University of Technology
Laser & Optoelectronics Engineering Department

Digital Signal Processing II

Lec. Dr. Taif Alawsi

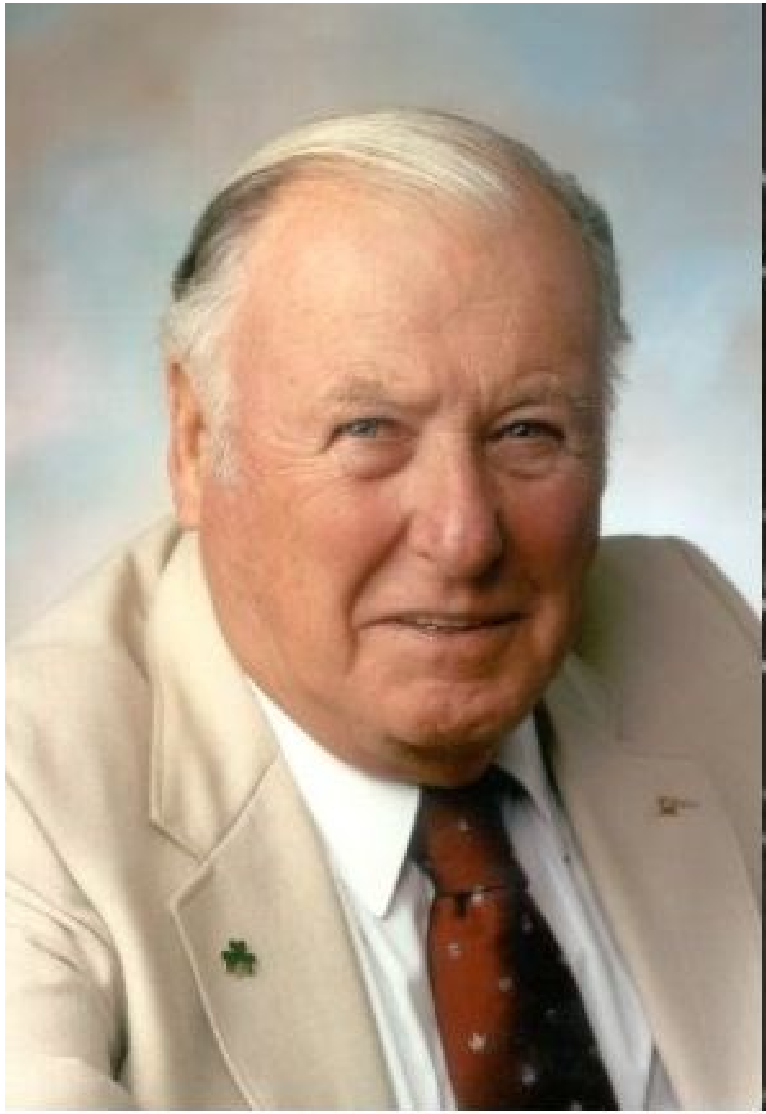
Lec. 1: Fast Fourier Transform 1: 2025-Jan-19

Course Outline

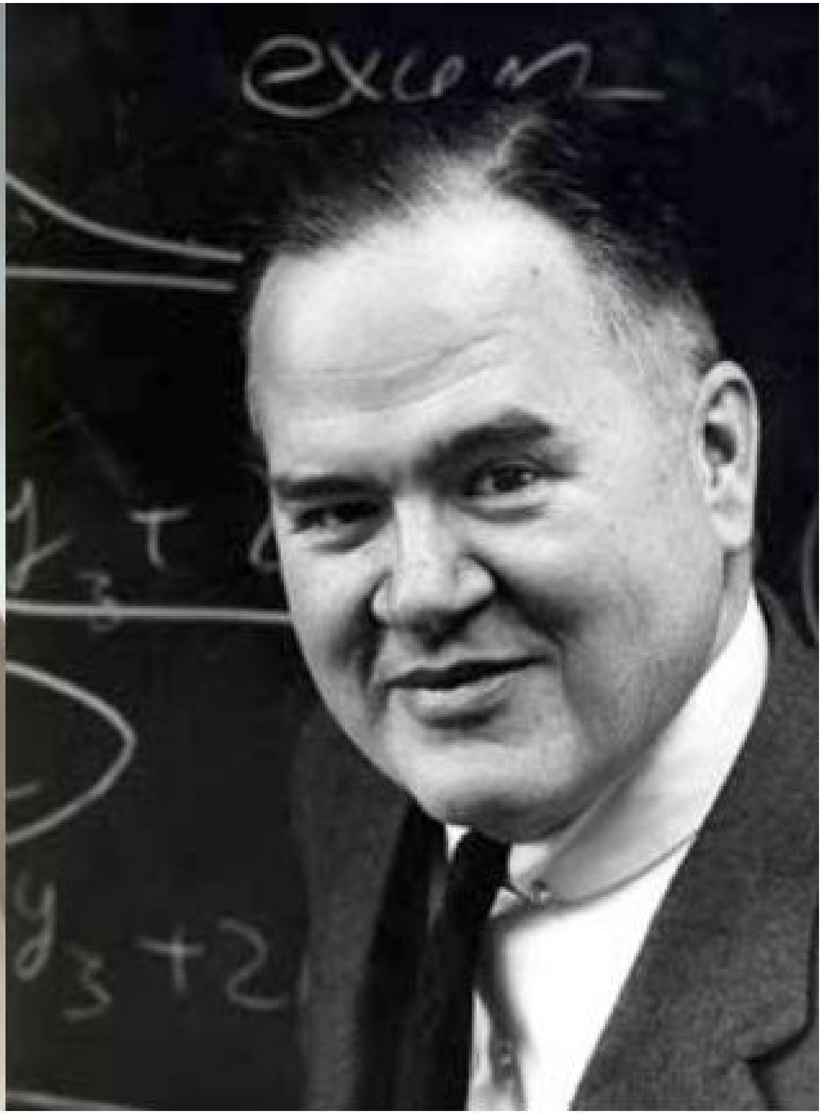
1. Fast Fourier Transform
2. Implementation of Discrete-Time Systems
3. Filters
4. Analog-to-Digital Conversion
5. MATLAB

