Definite Integral

$\int_a^b f'(x).dx = f(x)\Big|_a^b = f(b) - f(a)$, *a & b are Constant*

Ex.:

$\int_1^2 3x^2.dx = 3\frac{x^3}{3}\Big|_1^2 = x^3\Big|_1^2 = (2)^3 - (1)^3 = 8 - 1 = 7$

Properties of Definite Integral:

1. $\int_a^a f(x).dx = 0$

   Ex.: $\int_2^2 x.dx = \frac{x^2}{2}\Big|_2^2 = 2 - 2 = 0$

2. $\int_a^b f(x).dx = -\int_b^a f(x).dx$ قلب الحدود

   Ex.: $\int_2^3 x^3.dx = -\int_3^2 x^3.dx$

3. $\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$ 1

4. $\int_a^b k.dx = k\,x\Big|_a^b = k\,(b - a)$, *k Constant*

   Ex.: $\int_3^{13} 4.dx = 4\,x\Big|_3^{13} = 4(13 - 3) = (4)(10) = 40$

5. $\int_a^b [f(x) \pm g(x)].dx = \int_a^b f(x).dx \pm \int_a^b g(x).dx$

Ex.: *If we have*

$\int_0^{16} f(x).dx = 12$ & $\int_{19}^{16} 12f(x).dx = 48$

Find $\int_0^{19} f(x).dx$

Sol.:

$$\int_0^{16} f(x).dx + (-)\int_{16}^{19} 12f(x).dx$$

$$-\int_{16}^{19} \frac{12}{12} f(x).dx = \frac{48}{12} = -\int_{16}^{19} f(x).dx = 4$$

$$\int_{16}^{19} f(x).dx = -4$$

$$\int_0^{16} f(x).dx + (-)\int_{16}^{19} 12f(x).dx = \int_0^{16} 12.dx + (-)\int_{16}^{19} (4).dx$$

$$= 12\,x\Big|_0^{16} - 4\,x\Big|_{16}^{19} = 12(16-0) - 4(19-16) = 192 - 12 = 180$$

Ex.: *If* $f(x) = \begin{cases} 2x & , \quad 0 \le x \le 2 \\ x^2 + 1 & , \quad 2 < x \le 3 \end{cases}$

Find $\int_0^3 f(x).\,dx$

Sol.:

$$\int_0^3 f(x).\,dx = \int_0^2 2x.\,dx + \int_2^3 (x^2+1).\,dx$$

$$= \frac{2x^2}{2}\Big|_0^2 + \left(\frac{x^3}{3} + x\right)\Big|_2^3 = (4-0) + \left[\left(\frac{27}{3} + 3\right) - \left(\frac{8}{3} + 2\right)\right]$$

$$= 4 + \left[\frac{27}{3} + 3 - \frac{8}{3} - 2\right] = 4 + \left[\frac{19}{3} + 1\right] = 4 + \frac{19+3}{3}$$

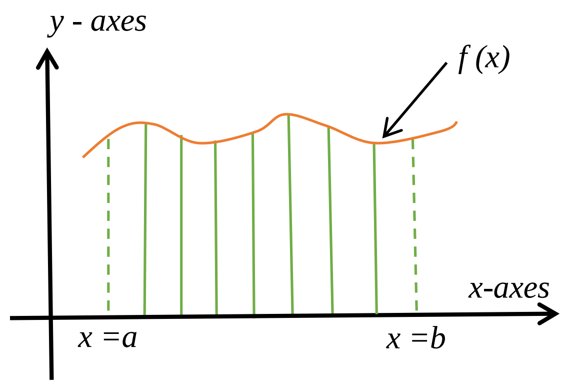
$$= 4 + \frac{22}{3} = \frac{12+22}{3} = \frac{34}{3}$$

Integral Applications

* من أهم التطبيقات في التكامل هو أيجاد المساحات، منها أيجاد المساحة تحت المنحني أو ما بين منحني وأحد المحاور $(x$ أو $y)$ أو ما بين منحنيين أو ما بين منحني وخط.

