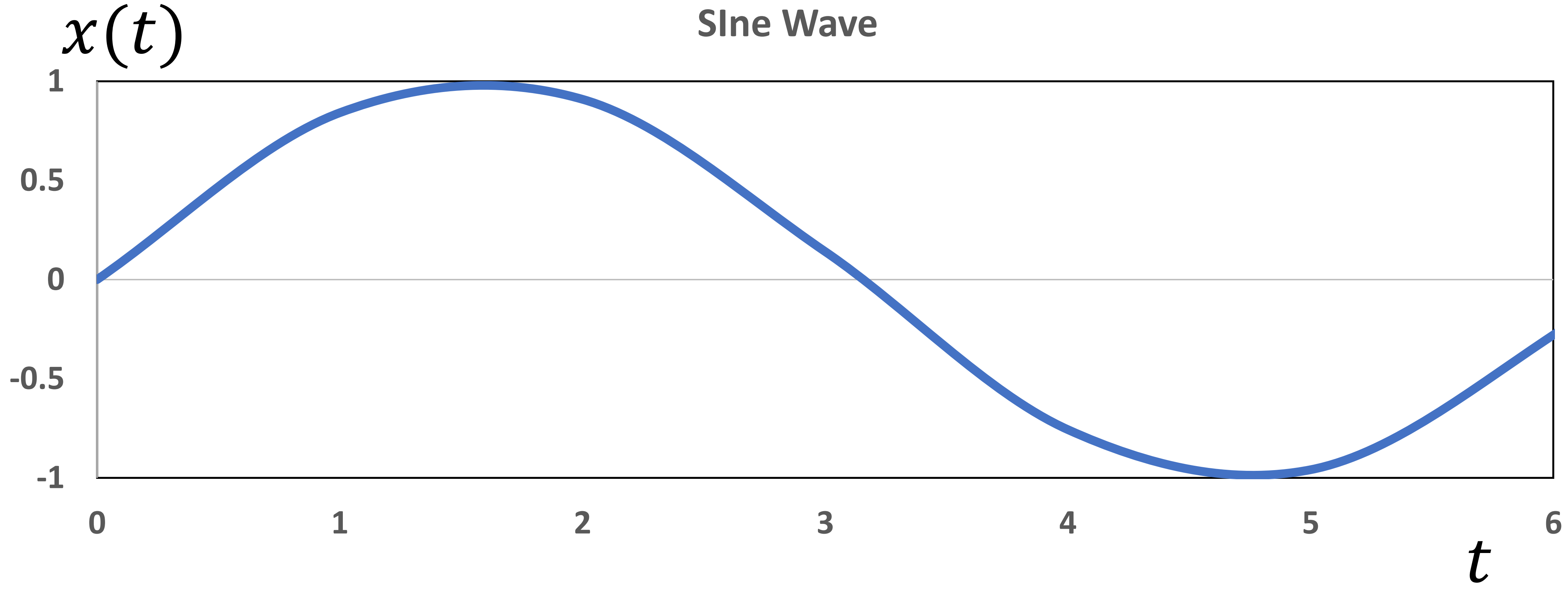
Q1: Sketch with numbers the sampling signal for $T_s = \frac{\pi}{2}$ sec



5M

Home Work

Q2: For the Analog Signal X(t) find the Quantized Digital Signal for Bit depth of 4 and Sampling Time = 0.25 Sec 10M



University of Technology
Laser & Optoelectronics
Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

Lec. 8: Digital Filtering: 2024-Nov-17

Lecture Outline

1. Digital Filtering
2. Filter Types
3. Window Functions
4. Low Pass Filter
5. Kaiser Filter
6. Butterworth Filter
7. Chebyshev Filter

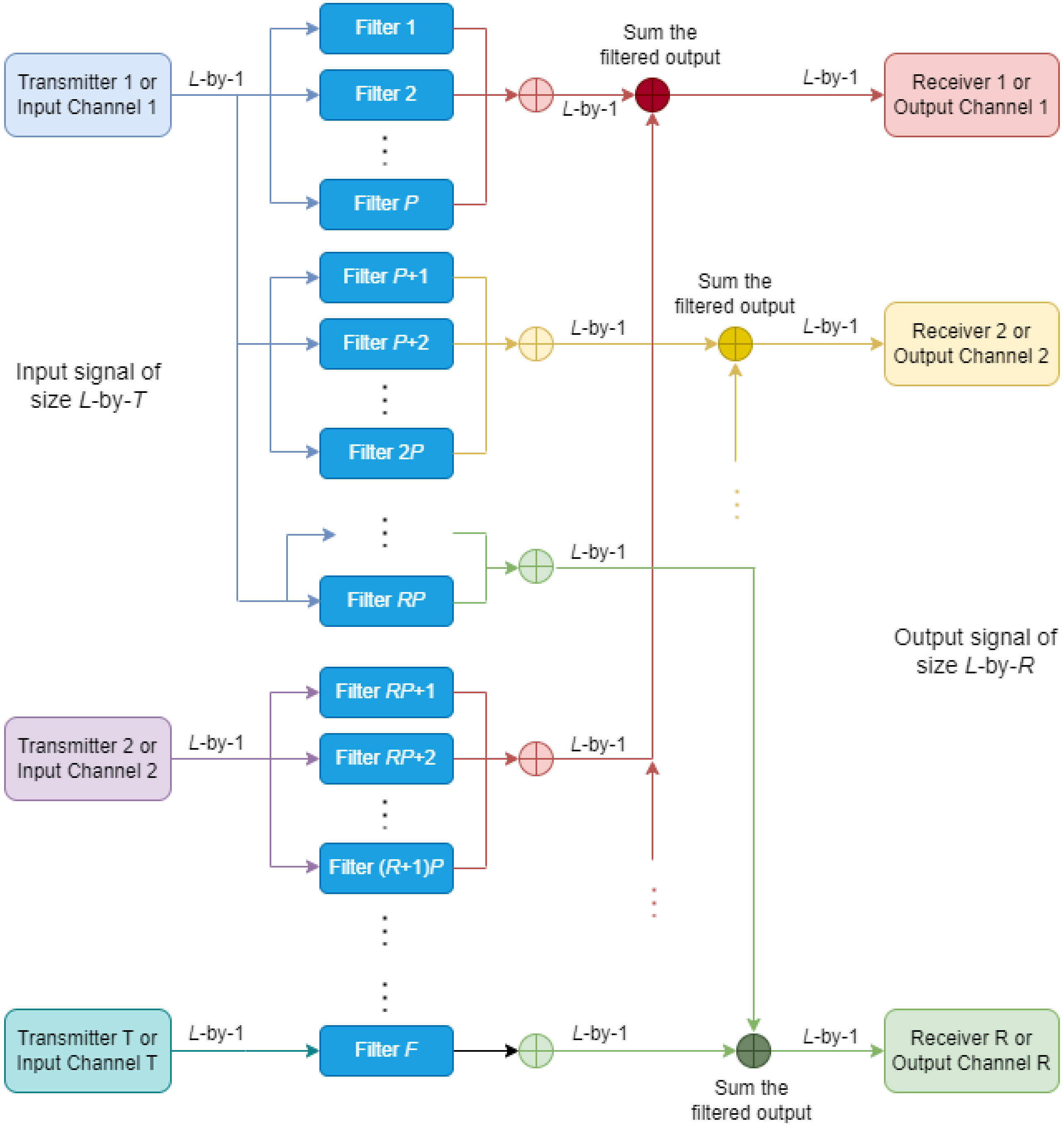
Digital Filtering

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

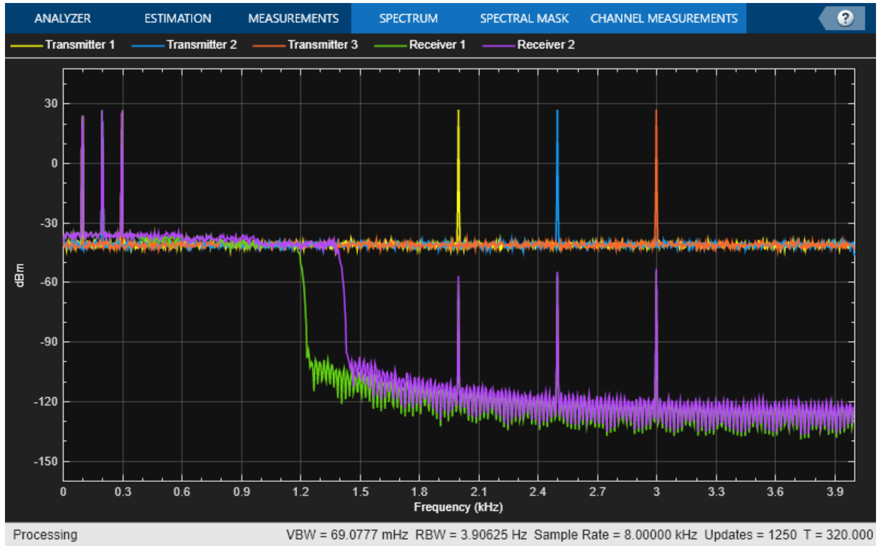
$$h_d(n) = \frac{\sin(n-\alpha)\omega_c}{\pi(n-\alpha)}$$

$$1-\delta_p < |H(e^{j\omega})| \le 1+\delta_p \qquad 0 \le |\omega| < \omega_p$$

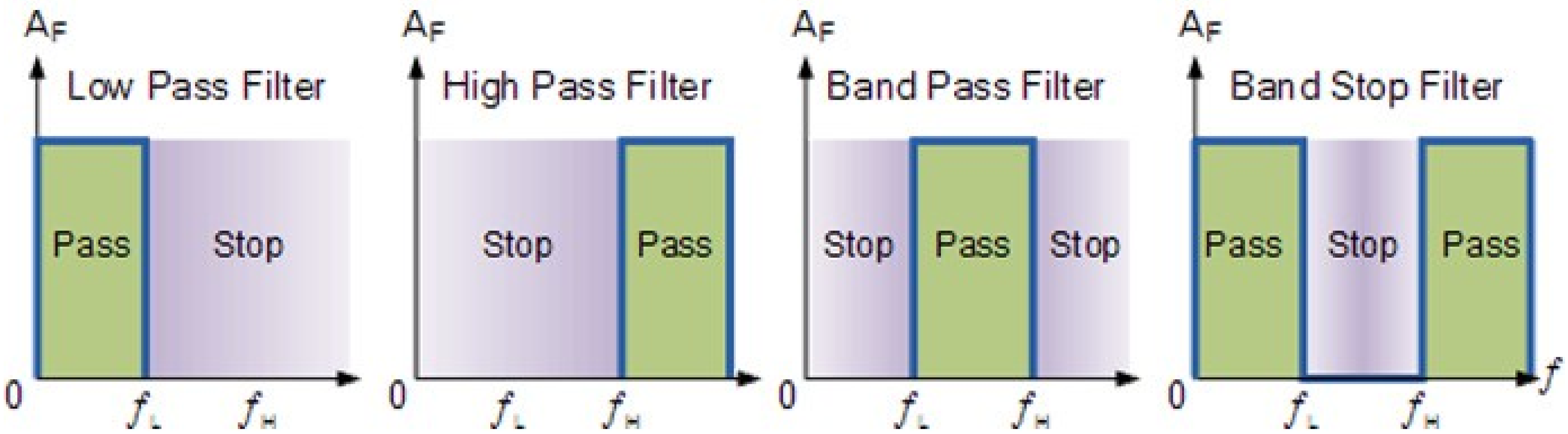
$$|H(e^{j\omega})| \le \delta_s \qquad \omega_s \le |\omega| < \pi$$


Transmitter 1 or Input Channel 1
L-by-1
Filter 1
Filter 2
Filter P
Sum the filtered output
L-by-1
L-by-1
Receiver 1 or Output Channel 1
Filter P+1
Filter P+2
Filter 2P
Sum the filtered output
L-by-1
L-by-1
Receiver 2 or Output Channel 2
Input signal of size L-by-T
Filter RP
L-by-1
Output signal of size L-by-R
Transmitter 2 or Input Channel 2
L-by-1
Filter RP+1
Filter RP+2
Filter (R+1)P
L-by-1
Transmitter T or Input Channel T
L-by-1
Filter F
L-by-1
L-by-1
Receiver R or Output Channel R
Sum the filtered output