FFT

- **A fast Fourier transform (FFT) is an algorithm that computes the Discrete Fourier Transform (DFT) of a sequence, or its inverse (IDFT).**
- **A Fourier transform converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa.**
- **The DFT is obtained by decomposing a sequence of values into components of different frequencies.**
- **This operation is useful in many fields.**



James William Cooley
(1926-)



John Wilder Tukey
(1915-2000)

Sequence Length (N)

x(n)={ 1, 2, 3, 4}   N=4

x(n)={ 1, 3, 6, 7, 9, 17}   N=6

In FFT $\mathrm{N} = 2^{x}$

$where\ x\ is\ integer$

FFT

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$g(n) = x(2n) \qquad n = 0, 1, \ldots, \frac{N}{2} - 1$$

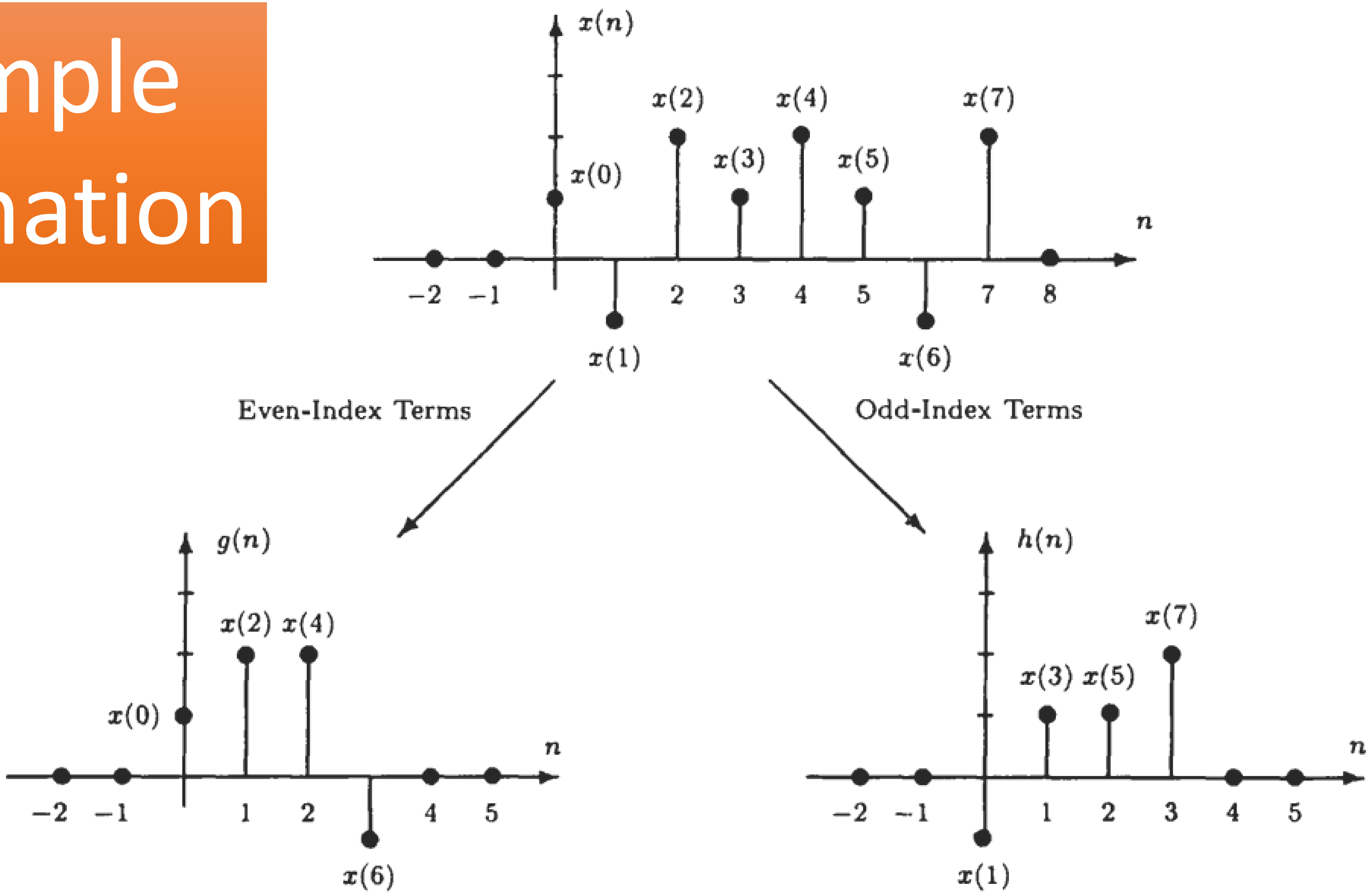
$$h(n) = x(2n+1) \qquad n = 0, 1, \ldots, \frac{N}{2} - 1$$