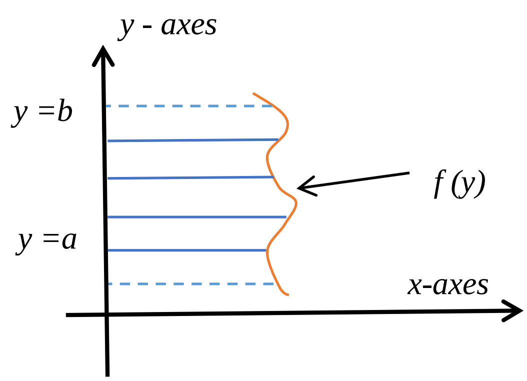
1. Area between curve and $x - axes$:

$$A = \int_{x=a}^{x=b} f(x).dx$$



2. Area between curve and $y - axes$:

$$A = \int_{y=a}^{y=b} f(y).dy$$



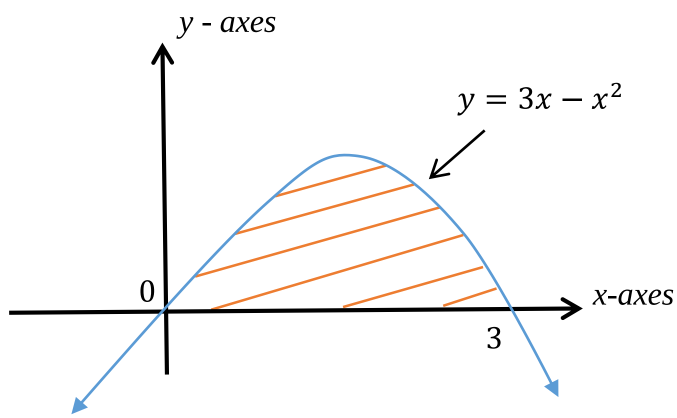
Ex.: *Find the area of the region bounded by* $y = 3x - x^2$ *and the* $x - axes$.

Sol.:

* لغرض حل أي سؤال، نتبع الخطوات التالية:

1. رسم الدالة. 2. تثبيت حدود التكامل. 3. حل التكامل.

x	$y = 3x - x^2$
-2	- 10
-1	- 4
0	0
1	2
2	2
3	0
4	- 4
5	- 10



$$y = 3x - x^2 = x(3 - x) \quad \rightarrow \quad x = 0 \quad , \quad x = 3$$

$$A = \int_0^3 (3x - x^2).\,dx = \left[3\frac{x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$= \left[3\frac{x^2}{2} - \frac{x^3}{3}\right] - \left[3\frac{(0)^2}{2} - \frac{(0)^3}{3}\right] = \left[\frac{27}{2} - \frac{27}{3}\right] - [0 - 0]$$

$$= 4.5 \ unit^2$$

Ex.: *Find the area between* $y = 2 - x^2$ *and the line* $y = -x$.

Sol.:

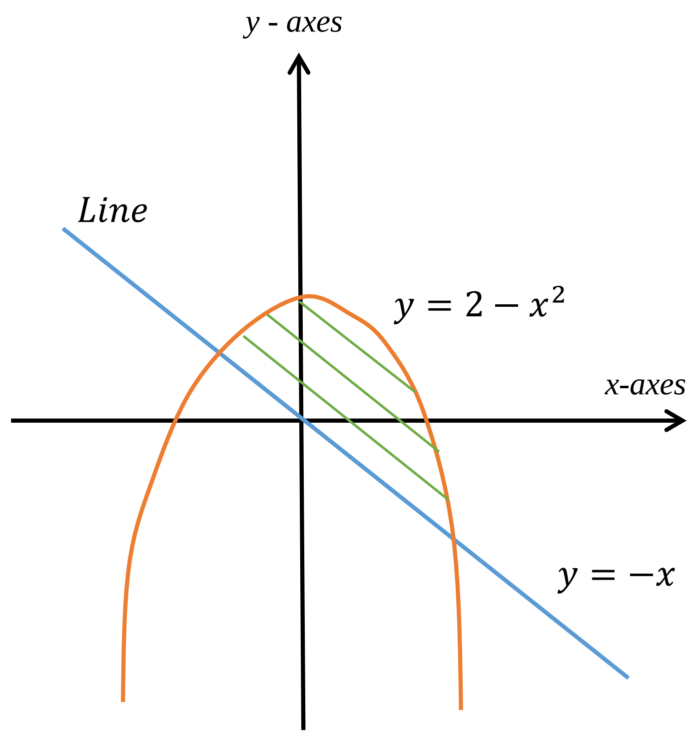
* عندما نرسم الشكل، نستفاد من الرسم للحصول على حدود التكامل (النقاط).