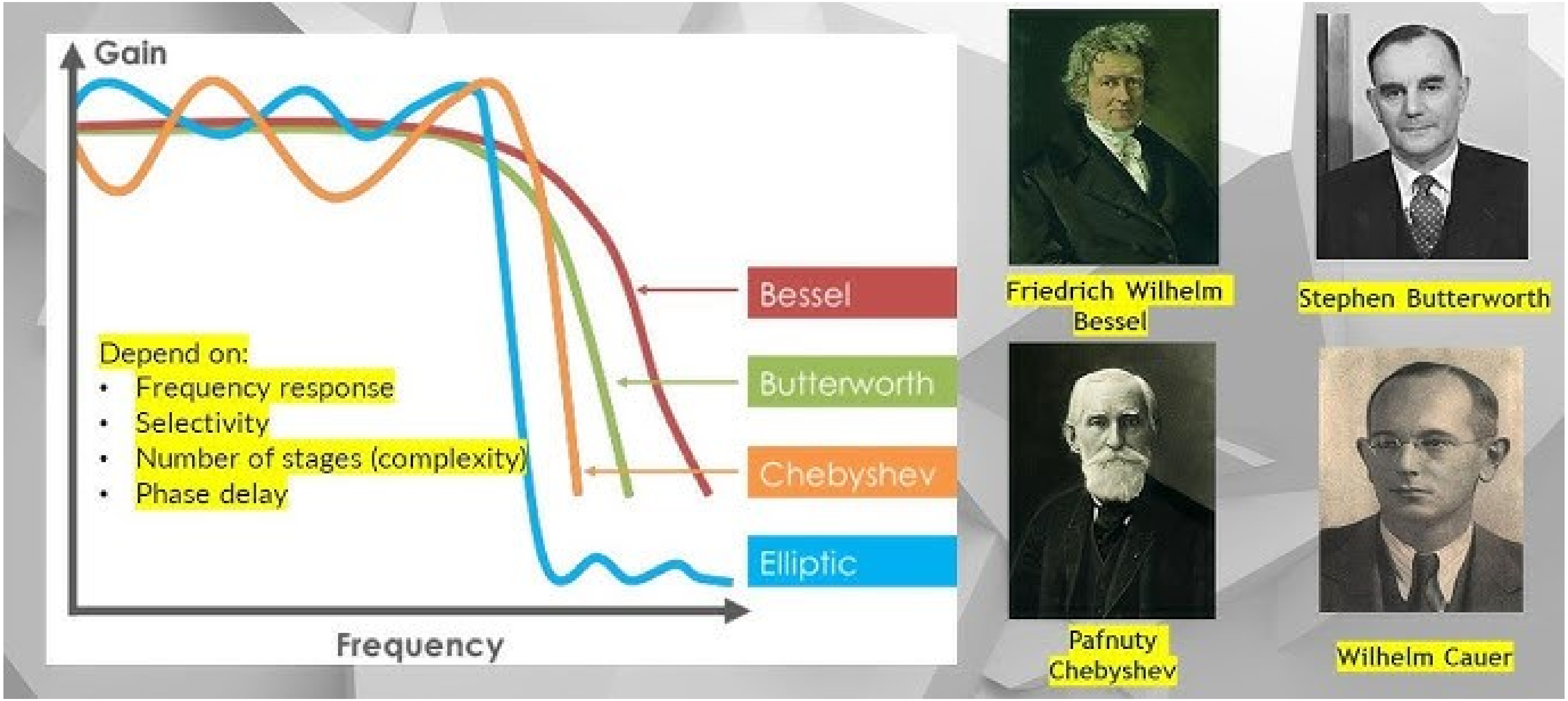
MATLAB

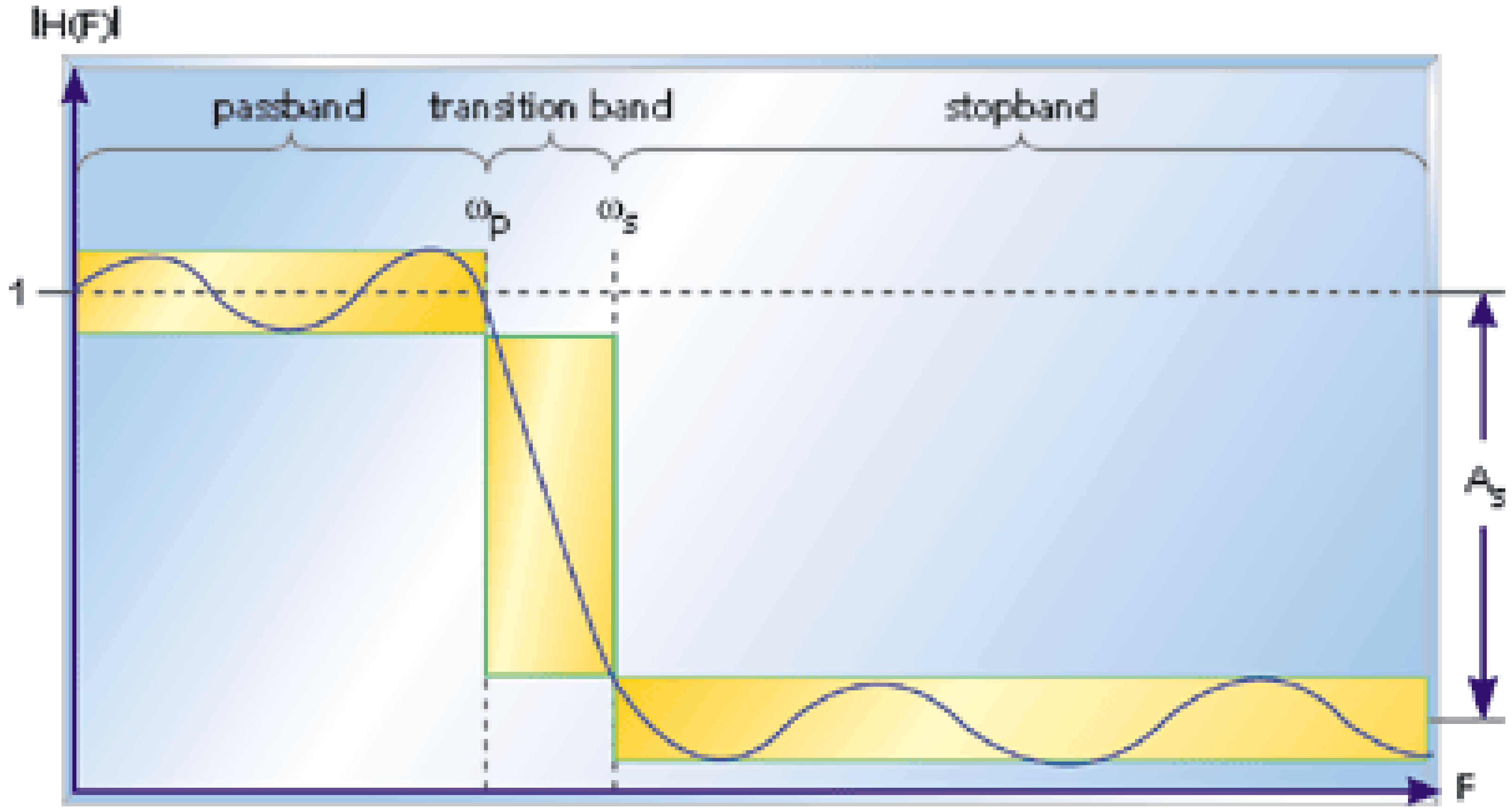

ANALYZER
ESTIMATION
MEASUREMENTS
SPECTRUM
SPECTRAL MASK
CHANNEL MEASUREMENTS
Transmitter 1
Transmitter 2
Transmitter 3
Receiver 1
Receiver 2
dBm
30
0
-30
-60
-90
-120
-150
0
0.3
0.6
0.9
1.2
1.5
1.8
2.1
2.4
2.7
3
3.3
3.6
3.9
Frequency (kHz)
Processing
VBW = 69.0777 mHz RBW = 3.90625 Hz Sample Rate = 8.00000 kHz Updates = 1250 T = 320.000