FFT Equation

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n \text{ even}} x(n) W_N^{nk} + \sum_{n \text{ odd}} x(n) W_N^{nk}$$

$$= \sum_{l=0}^{\frac{N}{2}-1} g(l) W_N^{2lk} + \sum_{l=0}^{\frac{N}{2}-1} h(l) W_N^{(2l+1)k}$$

Example Decimation



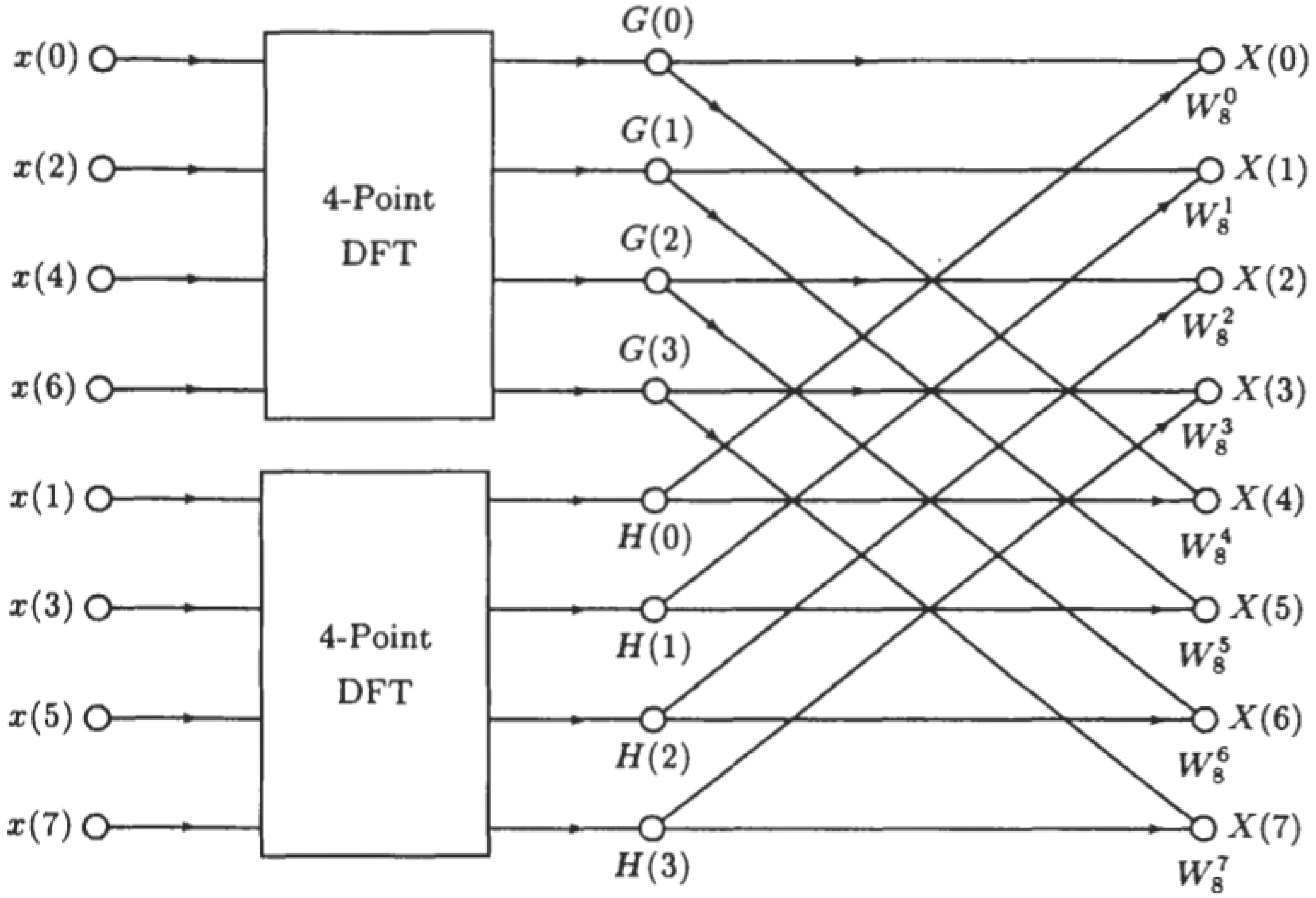
Functions

$$W_N^{2lk} = W_{N/2}^{lk}$$