$y = 2 - x^2$

If $x = 0 \Rightarrow y = 2$
$x = 1 \Rightarrow y = 1$
$x = 2 \Rightarrow y = -2$
$x = 3 \Rightarrow y = -7$
$x = -1 \Rightarrow y = 1$
$x = -2 \Rightarrow y = -2$
$x = -3 \Rightarrow y = -7$

$y = -x$ *(Plot line)*

If $x = 0 \Rightarrow y = 0$
$x = 1 \Rightarrow y = -1$
$x = 2 \Rightarrow y = -2$
$x = 3 \Rightarrow y = -3$
$x = -1 \Rightarrow y = 1$
$x = -2 \Rightarrow y = 2$
$x = -3 \Rightarrow y = 3$



* هناك طريقة اخرى لإيجاد حدود التكامل، نساوي الدالتين:

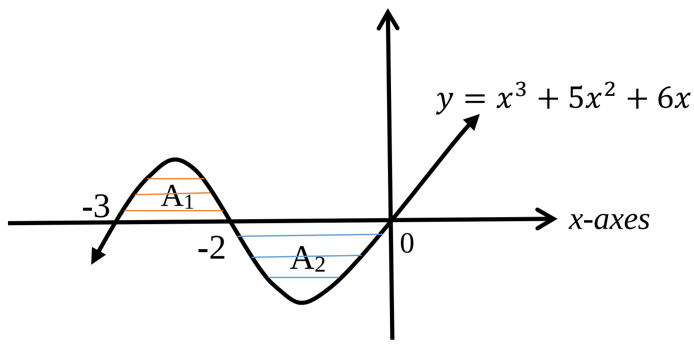
$$2 - x^2 = -x \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x+1)(x-2) = 0$$

$$x = -1 = a \quad , \quad x = 2 = b$$

$$A = \int_a^b [y_1(x) - y_2(x)].dx = \int_{-1}^{2} [2 - x^2 + x]_{-1}^{2}.dx = \frac{9}{2}$$

$$= 4.5 \ unit\ area.$$

Ex.: *Find the area of the region bounded by* $y = x^3 + 5x^2 + 6x$ *and the* $x - axes$.



Sol.:

$$y = x^3 + 5x^2 + 6x$$
$$= x(x^2 + 5x + 6)$$
$$= x(x + 2)(x + 3)$$

$x = 0$, $x = -2$, $x = -3$

$$A = A_1 + A_2$$

$$A_1 = \int_{-3}^{-2} (x^3 + 5x^2 + 6x).dx = \left[\frac{x^4}{4} + 5\frac{x^3}{3} + 6\frac{x^2}{2}\right]_{-3}^{-2}$$

$$= \left[\frac{(-2)^4}{4} + 5\frac{(-2)^3}{3} + 6\frac{(-2)^2}{2}\right] - \left[\frac{(-3)^4}{4} + 5\frac{(-3)^3}{3} + 6\frac{(-3)^2}{2}\right]$$

$$A_1 = \frac{5}{12} \text{ unit area.}$$

$$|A_2| = \int_{-2}^{0} (x^3 + 5x^2 + 6x).dx = \left[\frac{x^4}{4} + 5\frac{x^3}{3} + 6\frac{x^2}{2}\right]_{-2}^{0}$$

$$= \left[\frac{(0)^4}{4} + 5\frac{(0)^3}{3} + 6\frac{(0)^2}{2}\right] - \left[\frac{(-2)^4}{4} + 5\frac{(-2)^3}{3} + 6\frac{(-2)^2}{2}\right]$$

$$|A_2| = \frac{8}{3} \text{ unit area.}$$

$$A = A_1 + A_2 = \frac{5}{12} + \frac{8}{3} = \frac{37}{12} \text{ unit area.}$$