Filter Types



Filter Shapes

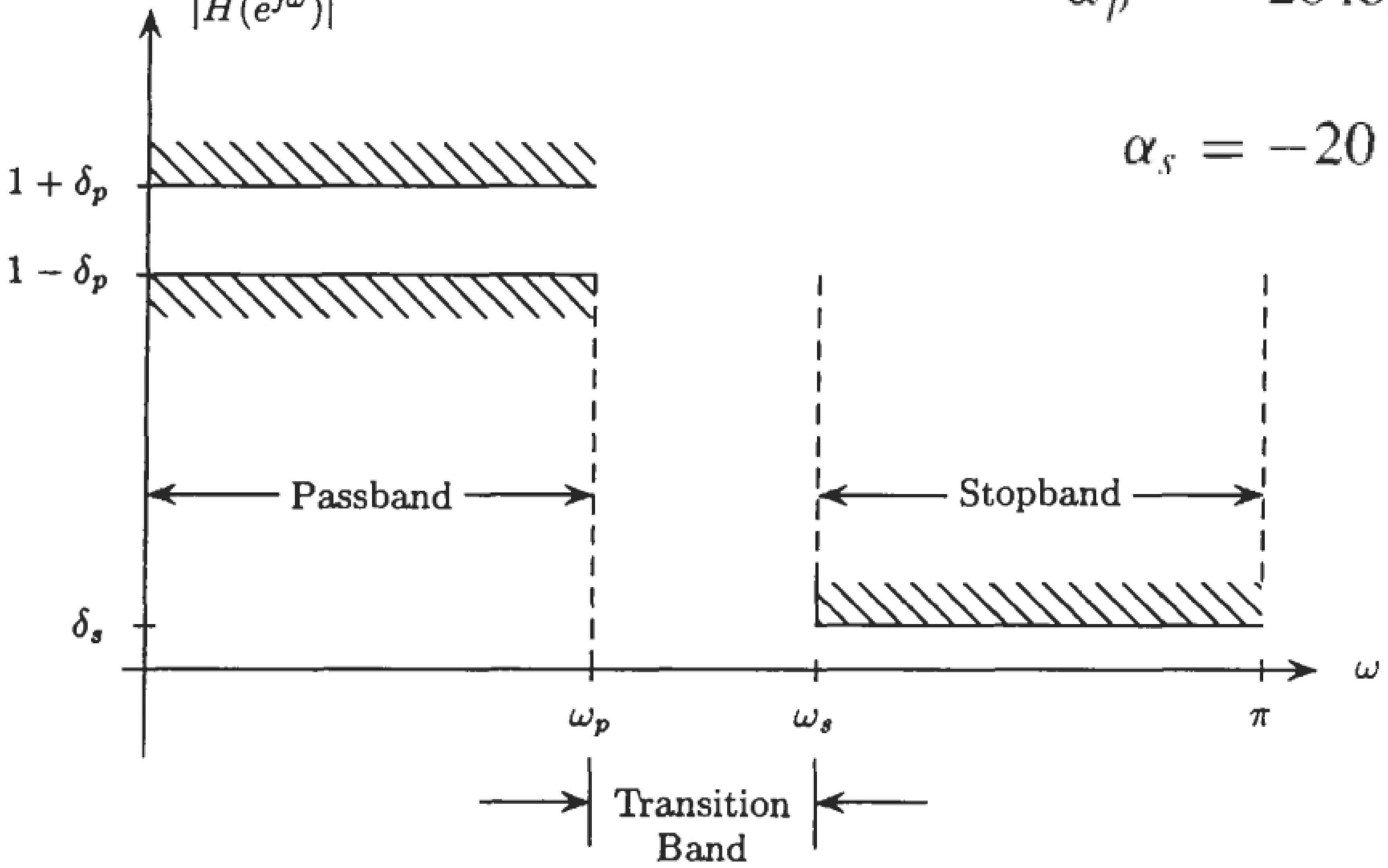



|H(F)|
passband
transition band
stopband
ω_p
ω_s
1
A_s
F

Filter Equations

$$\alpha_p = -20\log(1-\delta_p)$$

$$\alpha_s = -20\log(\delta_s)$$



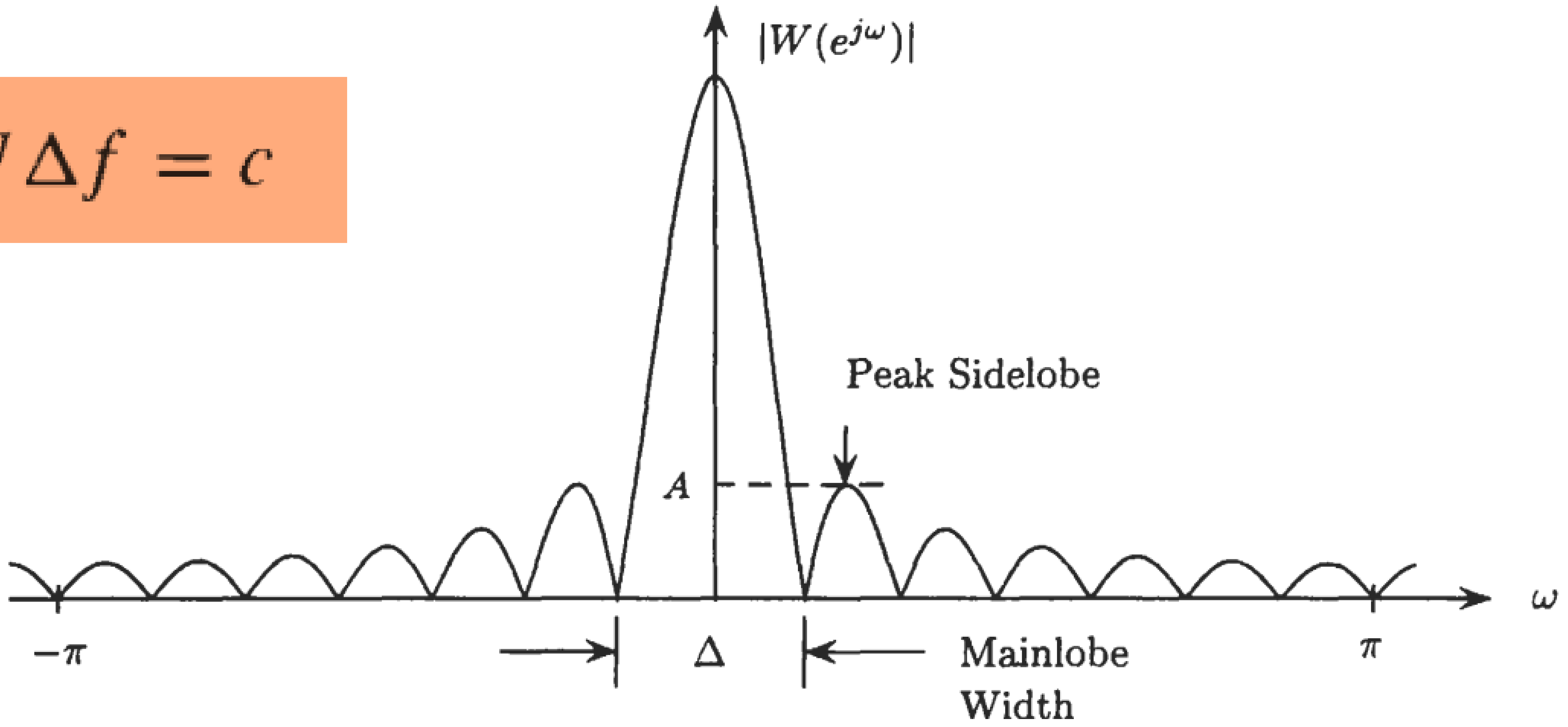
$$H(e^{j\omega}) = \sum_{n=0}^{N} h(n)e^{-jn\omega}$$

$$H_d(e^{j\omega}) = A(e^{j\omega})e^{-j(\alpha\omega-\beta)}$$

$$h(n) = h_d(n)w(n)$$

$$H(e^{j\omega}) = \frac{1}{2\pi}H_d(e^{j\omega}) * W(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})W(e^{j(\omega-\theta)})\,d\theta$$

$$N \Delta f = c$$



Digital Windows

Rectangular	$w(n) = \begin{cases} 1 & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hanning[1]	$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Hamming	$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$
Blackman	$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{N}\right) + 0.08\cos\left(\frac{4\pi n}{N}\right) & 0 \le n \le N \\ 0 & \text{else} \end{cases}$

Window Parameters

Window	Side-Lobe Amplitude (dB)	Transition Width (Δf)	Stopband Attenuation (dB)
Rectangular	−13	$0.9/N$	−21
Hanning	−31	$3.1/N$	−44
Hamming	−41	$3.3/N$	−53
Blackman	−57	$5.5/N$	−74

Ex-1 Suppose that we would like to design an FIR linear phase low-pass filter according to the following specifications:

$$0.99 \le |H(e^{j\omega})| \le 1.01 \qquad 0 \le |\omega| \le 0.19\pi$$
$$|H(e^{j\omega})| \le 0.01 \qquad 0.21\pi \le |\omega| \le \pi$$

For a stopband attenuation of $20\log(0.01) = -40$ dB, we may use a Hanning window. Although we could also use a Hamming or a Blackman window, these windows would overdesign the filter and produce a larger stopband attenuation at the expense of an increase in the transition width. Because the specification calls for a transition width of $\Delta\omega = \omega_s - \omega_p = 0.02\pi$, or $\Delta f = 0.01$, with

$$N\,\Delta f = 3.1$$

for a Hanning window (see Table 9.2), an estimate of the required filter order is

$$N = \frac{3.1}{\Delta f} = 310$$

The last step is to find the unit sample response of the ideal low-pass filter that is to be windowed. With a cutoff frequency of $\omega_c = (\omega_s + \omega_p)/2 = 0.2\pi$, and a delay of $\alpha = N/2 = 155$, the unit sample response is

$$h_d(n) = \frac{\sin[0.2\pi(n - 155)]}{(n - 155)\pi}$$

Kaiser Window

$$w(n) = \frac{I_0[\beta(1 - [(n-\alpha)/\alpha]^2)^{1/2}]}{I_0(\beta)} \qquad 0 \le n \le N$$

Design Equations

$$I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{(x/2)^k}{k!}\right]^2 \qquad \alpha = N/2$$

$$\beta = \begin{cases} 0.1102(\alpha_s - 8.7) & \alpha_s > 50 \\ 0.5842(\alpha_s - 21)^{0.4} + 0.07886(\alpha_s - 21) & 21 \le \alpha_s \le 50 \\ 0.0 & \alpha_s < 21 \end{cases} \qquad \alpha_s = -20\log(\delta_s)$$

The second relates N to the transition width Δf and the stopband attenuation α_s,

$$N = \frac{\alpha_s - 7.95}{14.36\Delta f} \qquad \alpha_s \ge 21$$

Note that if $\alpha_s < 21$ dB, a rectangular window may be used ($\beta = 0$), and $N = 0.9/\Delta f$.

Table 9-3 Characteristics of the Kaiser Window as a Function of β

Parameter β	Side Lobe (dB)	Transition Width ($N\Delta f$)	Stopband Attenuation (dB)
2.0	−19	1.5	−29
3.0	−24	2.0	−37
4.0	−30	2.6	−45
5.0	−37	3.2	−54
6.0	−44	3.8	−63
7.0	−51	4.5	−72
8.0	−59	5.1	−81
9.0	−67	5.7	−90
10.0	−74	6.4	−99

Ex-2 Suppose that we would like to design a low-pass filter with a cutoff frequency $\omega_c = \pi/4$, a transition width $\Delta\omega = 0.02\pi$, and a stopband ripple $\delta_s = 0.01$. Because $\alpha_s = -20\log(0.01) = +40$, the Kaiser window parameter is

$$\beta = 0.5842(40-21)^{0.4} + 0.07886(40-21) = 3.4$$

With $\Delta f = \Delta\omega/2\pi = 0.01$, we have

$$N = \frac{40 - 7.95}{14.36 \cdot (0.01)} = 224$$

Therefore,

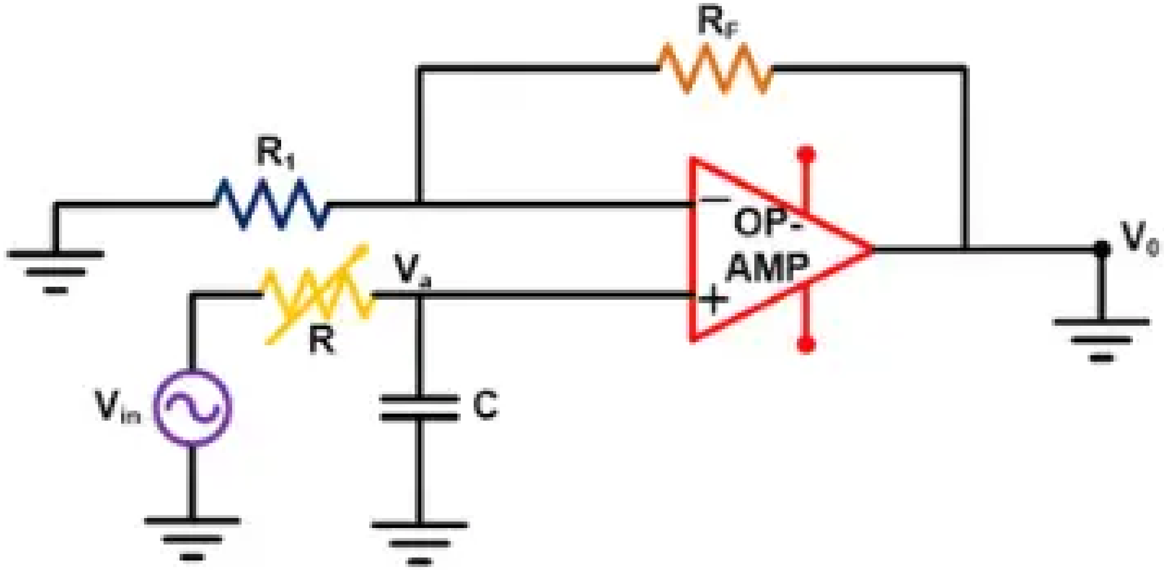
$$h(n) = h_d(n)w(n)$$

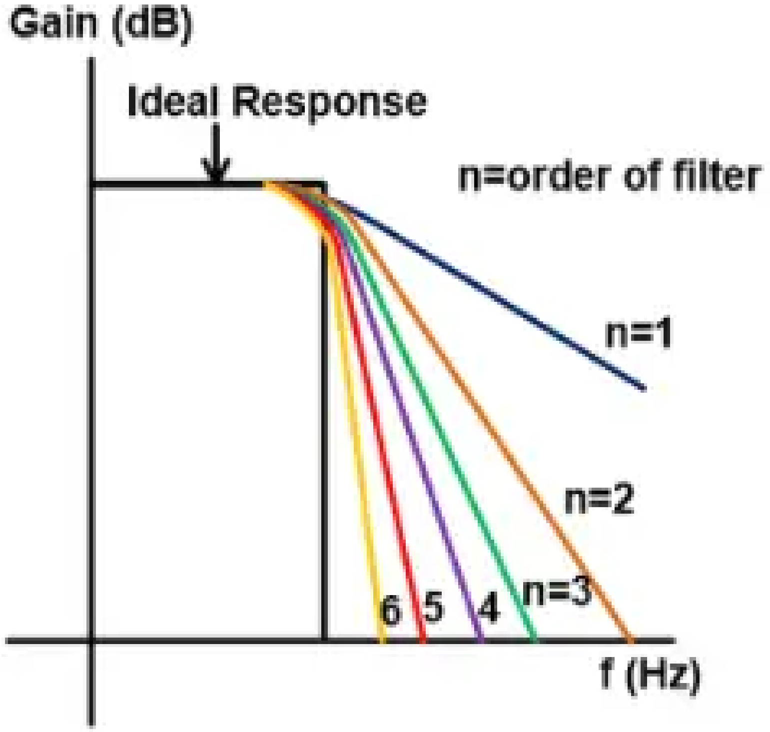
where

$$h_d(n) = \frac{\sin[(n-112)\pi/4]}{(n-112)\pi}$$

is the unit sample response of the ideal low-pass filter.