$$X(k) = \sum_{l=0}^{\frac{N}{2}-1} g(l) W_{N/2}^{lk} + W_N^k \sum_{l=0}^{\frac{N}{2}-1} h(l) W_{N/2}^{lk}$$

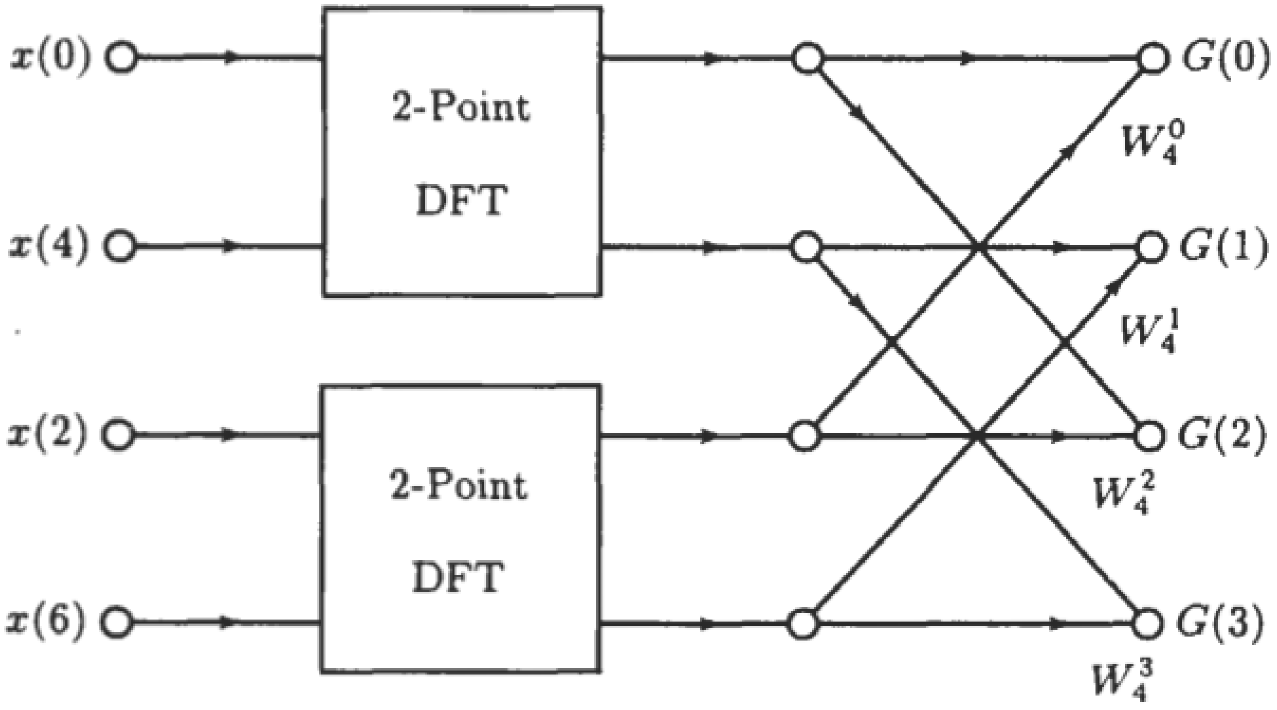
$$G(k) = \sum_{n=0}^{\frac{N}{2}-1} g(n) W_{N/2}^{nk} = \sum_{n \text{ even}}^{\frac{N}{2}-1} g(n) W_{N/2}^{nk} + \sum_{n \text{ odd}}^{\frac{N}{2}-1} g(n) W_{N/2}^{nk}$$