H.W.: *Find the definite integrals for the following functions:*

1. $\int_0^1 \sqrt{x}\,.dx$

2. $\int_0^1 (x + x^2)\,.dx$

3. $\int_{-1}^3 \frac{1}{\sqrt{3x+4}}\,.dx$

4. $\int_1^2 (-2x + 3)\,.dx$

5. $\int_{-1}^2 (x^2 - 2x)\,.dx$

6. $\int_1^3 \frac{1}{\sqrt{5x-1}}\,.dx$

7. $\int_4^9 \frac{t-8}{\sqrt{t}}\,.dt$

8. $\int_1^2 \left(\frac{4}{x} - \frac{2}{x^2}\right).dx$

9. $\int_0^1 \frac{x}{x+3}\,.dx$

10. $\int_0^2 \sqrt{x+3}\,.dx$

11. $\int_0^{\pi/3} (1 - \sin\theta)\,.d\theta$

12. If $f(x) = \begin{cases} 1 + x & x < 0 \\ 2 - x & 0 \le x \le 2 \\ \frac{3}{2} & x \ge 2 \end{cases}$

    Find $\int_{-1}^3 f(x)\,.dx$

13. If $f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 2 & 1 < x \le 2 \end{cases}$

    Find $\int_0^2 f(x)\,.dx$

14. If $f(x) = \begin{cases} 1 & 0 < x \le 1 \\ -3 & 1 < x \le 2 \end{cases}$

    Find $\int_0^2 5f(x)\,.dx$

15. If $f(x) = \begin{cases} \frac{1}{x^2} & \frac{1}{2} \le x < 1 \\ \sqrt{x} & 1 \le x \le 2 \end{cases}$

    Find $\int_{\frac{1}{2}}^2 f(x)\,.dx$

16. If $f(x) = \begin{cases} 1 + x & 0 \le x \le 1 \\ -x + 2 & 1 < x \le 2 \end{cases}$

    Find $\int_0^2 f(x)\,.dx$

17. If $f(x) = \begin{cases} x & 0 \le x \le 1 \\ x^2 & 1 < x \le 2 \end{cases}$

    Find $\int_0^2 f(x)\,.dx$

18. If $f(x) = \begin{cases} -x & 0 \le x < 1 \\ -1 + x & 1 \le x \le 4 \end{cases}$

    Find $\int_0^4 f(x)\,.dx$

19. If $f(x) =$

    $\begin{cases} x^2 - 1 & -2 \le x \le 1 \\ x - 1 & 1 < x \le 3 \end{cases}$

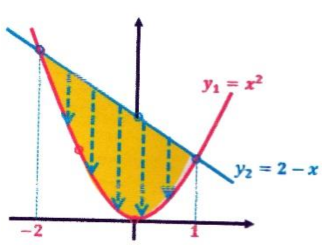
    Find $\int_{-2}^3 f(x)\,.dx$

20. If $f(x) =$

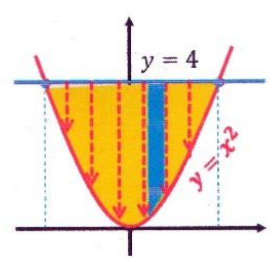
    $\begin{cases} \sqrt{x} & 1 \le x \le 4 \\ 2x^2 - 6x & 4 < x \le 5 \end{cases}$

    Find $\int_1^5 f(x)\,.dx$

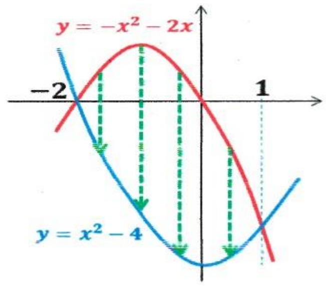
21. Find the area bounded by $y = x^2$ & $y = 2 - x$