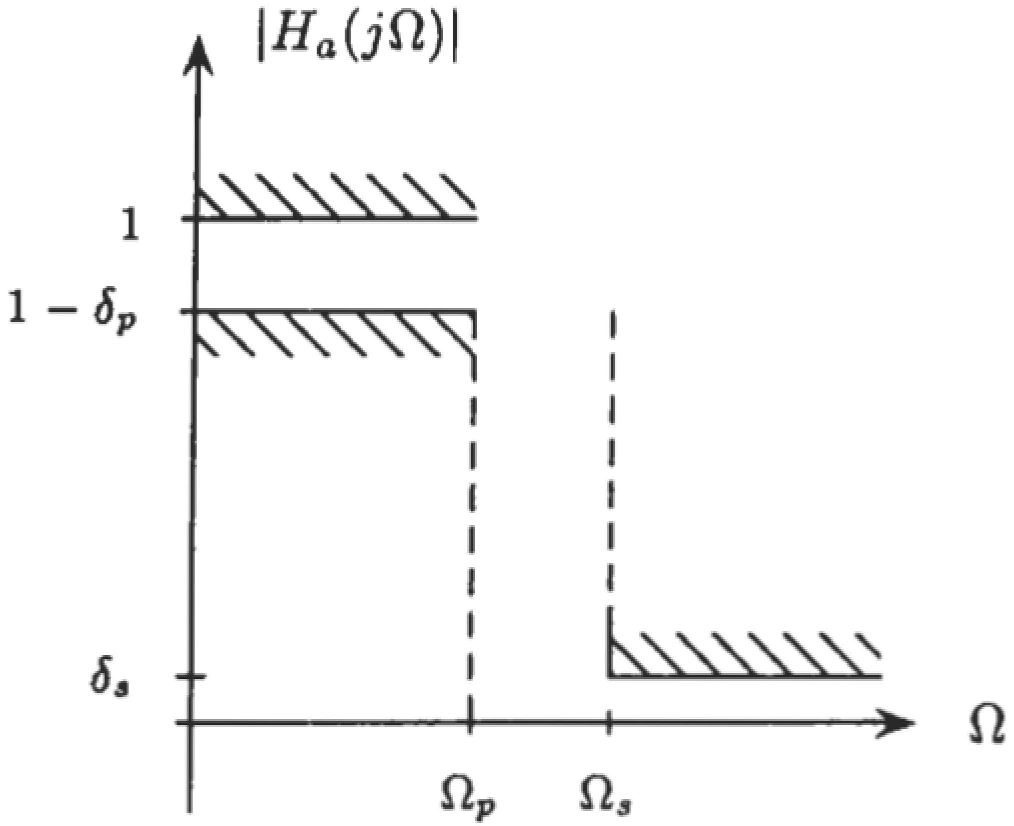
Butterworth Filter





Design Equations

$$1 - \delta_p \le |H_a(j\Omega)| \le 1$$



$$d = \left[\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}\right]^{1/2} = \frac{\epsilon}{\sqrt{A^2 - 1}}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

$$\epsilon = \left(\frac{\Omega_p}{\Omega_c}\right)^N$$

Design Equations

Procedure

1. Find the values for the selectivity factor, k, and the discrimination factor, d, from the filter specifications.
2. Determine the order of the filter required to meet the specifications using the design formula

$$N \geq \frac{\log d}{\log k}$$

3. Set the 3-dB cutoff frequency, Ω_c, to any value in the range

$$\Omega_p[(1-\delta_p)^{-2}-1]^{-1/2N} \leq \Omega_c \leq \Omega_s\left[\delta_s^{-2}-1\right]^{-1/2N}$$

4. Synthesize the system function of the Butterworth filter from the poles of

Table 9-4 The Coefficients in the System Function of a Normalized Butterworth Filter ($\Omega_c = 1$) for Orders $1 \leq N \leq 8$

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.0000							
2	1.4142	1.0000						
3	2.0000	2.0000	1.0000					
4	2.6131	3.4142	2.6131	1.0000				
5	3.2361	5.2361	5.2361	3.2361	1.0000			
6	3.8637	7.4641	9.1416	7.4641	3.8637	1.0000		
7	4.4940	10.0978	14.5918	14.5918	10.0978	4.4940	1.0000	
8	5.1258	13.1371	21.8462	25.6884	21.8462	13.1372	5.1258	1.0000

Ex-3

Let us design a low-pass Butterworth filter to meet the following specifications:

$$f_p = 6\text{ kHz} \qquad f_s = 10\text{ kHz} \qquad \delta_p = \delta_s = 0.1$$

First, we compute the discrimination and selectivity factors:

$$d = \left[\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}\right]^{1/2} = 0.0487 \qquad k = \frac{\Omega_p}{\Omega_s} = \frac{f_p}{f_s} = 0.6$$

Because

$$N \geq \frac{\log d}{\log k} = 5.92$$

it follows that the minimum filter order is $N = 6$. With

$$f_p[(1-\delta_p)^{-2} - 1]^{-1/2N} = 6770$$

and

$$f_s\left[\delta_s^{-2} - 1\right]^{-1/2N} = 6819$$

the center frequency, f_c, may be any value in the range

$$6770 \leq f_c \leq 6819$$

The system function of the Butterworth filter may then be found using Eq. (9.8) by first constructing a sixth-order normalized Butterworth filter from Table 9-4,

$$H_a(s) = \frac{1}{s^6 + 3.8637s^5 + 7.4641s^4 + 9.1416s^3 + 7.4641s^2 + 3.8637s + 1}$$

Design Equations

Procedure 2 Poles

$$\alpha_s = -20\log(\delta_s)$$

$$N = \frac{\log[(1/\delta_2^2) - 1]}{2\log(\Omega_s/\Omega_c)}$$

$$s_k = \Omega_c e^{j\pi/2} e^{j(2k+1)\pi/2N} \qquad k = 0, 1, \ldots, N-1$$

Ex-4

Determine the order and the poles of a lowpass Butterworth filter that has a −3-dB bandwidth of 500 Hz and an attenuation of 40 dB at 1000 Hz.

Solution The critical frequencies are the −3-dB frequency Ω_c and the stopband frequency Ω_s, which are

$$\Omega_c = 1000\pi$$

$$\Omega_s = 2000\pi$$

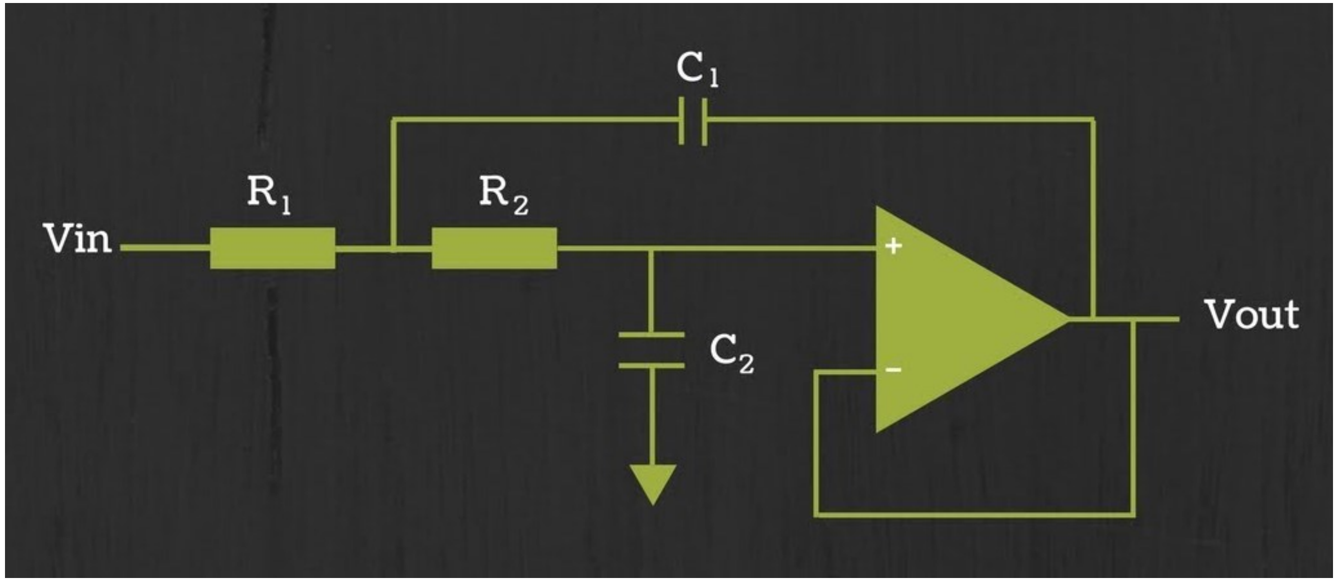
For an attenuation of 40 dB, $\delta_2 = 0.01$.

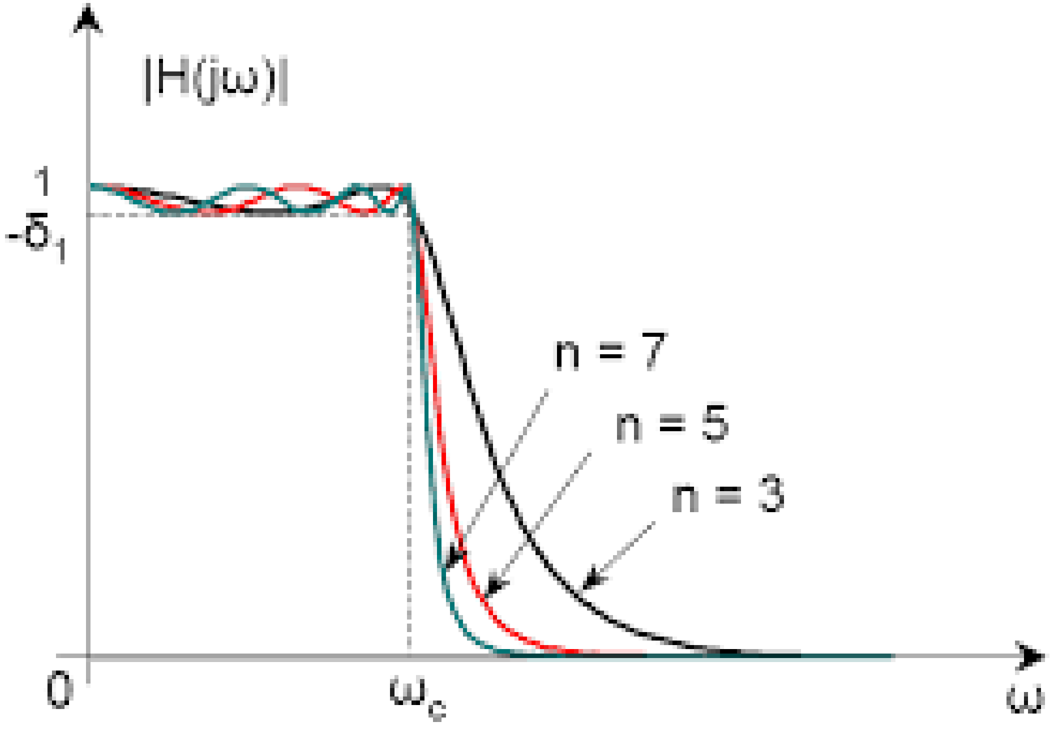
$$N = \frac{\log_{10}(10^4 - 1)}{2\log_{10} 2} = 6.64$$

To meet the desired specifications, we select $N = 7$. The pole positions are

$$s_k = 1000\pi e^{j[\pi/2+(2k+1)\pi/14]} \qquad k = 0, 1, 2, \ldots, 6$$

Chebyshev filter





Design Equations

$$N = \frac{\log\left[\left(\sqrt{1-\delta_2^2} + \sqrt{1-\delta_2^2(1+\epsilon^2)}\right)/\epsilon\delta_2\right]}{\log\left[(\Omega_s/\Omega_p) + \sqrt{(\Omega_s/\Omega_p)^2 - 1}\right]}$$

$$\beta = \left[\frac{\sqrt{1+\epsilon^2}+1}{\epsilon}\right]^{1/N}$$

Design Equations

$$r_1 = \Omega_p \frac{\beta^2 + 1}{2\beta} \qquad r_2 = \Omega_p \frac{\beta^2 - 1}{2\beta}$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k+1)\pi}{2N}$$

$$x_k = r_2 \cos\phi_k, \qquad k = 0, 1, \ldots, N-1$$

$$y_k = r_1 \sin\phi_k, \qquad k = 0, 1, \ldots, N-1$$

Ex-5

Determine the order and the poles of a type I lowpass Chebyshev filter that has a 1-dB ripple in the passband, a cutoff frequency $\Omega_p = 1000\pi$, a stopband frequency of 2000π, and an attenuation of 40 dB or more for $\Omega \geq \Omega_s$.