An eight-point decimation-in-time FFT algorithm after the first decimation



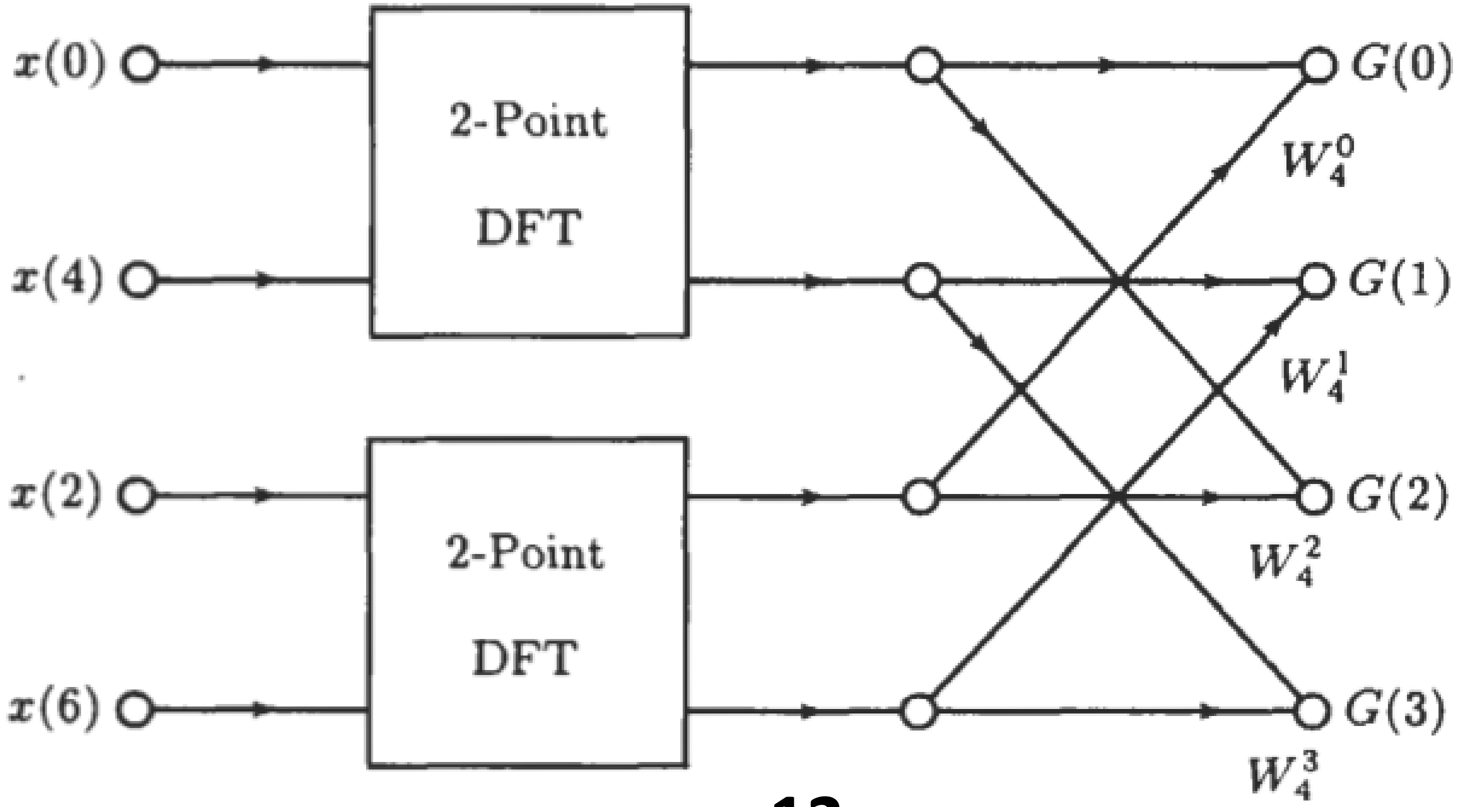
Decimation of the four-point DFT into two two-point DFTs in the decimation-in-time FFT.

$$G(k) = \sum_{n=0}^{\frac{N}{4}-1} g(2n) W_{N/4}^{nk} + W_{N/2}^{k} \sum_{n=0}^{\frac{N}{4}-1} g(2n+1) W_{N/4}^{nk}$$