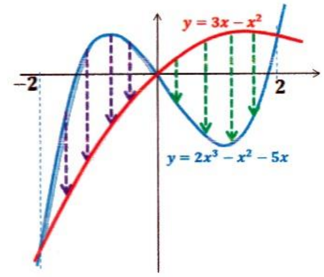
22. Find the area bounded by $y = x^2$ & $y = 4$



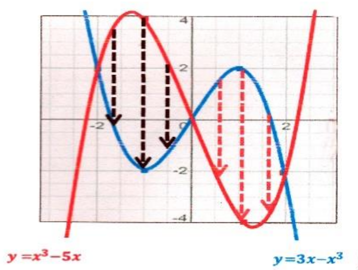
23. *Find the area bounded by* $y = -x^2 - 2x$ & $y = x^2$-4



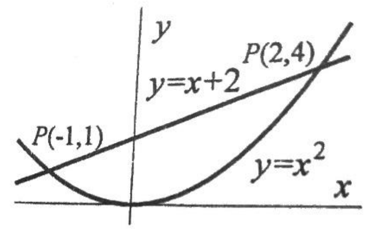
24. *Find the area bounded by* $y = 3x - x^2$ & $y = 2x^3 - x^2 - 5x$



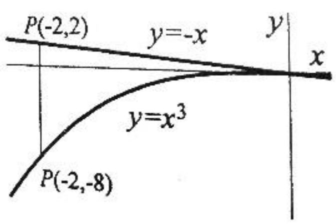
25. *Find the area bounded by* $y = x^3 - 5x$ & $y = 3x - x^3$



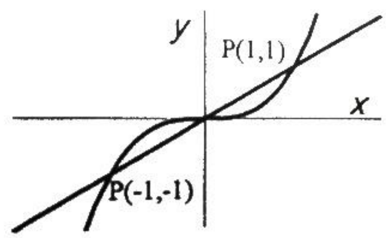
26. *Find the area bounded by* $y = x^2$ & $y = x + 2$



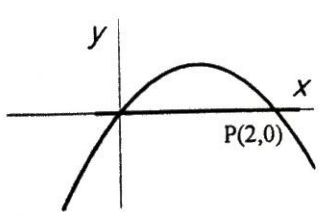
27. *Find the area bounded by* $y = x^3$ & $y = -x$ *in period* $[-2,0]$



28. *Find the area bounded by* $y = x^3$ & $y = x$



29. *Find the area bounded by* $y = 2x - x^2$ & $y = 0$



30. *Find the area bounded by* $y = 5x - x^3$ & $y = 0$

31. *Find the area bounded by* $y = x^3 - 3x$ & $y = x$

32. *Find the area bounded by* $y = x^3 - 6x$ & $y = -2x$

Indefinite Integral

التكامل غير المحدد: هو العملية العكسية لعملية الاشتقاق.

ملاحظة: في التكامل غير المحدد يجب ان نضيف C (integral constant) الى ناتج التكامل.

Properties of the indefinite integral

Rule. 1:

$$\int k\,dx = kx + c$$

$c = integral\ constant$

$k = number$

Ex.:

1. $\int 4\,dx = 4x + c$
2. $\int \pi\,dx = \pi x + c$
3. $\int \frac{dx}{2} = \int \frac{1}{2}dx = \frac{1}{2}x + c$
4. $\int \frac{1}{3}dm = \frac{1}{3}m + c$

Rule. 2:

$$\int \mathrm{x}^n\,dx = \frac{\mathrm{x}^{n+1}}{n+1} + \mathrm{c}$$

Ex.:

1. $\int x^2\,dx = \frac{1}{3}x^3 + c$
2. $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + c = \frac{-1}{x} + c$

3. $\int \sqrt[3]{x}\,dx = \int x^{\frac{1}{3}}\,dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3}{4}x^{\frac{4}{3}} + c$

4. $\int \frac{1}{\sqrt{x^3}}\,dx = \int x^{\frac{-3}{2}}\,dx = \frac{x^{\frac{-1}{2}}}{\frac{-1}{2}} + c = \frac{-2}{\sqrt{x}} + c$

Rule. 3:

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)(a)} + c$$

a: تمثل مشتقة داخل القوس.