Solution First, we determine the order of the filter. We have

$$10\log_{10}(1+\epsilon^2) = 1$$

$$1+\epsilon^2 = 1.259$$

$$\epsilon^2 = 0.259$$

$$\epsilon = 0.5088$$

Log equation → Exponential equation

$$\log_b(M) = N \quad \Rightarrow \quad M = b^N$$

Also,

$$20\log_{10}\delta_2 = -40$$

$$\delta_2 = 0.01$$

$$N = \frac{\log_{10}196.54}{\log_{10}(2+\sqrt{3})}$$

$$= 4.0$$

Thus a type I Chebyshev filter having four poles meets the specifications.

The pole positions are determined from the relations

First, we compute β, r_1, and r_2. Hence

$$\beta = 1.429$$

$$r_1 = 1.06\Omega_p$$

$$r_2 = 0.365\Omega_p$$

The angles $\{\phi_k\}$ are

$$\phi_k = \frac{\pi}{2} + \frac{(2k+1)\pi}{8} \qquad k = 0, 1, 2, 3$$

Therefore, the poles are located at

$$x_1 + jy_1 = -0.1397\Omega_p \pm j0.979\Omega_p$$

$$x_2 + jy_2 = -0.337\Omega_p \pm j0.4056\Omega_p$$

Home Work

Q1: Design a Low-Pass Butterworth Filter with $\delta_p = \delta_s = 0.2$ and fs = 35 KHz, fp=5 KHz — 10 M

Q2: Determine the poles of type I low pass Chebyshev Filter that has 0.2 dB ripple in the bandpass, cutoff frequency 200 π stopband frequency 400 π and attenuation of 60 dB for $\Omega \geq \Omega_s$ — 10 M

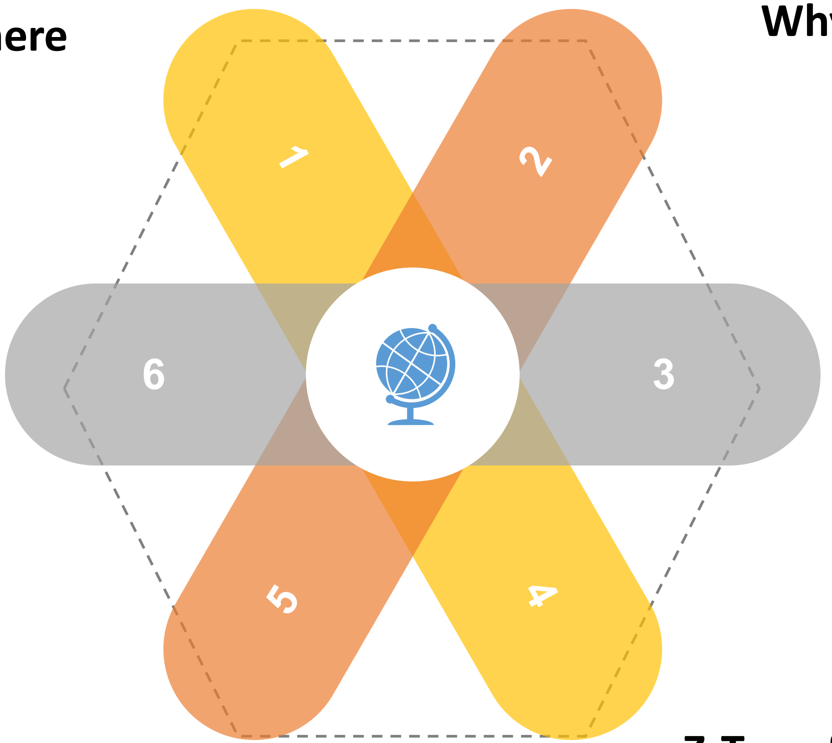
University of Technology
Laser & Optoelectronics Engineering Department

Digital Signal Processing I

Lec. Dr. Taif Alawsi

Lec. 9: DSP with MATLAB: 2024-Nov-24

Contents

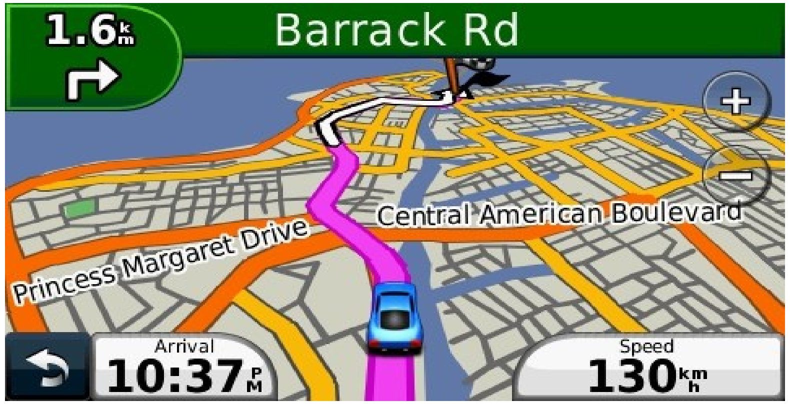


Signal processing is everywhere

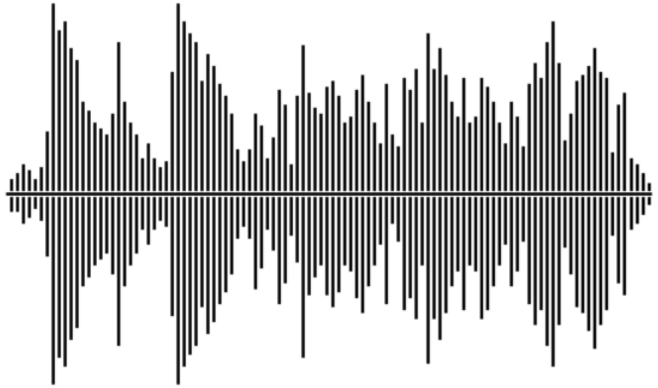
Communication &Radar

Medical Devices

GPS



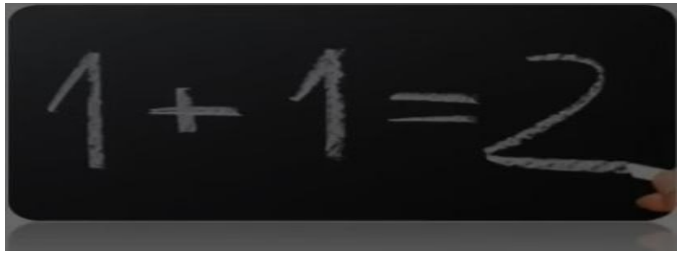
Audio



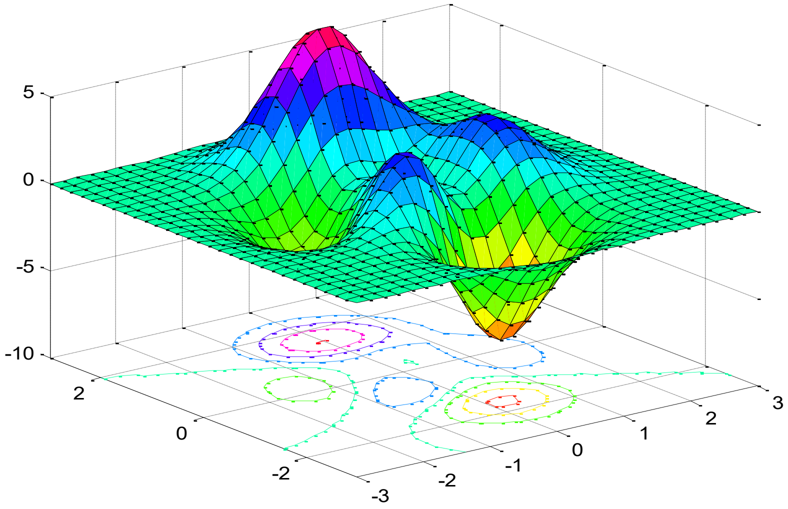
Financial Analytics

Why MATLAB?

Easy to use



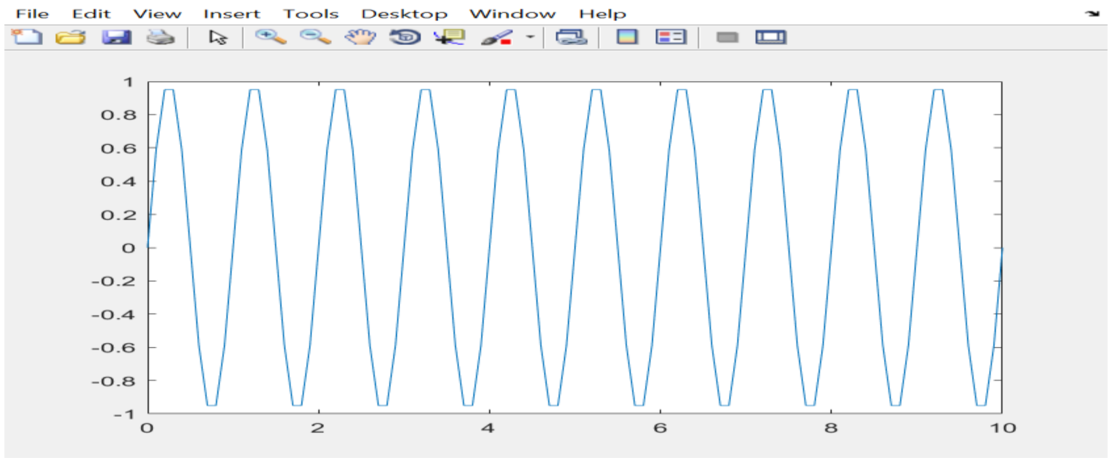
Visualization



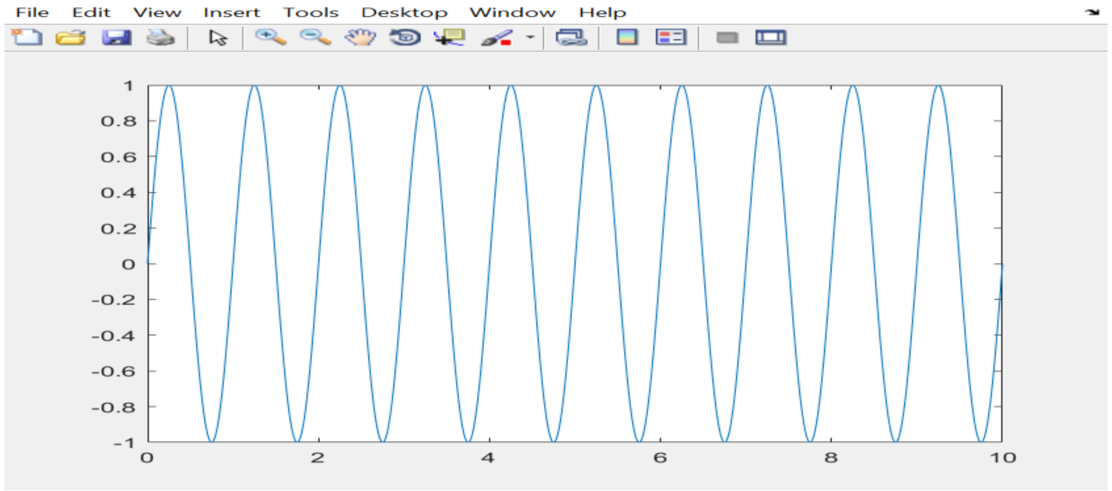
DSP Toolbox

Signal Generation

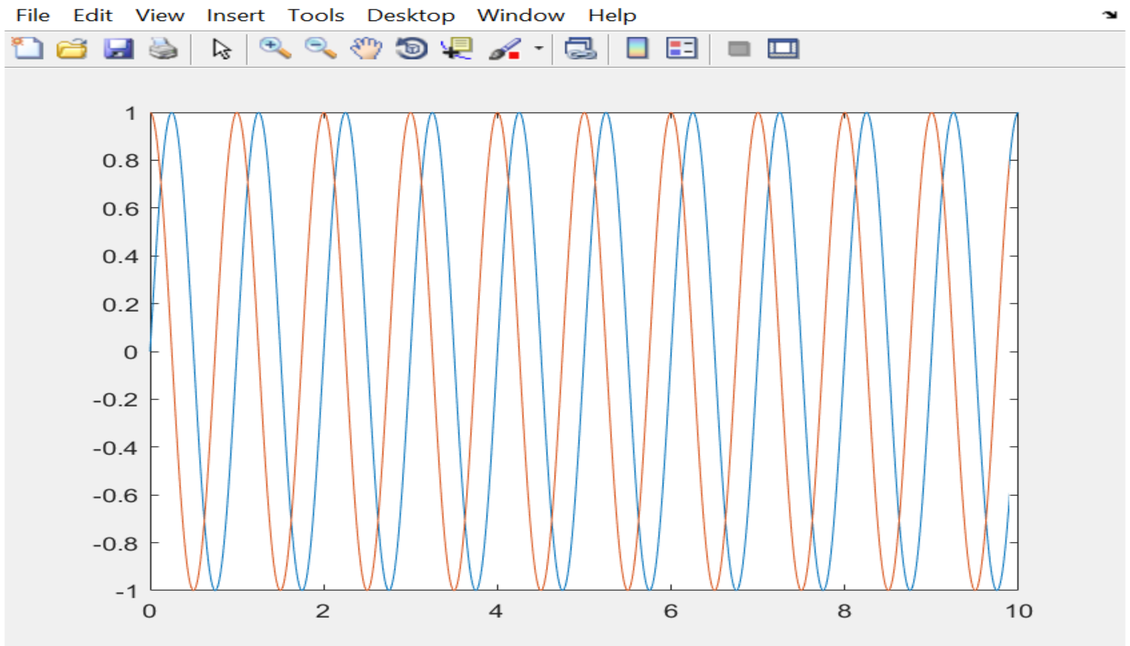
```
t=0:1:10
xt=sin (2*pi*t);
plot(t,xt);
```



