Textbooks:

1. Digital Design, Morris M. Mano, (3rd Edition), Prentice Hall, 2002

2. Digital Fundamentals, Thomas L. Floyd, (9th Edition), Prentice Hall, 2006

3. Microprocessor Architecture, Programming, and Applications with the 8085, by R. Gaonkar.

Overview

- The design of computers
  - •It all starts with numbers
  - •Building circuits
  - •Building computing machines
- Digital systems
- Understanding decimal numbers
- Binary and octal numbers
  - •The basis of computers!
- Conversion between different number systems

Digital Systems

- Digital systems consider *discrete* amounts of data.

Examples

- 26 letters in the alphabet
- 10 decimal digits

- Larger quantities can be built from discrete values:
  - Words made of letters
  - Numbers made of decimal digits (e.g. 239875.32)

- Computers operate on *binary* values (0 and 1)

- Easy to represent binary values electrically
  - Voltages and currents.
  - Can be implemented using circuits
  - Create the building blocks of modern computers

Understanding Decimal Numbers

- Decimal numbers are made of decimal digits: (0,1,2,3,4,5,6,7,8,9)
- But how many items does a decimal number represent?

  $8653 = 8*10^3 + 6*10^2 + 5*10^1 + 3*10^0$
- What about fractions?

  $97654.35 = 9*10^4 + 7*10^3 + 6*10^2 + 5*10^1 + 4*10^0 + 3*10^{-1} + 5*10^{-2}$
- In formal notation -> $(97654.35)_{10}$

Understanding Binary Numbers

- Binary numbers are made of binary digits (bits): 0 and 1
- How many items does a binary number represent?

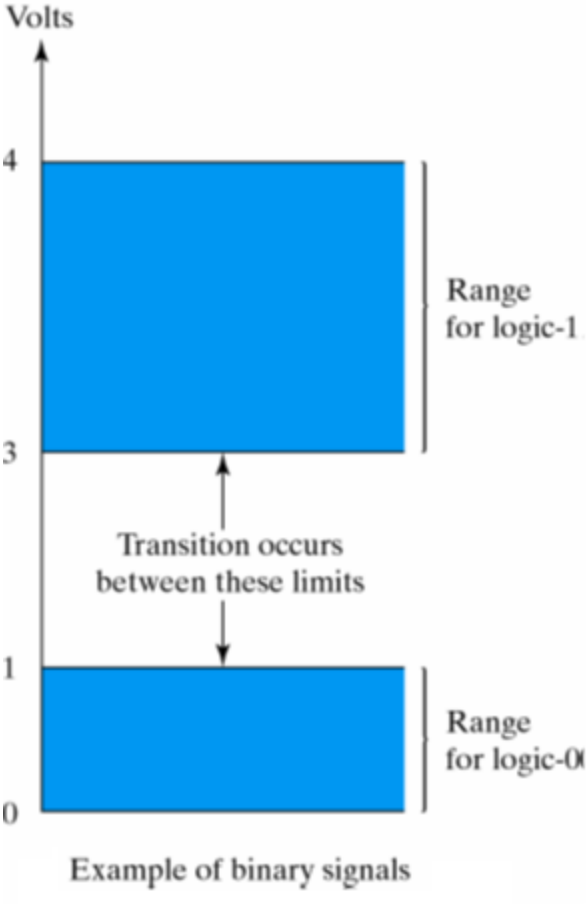
  $(1011)2 = 1*2^3 + 0*2^2 + 1*2^1 + 1*2^0 = (11)_{10}$
- What about fractions?

  $(110.10)2 = 1*2^2 + 1*2^1 + 0*2^0 + 1*2^{-1} + 0*2^{-2}$
- Groups of eight bits are called a *byte*

  $(11001001)_2$
- Groups of four bits are called a *nibble.*

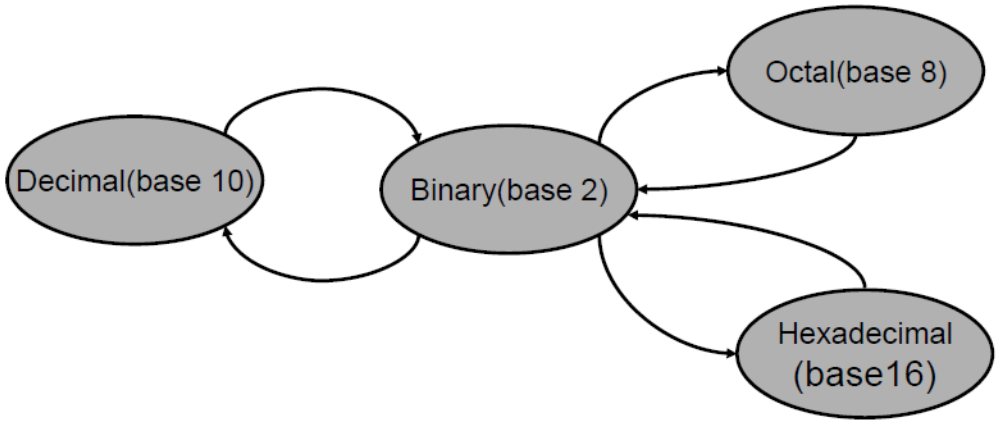
  $(1101)_2$

Why Use Binary Numbers?



- Easy to represent 0 and 1 using electrical values.
- Possible to tolerate noise.
- Easy to transmit data
- Easy to build binary circuits.

Conversion Between Number Bases



- Learn to convert between bases.
- Already demonstrated how to convert from binary to decimal.