Ex.:

1. $\int (1-2x)^3\,dx = \frac{(1-2x)^4}{(4)(-2)} + c = \frac{-1}{8}(1-2x)^4 + c$

2. $\int \frac{1}{\sqrt[3]{x-1}}\,dx = \int (x-1)^{\frac{-1}{3}}dx = \frac{(x-1)^{\frac{2}{3}}}{\left(\frac{2}{3}\right)(1)} + c = \frac{3}{2}(x-1)^{\frac{2}{3}} + c$

Rule. 4:

$$\int K * f(x)\,dx = K\int f(x)\,dx$$

رقم مضروب في دالة

Ex.:

$$\int 2x^3\,dx = 2\int x^3 dx = 2\frac{x^4}{4} + c = \frac{1}{2}x^4 + c$$

Rule. 5:

$$\int [f(x) \mp g(x)]dx = \int f(x)dx \mp \int g(x)dx$$

تستخدم هذه الصيغة في حالتي الجمع والطرح.

Ex.:

1. $\int (x^2 + 6x - 3)dx = \int x^2 dx + \int 6x\, dx - \int 3dx$

$$= \frac{1}{3}x^3 + 3x^2 - 3x + c$$

2. $\int \left(x^6 + \frac{1}{x^3} - \sqrt[5]{x^2}\right) dx = \int \left(x^6 + x^{-3} - x^{\frac{2}{5}}\right) dx$

$$= \frac{x^7}{7} + \frac{x^{-2}}{-2} - \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + c = \frac{1}{7}x^7 - \frac{1}{2x^2} - \frac{5}{7}x^{\frac{7}{5}} + c$$

Ex.:

* يجب تبسيط المعادلات (اذا كان هناك تبسيط) قبل اجراء التكامل.

1. $\int \frac{x^3 \cdot \sqrt{x}}{x}\, dx = \int x^2 \cdot x^{\frac{1}{2}}\, dx = \int x^{\frac{5}{2}}\, dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + c = \frac{2}{7}x^{\frac{7}{2}} + c$

2. $\int (1 - x)(\sqrt{x})dx = \int \left(\sqrt{x} - x^{\frac{3}{2}}\right) dx = \int \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c = \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + c$$

3. $\int \frac{x^3 + 5x^2 - 4}{x^2}\, dx = \int \left(\frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4}{x^2}\right) dx = \int \left(x + 5 - \frac{4}{x^2}\right) dx$

$$= \int (x + 5 - 4x^{-2})\, dx = \frac{x^2}{2} + 5x + 4\frac{1}{x} + c$$

4. $\int \left(\frac{x^2-4}{x-2}\right) dx = \int \left(\frac{(x-2)(x+2)}{(x-2)}\right) dx = \int (x+2)\, dx = \frac{x^2}{2} + 2x + c$

5. $\int \frac{1}{x^2+6x+9}\, dx = \int \frac{1}{(x+3)(x+3)} dx = \int \frac{1}{(x+3)^2} dx = \int (x+3)^{-2} dx$

$$= \frac{(x+3)^{-1}}{(-1)(1)} + c = \frac{-1}{x+3} + c$$

بعض المتطابقات المهمة:

$secx = \frac{1}{cosx}$. $cscx = \frac{1}{sinx}$. $tanx = \frac{sinx}{cosx}$

$$cotx = \frac{1}{tanx} = \frac{cosx}{sinx}$$

$sin^2(x) + cos^2(x) = 1$, $tan^2(x) = sec^2(x) - 1$

$cot^2(x) = csc^2(x) - 1$, $sin(2x) = 2sin(x)cos(x)$

$$cos(2x) = cos^2(x) - sin^2(x) = 2cos^2(x) - 1 = 1 - 2sin^2(x)$$

Rule. 6:

$$\int sin(ax+b)\,dx = \frac{-cos(ax+b)}{a} + c$$

$$\int cos(ax+b)\,dx = \frac{sin(ax+b)}{a} + c$$

$$\int sec^2(ax+b)\,dx = \frac{tan(ax+b)}{a} + c$$

$$\int csc^2(ax+b)\,dx = \frac{-cot(ax+b)}{a} + c$$

$$\int sec(ax+b)\cdot\ tan(ax+b)dx = \frac{sec(ax+b)}{a} + c$$

$$\int csc(ax+b)\cdot\ cot(ax+b)dx = \frac{-csc(ax+b)}{a} + c$$

ملاحظة:

tan دائماً ترتبط بـ sec . و cot دائماً ترتبط بـ csc .