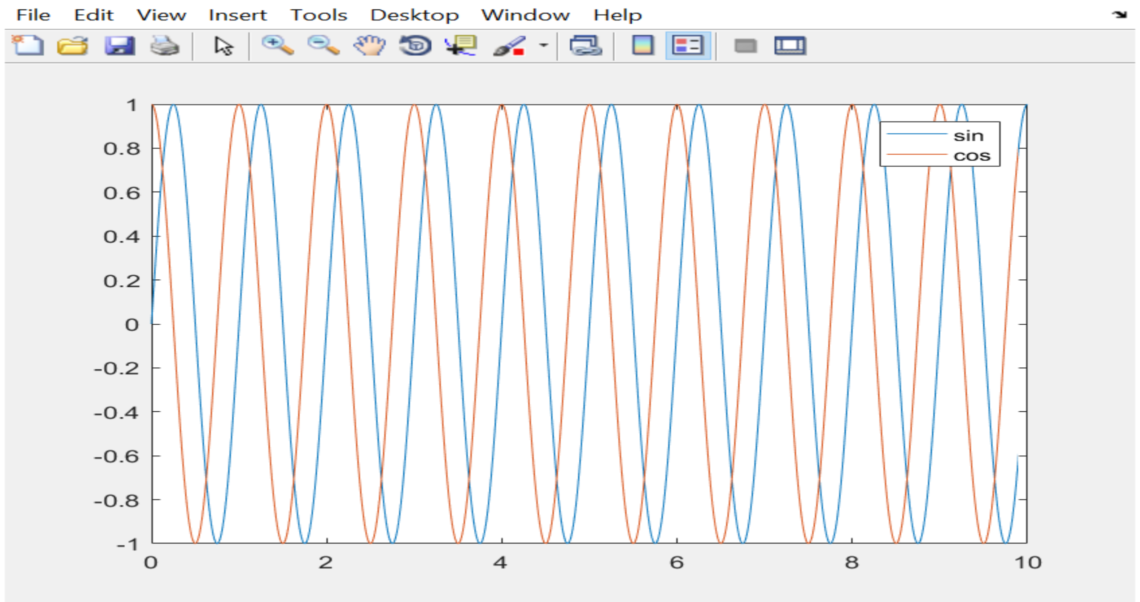

```
t=0:0.01:10
xt=sin (2*pi*t);
plot(t,xt);
```



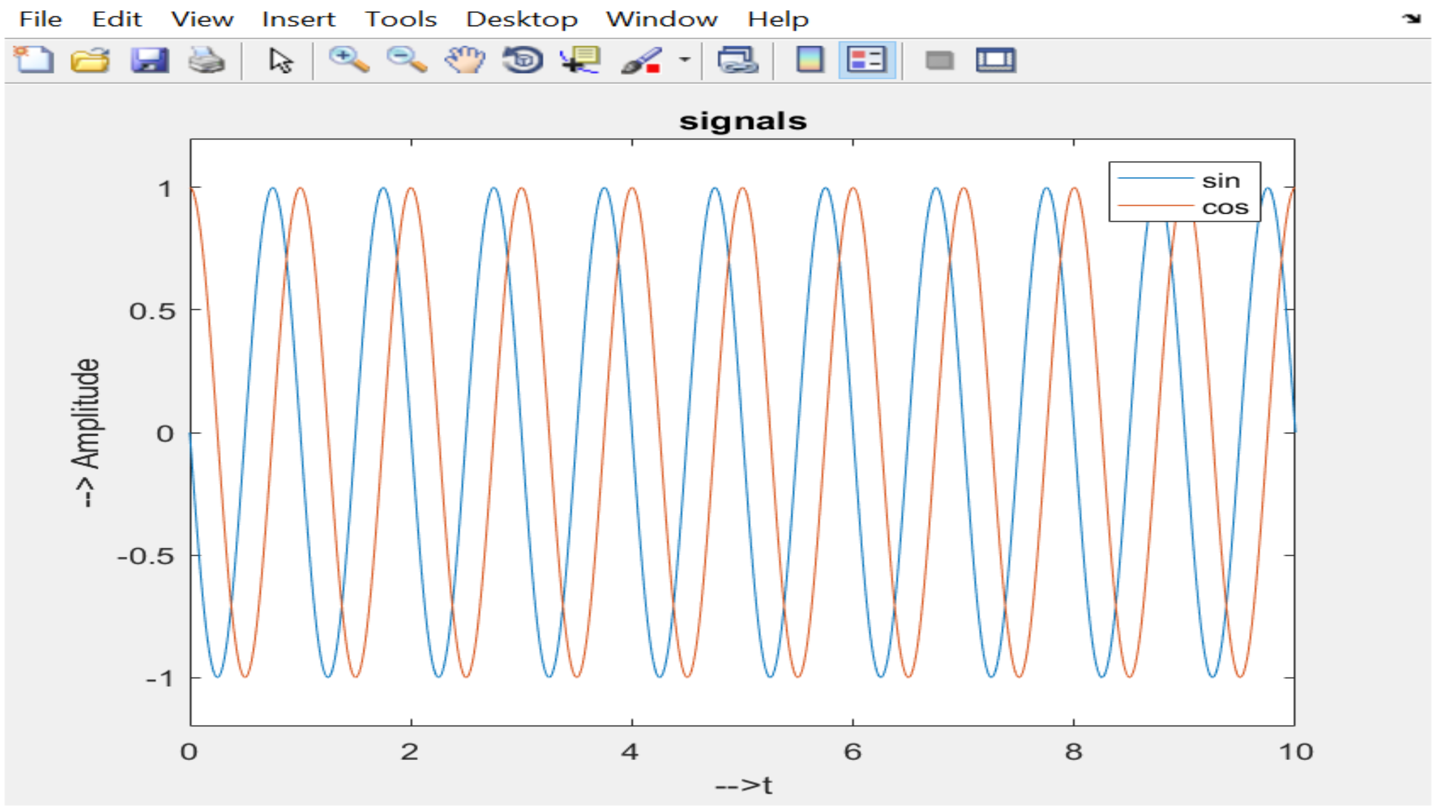
```
t=0:0.01:10
xt=sin (2*pi*t);
yt=cos (2*pi*t);
plot(t,xt);
hold on
plot(t,yt);
hold off
```




```
t=0:0.01:10
xt=sin (2*pi*t);
yt=cos (2*pi*t);
plot(t,xt);
hold on
plot(t,yt);
legend('sin', 'cos');
hold off
```



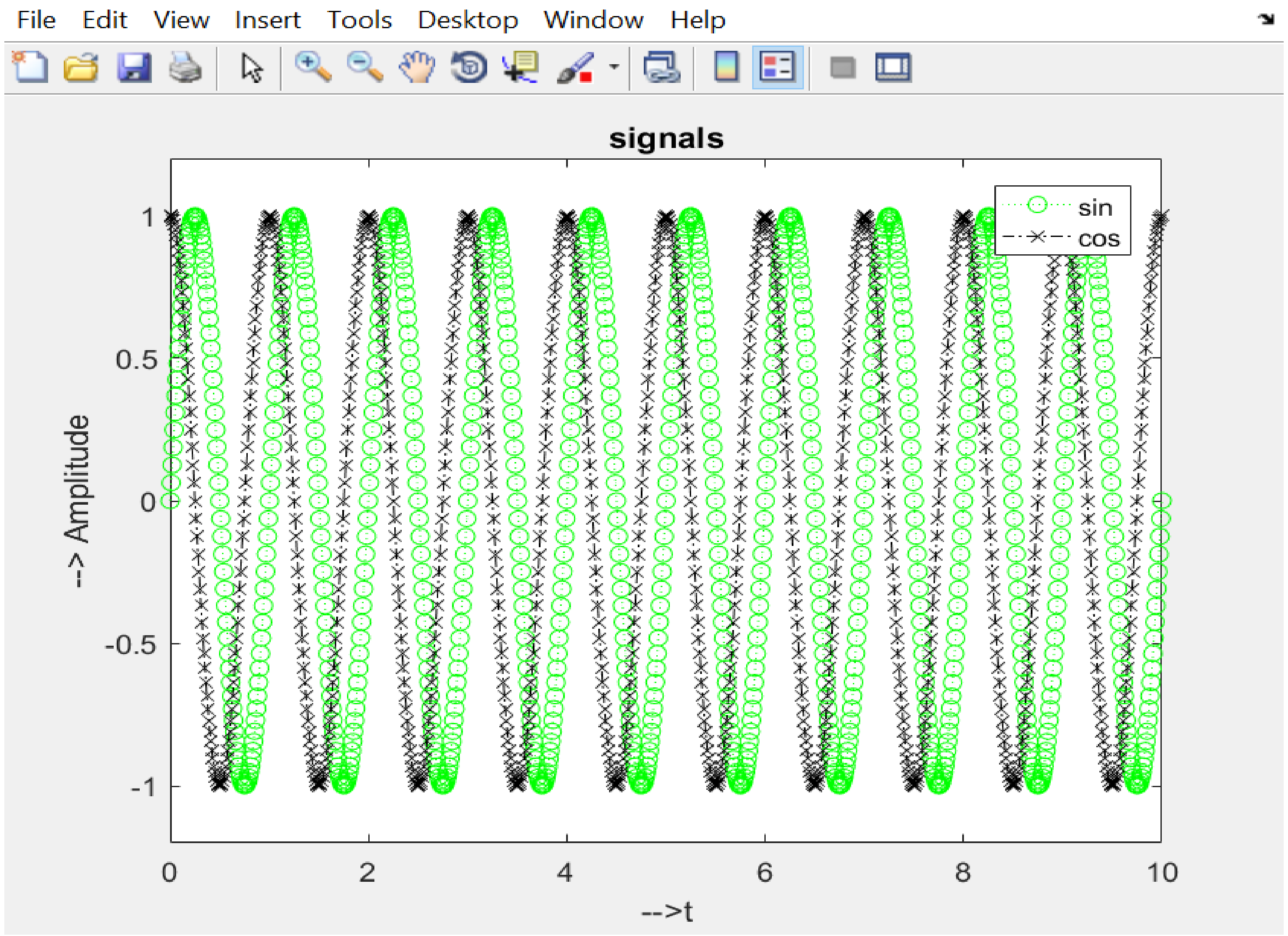
```
t=0:0.01:10
xt=sin (2*pi*t+pi);
yt=cos (2*pi*t);
plot(t,xt);
hold on
plot(t,yt);
xlabel('-->t');
ylabel('--> Amplitude');
title('signals');
legend('sin', 'cos');
axis([t(1) t(end) -1.2 1.2]);
hold off
```