Convert an Integer *from* Decimal *to* another Base

For each digit position:

1. Divide decimal number by the base (e.g. 2)
2. The remainder is the lowest-order digit3.Repeat first two steps until no divisor remains.

Example for $(13)_{10}$:

	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	½	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$
3/2 =	1	+	½	$a_2 = 1$
1/2 =	0	+	½	$a_3 = 1$

Answer $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

Convert an Fraction *from* Decimal *to* another Base1.

1. Multiply decimal number by the base (e.g. 2)
2. The *integer* is the highest-order digit
3. Repeat first two steps until fraction becomes zero.

Example for $(0.625)_{10}$:

	Integer		Fraction	Coefficient
0.625 x 2 =	1	+	0.25	$a_{-1} = 1$
0.250 x 2 =	0	+	0.50	$a_{-2} = 0$
0.500 x 2 =	1	+	0	$a_{-3} = 1$

Answer $(0.625)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_2 = (0.101)_2$

The Growth of Binary Numbers

n	2^n
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
4	$2^4=16$
5	$2^5=32$
6	$2^6=64$
7	$2^7=128$

n	2^n	
8	$2^8=256$	
9	$2^9=512$	
10	$2^{10}=1024$	
11	$2^{11}=2048$	
12	$2^{12}=4096$	
20	$2^{20}=1M$	Mega
30	$2^{30}=1G$	Giga
40	$2^{40}=1T$	Tera

Binary Addition

- Binary addition is very simple.
- This is best shown in an example of adding two binary numbers…

```
1   1   1   1   1   1 <-------- carries
    1   1   1   1   0   1
+       1   0   1   1   1
-----------------------
1   0   1   0   1   0   0
```

Binary Subtraction

- We can also perform subtraction (with borrows in place of carries).
- Let's subtract $(10111)_2$ from $(1001101)_2$…

```
        1           10        <-------- borrows
    0   1̸0̸  10   0   0̸  10

    1̸   0̸   0̸   1̸   1̸   0̸   1
-           1   0   1   1   1
---------------------------
        1   1   0   1   1   0
```

Binary Multiplication

Binary multiplication is much the same as decimal multiplication, except that the multiplication operations are much simpler…

```
            1  0  1  1  1
X              1  0  1  0
-------------------------
            0  0  0  0  0
         1  0  1  1  1
      0  0  0  0  0
   1  0  1  1  1
-------------------------
   1  1  1  0  0  1  1  0
```

Understanding Octal Numbers

- Octal numbers are made of octal digits: (0,1,2,3,4,5,6,7)
- How many items does an octal number represent?
  - $(4536)_8 = 4x8^3 + 5x8^2 + 3x8^1 + 6x8^0 = (1362)_{10}$
- What about fractions?
  - $(465.27)_8 = 4x8^2 + 6x8^1 + 5x8^0 + 2x8^{-1} + 7x8^{-2}$
- Octal numbers don't use digits 8 or 9

Convert an Integer *from* Decimal *to* Octal

1. Divide decimal number by the base (8)
2. The *remainder* is the lowest-order digit
3. Repeat first two steps until no *divisor* remains.

Example for $(175)_{10}$:

	Integer Quotient		Remainder	Coefficient
175/8 =	21	+	7/8	$a_0 = 7$
21/8 =	2	+	5/8	$a_1 = 5$
2/8 =	0	+	2/8	$a_2 = 2$

Answer $(175)_{10} = (a_2\, a_1\, a_0)_2 = (257)_8$

Convert an Fraction *from* Decimal *to* Octal

1. Multiply decimal number by the base (e.g. 8)
2. The *integer* is the highest-order digit
3. Repeat first two steps until fraction becomes zero.

Example for $(0.3125)_{10}$:

	Integer		Fraction	Coefficient
0.3125 x 8 =	2	+	5	$a_{-1} = 2$
0.5000 x 8 =	4	+	0	$a_{-2} = 4$

Answer $(0.3125)_{10} = (0.24)_8$

Understanding Hexadecimal Numbers

- Hexadecimal numbers are made of 16 digits:
  - (0,1,2,3,4,5,6,7,8,9,A, B, C, D, E, F)
- How many items does an hex number represent?
  - $(3A9F)_{16} = 3x16^3 + 10x16^2 + 9x16^1 + 15x16^0 = 14999_{10}$
- What about fractions?
  - $(2D3.5)_{16} = 2x16^2 + 13x16^1 + 3x16^0 + 5x16^{-1} = 723.3125_{10}$
- Note that *each* hexadecimal digit can be represented with four bits.
  - $(1110)_2 = (E)_{16}$
- Groups of four bits are called a *nibble.*
  - $(1110)_2$

Putting It All Together

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

- Binary, octal, and hexadecimal similar
- Easy to build circuits to operate on these representations
- Possible to convert between the three formats

Converting Between Base 16 and Base 2

$$3A9F_{16} = \underset{3}{0011}\ \underset{A}{1010}\ \underset{9}{1001}\ \underset{F}{1111}_2$$

°Conversion is easy!

- Determine 4-bit value for each hex digit

°Note that there are 24= 16 different values of four bits

°Easier to read and write in hexadecimal.

°Representations are equivalent!

Converting Between Base 16 and Base 8

$$3A9F_{16} = \underset{3}{0011}\ \underset{A}{1010}\ \underset{9}{1001}\ \underset{F}{1111}_2$$

$$\downarrow$$

$$35237_8 = \underset{3}{011}\ \underset{5}{101}\ \underset{2}{010}\ \underset{3}{011}\ \underset{7}{111}_2$$

1. Convert from Base 16 to Base 2
2. Regroup bits into groups of three starting from right
3. Ignore leading zeros
4. Each group of three bits forms an octal digit.