Ex.:

1. $\int (sinx - 3cosx)\, dx = -\, cosx - 3sinx \; + c$

2. $\int cos(3x-1)\, dx = \frac{sin(3x-1)}{3} + c = \frac{1}{3} sin(3x-1) + c$

3. $\int sec(2x) \cdot \; tan(2x) dx = \frac{sec(2x)}{2} + c = \frac{1}{2} sec(2x) + c$

4. $\int sin^2(x) + cos^2(x)\, dx = \int 1\, dx = x + c$

5. $\int \frac{secx}{cosx}\, dx = \int secx\, secx\, dx = \int sec^2 x\, dx = tanx + c$

6. $\int \frac{5}{sin^2 x}\, dx = \int 5\, csc^2 x\, dx = 5(-cotx) + c = -5cotx + c$

7. $\int cos(x) \cdot \; tan(x) dx = \int cosx \frac{sinx}{cosx} dx = \int sin(x) dx = -cosx + c$

8. $\int \frac{cos^3(x)-5}{1-sin^2(x)}\, dx = \int \frac{cos^3(x)-5}{cos^2(x)}\, dx = \int \left(\frac{cos^3(x)}{cos^2(x)} - \frac{5}{cos^2(x)}\right). dx$

$= \int \left(cos(x) - 5\, sec^2(x)\right). dx = sin(x) - 5tan(x) + c$

9. $\int tan^2(x) dx = \int (sec^2(x) - 1)\, dx = tan(x) - x + c$

10. $\int cot^2(x) dx = \int (csc^2(x) - 1)\, dx = -cot(x) - x + c$

11. $\int \frac{1}{1+sin(x)}\, dx = \int \frac{1}{1+sin(x)} \cdot \frac{1-sin(x)}{1-sin(x)} dx = \int \frac{1-sin(x)}{1-sin^2(x)} dx$

$$= \int \frac{1-sin(x)}{cos^2(x)} dx = \int \frac{1}{cos^2(x)} - \frac{sin(x)}{cos^2(x)} dx$$

$$= \int (sec^2(x) - tanx \cdot secx)\, dx = tan(x) - sec(x) + c$$

- نضرب في مرافق المقام.

- عندما لا توجد حلول مباشرة، نلجأ الى المتطابقات في الحل.

Rule. 7:

$$\int (k)^{(ax+b)}\, dx = \frac{(k)^{(ax+b)}}{(a)\cdot \ln k} + c$$

$$\int e^{(ax+b)}\, dx = \frac{e^{(ax+b)}}{a\cdot \ln e} + c$$

a: تمثل مشتقة داخل القوس.

$\ln e = 1$

Ex.:

1. $\int (2)^{4x+5}\, dx = \frac{(2)^{4x+5}}{4\cdot \ln 2} + c$
2. $\int e^{x}\, dx = e^{x} + c$
3. $\int e^{(2-5x)}\, dx = \frac{e^{(2-5x)}}{(-5)} + c$
4. $\int 2^{x}\cdot e^{x}\, dx = \int (2e)^{x}\, dx = \frac{(2e)^{x}}{(1)\cdot \ln 2e} + c$

ملاحظة: عند وجود عملية ضرب بين رقمين لنفس الاس، فيمكن ضرب الاساس مرفوع للاس المشترك.

Rule. 8:

$$\int \frac{f'(x)}{f(x)} dx = ln|f(x)| + c$$

* عندما يكون البسط هو مشتقة المقام، فالتكامل هو ln المقام.