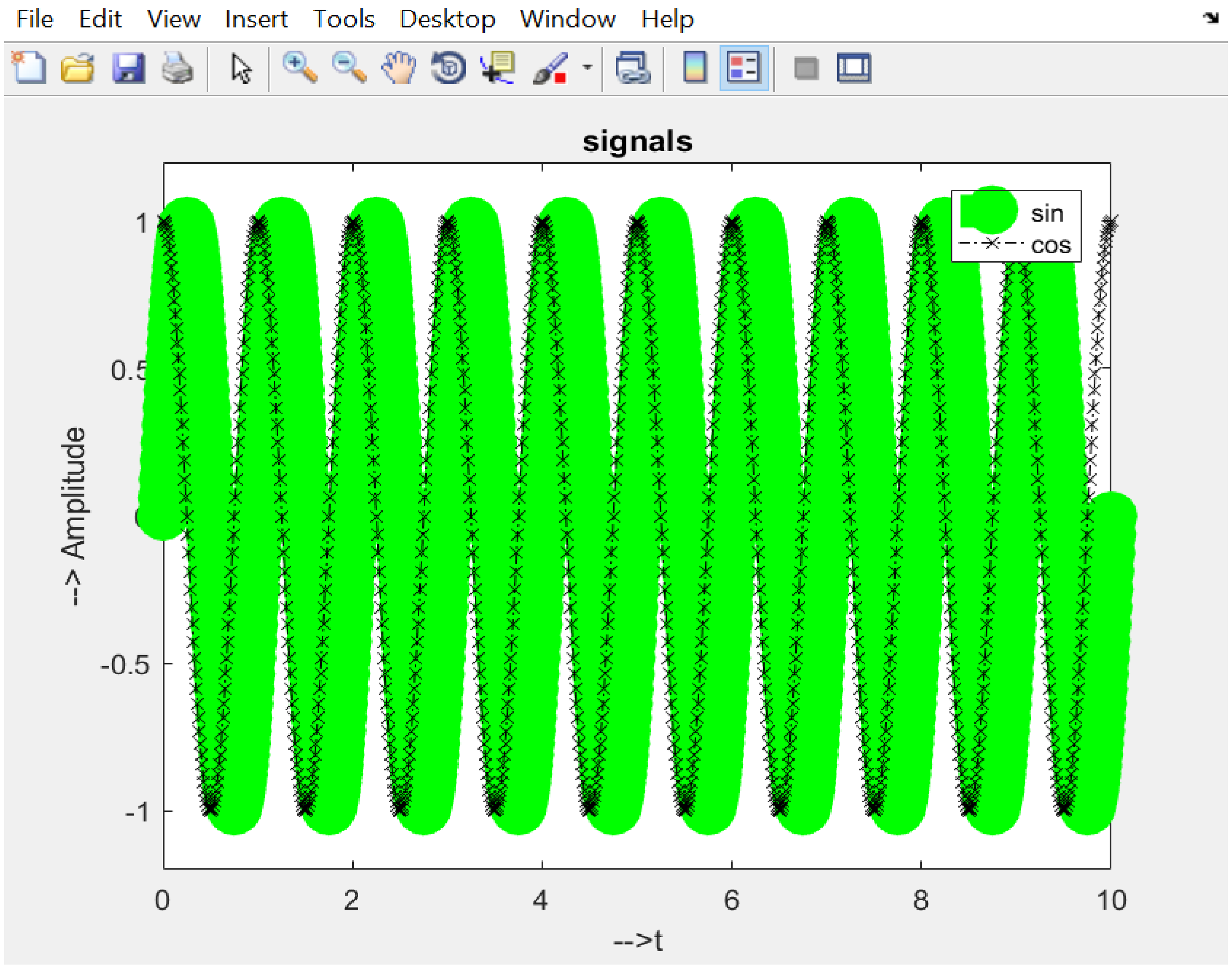
Plot symbols and colors may be obtained with PLOT(X,Y,S) where S is a character string made from one element from any or all the following 3 columns:

b	blue	.	point	-	solid
g	green	o	circle	:	dotted
r	red	x	x-mark	-.	dashdot
c	cyan	+	plus	--	dashed
m	magenta	*	star	(none)	no line
y	yellow	s	square		
k	black	d	diamond		
w	white	v	triangle (down)		
		^	triangle (up)		
		<	triangle (left)		
		>	triangle (right)		
		p	pentagram		
		h	hexagram		

```
t=0:0.01:10
xt=sin (2*pi*t);
yt=cos (2*pi*t);
plot(t,xt,'go:');
hold on
plot(t,yt,'kx-.');
xlabel('-->t');
ylabel('--> Amplitude');
title('signals');
legend('sin', 'cos');
axis([t(1) t(end) -1.2 1.2]);
hold off
```

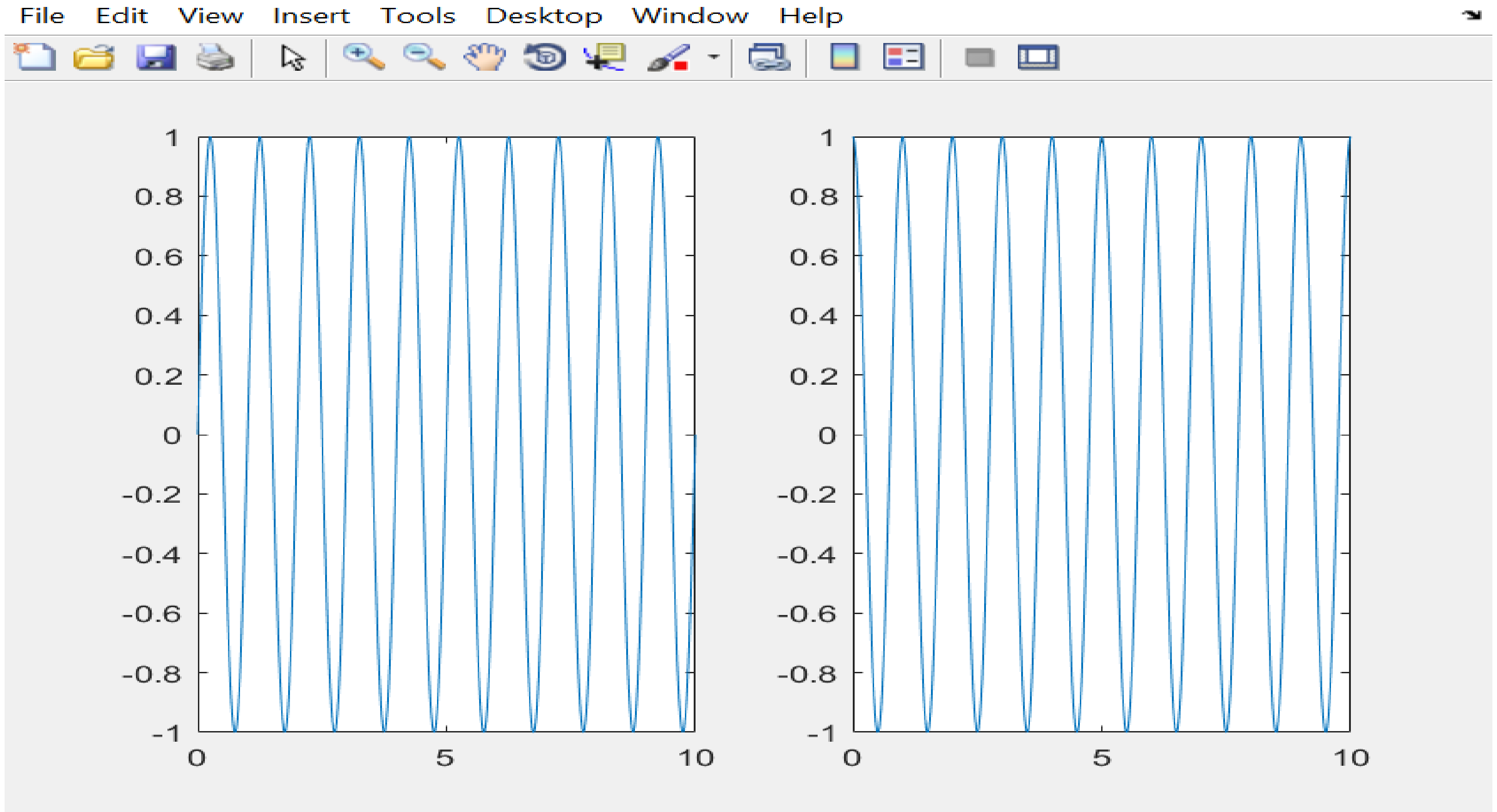


```
t=0:0.01:10
xt=sin (2*pi*t);
yt=cos (2*pi*t);
plot(t,xt,'go:','Linewidth',12);
hold on
plot(t,yt,'kx-.');
xlabel('-->t');
ylabel('--> Amplitude');
title('signals');
legend('sin', 'cos');
axis([t(1) t(end) -1.2 1.2]);
hold off
```



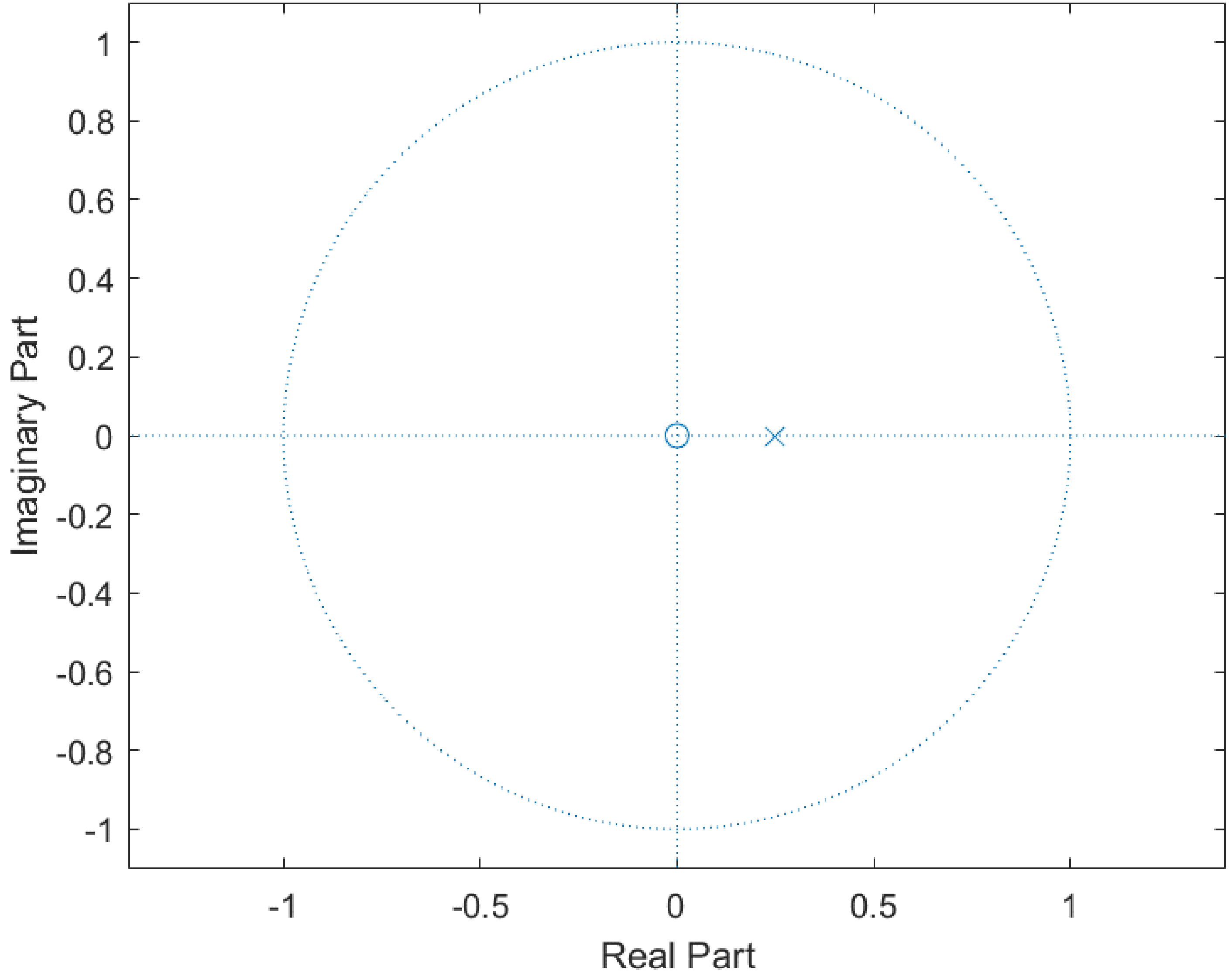
Subplot(m,n,p)

```
t=0:0.01:10
xt=sin (2*pi*t);
yt=cos (2*pi*t);
subplot(1,2,1);
plot(t,xt);
subplot(1,2,2);
plot(t,yt);
```