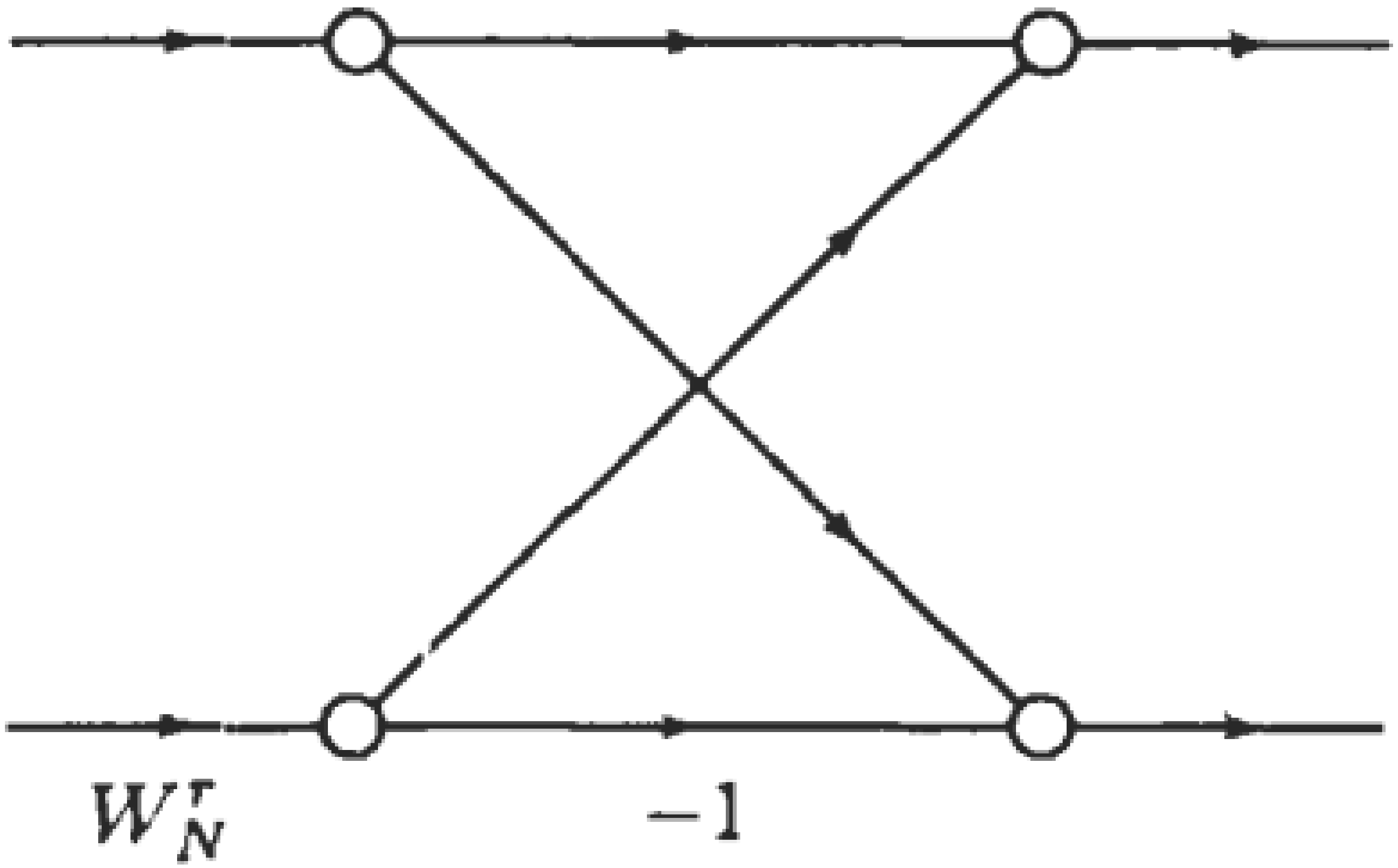


Decimation of the four-point DFT into two two-point DFTs in the decimation-in-time FFT.