Ex.:

1. $\int x^{-1} dx = \int \frac{1}{x} dx = ln|x| + c$

2. $\int \frac{4}{x} dx = 4 \int \frac{1}{x} dx = 4ln|x| + c$

3. $\int \frac{2}{x+2} dx = 2 \int \frac{1}{x+2} dx = 2ln|x+2| + c$

4. $\int \frac{2x}{x^2+5} dx = ln|x^2 + 5| + c$

5. $\int \frac{x}{x^2+5} dx = \frac{1}{2} \int \frac{2x}{x^2+5} dx = \frac{1}{2} ln|x^2 + 5| + c$

6. $\int \frac{2}{4x+2} dx = \frac{1}{2} \int \frac{4}{4x+2} dx = \frac{1}{2} ln|4x + 2| + c$

7. $\int tan(x)\, dx = \int \frac{sin(x)}{cos(x)} dx = - \int \frac{-sin(x)}{cos(x)} dx = -ln|cosx| + c$

   $= ln|secx| + c$

8. $\int cot(x)\, dx = \int \frac{1}{tan(x)} dx = \int \frac{cos(x)}{sin(x)} dx = ln|sin(x)| + c$

H.W.: *Find the indefinite integrals for the following functions:*

1. $\int \left(x - \frac{1}{cos^2 x}\right) \cdot dx$
2. $\int cos(2x) \cdot dx$
3. $\int e^{-3x} \cdot dx$
4. $\int 2x \cdot dx$
5. $\int 3\sqrt{x} \cdot dx$
6. $\int (x-2)^2 \cdot dx$
7. $\int \left(x + \frac{1}{x}\right) \cdot dx$
8. $\int \left(\frac{1}{x^7} + \frac{1}{x^2}\right) \cdot dx$
9. $\int 10 \cdot dx$
10. $\int (x^2 + cos\, x) \cdot dx$
11. $\int sec^2(x-1) \cdot dx$
12. $\int sin(4x) \cdot dx$
13. $\int \left(x^{3/2} + x^{3/5}\right) \cdot dx$
14. $\int \frac{1}{\sqrt{x+1}} \cdot dx$
15. $\int \left(\sqrt{t} - sec^2 t + 1\right) \cdot dt$
16. $\int tan^2 x \cdot dx$
17. $\int \left(\frac{1}{csc\, x \cdot cot\, x \cdot cos x}\right) dx$
18. $\int \left(x^{3/2} + x^{5/2}\right) \cdot dx$
19. $\int \frac{x^2 - 2x + 1}{\sqrt{x}} \cdot dx$
20. $\int \left(y + \frac{1}{y}\right)^2 \cdot dy$
21. $\int \left(x - \frac{1}{\sqrt{x}}\right)^3 \cdot dx$
22. $\int \left(\frac{4}{x^3} + \frac{5}{x^2} - \frac{7}{x}\right) \cdot dx$
23. $\int \left(\sqrt{x} + 2csc^2 x\right) \cdot dx$
24. $\int \left[x^{-1}\left(\sqrt{x} + 5x\right)\right] \cdot dx$
25. $\int \left[x^{\frac{-1}{3}}\left(\sqrt{x} + \frac{5}{x}\right)\right] \cdot dx$
26. $\int \left(\sqrt{x+1} - \sqrt{x+2}\right) \cdot dx$
27. $\int cot^2 x \cdot dx$
28. $\int (e^x + e^{-x})^2 \cdot dx$
29. $\int \left(\frac{(x+1)^2 - 1}{(x+1)^3}\right) \cdot dx$
30. $\int \frac{(1 + secx)cotx}{cscx} \cdot dx$
31. $\int \frac{7^{2x} - e^{-5x}}{3} \cdot dx$
32. $\int \left(\frac{4^{2x} - e^{-5x}}{4}\right) \cdot dx$
33. $\int \sqrt{2x - x^2} \cdot dx$
34. $\int \frac{x^2 + 2}{1 + x^2} \cdot dx$
35. $\int \frac{cot\, x}{\sqrt{1 - cos^2 x}} \cdot dx$