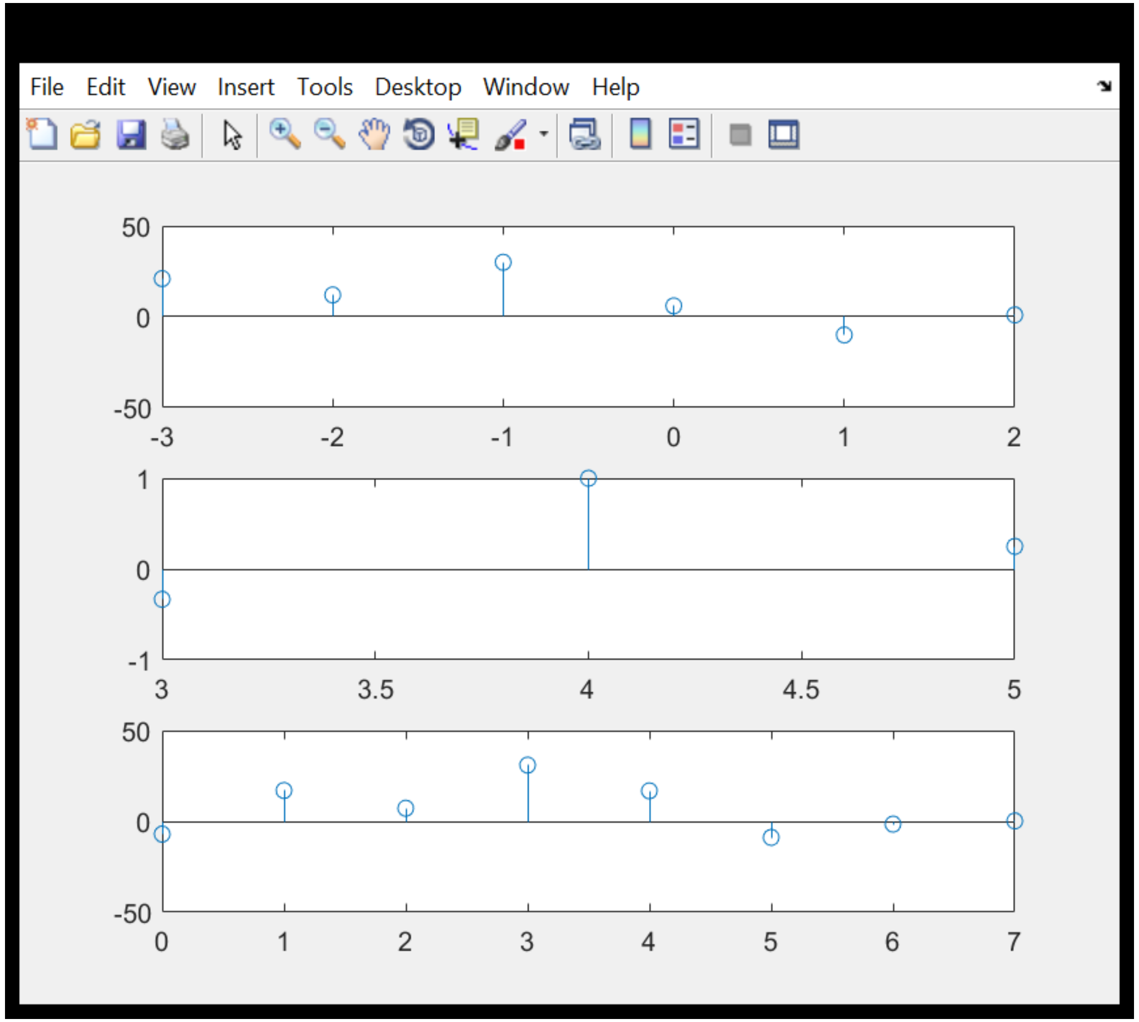
Z-Transform

```
clc
syms z
F = 1/4^n;
X=ztrans(F)
%x=z/(z - 1/4)
%a=[1 0]
%b=[1 -1/4]
%zplane(a,b)
```

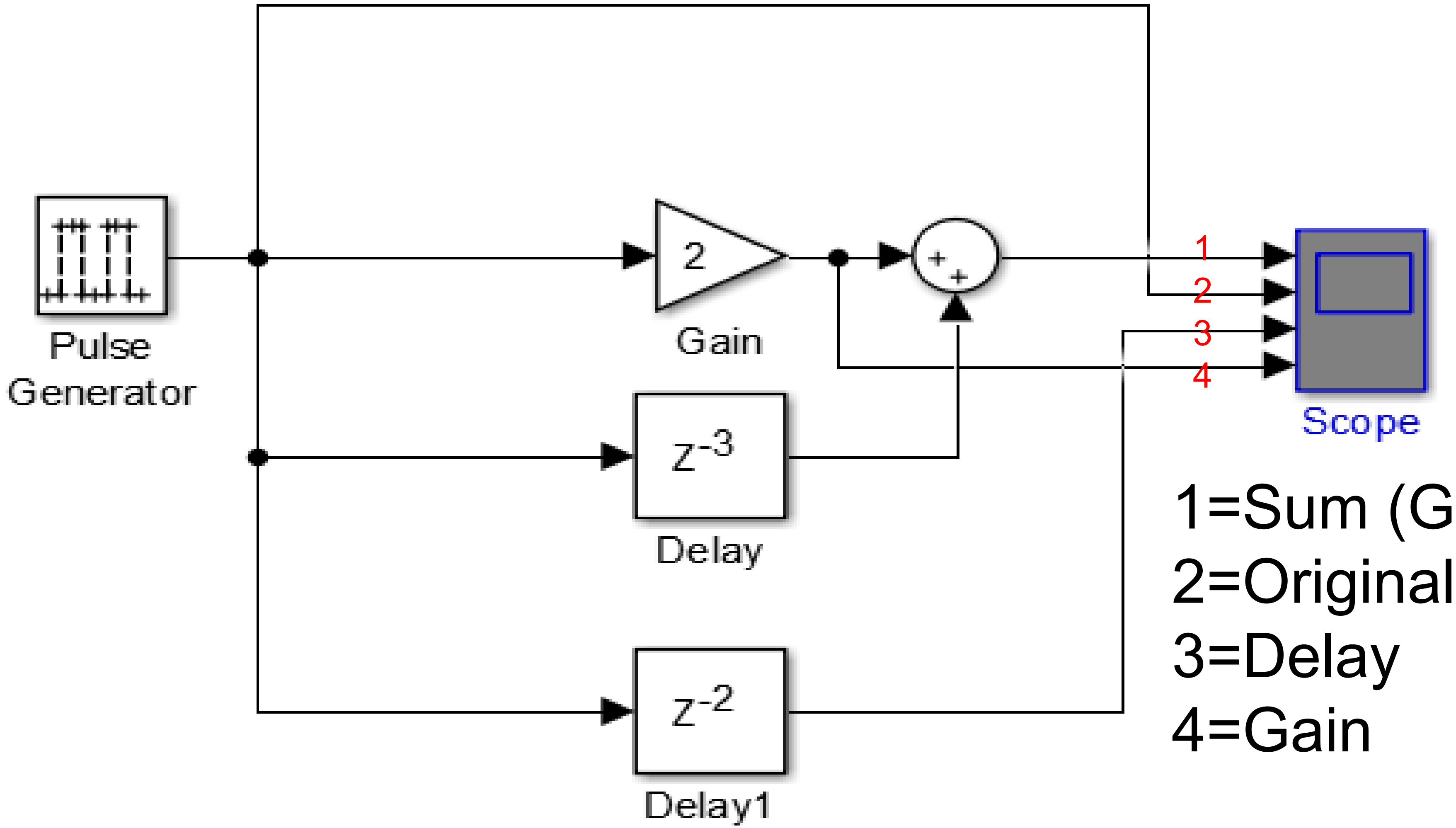


Convolution

```
x=[21 12 30 6 -10 1];
x_x=-3:2; % x_x=Min:Max value
h=[-1/3 1 1/4];
x_h=3:5; % x_h=Min:Max value
y=conv(x,h);
x_y= (x_x(1)+x_h(1)):(x_x(end)+x_h(end));
figure
subplot(3,1,1)
stem(x_x,x)
subplot(3,1,2)
stem(x_h,h)
subplot(3,1,3)
stem(x_y,y)
```

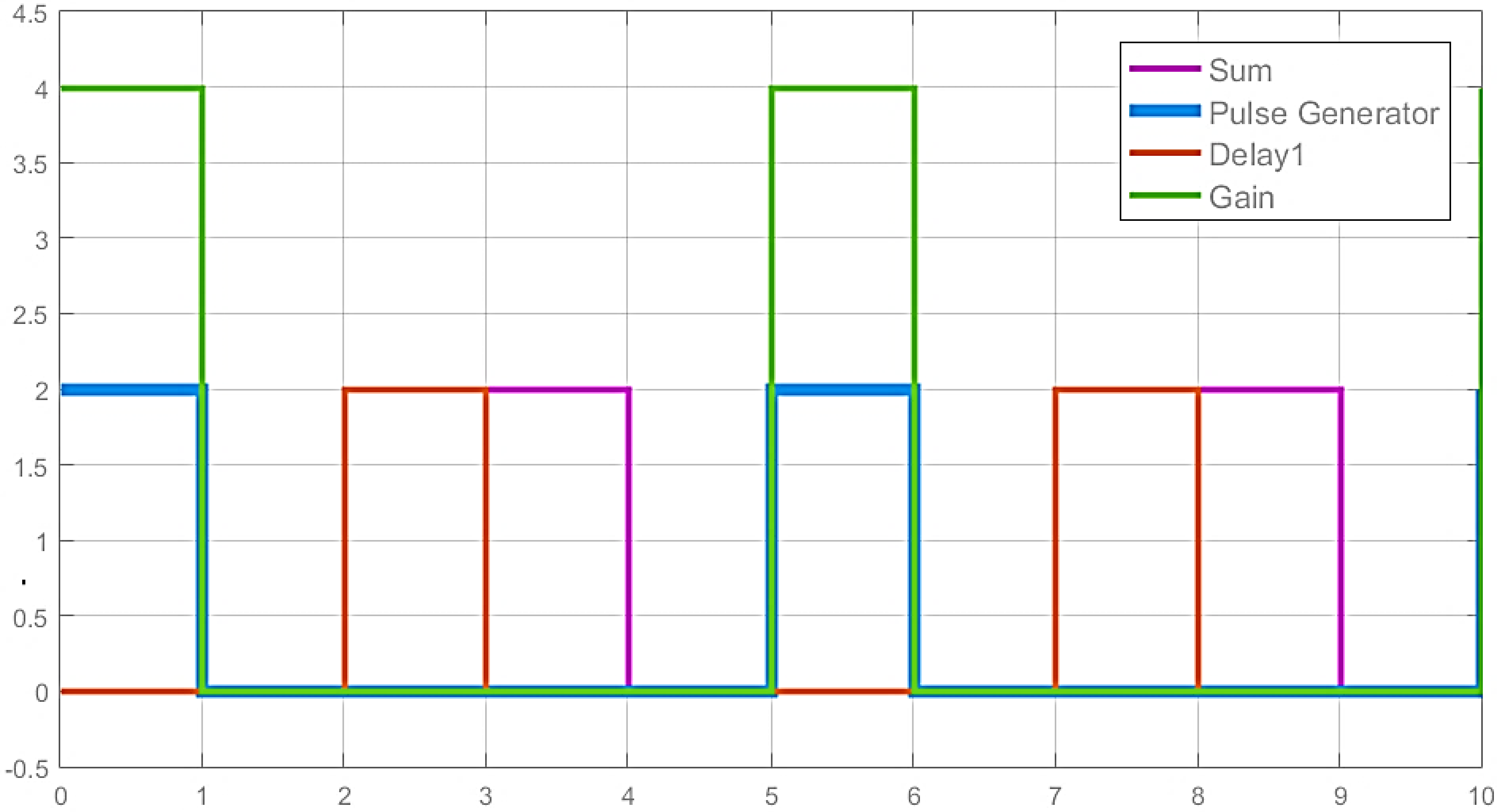


MATLAB Simulink



1=Sum (Gain & Delay)
2=Original Signal
3=Delay
4=Gain

Simulink Result



Home Work

1- Use MATLAB Code to Generate a Convolution Operation

x={25 16 -30 6 -1 19 5}

↑

h={1/3 0 -1/4}

↑

2- Use MATLAB Code to Generate Z-Transform Operation

F = 1/3^n

3- Use MATLAB Code to Generate Inverse Z-Transform

$$X(z) = \frac{-2z}{z-2} + \frac{3z}{z-3}$$