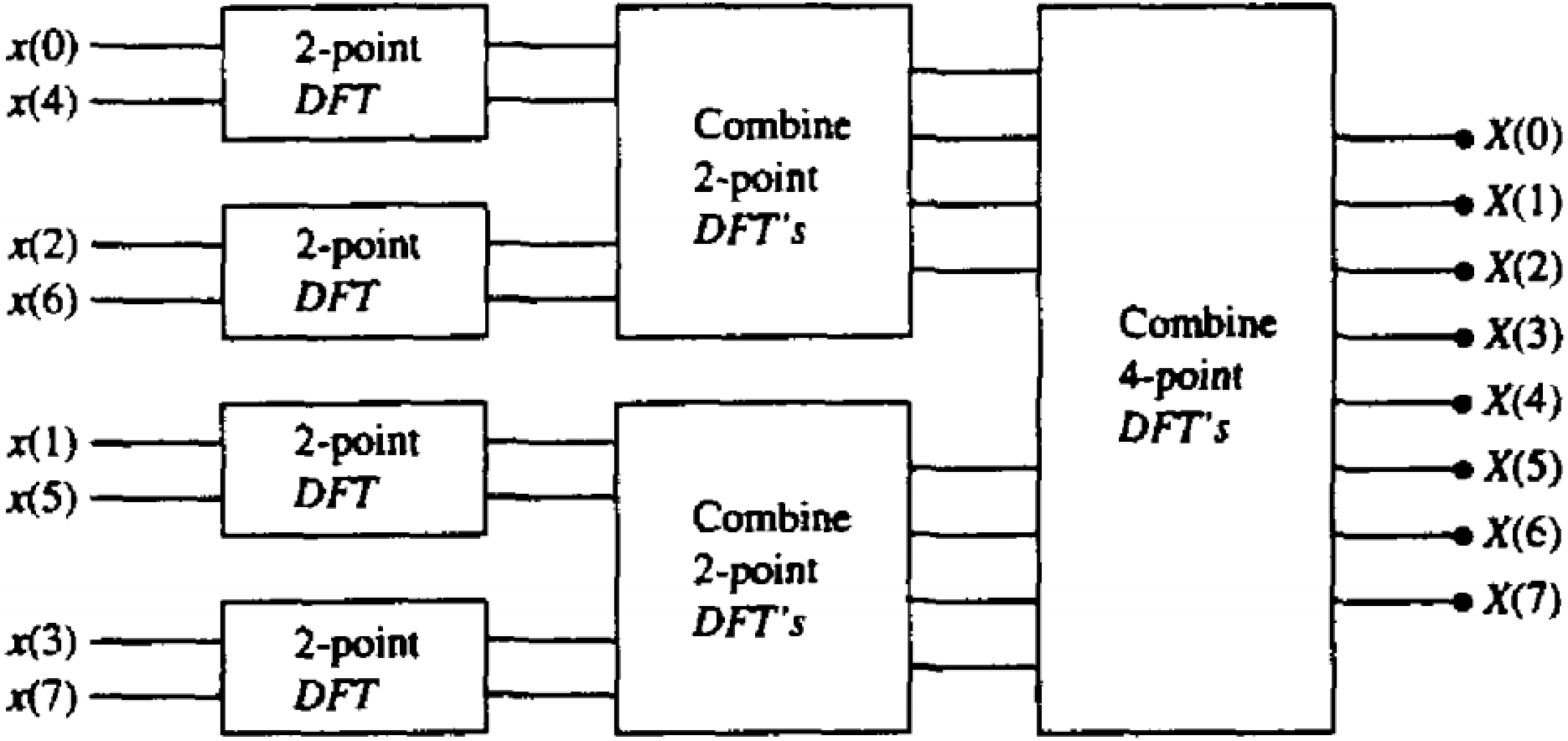
$$G(k) = \sum_{n=0}^{\frac{N}{4}-1} g(2n) W_{N/4}^{nk} + W_{N/2}^{k} \sum_{n=0}^{\frac{N}{4}-1} g(2n+1) W_{N/4}^{nk}$$



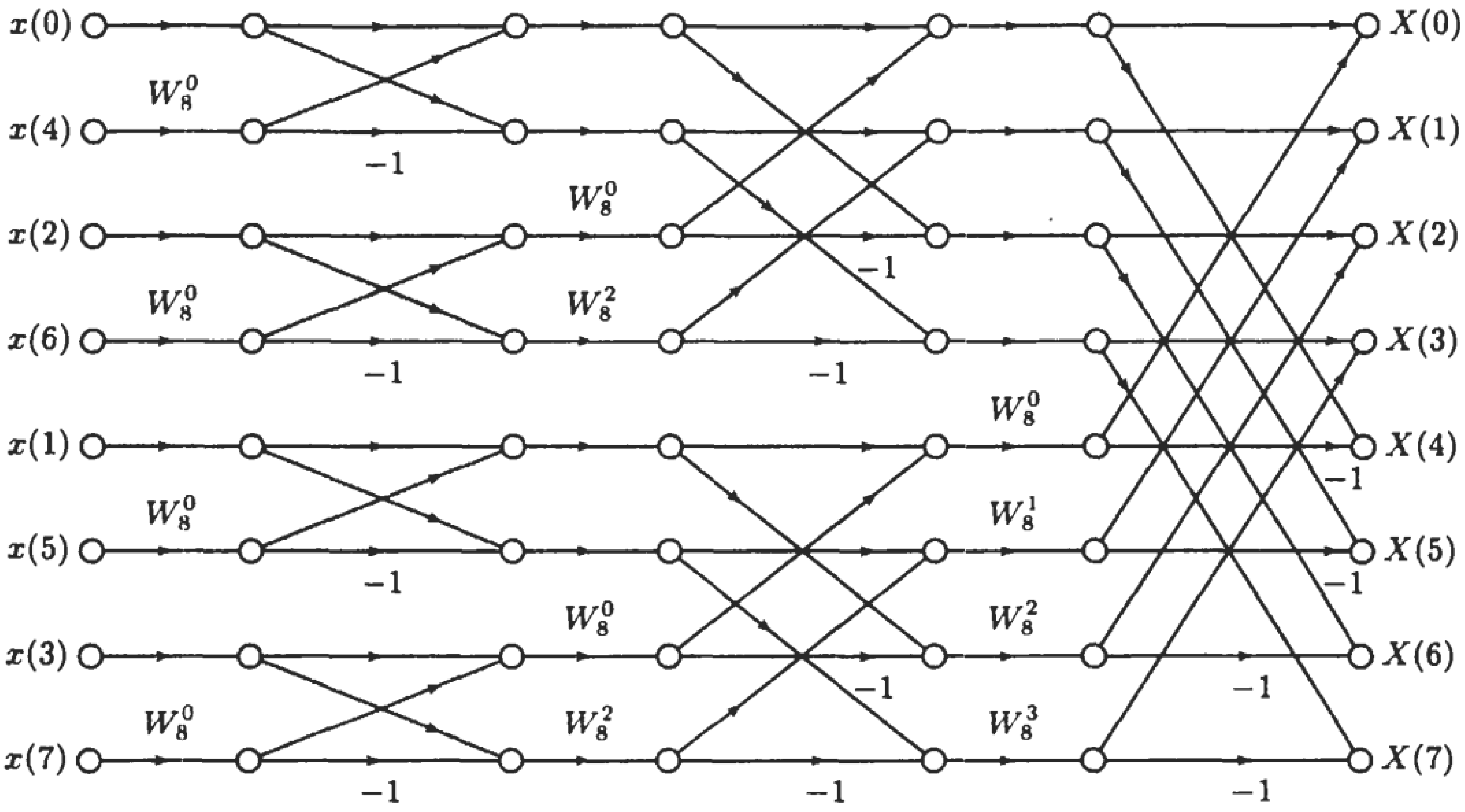
Simplified Butterfly Structure



Eight-point radix-2 decimation-in-time FFT



Eight-point radix-2 decimation-in-time FFT



Calculation of Speed Improvement Factor (S)

$$\mathrm{N} = 2^x$$

Complex Multiplication in Direct Method (D) = N^2

Complex Multiplication in FFT = $\frac{N}{2}\log_2 N$

Speed Improvement Factor (S) = $\frac{D}{FFT}$

Example

If x=6, what is the Speed improvement factor?

Solution

$\mathrm{N} = 2^{x} = 2^{6} = 64$

D = $N^{2} = 64^{2} = 4096$

FFT = $\frac{N}{2}\log_2 N = \frac{64}{2}\log_2 64$

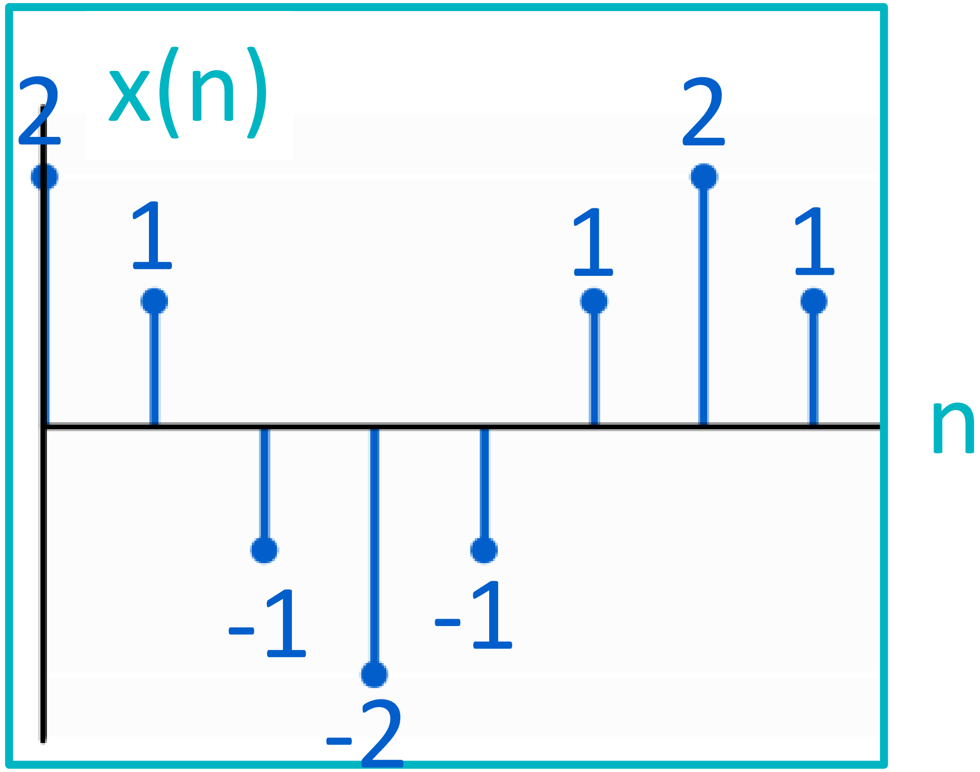
$= 32 * \log_2 2^{6} = 32 * 6 = 192$

$\therefore s = \frac{D}{FFT}$

$= \frac{4096}{192} = 21.3$

Homework

Q1:Decimate the time signal by 2



Q2:Construct a 16-point decimation-in-time FFT algorithm

Q3:Calculate the speed improvement Factor if x=9